Remarks on the paper of A. De Visscher "What does the *g*-index really measure?"

by

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ABSTRACT

This paper presents a different view on properties of impact measures than given in the paper of De Visscher. We argue that a good impact measure should prefer concentrated (unequal) situations of citations per paper rather than situations where citations are more equally spread out over papers.

We also present theoretical evidence that the *g*-index and the *R*-index can be close to the square root of the total number of citations, while this is not the case for the *A*-index. Here we confirm an assertion of De Visscher.

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Introduction

The most famous impact measure is Hirsch's index (also called the *h*-index) and is defined as follows. Arrange the papers of (e.g.) an author in decreasing order of number of received citations. Then this author has *h*-index *h* if *r=h* is the highest rank such that all papers on ranks 1,2,…, *h* have received at least *h* citations (Hirsch (2005)). We do not go into the vast literature that studies this *h*-index and its variants. For this we refer to the review paper Egghe (2010).

Here we just mention one disadvantage of the *h*-index. Once a paper is in the *h*-core (i.e. a paper with rank $r \in \{1,...,h\}$) it does not matter how many citations this paper received (as long as it is *h* or more). Many authors (including myself) think that a good impact measure should reward high numbers of citations but the *h*-index does not really use the actual number of citations to papers.

That is why in Egghe (2006) the *g*-index was introduced as follows. Again, order the papers of an author in decreasing order of number of received citations. Then this author has *g*-index *g* if *r=g* is the highest rank such that the papers on ranks 1,2,…, *g* have at least g^2 citations <u>together</u>. Denote by c_r the number of received citations of the paper on rank $r \in \{1,...,T\}$ where *T* denotes the total number of papers. Hence this author has *g*-index *g* is *r=g* is the highest rank such that

$$\sum_{r=1}^{g} c_r \geq g^2 \qquad (1)$$

This also implies

$$\frac{1}{g}\sum_{r=1}^{g} c_r \geq g \qquad (2)$$

so that *r=g* is the highest rank such that the papers on ranks 1,2,…. *g* have on average at least *g* citations.

The *g*-index was defined as in (1) since the *h*-index also satisfies (1) with *g* replaced by *h*, but *g* is the largest rank with this property, so $g \geq h$. By using the actual number of citations (in (1) or (2)), the *g*-index is much more dependent on these numbers than is the *h*-index.

The *R*-index was introduced in Jin et al. (2007) with the same purpose as the *g*-index: taking more into account the actual number of citations. It is defined as follows:

$$R = \sqrt{\sum_{r=1}^{h} c_r} \tag{3}$$

Hence the *R*-index uses the *h*-index but also the number of citations $c_1, c_2, ..., c_h$. Note that also $R \geq h$, by definition of the *h*-index.

Of the same nature is the *A*-index, introduced in Jin (2006). It is the average number of citations in the *h*-core:

$$A = \frac{1}{h} \sum_{r=1}^{h} c_r \tag{4}$$

So also this measure uses the number of citations $c_1, c_2, ..., c_h$. But this measure has a disadvantage: one can give examples where the number of citations increases and where the *A*-index decreases because the *h*-index increases. One such a simple example is the sequence of number of citations (see Egghe (2010), p.74) 6,5,4,3. So here *h=3* and $A = \frac{1}{3}(6+5+4) = 5$ Now increase the last number by one citation. Hence we have the sequence 6,5,4,4. Now *h=4* and $A = \frac{1}{4}(6+5+4+4) = \frac{19}{4} < 5$. A similar example was given in Jin et al. (2007). Note that *h*, *g* and *R* do not have this undesirable property.

Since they are used in De Visscher (2011) we also present the measures

$$\sqrt{C} = \sqrt{\sum_{r=1}^{T} c_r} \qquad (5)$$

, the square root of the overall total number of citations *C* and

$$\frac{C}{T} = \frac{1}{T}\sum_{r=1}^{T} c_r \qquad (6)$$

, the overall average number of citations (*T* = total number of papers).

A vision on impact measures, divergent from De Visscher's vision.

In De Visscher (2011) it is stated "The main difference between the *g*-index and the *h*-index is that the former penalizes consistency of impact whereas the latter rewards such consistency" and he concludes that because of this fact "the *h*-index is a better bibliometric tool than the *g*-index". Although not explicitly defined, consistency here means that an author keeps up an almost fixed level of impact (through citations) in his/her papers.

There is some truth in De Visscher's quote on the difference between the *g*-index and the *h*-index but I would reformulate it as follows: the (main) difference between the *g*-index and the *h*-index is that the former awards non-consistency of impact whereas the latter penalizes non-consistency. This is a subtle different formulation: in case of perfect consistency (all papers have the same number of received citations) the *h*-index and the *g*-index are the same (so no penalization of consistency by the *g*-index), while in non-consistent situations (unequal distribution of citations to the papers), the *g*-index is larger than the *h*-index (so the *g*-index awards non-consistency).

We agree with this "reformulation" of the statement of De Visscher but we conclude in an opposite way: for this reason we think that the *g*-index is a better impact measure than the *h*-index: in two situations with an equal total number of citations and an equal total number of papers, the case where the citations are more concentrated (over the papers) has the higher impact. I agree that this is a "philosophical" view on impact measures but it is fed by the idea that impact increases faster than linear with the number of citations of a paper.

One referee asks me to elaborate on this further since he/she is not convinced that "impact increases faster than linear with the number of citations per paper". As said above it is very much linked with the "definition" of impact measure which is non-existent. Every researcher on this topic has his/her own intuitive insight in this (also this referee since he/she uses terminology such as "to me, it seems" that the above assertion is not valid. The argument given by this referee is that "highly cited papers get additional citations because everyone else cites them, irrespective of the actual influence of the paper on the citing author's thinking".

While this may be partially true, if the above statement would be true in a majority of cases, this would mean the total impossibility of the use of citation analysis in research evaluation. This discussion goes beyond the scope of this paper.

One argument for my statement that impact increases faster than linear with the number of citations per paper is by looking at price winning researchers such as Nobel price winners or Fields medalist winners. It is clear that their influence (impact) on the sciences is incomparable with, say, ten other researchers in the same field with papers that are cited around ten times less than these price winning researchers. That the g-index is the better impact measure on this account is also supported in Schreiber (2008 a,b), Tol (2008) and Costas and Bordons (2008).

In Egghe (2009) these ideas are elaborated in terms of transfer principles and Lorenz curves in econometrics. Everything is based on the definition of "an elementary transfer" which we will describe now. Let $\left(x_1, x_2, ..., x_T\right)$ be a citation vector of an author, i.e. x_r is the number of citations to the paper on rank $r \in \{1, 2, ..., T\}$ where we suppose that the vector is ranked in decreasing order (as is needed for the definition of the *h*- and *g*-index). An elementary transfer is taking away an amount of a smaller x_j and add it to a larger x_i yielding the new vector

$$\left(x_1, ..., x_i + k, ..., x_j - k, ..., x_T\right)$$

(for $k \le x_j$). In De Visscher's vision this new situation has less impact; in my vision this new situation has more impact. By extension this is also true for a vector that results from another vector by a finite repetition of elementary transfers. An example: let $X = \left(5, 5, 5, 5, 4, 2\right)$ and $Y = \left(9, 5, 5, 3, 3, 1\right)$. Then *Y* follows from *X* by 4 elementary transfers (the last three coordinates of *X*, (5,4,2) reduce to (3,3,1) and the lost 4 units are added to the first coordinate of *X*, 5, yielding 9 for the first coordinate of *Y*).

In my vision the *Y*-situation has more impact since we have a paper with 9 citations. This is indeed measured by the *g*-index: $g(X) = 4$ and $g(Y) = 5$. Both the *h*-index and the *R*-index give lower values of impact of *Y*: $h(X) = 4$ and $h(Y) = 3$ and $R(X) = \sqrt{20}$ and $R(Y) = \sqrt{19}$.

The measures $\sqrt{C}$ (formula (5)) and C/T (formula (6)) do not give different values for the *X*- or *Y*-case since the total number of citations and papers remained constant.

I think that both De Visscher's vision and my vision have their values. This shows that it is not evident to formulate "what is a good property for an impact measure", let alone to conclude on which of two impact measures is the better one. As De Visscher states, the *g*-index has been received favorably in the bibliometrics literature. Its greater discriminating power has been recognized (Schreiber (2008 a,b), Tol (2008), Costas and Bordons (2008)). This is clear by the example given in Egghe (2006) where we found $h=27$ for both Garfield and Narin but where Garfield had 14 papers with more than 100 citations while Narin had only 1 paper with more than 100 citations! The *g*-index rewarded this for Garfield: $g=59$ while $g=40$ for Narin.

In the next section we will partially confirm another finding of De Visscher.

A partial confirmation of an empirical finding of De Visscher

In De Visscher (2011) it is stated that "the *g*-index does not differ from the square root of the total number of citations in a bibliometrically meaningful way when the entire publication list is considered". While it is unclear what is exactly meant by "a bibliometrically meaningful way", I can give, in some cases, theoretical evidence for this statement. For this we will assume that our publication-citation framework conforms with Lotka's law. This law states that the number of papers with *n* citations is a decreasing power law $f(n)$:

$$f(n)=\frac{D}{n^{\alpha}} \qquad (7)$$

where $D>0$ and $\alpha>1$ (α is called Lotka's exponent). Egghe (2005) is a book devoted to the study of Lotka's law, a law first stated in Lotka (1926) and it is a basic law in informetrics.

In this framework the following formulae were proved, if $\alpha > 2$ (notation as in the previous section)

$$\frac{C}{T} = \frac{\alpha - 1}{\alpha - 2} \qquad (8)$$

$$\sqrt{C} = \sqrt{\frac{\alpha - 1}{\alpha - 2}}\sqrt{T} \qquad (9)$$

$$h = T^{1/\alpha} \qquad (10)$$

$$g = \left(\frac{\alpha - 1}{\alpha - 2}\right)^{\frac{\alpha - 1}{\alpha}} T^{1/\alpha} \qquad (11)$$

$$R = \sqrt{\frac{\alpha - 1}{\alpha - 2}}T^{1/\alpha} \qquad (12)$$

$$A = \frac{\alpha - 1}{\alpha - 2}T^{1/\alpha} \qquad (13)$$

Formula (8) can be found in Egghe (2005), p.115, from which (9) follows. Formula (10) is proved in Egghe and Rousseau (2006). Formula (11) is proved in Egghe (2006) and formulae (12) and (13) are proved in Jin et al. (2007).

Now let α be close to 2 (but larger than 2). Then it follows from (11) that

$$g \approx \sqrt{\frac{\alpha - 1}{\alpha - 2}}\sqrt{T} = \sqrt{C} \qquad (14)$$

and it follows from (12) that

$$R \approx \sqrt{\frac{\alpha - 1}{\alpha - 2}}\sqrt{T} = \sqrt{C} \qquad (15)$$

from (13) we have

$$A \approx \frac{\alpha - 1}{\alpha - 2}\sqrt{T} \neq \sqrt{C} \tag{16}$$

This confirms the finding in De Visscher (2011) that *g* and *R* are close to the square root of the total number of citations. Hence *g* and *R* are measures of overall impact while *A* is not (as confirmed by (16)).

Remark:

One referee asked to comment on these findings outside the Lotkaian model. We can give a heuristic argument.

Since, by definition of the R-index,

$$R = \sqrt{\sum_{i=1}^{h} c_i} \tag{17}$$

and, since, by definition of the g-index,

$$g \approx \sqrt{\sum_{i=1}^{g} c_i} \tag{18}$$

and since the sequence $(c_i)_{i=1,\ldots,T}$ decreases, we can say that (17) and (18) approximate

$$\sqrt{C} = \sqrt{\sum_{i=1}^{T} c_i} \tag{19}$$

This argument cannot be given for A:

$$A = \frac{\sum_{i=1}^{h} c_i}{h} \approx \frac{C}{h} > \sqrt{C} \tag{20}$$

since $h < C$. So, heuristically, we obtain the same results as in the Lotkaian case.

Final remark : in De Visscher (2011) it is stated that $R \le g$. This can be proved from (11) and (12): $R \le g$ if and only if

$$\sqrt{\frac{\alpha-1}{\alpha-2}}T^{1/\alpha} \le \left(\frac{\alpha-1}{\alpha-2}\right)^{\frac{\alpha-1}{\alpha}} T^{1/\alpha}$$

if and only if

$$\frac{\alpha-1}{\alpha-2} \le \left(\frac{\alpha-1}{\alpha-2}\right)^{2\left(\frac{\alpha-1}{\alpha}\right)}$$

which is true if and only if

$$2\left(\frac{\alpha-1}{\alpha}\right) \ge 1$$

and this is true since $\alpha > 2$.

However there are some practical cases where $R > g$. Example: take the citation vector (4,3,3,1,1). Then $h = 3$, $g = 3$ and $R = \sqrt{10} > 3 = g$.

IV. Conclusions

We expressed a divergent vision from De Visscher's vision in that we think an impact measure should award non-consistency: the more citations over papers are unequally distributed, the higher the impact should be.

We gave theoretical evidence that, in some cases, the *g*-index and *R*-index are close to the square root of the total number of citations, an empirical finding of De Visscher.

References

R. Costas and M. Bordons (2008). Is *g*-index better than *h*-index? An exploratory study at the individual level. Scientometrics 77(2), 267-288.

A. De Visscher (2011). What does the *g*-index really measure? Journal of the American Society for Information Science and Technology 62(11), 2290-2293.

L. Egghe (2005). Power laws in the Information Production Process: Lotkaian Informetrics. Elsevier, Oxford, UK.

L. Egghe (2006). Theory and practice of the *g*-index. Scientometrics 69(1), 131-152.

L. Egghe (2009). An econometric property of the *g*-index. Information Processing and Management 45(4), 484-489.

L. Egghe (2010). The Hirsch-index and related impact measures. Annual Review of Information Science and Technology, Volume 44 (B. Cronin, ed.), 65-114, Information Today, Inc., Medford, New Jersey, USA.

L. Egghe and R. Rousseau (2006). An informetric model of the Hirsch-index. Scientometrics 69(1), 121-129.

J.E.Hirsch (2005). An index to quantify an individual's scientific research output. Proceedings of the National Academy of Sciences of the United States of America 102, 16569-16572.

B. Jin (2006). *h*-index: An evaluation indicator proposed by scientist. Science Focus 1(1), 8-9.

B.H. Jin, L.M. Liang, R. Rousseau and L. Egghe (2007). The *R*- and *AR*-indices: Complementing the *h*-index. Chines Science Bulletin 52, 855-863.

A.J. Lotka (1926). The frequency distribution of scientific productivity. Journal of the Washington Academy of Sciences 16(12), 317-324.

M. Schreiber (2008a). The influence of self-citations on Egghe's *g*-index. Scientometrics 76(1), 187-200.

M. Schreiber (2008b). An empirical investigation of the *g*-index for 26 physicists in comparison with the *h*-index, the *A*-index and the *R*-index. Journal of the American Society for Information Science and Technology 59(9), 1513-1522.

R.S.J. Tol (2008). A rational, successive *g*-index applied to economics departments in Ireland. Journal of Informetrics 2(2), 149-155.