Loading constraints in vehicle routing problems:
a focus on axle weight limits

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Chapter 1

Introduction and problem statement

1.1 Introduction

Efficient freight transportation plays a crucial role in modern society. Transporting goods safely, quickly and cost-efficiently is essential for international trade and economic development (Eurostat, 2016a). The need for transportation has increased due to increasing consumption, economic growth and globalization (Hoff et al., 2010). In the EU, the transportation sector employed more than 10.5 million people in 2013 (Eurostat, 2016b). Especially in Belgium, the transportation sector is very important. Many companies have set up a logistics base or distribution center in Belgium because of its central location in Europe and its dense road network. The transportation and logistics sector was in 2014 responsible for 5% of the Belgian GDP (Vlaamse overheid, 2016).

Strong competition in the transportation sector enables customers to demand cost reduction, customer service, timeliness and reactivity (Hoff et al., 2010). To meet these demands, transportation companies are forced to improve their efficiency. Climate change and other environmental concerns also encourage transportation companies to plan their routes more efficiently in order to decrease emissions. Additionally, by reducing unnecessary long routes, pressure on the road infrastructure diminishes and the traffic flow improves, for freight as well as passenger transportation (Drexl, 2012). The cost savings of the use of computerized procedures for route scheduling have been shown in numerous real-world applications (Toth and Vigo, 2002). Benefits of automated route scheduling include operational cost savings, reduced scheduling time and exclusion of human error from the routing schedule (Drexl, 2012).

Due to its economic importance and because it poses interesting methodological challenges, vehicle routing has received considerable research attention in the Operational Research community (Laporte et al., 2013). The Capacitated Vehicle Routing Problem (CVRP) is introduced in 1959 by Dantzig and Ramser. It considers the delivery of goods from a depot to customer locations with a homogeneous fleet of vehicles with a fixed capacity (in terms of weight or number of items) (Toth and Vigo, 2002). Common extensions of the CVRP that have been studied extensively include the CVRP with Time Windows in which intervals may be specified in which deliveries need to take place and the CVRP with Pickups and Deliveries in which orders may be picked up and delivered at customer places (Parragh et al., 2008).

The aforementioned routing problems do not correspond to the routing problems that transportation companies are currently faced with. In real-life, companies need to consider several additional constraints in their route scheduling which greatly increases the complexity of the problem. Therefore, there is an increasing scientific focus on the integration of *Rich* constraints in vehicle routing problems. *Rich* vehicle routing problems (RVRP) refer to those problems taking these additional realistic constraints into account (Battarra et al., 2009). Examples of rich constraints include time-dependent travel times and legislation concerning driving, working, break and rest times for drivers (Drexl, 2012). Furthermore, in real-life applications, the vehicle fleet of a transportation company is generally not homogeneous but consists of several types of vehicles in order to meet varying customer demands. Vehicles in the fleet may differ in terms of capacity, costs and other factors such as speed and product compatibility. Transporters are also faced with loading problems in their route scheduling. A feasible loading plan is not guaranteed when only total capacity (in terms of number of pallets or weight) of a vehicle is considered.

In the following section, an overview of loading problems that distributors are faced with is provided (Section 1.2). In Section 1.3 the research objectives of the thesis will be discussed. Section 1.4 presents the outline of the thesis.

1.2 Loading problems in vehicle routing

Loading problems arise when goods cannot be placed freely in a container or vehicle because several constraints have to be taken into account. Current commercial route scheduling programs do not take into account most loading constraints, which makes the routing schedules often not feasible in practice. This gives rise to last-minute changes which may result in additional costs. The development of vehicle routing models incorporating loading constraints is therefore critical to more efficient route scheduling.

Common loading problems that are faced by distributors include multi-dimensional packing constraints, sequence-based loading, stability constraints and weight distribution constraints. Multi-dimensional packing constraints entail that items cannot overlap and should be completely packed inside the vehicle. In a three-dimensional loading problem the length, width and height of the vehicle need to be considered to check this constraint. In a one-dimensional or two-dimensional loading problem only a single or two dimensions are taken into consideration respectively. Sequence-based loading ensures that no consignment is placed in such a way that it blocks the removal of items to be delivered earlier on the route. Stability constraints guarantee vertical as well as horizontal stability of the cargo in the vehicle. When items are stacked on top of each other in the vehicle, the items have to be supported by other items or by the floor to ensure the vertical stability of the cargo. Horizontal stability of the cargo refers to the support of the lateral faces of items in the container to prevent items moving around in the container.

Weight distribution constraints ensure the stability of the vehicle by balancing the cargo weight aboard. Axle weight limits in particular impose a huge challenge for transportation companies since they are faced with high fines (up to € 75,000 for a violation of more than 3 tonnes) when violating these limits. The axle weight is the gross weight (cargo weight plus tare weight of the truck) placed on an axle. This is illustrated in Figure 1.1. When item j is placed onto a vehicle, the weight of the item is divided over the axles of the tractor and those of the semi-trailer. a_j^F represents the weight of item j placed on the first two axles, which are the axles of the tractor. a_j^R represents the weight of item j on the rear axles, which are the axles of the semi-trailer. Since the weight on the axles changes when items are loaded and unloaded, it is important that axle weights are considered during the entire trip of the vehicle and not only when the vehicle departs from the depot.

Weigh-In-Motion (WIM) systems on highways increase the chances that axle weight violations are detected. A WIM system monitors axle weight violations of trucks while driving. The authorities can thereby focus on trucks with violations according to the WIM system, to do a precise measurement. This leads to an enormous efficiency gain of the controls (Jacob and Feypell-de La Beaumelle, 2010). In Flanders, WIM systems are installed on 10 locations. Since the WIM systems are used as a preselection system, the efficiency gain of the controls has increased from the

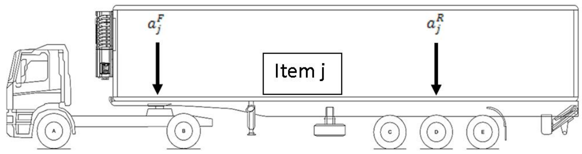

Figure 1.1: Axle weight tractor and semi-trailer (figure adapted from TruckScience)

usual 15 % - 20 % based on visual inspection to 83 % (Agentschap Wegen en Verkeer, 2016). In 2013, 1,492 overloaded trucks were fined in Flanders, with an average fine of more than 2000 euro (Vlaams Parlement, 2014).

Axle weight violations are not only important for transportation companies because of the fines they are faced with, it may also damage the vehicle and put the truck driver and other road users at risk. Overloading of the axles puts strain on the tyres of the vehicle which will lead to premature failure. Furthermore, overloading will make the vehicle less stable and difficult to steer and increases the braking distance (Driver and Vehicle Standards Agency UK, 2013). This results in more accidents and since overloading is illegal, the insurance may not intervene in the costs.

Besides transportation companies, the society as a whole may benefit from the integration of axle weight constraints in route scheduling. External costs of axle weight violations include accident costs since overloaded axles represent a significant threat for traffic safety. Furthermore, axle weight violations may cause damage to roads, bridges and pavements which leads to road infrastructure costs. Finally, congestion costs will increase due to the increase in road works and accidents caused by vehicles with overloaded axles.

1.3 Research objectives

Axle weight constraints have not yet been considered in a VRP model. Therefore, little is known about the impact on solution cost of the integration of axle weight constraints. Also the effect of the characteristics of the demand and of the vehicle fleet on the integration of axle weight constraints in route scheduling is not known. Therefore, this thesis addresses the following central research question.

> What is the effect of the integration of axle weight constraints in vehicle routing problems under varying demand characteristics, vehicle fleet characteristics and objective functions?

The *primary objective* of this thesis is directly derived from the central research question and aims to analyze the effect of axle weight constraints on the objective value of a vehicle routing problem for varying demand characteristics, vehicle fleet characteristics and objective functions. Demand characteristics consist of the weight and size of the customer demand. Fleet characteristics refer to the measurements and tare weight of the vehicle and to the fleet composition (homogeneous or heterogeneous). Finally, the objective functions that are considered are distance minimization and transport cost minimization. Distance minimization is traditionally used in VRP literature while the minimization of total transport cost corresponds more to the objective that transporters are faced with in real-life. In order to perform this analysis, the *secondary objective* of this dissertation aims at the introduction of a vehicle routing problem with axle weight constraints. More specifically, the CVRP with sequence-based pallet loading and axle weight constraints is introduced. To analyze the effect of a heterogeneous vehicle fleet, the Fleet Size and Mix VRP with sequence-based pallet loading and axle weight constraints is introduced, in which an unlimited heterogeneous vehicle fleet is considered. The *tertiary objective* is to develop a heuristic solution approach to solve both problems on instances of a realistic size. For an overview of the existing vehicle routing models with loading constraints and the solution techniques that are used to tackle these models, the *final objective* is to give a state-of-the-art of the combination of routing and loading. Based on this overview, research gaps and opportunities for future research are identified.

1.4 Thesis outline

The outline of the thesis is presented in Figure 1.2. In Chapter 2, the state-of-the-art of vehicle routing problems with loading constraints is discussed. Although the combination of routing and loading problems is a fairly recent domain of research, contributions to this field have increased immensely over the last couple of years. A discussion of loading constraints is presented, based on the classification of Bortfeldt and Wäscher (2013). In case *rich* (other than loading) constraints are considered, this is mentioned in the description of the papers. This chapter offers a broad perspective as it considers road, maritime and air transport as well as the routing of automated guided vehicles. It discusses the various papers in comparative perspective and identifies future research directions.

Chapters 3, 4, 5 and 6 introduce and analyze the integration of axle weight constraints in a CVRP. The problem considers the delivery of pallets to customer lo-

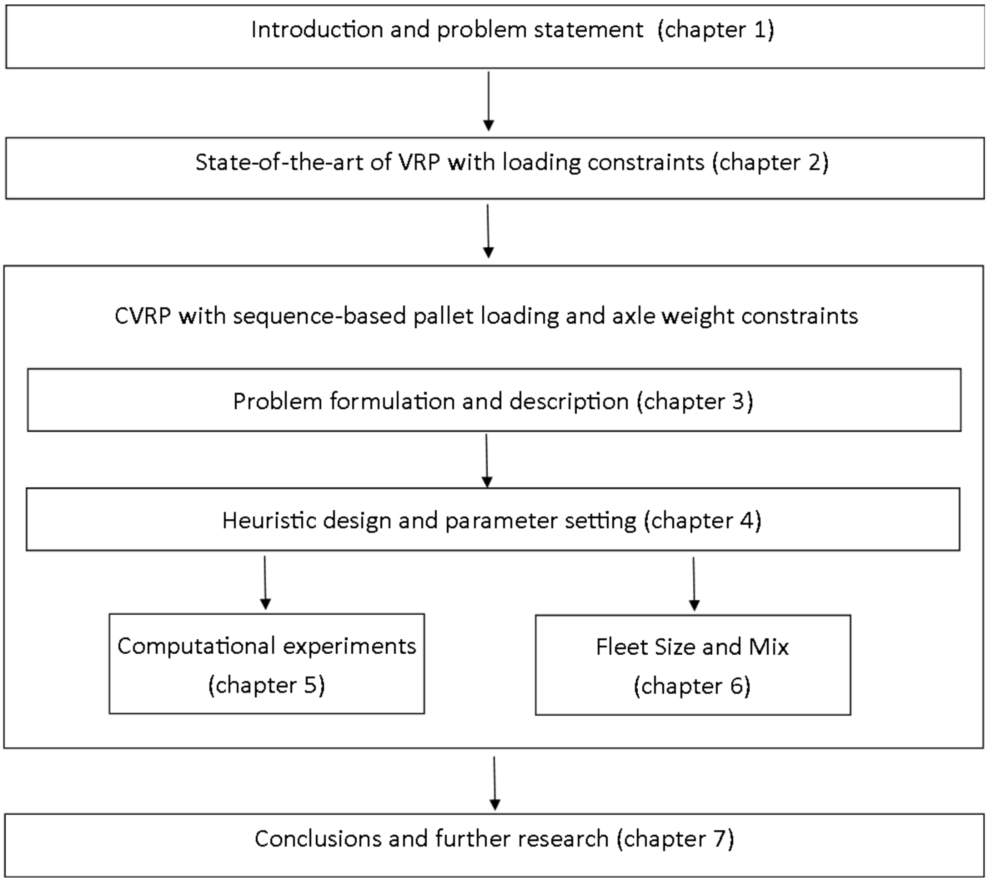

Figure 1.2: Outline of the thesis

cations. To avoid moving pallets of other customers when arriving at a customer, sequence-based loading is imposed. Pallets may be placed in two rows inside the vehicle but cannot be stacked on top of each other because of their weight, fragility or customer preferences. The resulting problem is the CVRP with sequence-based pallet loading and axle weight constraints.

In Chapter 3, the calculation of the weight on the axles is described. The calculation depends on the center of gravity of the pallets inside the truck and on vehicle-specific parameters. Furthermore, two problem formulations for the CVRP with sequence-based pallet loading and axle weight constraints are presented in this chapter. The first formulation is a Mixed Integer Linear Programming (MILP) formulation. In the second formulation, the problem is formulated as a Set Partitioning (SP) model.

In order to solve realistic-size instances for the CVRP with sequence-based pallet loading and axle weight constraints, an Iterated Local Search (ILS) metaheuristic is developed. In Chapter 4, the design of the ILS is presented. The parameters of the metaheuristic are tuned with an automatic algorithm configuration software. To analyze the impact of the values of the parameters on the solution quality of the ILS, a sensitivity analysis is performed.

In Chapter 5, the results of the MILP model, the SP model and the ILS are compared on instances with up to 50 customers. Furthermore, the ILS is used to analyze the effect of introducing axle weight constraints in a CVRP on total routing cost in instances with networks consisting of up to 100 customers. Four problem classes are defined based on the size of the demand in terms of number of pallets and total weight to analyze the impact of axle weight constraints for varying demand characteristics.

Chapter 6 considers the integration of a heterogeneous fleet in the CVRP with sequence-based pallet loading and axle weight constraints. The resulting problem is defined as the Fleet Size and Mix CVRP with sequence-based pallet loading and axle weight constraints. To measure the impact of the vehicle fleet on the integration of axle weight constraints in a VRP, a heterogeneous fleet with 30-foot and 45-foot trucks is compared to a homogeneous fleet with 30-foot trucks and a homogeneous fleet with 45-foot trucks. Furthermore, two scenarios are analyzed for which the objective function differs: in the first scenario the objective is the minimization of total distance while in the second scenario the objective is the minimization of total transport costs. Both objectives are considered to examine whether the objective function influences the results of the analysis.

Finally, Chapter 7 presents the conclusions of this thesis and future research opportunities.

Chapter 2

State-of-the-art of VRP with loading constraints

2.1 Introduction

The vehicle routing problem is the most studied combinatorial optimization problem in transport and logistics. The issue concerns the distribution of goods between depots and customers (Toth and Vigo, 2002) along a set of routes for a fleet of vehicles where an objective function (e.g. total distance, total routing cost) is optimized. Customer demand must be met and vehicle capacities respected. Solving a basic vehicle routing problem involves two elements: the assignment of all customers to a trip and the sequence in which each are visited. The basic version of the vehicle routing problem is called the Capacitated Vehicle Routing Problem (CVRP). The CVRP considers a homogeneous fleet of vehicles with a fixed capacity (in terms of weight or number of items) which delivers goods from a depot to customer locations. Split deliveries are not allowed. The CVRP can be extended to the VRP with time windows (VRPTW) by specifying time windows in which deliveries need to take place. Another variant is the VRP with Pickups and Deliveries (VRPPD) in which orders may be picked up and delivered. For each order, an origin (pickup location) and a destination (delivery location) are specified (Parragh et al., 2008) while both operations may occur at a same location. When only a single vehicle is considered, the VRPPD reduces to the Traveling Salesman Problem with Pickup and Delivery (TSPPD). A third common extension of the basic CVRP is the VRP with backhauls (VRPB), in which goods are transported from the depot to linehaul customers and from backhaul customers

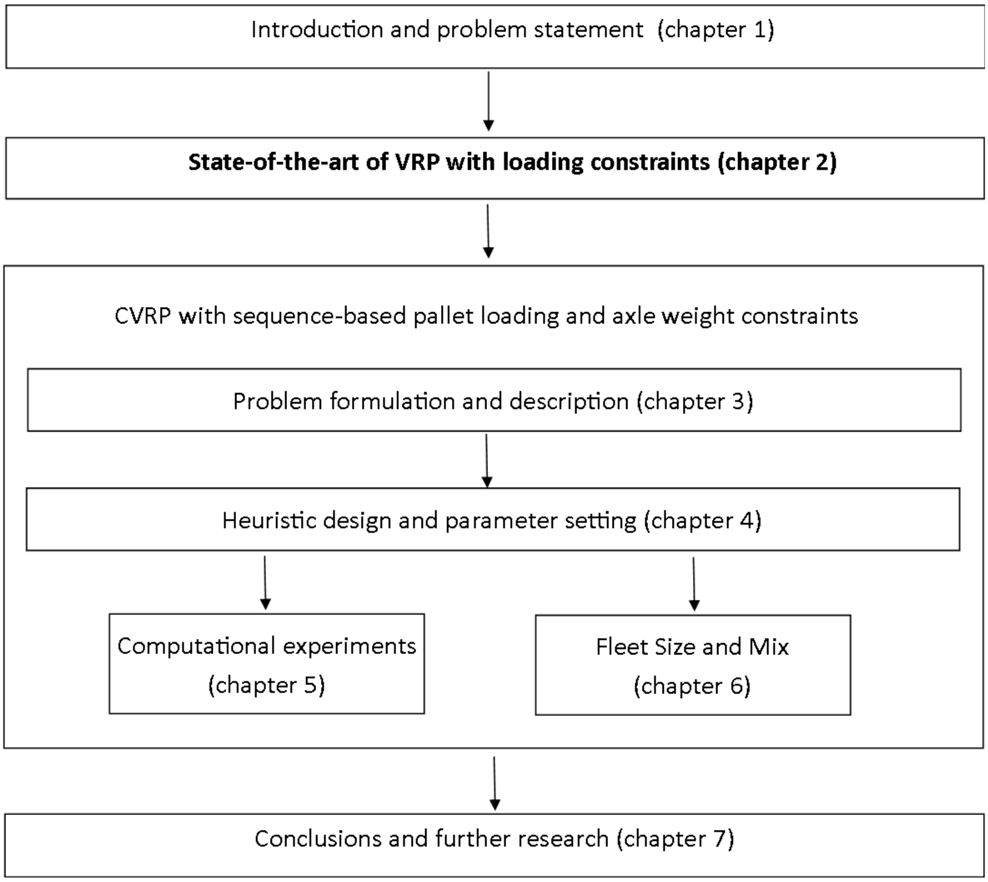

Figure 2.1: Outline of the thesis

to the depot (Parragh et al., 2008; Toth and Vigo, 2002).

The *classic* vehicle routing problem, described in the previous paragraph, has been studied extensively in the last decades. A review of solution methods can be found in (Laporte, 2009). In real-life, companies are faced with several additional constraints which greatly increase the complexity of the problem. Examples of such complicating constraints or attributes include maximum route length and duration, a heterogeneous vehicle fleet, incompatibilities between goods and vehicles and loading constraints. *Rich* vehicle routing problems (RVRP) refer to those problems taking some of these additional realistic constraints into account (Battarra et al., 2009). Vidal et al. (2013) provide a synthesis and analysis of solution methods dealing with rich vehicle routing problems.

This chapter [1] focuses on the integration of loading constraints in vehicle routing problems and reviews the relevant literature (Figure 2.1). The combination of routing and loading problems is a recent domain of research. Since lori and Martello's review (Iori and Martello, 2010) up to 2010 of 31 papers concerning vehicle routing and loading constraints, contributions to this field have soared over the last couple of years. This chapter extends these authors' review by covering 84 papers (including the 31 papers considered in (Iori and Martello, 2010)). It also discusses the loading constraints more thoroughly and uses the classification of Bortfeldt and Wäscher (2013) to identify them. In case rich (other than loading) constraints are included, mention is made in the description of the models. Besides, this chapter offers a broad perspective as it does not only focus on road transport, but it also considers maritime and air transport as well as automated guided vehicles.

Section 2.2 describes relevant problem characteristics for the VRP. Section 2.3 identifies loading problems that may be considered in combination with routing problems. Section 2.4 provides an overview of the literature concerning vehicle routing problems in combination with loading problems. Section 2.5 presents conclusions and opportunities for further research.

2.2 Problem characteristics of VRP

This section describes the main characteristics likely to influence the solution of a vehicle routing problem, i.e. characteristics of the vehicle fleet and of the cargo, (time dependent) travel times, the legal framework, transportation requests and the objective function. The reader is referred to Toth and Vigo (2002), Cordeau et al. (2007) and Golden et al. (2008) for a general discussion of the VRP.

Characteristics of the vehicle fleet such as vehicle capacity, configuration of the loading space and unloading possibilities largely determine the solution to the problem. The capacity of vehicles may be specified in terms of weight, number of items or volume. The loading space of the vehicle often influences the capacity. The loading space is determined by the measurements of the vehicle (length, width and height) and may have a specific configuration. For example, vehicles may be divided into multiple compartments allowing for the transportation of goods that need to be kept segregated. Besides, a tank truck may be divided into compartments to prevent the liquid accumulating in the front of the truck when this comes to a halt (due to *mass in motion*). The configuration of the loading space may also make it possible

[1] This chapter is based on Pollaris et al. (2015).

to load goods into several piles. Finally, vehicles differ in the ways in which they can be loaded or unloaded. A vehicle may be loaded via the rear (rear loading), the long side, and/or via the top side. A homogeneous vehicle fleet consists of vehicles having the same vehicle characteristics. In a heterogeneous fleet, vehicles may differ in terms of capacity, loading space or other relevant vehicle characteristics.

Characteristics of the cargo include the measurements and fragility of the items as well as orientation issues. Measurements may determine whether an item fits into a container or not. Often items are assumed to have a rectangular shape in two dimensions and a cuboid shape in three dimensions to make the loading process easier. Items can be fragile (e.g. porcelain) or non-fragile (e.g. newspapers) which may bear on the loading possibilities into a container. They may have specific orientation constraints, e.g. several require a fixed orientation with respect to height. This means they cannot be placed upside-down but have a pre-determined top. Cargo may consist of homogeneous or heterogeneous items. In the latter case, compatibility issues of product pairs may arise. More specifically, certain products are not allowed to be transported together in the same vehicle or vehicle compartment. Furthermore, some product types (e.g. frozen or refrigerated items) require to be transported in adapted containers or container compartments.

Travel time on a certain route may vary at different times of travel (e.g. due to traffic congestion).

Legal limitations on driving time specify the maximum time a truck driver may drive each day as well as the minimum duration and frequency of breaks during his working shift. Next, rules concerning the loading of vehicles (e.g. European Best Practice Guidelines on Cargo Securing for Road Transport [2]) may apply. Road speed limits are used to regulate the speed of the trucks and may therefore influence the solution of the VRP.

Transportation requests are either for a pickup, a delivery or both. Split deliveries or split pickups are mostly not allowed, which implies that each customer is only visited once. Customers may specify time windows within which the delivery or pickup must take place. These time windows may be *hard* or *soft*. Soft time windows imply that deliveries may occur outside the time windows, in which case a penalty cost is incurred by the transportation company, while hard time windows do not allow delivery outside the time windows.

Multiple **objectives** are relevant when considering the VRP: the minimization of the number of vehicles, total cost, total route length and total time are often con-

[2] `http://ec.europa.eu/transport/road_safety/vehicles/doc/cargo_securing_guidelines_en.pdf`

sidered. In addition, balancing the workload of drivers in terms of required time or distance traveled and maximization of volume utilization may also feature as objectives.

2.3 Loading constraints

Loading problems arise when goods cannot be placed freely in a container or vehicle because several constraints have to be taken into account. An overview of packing problems discussed in the literature can be found in Wäscher et al. (2007). In a state-of-the-art review of container loading problems, Bortfeldt and Wäscher (2013) identify several types of loading constraints which are cargo-related, item-related, container-related or load-related. Cargo-related constraints address a subset of items whereas item-related constraints refer to individual items. Container-related constraints concern the container or vehicle in which the items are placed. Load-related constraints relate to the result of the packing process. The following paragraphs briefly discuss loading constraints that may be relevant in combination with vehicle routing problems. The classification is mainly based on the taxonomy of Bortfeldt and Wäscher (2013). The first category 'Classical (multi-) dimensional packing constraints' is added to the taxonomy in this thesis.

2.3.1 Classical (multi-) dimensional packing constraints

This constraint entails that items cannot overlap and should be thoroughly packed inside the vehicle. In a three-dimensional loading problem the length, width and height of the vehicle need to be verified to satisfying the constraint. In a one-dimensional or two-dimensional loading problem, respectively, a single or two dimensions are taken into consideration. In a Strip Packing Problem (SPP), items are placed in an open-ended rectangle with infinite height with the objective to minimize total height. In a Bin Packing Problem (BPP), items are placed into a minimum number of identical bins (=vehicles) (Wäscher et al., 2007). In a combined vehicle routing and container loading problem, the loading problem may be defined as a BPP Bortfeldt and Wäscher (2013). The loading feasibility of a route is checked by solving a BPP.

2.3.2 Cargo-related constraints

Complete-Shipment constraints

In case the vehicle capacity cannot accommodate all items, some items will be left behind. Complete-shipment constraints may be specified when a subset of items needs to be shipped together, i.e. either all or none can be loaded (Bortfeldt and Wäscher, 2013). Shipping companies that operate in the tramp market face complete-shipment constraints in ship scheduling. Tramp shipping companies select cargoes at the spot market and construct routes to maximize profit (Fagerholt et al., 2013). A single order on the spot market may consist of several cargoes from different origins, i.e. the service company must service all of these or none at all.

Allocation constraints

Allocation constraints may be specified when multiple vehicles or containers are considered. Two types of such constraints have been identified: connectivity and separation constraints (Bortfeldt and Wäscher, 2013). **Connectivity constraints** require that the items of a certain subset are shipped in the same container or vehicle. In the VRP literature each customer is usually visited only but once and by a single vehicle (split deliveries are not allowed). All items requested by a customer, therefore, need to be shipped in the same vehicle. As a result, connectivity constraints are incorporated in most VRP models (e.g. Gendreau et al., 2006; Tarantilis et al., 2009; Fuellerer et al., 2010; Ruan et al., 2013). **Separation constraints** may be specified to prevent certain types of products being shipped in the same container or vehicle. Separation constraints are relevant when different types of goods (e.g. food and toxic items) may not be transported together in the same vehicle. An example may be found in Battarra et al. (2009) where a distinction is made between three types of commodities: vegetables, fresh products (e.g. milk and meat) and non-perishable items. A variation of this constraint has been investigated in the multi-compartment VRP. The multi-compartment VRP allows the transport of different types of goods in separate compartments of the same vehicle. Applications of VRPs with multiple compartments can be found in the distribution of fuel (different types of petroleum products transported within the same vehicle) (e.g. Brown and Graves, 1981; Cornillier et al., 2008a), distribution of food (e.g. a refrigerated compartment and a regular compartment within the same vehicle) (Chajakis and Guignard, 2003), waste collection (Muyldermans and Pang, 2010), on-farm milk collection (Dooley et al., 2005) and ship scheduling (Fagerholt and Christiansen, 2000a).

Positioning constraints

The location of the items inside the vehicle may be restricted by positioning constraints. Absolute as well as relative positioning restrictions may be specified (Bortfeldt and Wäscher, 2013). Relative constraints allow or restrict the placement of the item relative to the positions of other items. An example of relative constraints may be found in Lurkin and Schyns (2015), who present an airline container loading problem in which they specify a minimum distance required within the airplane between dangerous goods and other goods. In case a vehicle makes use of multiple drop-off points in a single trip, it requires sequence-based loading which can be seen as a combination of relative and absolute constraints. Sequence-based loading ensures that no consignment is placed in such a way that it blocks the removal of items to be delivered earlier on the route. This constraint is commonly used in VRPs (e.g. Iori et al., 2007; Gendreau et al., 2006; Moura, 2008; Doerner et al., 2007) and is in the literature also referred to as a Last-In-First-Out (LIFO) constraint. The LIFO policy is relevant only for the case of a single dimension. In case of higher dimensions, items can be placed next to each other or on top of other items.

2.3.3 Item-related constraints

Loading priorities

Loading priorities play a role in the packing process when vehicle capacity is not sufficient to accommodate all items. The decision as to which items are shipped or left behind may depend on factors such as product shelf life and delivery deadlines (Bischoff and Ratcliff, 1995). Several papers in the literature on aircraft loading (e.g. Fok and Chun, 2004; Chan et al., 2006; Vancroonenburg et al., 2014) consider loading priorities to select the items to be loaded.

The incorporation of priorities in vehicle routing problems is considered in orienteering problems where a score or priority is assigned to each location. Since the literature on orienteering problems does not consider any other loading constraints, those papers dealing with the orienteering problem are not considered in what follows. For a survey of research on the orienteering problem, the reader is referred to Vansteenwegen et al. (2011) and Gunawan et al. (2016).

Orthogonality constraints

In the literature devoted to packing, it is often assumed that items have a rectangular shape. When a rectangular shape is assumed (e.g. Gendreau et al., 2006; Moura and Oliveira, 2009; Iori et al., 2007; Fuellerer et al., 2010), the edges of the items are assumed to be packed orthogonal or parallel with the edges of the vehicle. This constraint is often used in combination with two- and three-dimensional loading constraints.

Orientation constraints

The orientation of items may be fixed with respect to the height, width and length of the vehicle. The vertical orientation is often fixed to prevent the item being damaged when put upside down in the container. A fixed vertical orientation constraint is also denoted as a “this-way-up!” constraint, referring to items that are marked with a “this-way-up!” label (Bortfeldt and Homberger, 2013). The horizontal orientation of the items may be fixed as well (e.g. Junqueira et al., 2012). This may be necessary when items can only be accessed via a particular side (e.g. pallets that need to be accessed by forklifts) (Bortfeldt and Wäscher, 2013). However, in most papers incorporating orientation constraints, it is allowed to rotate the items 90 degrees on the width-length (horizontal) plane (e.g. Gendreau et al., 2006; Tarantilis et al., 2009; Fuellerer et al., 2010; Zhu et al., 2012; Ruan et al., 2013). This constraint is frequently used in VRPs with two- and three-dimensional loading constraints.

Stacking constraints

When items are placed on top of each other in the vehicle, items may be damaged by the pressure of items placed above them. Stacking constraints (also denoted as load-bearing strength constraints or fragility constraints) prevent this from happening. The load-bearing strength of an item is defined as the maximum pressure that can be applied on the item before damage takes place (Junqueira et al., 2013). The load-bearing strength may vary across different vertical orientations of the item (Ratcliff and Bischoff, 1998). The box contents (solid contents vs. less solid contents) and loading conditions (humidity, duration of loading, way of stacking ...) may also influence the load-bearing strength of an item (Bortfeldt and Wäscher, 2013). Fragile items may be defined as items that cannot bear any pressure from other items, indicating that no item may be placed upon the item. Some models in the literature (e.g. Gendreau et al., 2006; Tarantilis et al., 2009; Fuellerer et al., 2010; Ruan et al., 2013) allow for fragile items being placed upon other such items, but forbid non-fragile items to be placed upon fragile ones. Stacking constraints have been considered in several papers concerning three-dimensional loading VRPs (e.g. Gendreau et al., 2006; Tarantilis et al., 2009; Fuellerer et al., 2010; Ruan et al., 2013; Junqueira et al., 2013).

2.3.4 Container-related constraints

Weight limits

The total weight of items in the vehicle or container should not exceed the weight capacity of the vehicle. Weight limits are a standard feature in VRPs. In several types of vehicles (truck, airplane, ship) weight capacity may be an important restriction when transporting heavy cargo.

Weight distribution constraints

To ensure the stability of the vehicle, it is important to balance the cargo weight aboard. Several authors propose to achieve an uniform weight distribution by ensuring that the center of gravity (CG) of the load should be close to the midpoint of the container (e.g. Amiouny et al., 1992; Gehring and Bortfeldt, 1997; Davies and Bischoff, 1999; Bortfeldt and Wäscher, 2013; Paquay et al., 2016). Limbourg et al. (2012) propose an approach for loading ULDs (Unit Loading Devices) into an aircraft. To ensure its balance, the authors not only take the center of gravity into consideration but also minimize the moment of inertia. The minimization of the moment of inertia leads to a more dense packing of the load around the CG, which reduces stress on the aircraft structure and leads to better aircraft manoeuvrability (Limbourg et al., 2012). Although weight distribution is an important issue in practice (Davies and Bischoff, 1999), to our knowledge, it is only considered once in combination with routing problems. Øvstebø et al. (2011) introduce weight distribution constraints in a maritime transportation problem. To ensure the stability of the ship, the torque from the cargo on the ship (making the ship lean sideways) and the distance between the bottom of the ship and its center of gravity are considered.

Closely related to weight balance aboard is the distribution of the cargo over the axles of the vehicle. Lim et al. (2013) address axle weight constraints in a container loading problem. They develop a heuristic method to tackle the single container loading problem with axle weight constraints. Alonso et al. (2017) develop integer linear programming models to tackle multi-container loading problems with axle weight constraints in which items are first packed on pallets and afterwards, pallets are placed onto trucks.

2.3.5 Load-related constraints

Stability constraints

Cargo stability may be defined as the ability of each box to maintain the loading position without a significant change during cargo loading and transportation operations (Ramos, A. G., 2015). When items are stacked on top of each other in the vehicle, the items have to be supported by other items or by the floor to ensure vertical (or static) stability of the cargo. Vertical stability constraints specify the minimum supporting area of each item (e.g. as a percentage of the base area of the item). Horizontal (or dynamic) stability of the cargo refers to the support of the lateral faces of items in the container to prevent items moving around in the container (Junqueira et al., 2013). For more information on cargo stability in container loading problems, the reader is referred to Ramos, A. G. (2015). In this work, static as well as dynamic stability in container loading is considered. The literature concerning three-dimensional VRPs often takes vertical stability constraints into account (e.g. Gendreau et al., 2006; Fuellerer et al., 2010; Bortfeldt, 2012; Zhu et al., 2012; Ruan et al., 2013). According to our knowledge, horizontal stability constraints have not yet been considered explicitly in routing models in literature.

2.4 Integration of loading constraints in vehicle routing problems

The integration of loading constraints in VRPs is a recent domain of research. Both problems belong to the NP-hard type of optimization problems. Combining these problems is therefore very challenging but leads to a better overall logistical solution. This section reviews the literature on the integration of vehicle routing and loading problems. Since loading constraints also apply in a maritime transport context, papers introducing these constraints in routing problems for maritime transport are also included. To our knowledge, no literature exists on the integration of loading constraints in a routing model in an air transport context.

The papers dealing with the combination of routing and loading problems may be categorized in one of the following categories defined on the basis of the type of routing problem and the loading characteristics dealt with: Two-Dimensional Loading CVRP (2L-CVRP), Three-Dimensional Loading CVRP (3L-CVRP), multi-pile VRP, multi-compartments VRP, Pallet Packing VRP (PPVRP), Minimum Multiple Trip VRP (MMTVRP) with incompatible commodities, Traveling Salesman Problem with Pickups and Deliveries (TSPPD) with LIFO/FIFO constraints, Double TSP with Pickups and Deliveries with Multiple Stacks (DTSPMS) and Vehicle Routing

Problem with Pickups and Deliveries (VRPPD) with additional loading constraints. This is a classification similar to the classification used by Iori and Martello (2010). For each category, we give an overview of the loading constraints using the classification of Bortfeldt and Wäscher (2013). Table 2.1 overviews the papers on 2L-CVRP and on 3L-CVRP. Table 2.2 overviews those concerning the multi-pile VRP, multi-compartments VRP, PPVRP and the MMTVRP with incompatible commodities. Table 2.3 overviews the papers on the TSPPD with LIFO/FIFO constraints, the DTSPMS and the VRPPD with additional loading constraints.

Except for one paper (Fagerholt et al., 2013), complete-shipment constraints and loading priorities do not apply in the models since the capacity of the vehicle fleet is assumed to be sufficient to accommodate all items. Connectivity constraints, on the other hand, are standard features in routing models with multiple vehicles since it is often assumed that all the items of a customer have to be shipped in the same vehicle. Vertical stability constraints and stacking constraints are only relevant when the height dimension is taken into account. Orthogonality and orientation constraints only apply when at least two dimensions are considered.

The papers of each category are discussed below. It generally appears that few other rich constraints (besides loading constraints) are included in the current VRP models with loading constraints. When models do include other real-life characteristics (such as time windows or heterogeneity of the vehicle fleet), they are mentioned. In most papers described in this survey, the objective function is to minimize total routing costs or travel distance. If not, the objective function is mentioned in the description of the problem. Another observation is that problems in which more than one dimension is considered (2L-CVRP, 3L-CVRP, pallet packing VRP) are mostly solved by means of a two-stage approach. The routing problem acts as the master problem and iteratively calls exact or heuristic methods to solve the packing subproblem (Tao and Wang, 2015). The methods for solving the packing problem are mostly based on the the bin packing literature (e.g. Baker et al., 1980; Lodi et al., 1999; Martello and Vigo, 1998). Maximum touching perimeter (or touching area in the three-dimensional case) and bottom-left-fill are often used to solve two- and three dimensional packing problems heuristically (e.g. Iori et al., 2007; Gendreau et al., 2006; Tarantilis et al., 2009; Tao and Wang, 2015; Dominguez et al., 2014), while branch-and-bound methods and lower bounds are usually employed to deal with packing problems to obtain exact solutions (e.g. Iori et al., 2007; Fuellerer et al., 2009; Gendreau et al., 2008). For each category with multi-dimensional loading, a paragraph describes how the packing problem is generally dealt with. For the other categories, the loading part is usually less complex, which does not make it necessary to apply heuristics for the packing problem. In the latter case, the loading constraints are usually incorporated in the vehicle routing problem (e.g. Cordeau et al., 2010b; Petersen and Madsen, 2009; Cherkesly et al., 2015a).

2.4.1 Two-dimensional loading CVRP

In the Two-Dimensional Loading CVRP (2L-CVRP), the customers' requests and the measurements of the vehicles are expressed in two dimensions. Width and length are usually taken into account whereas height is not. In real-life applications, this problem arises in distribution logistics when items cannot be stacked on top of each other because of their weight, fragility or large dimensions (Strodl et al., 2010). Examples of applications may be found in the distribution of large kitchen appliances such as refrigerators, large mechanical components or fragile items. Two papers propose an exact method (Iori et al., 2007; Martinez and Amaya, 2013). Most papers assume sequence-based loading and multiple vehicles (see Table 2.1). When height is not considered, stacking constraints and vertical stability constraints are not applicable in the problems. Two papers assume a heterogeneous fleet (Leung et al., 2013; Dominguez et al., 2016) and three papers consider time windows (Attanasio et al., 2007; Khebbache-Hadji et al., 2013; Martinez and Amaya, 2013). A mathematical formulation for a 2L-CVRP is presented by Martinez and Amaya (2013) and Dominguez et al. (2014).

Iori et al. (2007) are the first to address a 2L-CVRP. They develop a branch-and-bound algorithm and solve the problem to optimality for up to 35 customers. Fuellerer et al. (2009) employ an Ant Colony Optimisation (ACO) method for a similar problem, with a small alteration in the loading constraints. The items are allowed to rotate 90 degrees on the horizontal plane. Attanasio et al. (2007) consider a variant of the 2L-CVRP based on a consolidation and dispatching problem of a multinational chemical company. Each shipment must take place within a multi-day time window, spanning from the manufacturing date to a given deadline. Attanasio et al. (2007) develop a heuristic based on a cutting plane framework in which a simplified Integer Linear Program (ILP) is solved. Items are allowed to rotate and sequence-based loading is assumed. Strodl et al. (2010) develop a Variable Neighborhood Search (VNS) to address the routing problem and formulate a heuristic and an exact procedure for the two-dimensional loading problem. Items have a fixed orientation and sequence-based loading is not considered. Duhamel et al. (2011) address the 2L-CVRP without sequence-based loading. They solve the problem using a two-stage approach. First, the 2L-CVRP is converted into a Resource Constraint Project Scheduling Problem

Table 2.1: Papers on 2L-CVRP

	Ex	HV	TW	CP	CS	Co	Se	Po(*)	WL	WD	LP	Orth	Or	St	VS	HS
2L-CVRP																
Iori et al. (2007)	x			x	-	x		x	x		-	x	x	-	-	
Attanasio et al. (2007)			x	x	-	x		x	x		-		x	-	-	
Gendreau et al. (2008)				x	-	x		x	x		-	x	x	-	-	
Fuellerer et al. (2009)				x	-	x		x	x		-	x	x	-	-	
Zachariadis et al. (2009)				x	-	x		x	x		-	x	x	-	-	
Strodl et al. (2010)				x	-	x			x		-	x	x	-	-	
Leung et al. (2011)				x	-	x		x	x		-	x	x	-	-	
Duhamel et al. (2011)				x	-	x			x		-	x	x	-	-	
Leung et al. (2013)		x		x	-	x		x	x		-	x	x	-	-	
Khebbache-Hadji et al. (2013)			x	x	-	x			x		-	x	x	-	-	
Zachariadis et al. (2013b)				x	-	x		x	x		-	x	x	-	-	
Martinez and Amaya (2013) (1)	x		x	x	-	x					-			-	-	
Martinez and Amaya (2013) (2)			x	x	-	x					-			-	-	
Dominguez et al. (2014)				x	-	x			x		-	x	x	-	-	
Wei et al. (2015)				x	-	x		x	x		-	x	x	-	-	
Dominguez et al. (2016)		x		x	-	x			x		-	x	x	-	-	
3L-CVRP																
Gendreau et al. (2006)				x	-	x		x	x		-	x	x	x	x	
Aprile et al. (2007)				x	-	x					-					
Moura (2008)			x	x	-	x		x			-	x	x	x		
Moura and Oliveira (2009)			x	x	-	x		x			-	x	x	x		
Tarantilis et al. (2009)				x	-	x		x	x		-	x	x	x	x	
Fuellerer et al. (2010)				x	-	x		x	x		-	x	x	x	x	
Ren et al. (2011)				x	-	x		x	x		-	x	x	x	x	
Massen et al. (2012)				x	-	x		x	x		-	x	x	x	x	
Bortfeldt (2012)				x	-	x		x	x		-	x	x	x	x	
Wisniewski et al. (2012)				x	-	x		x	x		-	x	x	x	x	
Zhu et al. (2012)				x	-	x		x	x		-	x	x	x	x	
Miao et al. (2012)				x	-	x		x	x		-	x	x	x	x	
Ruan et al. (2013)				x	-	x		x	x		-	x	x	x	x	
Bortfeldt and Homberger (2013)			x	x	-	x		x	x		-	x	x	x	x	
Ceschia et al. (2013)		x		x	-	x		x	x		-	x	x	x	x	
Tao and Wang (2015)				x	-	x		x	x		-	x	x	x	x	
Junqueira et al. (2013)	x			x	-	x		x	x		-	x	x	x	x	
Bortfeldt et al. (2015)				x	-	x		x	x		-	x	x	x	x	
Zhang et al. (2015)				x	-	x		x	x		-	x	x	x	x	

Ex = exact solution method, HV = heterogeneous vehicles, TW = time windows, CP = classical packing, CS = complete shipment, Co = connectivity, Se = separation constraint, Po = positioning, WL = weight limits, WD = weight distribution, LP = loading priorities, Orth = orthogonality, Or = orientation, St = Stacking (fragility), VS = vertical stability, HS = horizontal stability

x = considered in the reference, - = not applicable in the reference, ?= not mentioned in the reference

(*) positioning constraints refer in most papers to sequence-based loading (or LIFO loading)

Table 2.2: Papers on multi-pile VRP, multi-compartments VRP, Pallet-Packing VRP and MMTVRP with incompatible commodities

	Ex	HV	TW	CP	CS	Co	Se	Po(*)	WL	WD	LP	Orth	Or	St	VS	HS
Multi-pile VRP																
Doerner et al. (2007)				x	-	x		x			-	-	-			
Tricoire et al. (2011) (1)				x	-	x		x			-	-	-			
Tricoire et al. (2011) (2)	x			x	-	x		x			-	-	-			
Massen et al. (2012)				x	-	x		x			-	-	-			
Multi-compartments VRP																
Brown and Graves (1981)		x		x	-	x	x	-	x		x	-	-	-	-	-
Avella et al. (2004) (1)		x		x	-	x	x	-			-	-	-	-	-	-
Avella et al. (2004) (2)	x	x		x	-	x	x	-			-	-	-	-	-	-
Cornillier et al. (2008a)	x	x		x	-	x	x	-			-	-	-	-	-	-
Cornillier et al. (2008b)		x		x	-	x	x	-			?	-	-	-	-	-
Cornillier et al. (2009)		x	x	x	-	x	x	-			-	-	-	-	-	-
Cornillier et al. (2012)		x	x	x	-	x	x	-			-	-	-	-	-	-
Fagerholt and Christiansen (2000a)	x	x	x	x	x	x	x	-	x		-	-	-	-	-	-
Fagerholt and Christiansen (2000b)	x		x	x	x	x	x	-	x		-	-	-	-	-	-
Chajakis and Guignard (2003)	x		x	x	-	x	x	-	x		-	-	-	-	-	-
Dooley et al. (2005)				x	?	?	x	-			?	-	-	-	-	-
El Fallahi et al. (2008)				x	-		x	-	x		-	-	-	-	-	-
Mendoza et al. (2010)				x	-	x	x	-			-	-	-	-	-	-
Muyldermans and Pang (2010)				x	-	x	x	-			-	-	-	-	-	-
Lahyani et al. (2015)	x	x		x	-	x	x	-			-	-	-	-	-	-
Pallet Packing VRP																
Zachariadis et al. (2012)			x	x	-	x					-	x	x		x	
Zachariadis et al. (2013a)			x	x	-	x					-	x	x		x	
MMTVRP incomp. commodities																
Battarra et al. (2009)			x	x	-	x	x				-	-	-	-	-	-

Ex = exact solution method, HV = heterogeneous vehicles, TW = time windows, CP = classical packing, CS = complete shipment, Co = connectivity, Se = separation constraint, Po = positioning, WL = weight limits, WD = weight distribution, LP = loading priorities, Orth = orthogonality, Or = orientation, St = Stacking (fragility), VS = vertical stability, HS = horizontal stability

x = considered in the reference, - = not applicable in the reference, ?= not mentioned in the reference

x = considered in the reference, - = not applicable in the reference, ?= not mentioned in the reference

(*) positioning constraints refer in most papers to sequence-based loading (or LIFO loading)

Table 2.3: Papers on TSPPD with LIFO/FIFO constraints

	Ex	HV	TW	CP	CS	Co	Se	Po(*)	WL	WD	LP	Orth	Or	St	VS	HS
TSPPD with L/F constr.																
Ladany and Mehrez (1984)	x	-		x	-	-		x			-	-	-	-	-	-
Pacheco (1997)		-		x	-	-		x			-	-	-	-	-	-
Levitin and Abezgaouz (2003)	x	-		x	-	-		x			-	-	-	-	-	-
Carrabs et al. (2007a)		-		x	-	-		x			-	-	-	-	-	-
Carrabs et al. (2007b) (1)	x	-		x	-	-		x			-	-	-	-	-	-
Carrabs et al. (2007b) (2)	x	-		x	-	-		x$^{(a)}$			-	-	-	-	-	-
Erdoğan et al. (2009)		-		x	-	-		x$^{(a)}$			-	-	-	-	-	-
Arbib et al. (2009)	x	-		x	-	-		x			-	-	-	-	-	-
Cordeau et al. (2010b)	x	-		x	-	-		x			-	-	-	-	-	-
Cordeau et al. (2010a)	x	-		x	-	-		x$^{(a)}$			-	-	-	-	-	-
Li et al. (2011)		-		x	-	-		x			-	-	-	-	-	-
Øvstebø et al. (2011)(1)	x	-	x	x	-	-		x		**x**	-	-	-	-	-	-
Øvstebø et al. (2011)(2)		-	x	x	-	-		x		**x**	-	-	-	-	-	-
Côté et al. (2012b)		-		x	-	-		x			-	-	-	-	-	-
Côté et al. (2012a)	x	-		x	-	-		x			-	-	-	-	-	-
DTSPMS																
Petersen and Madsen (2009)		-		x	-	-		x			-	-	-	-	-	-
Felipe et al. (2009)		-		x	-	-		x			-	-	-	-	-	-
Lusby et al. (2010)	x	-		x	-	-		x			-	-	-	-	-	-
Petersen et al. (2010)	x	-		x	-	-		x			-	-	-	-	-	-
Lusby and Larsen (2011)	x	-		x	-	-		x			-	-	-	-	-	-
Alba et al. (2013)	x	-		x	-	-		x			-	-	-	-	-	-
Felipe et al. (2011)		-		x	-	-		x			-	-	-	-	-	-
Carrabs et al. (2013)	x	-		x	-	-		x			-	-	-	-	-	-
Iori and Riera-Ledesma (2015)	x	x		x	-	-		x			-	-	-	-	-	-
VRPPD with loading constr.																
Xu et al. (2003)		x	x	x	-	-	x	x			-	-	-	-	-	
Malapert et al. (2008)				x	-	x		x	x		-	x	x	-	-	
Cheang et al. (2012)				x	-	-		x			-	-	-	-	-	
Fagerholt et al. (2013)		x	x	x	x	x			x		-	-	-	-	-	
Cherkesly et al. (2015a)	x		x	x	-	x		x	x		-	-	-	-	-	
Cherkesly et al. (2015b)			x	x	-	x		x	x		-	-	-	-	-	
Cherkesly et al. (2016)	x		x	x	-	x		x	x		-	-	-	-	-	
Zachariadis et al. (2016)				x	-	x		x	x		-	x	x	-	-	

Ex = exact solution method, HV = heterogeneous vehicles, TW = time windows, CP = classical packing, CS = complete shipment, Co = connectivity, Se = separation constraint, Po = positioning, WL = weight limits, WD = weight distribution, LP = loading priorities, Orth = orthogonality, Or = orientation, St = Stacking (fragility), VS = vertical stability, HS = horizontal stability

x = considered in the reference, - = not applicable in the reference, ?= not mentioned in the reference

(*) positioning constraints refer in most papers to sequence-based loading (or LIFO loading)

$^{(a)}$: FIFO

- CVRP (RCPSP-CVRP) by relaxing the bin packing constraints. The items in the packing problem are represented by activities in the RCPSP. Each activity has a duration (length of item) and requirement of resource (width of item). A route is feasible if the makespan of the RCPSP does not exceed the length of the vehicle (Duhamel et al., 2011). In the second step, the feasibility of the best RCPSP-CVRP solutions with the 2L-CVRP constraints are checked by transforming the RCPSP-CVRP solutions into 2L-CVRP solutions. This approach saves a lot of computation time because a packing plan is computed only for the best RCPSP-CVRP solutions.

Leung et al. (2013) develop a Simulated Annealing (SA) model to solve the 2L-CVRP with heterogeneous fleet. The packing constraints that are considered in this model are the same as in Iori et al. (2007). The vehicles have different weight capacities and different measurements.

Martinez and Amaya (2013) consider a VRP with multi-trips, time windows and two-dimensional circular loading constraints. A homogeneous fleet is considered and sequence-based loading is not assumed. The problem is based on a real-life problem faced by a home-delivery service transporting perishable circular shaped products. A Mixed Integer Non-Linear Programming mathematical model (MINLP) is developed to solve small-size problems (up to 17 customers). Furthermore, a two-step heuristic method is proposed to handle instances of realistic size. In the first step, an initial solution is built using a sequential insertion heuristic. In the second step this solution is improved with a Tabu Search (TS) algorithm.

Dominguez et al. (2014) develop a biased-randomized algorithm for the 2L-CVRP with and without item rotations. The problem assumes a homogeneous vehicle fleet and sequence-based loading is not considered. The algorithm uses a multi-start approach and combines at each restart a biased randomization of a savings-based routing algorithm as proposed by Clarke and Wright (1964) for the routing part with a multi-start biased-randomized version of the best fit packing heuristic to check loading feasibility. In the first biased randomization process, the savings list of the edges is randomized using a biased probability distribution. For the loading feasibility check, first a biased randomization is applied on the list of items to be loaded. Next, the best fit heuristic is used. If after several iterations, the best fit heuristic does not find a feasible loading scheme, the proposed route is assumed to be infeasible and a new randomization is applied on the savings list of the edges which will again be followed by a loading feasibility check. Dominguez et al. (2016) use a similar algorithm for the 2L-CVRP with heterogeneous fleet with and without item rotations.

Finally, Khebbache-Hadji et al. (2013) develop a heuristic solution method to solve the 2L-CVRP with Time Windows (2L-CVRPTW) without sequence-based loading.

The packing feasibility check in the above papers consists of a mix of several types of solution methods (heuristic as well as exact). Commonly used methods include the bottom-left-fill heuristic (e.g. Iori et al., 2007; Zachariadis et al., 2009; Fuellerer et al., 2009), maximum touching perimeter (e.g. Zachariadis et al., 2009; Strodl et al., 2010; Khebbache-Hadji et al., 2013), lower bounds (e.g. Iori et al., 2007; Gendreau et al., 2006; Fuellerer et al., 2009) and branch-and-bound (e.g. Iori et al., 2007; Gendreau et al., 2006; Fuellerer et al., 2009; Strodl et al., 2010). If a combination of heuristic and exact algorithms is used, first the heuristics are applied and when they do not find a feasible solution, the exact method is used to solve the packing problem.

2.4.2 Three-dimensional loading CVRP

In the Three-Dimensional Loading CVRP (3L-CVRP), the three dimensions of the vehicle are taken into account and the customers' demand also consists of three-dimensional items. Since the height dimension is considered, additional loading constraints concerning fragility and vertical stability of the cargo may be specified. This problem is encountered in distribution logistics when items may be stacked on top of other items in a container. Examples of applications of the 3L-CVRP are found in the distribution of furniture, household appliances, soft drinks and staple goods (Ruan et al., 2013). Sequence-based loading is incorporated in most models as shown in Table 2.1. Most papers assume a homogeneous fleet, while only three papers consider time windows (Moura, 2008; Moura and Oliveira, 2009; Bortfeldt and Homberger, 2013). An exact solution method and a formulation of the 3L-CVRP is provided by Junqueira et al. (2013).

Gendreau et al. (2006) are the first to address the 3L-CVRP. Their model includes sequence-based loading, stacking and vertical stability constraints and a fixed vertical orientation of the items in the vehicles (it is allowed to rotate the items 90 degrees on the width-length plane). Tao and Wang (2015) use a TS method to solve the 3L-CVRP. They employ two mechanisms from the 3D bin packing literature to help exploiting the loading space better. First, a least waste packing heuristic (Wei et al., 2009) is employed which aims at minimizing the space wasted when packing an item into a vehicle. Second, the mechanism for updating new potential points or positions in the container at which items may be loaded is a combination of normal points and corner points. An item that is placed in a normal position touches with its bottom edge either the bottom of the bin or the top edge of an item in the truck, with its left edge either the left edge of the bin or the right edge of an item in the truck and with its front edge either the front edge of the bin or the right front of an item in the truck.

Corner points follow the concept of envelope and are introduced by Martello et al. (2000) for 3D bin packing. Zhang et al. (2015) consider a variant of the 3L-CVRP in which fuel consumption is minimized rather than total distance.

Junqueira et al. (2013) propose an exact method to solve the 3L-CVRP. They assume a homogeneous vehicle fleet, sequence-based loading, stacking constraints, orientation constraints and stability constraints. Junqueira et al. (2013) take into account the unloading pattern of the items at customer places. By specifying a maximum reach length of the worker or forklift, they avoid that items placed on top of items for other customers cannot be reached. An ILP is proposed to solve small-sized instances (number of customers < 15).

Bortfeldt and Homberger (2013) develop a two-stage method, called *Packing first - Routing second* for the 3L-CVRP with Time Windows (3L-CVRPTW). In the first stage, the packing problem is solved for each customer separately. The resulting packing plans minimize the total loading length of the boxes of each customer in a vehicle. In the second stage, vehicle routes are constructed with the constraint that the sum of the loading lengths (calculated in the first stage) may not exceed the length of the loading space of the vehicle. After these stages, a packing plan is determined for the previously generated routes. Moura (2008) develops a multi-objective Genetic Algorithm (GA) to solve the 3L-CVRPTW. The problem presented has three objectives: minimization of the number of vehicles, minimization of total distance traveled and maximization of volume utilization. The model considers sequence-based loading, orientation constraints and stability constraints. Moura and Oliveira (2009) develop a sequential and a hierarchical approach to solve the 3L-CVRPTW. The objectives are to minimize the number of vehicles and the total route time. In the hierarchical approach, the loading problem is seen as a subproblem of the routing problem. The routes are planned first and afterwards, for each route, the items are packed into the vehicles. As in Moura (2008), the model considers sequence-based loading, orientation constraints and stability constraints. In the sequential approach, the container loading and the vehicle routes are planned at the same time. The sequence-based loading constraint is relaxed in this solution approach.

Massen et al. (2012) develop a column generation based heuristic method for vehicle routing problems with black box feasibility (VRPBB). In the VRPBB the routes of the basic VRP need to satisfy a number of unknown constraints. A black box algorithm is used to verify the feasibility of a route. Their approach is tested on the 3L-CVRP as well as on the multi-pile VRP.

Ceschia et al. (2013) consider the 3L-CVRP with sequence-based loading and a (weakly) heterogeneous vehicle fleet. They consider stacking and stability constraints, orientation constraints, the maximum reach length of a worker or forklift as well as the possibility of split deliveries. Ceschia et al. (2013) solve the problem in one stage using a local search approach that combines SA and Large Neighborhood Search (LNS).

Bortfeldt et al. (2015) address the VRP with Clustered Backhauls (VRPCB) and 3D loading constraints. The VRPCB is a variant of the VRP with backhauls in which all linehaul customers are visited before the backhaul customers in every route. Bortfeldt et al. (2015) consider the problem combined with three-dimensional loading constraints, sequence-based loading, stacking and vertical stability constraints and a fixed vertical orientation of the items in the vehicles (it is allowed to rotate the items 90 degrees on the width-length plane). Two heuristic algorithms are proposed to solve the problem. In the first algorithm, the routing procedure is based on the Adaptive Large Neighborhood Search (ALNS) of Røpke and Pisinger (2006) while in the second algorithm a VNS algorithm is used for routing vehicles. Both algorithms integrate the 3D packing procedure by Bortfeldt (2012).

Maximum touching area and bottom-left-fill methods are often employed to check the loading feasibility in the 3L-CVRP literature (e.g. Gendreau et al., 2006; Fuellerer et al., 2010; Zhu et al., 2012; Wisniewski et al., 2012; Ruan et al., 2013). These heuristics are extensions of the bottom-left-fill and maximum touching perimeter methods from the 2D bin packing literature. Tao and Wang (2015) employ a least waste algorithm in combination with maximum touching area. Junqueira et al. (2013) solve the 3L-CVRP with an ILP in which they incorporate the 3D loading feasibility check.

2.4.3 Multi-pile VRP

The Multi-Pile Vehicle Routing Problem (MP-VRP) is introduced by Doerner et al. (2007) and is based on a real-world transportation problem regarding the transport of wooden chipboards. For every order, chipboards of the same type (small or large) are grouped into a unique item, which is placed onto a single pallet. The vehicle is divided into three piles on which pallets can be stacked. Pallets containing large chipboards can extend over multiple piles. The other pallets can be placed into a single pile. An example of a loading plan of a multi-pile vehicle is shown in Figure 2.2 where each tint represents a particular customer's items. Because of this specific configuration of pallets placed into multiple piles, the original problem in three dimensions can be reduced to a single-dimension problem. All papers on this problem that were found, assume a homogeneous vehicle fleet. Only a single paper by Tricoire et al. (2011) in our investigation proposes an exact solution method.

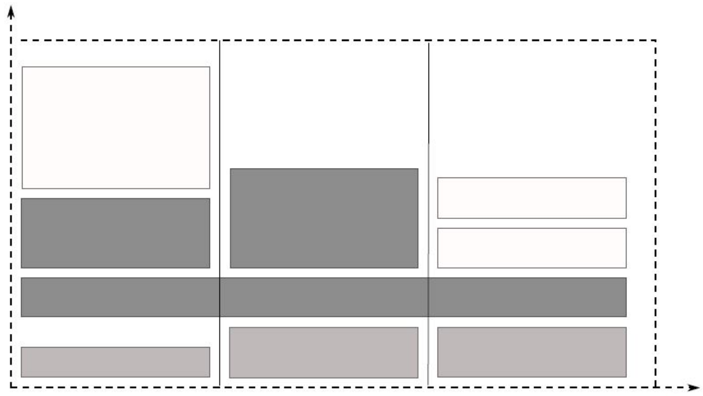

Figure 2.2: Example of a multi-pile vehicle (figure adapted from (Massen et al., 2012))

2.4.4 Multi-compartments VRP

In the multi-compartments VRP, vehicles with multiple compartments allow the transportation of heterogeneous products in separate compartments in the same vehicle. A compartment may not always be compatible with every type of product and certain product pairs cannot be loaded together into the same compartment (Derigs et al., 2011). Vehicle routing problems with compartments are encountered in industries like petroleum products and food distribution, waste collection, on-farm milk collection and ship scheduling. Note that these applications do not only consider the loading of individual items, but also include continuous loads such as petrol and milk. This section discusses papers dealing with the multi-compartments VRP. In several papers, a heterogeneous vehicle fleet and/or time windows are considered and various exact solution methods have been developed as shown in Table 2.2. El Fallahi et al. (2008) present a formulation for the multi-compartments VRP. Cornillier et al. (2008b), Cornillier et al. (2009) and Cornillier et al. (2012) provide formulations for respectively the Petrol Station Replenishment Problem (PSRP), the PSRP with Time Windows (PSRP-TW) and the multi-depot PSRP-TW. Lahyani et al. (2015) present a formulation for the multi-product, multi-period and multi-compartment VRP.

To our knowledge, Brown and Graves (1981) are the first to consider the dispatching of petroleum tank trucks. Each tank truck has several compartments which may carry different types of petroleum. An automated real-time dispatch system is developed for the distribution of petroleum products for a major US oil company. Each order includes several gasoline products, jointly constituting a full truckload.

Avella et al. (2004) also consider a real-life case of a company that supplies petroleum products to fuel pumps. Several less than truckload orders may be shipped in a single truck. They propose a solution method that uses a savings based routing algorithm for the generation of routes and a best fit decreasing heuristic for the packing problem. They also develop an exact method that uses a branch-and-price algorithm, based on a set partitioning formulation, which can solve instances with up to 60 stations. The PSRP has been studied by Cornillier et al. (2008a), Cornillier et al. (2008b), Cornillier et al. (2009) and Cornillier et al. (2012). The aim of the PSRP is to optimize the delivery of several petroleum products to petrol stations. Compartments can only hold a single type of product and since the compartments do not have flow meters, the content of one compartment may not be split between petrol stations. Cornillier et al. (2008b) consider the multi-period PSRP while Cornillier et al. (2012) consider the PSRP-TW with multiple depots. The exact algorithm of Cornillier et al. (2009) solves instances with up to 200 stations.

Fagerholt and Christiansen (2000a) consider the Ship Scheduling and Allocation Problem (SSAP) derived from a real-life case of the transport of mineral fertilizers by a bulk ship. The problem is similar to a pickup and delivery problem with time windows and multiple compartments. The compartments are flexible and are constructed by partitioning the loading space. Fagerholt and Christiansen (2000a) present a set partitioning approach to solve the problem exactly for instances with up to 70 customers. Fagerholt and Christiansen (2000b) focus on a subproblem of the SSAP studied by Fagerholt and Christiansen (2000a). More precisely, they consider the Traveling Salesman Problem with Allocation, Time Windows and Precedence Constraints (TSP-ATWPC). They develop a dynamic programming algorithm to solve the problem exactly for instances with up to 70 customers.

Chajakis and Guignard (2003) consider the distribution of goods to convenience stores in vehicles with multiple compartments. They develop two integer programming models for two possible cargo space layouts. Approximation schemes based on Lagrangian Relaxation are presented to solve these problems exactly for instances with up to 240 customers. Dooley et al. (2005) use a GA for the on-farm collection problem of milk. The model may be used to evaluate alternative transport management strategies with regards to milk collection.

El Fallahi et al. (2008) construct a memetic algorithm with a post-optimization phase based on path-relinking and a TS algorithm to solve the VRP with multiple compartments. El Fallahi et al. (2008) assume that each compartment is dedicated to a single product. The demand of a customer for a given product type cannot be split between vehicles, but different product types of the same customer order can be split between several vehicles. Since order splitting is allowed, connectivity constraints are not included in the model. Mendoza et al. (2010) consider the VRP with multiple compartments and a stochastic demand. Muyldermans and Pang (2010) consider a one-dimensional co-collection problem of waste. Homogeneous vehicles with multiple compartments are used to co-collect different types of waste. Lahyani et al. (2015) introduce and solve a multi-product, multi-period and multi-compartment VRP. The problem is based on a real-world application arising in the collection of olive oil in Tunisia. A heterogeneous vehicle fleet is considered with compartments of equal or different sizes. A branch-and-cut algorithm is proposed to solve the problem exactly.

2.4.5 Pallet packing VRP

The Pallet Packing VRP (PPVRP) is introduced by Zachariadis et al. (2012). Customer demand is for three-dimensional rectangular boxes which are first feasibly stacked onto pallets. These pallets are then loaded into the vehicles. The items demanded by a single customer must be stacked onto the same pallet. Many real-world applications of the PPVRP arise in distribution logistics. Examples may be found in the grocery and pharmaceutical industry. Distribution centers receive orders from grocery stores and manually pick and palletize the items of the orders for each store and send them to the store locations (Zachariadis et al., 2012). In the pharmaceutical industry, items are grouped into cardboard boxes which are palletized and transported from the production or distribution center to pharmacies (Zachariadis et al., 2012). To our knowledge, a formulation for the pallet packing VRP has not been provided yet.

Zachariadis et al. (2012) develop a local search metaheuristic strategy to solve the basic PPVRP and the PPVRP with time windows (PPVRPTW). They assume that every pallet can be unloaded at all times, without the need to move any other pallet. Because of this assumption, sequence-based loading of the pallets into the vehicle is not required. Sequence-based loading of the boxes onto the pallets is not assumed either. Orientation, orthogonality as well as vertical stability constraints are considered for the loading of the boxes onto the pallets. Zachariadis et al. (2013a) consider a variant of the PPVRP: the Pickup and Delivery Routing Problem with Time Windows and Pallet loading (PDRP-TWP). The key difference with the PPVRPTW is that two types of requests are considered in the PDRP-TWP namely delivery requests and paired pickup and delivery requests. Zachariadis et al. (2013a) extend the metaheuristic developed in Zachariadis et al. (2012) in order to deal with the paired pickup and delivery requests. The model takes into account the same routing and loading constraints as in Zachariadis et al. (2012).

With respect to the 3D loading feasibility check for the packing of boxes onto pallets, the above papers employ a heuristic that packs each box in the minimum volume cuboid that can accommodate this box in addition to the packing heuristics used in 3L-CVRP literature (bottom-left-fill and maximum touching area) (Zachariadis et al., 2012). This heuristic aims at finding a high degree of pallet volume utilization. The models also make use of a memory structure that keeps track of feasible and infeasible packing structures to avoid making the same feasibility check multiple times.

2.4.6 Minimum multiple trip VRP with incompatible commodities

Battarra et al. (2009) consider the minimum multiple trip VRP (MMTVRP) with time windows and incompatible commodities. Vehicles may perform multiple routes within a single trip (i.e. working shift) which is limited in total duration. The objective is to minimize the total number of multiple trips, which is the total number of vehicles. Three types of incompatible products (vegetables, fresh products and non-perishable items) are considered. Incompatibility means that the products cannot be transported together in a single vehicle. One-dimensional loading is considered. Battarra et al. (2009) propose a two-phase heuristic which decomposes the problem into two subproblems. In the first subproblem, a set of routes is determined using a VRPTW heuristic. In the second subproblem, the routes are aggregated into multiple trips by means of a packing heuristic. To the authors' knowledge, an exact method or a problem formulation have not yet been developed for the MMTVRP with incompatible commodities.

2.4.7 Traveling salesman problem with pickups and deliveries with LIFO/FIFO constraints

In a VRPPD, items can both be picked up at and delivered to customers, as opposed to the general VRPs in which items are only delivered or only picked up at customer locations. In the TSPPD a single route needs to be constructed. Applications of the TSPPD may be found in the routing of automated guided vehicles which move items between workstations, in dial-a-ride systems where passengers are transported between different pickup and delivery locations and in less-than-truckload transportation (Dumitrescu et al., 2010). The literature concerning the TSPPD includes exact methods as well as heuristics to solve the problem and all consider, to the author's knowledge, one-dimensional loading. The sequence-based loading constraint can therefore be reduced to a LIFO constraint. First-in-first-out (FIFO) is also sometimes assumed as can be seen in Table 2.3. Furthermore, various models include time windows. Orthogonality constraints, orientation constraints and stacking constraints are not relevant since only one-dimensional models have been developed. Formulations for the TSPPD with LIFO loading are presented by Arbib et al. (2009) and Cordeau et al. (2010b), while a formulation for the TSPPD with FIFO loading is presented by Erdoğan et al. (2009) and Cordeau et al. (2010a). Côté et al. (2012a) present a formulation for the TSPPD with multiple stacks and LIFO loading. Øvstebø et al. (2011) give a formulation for the TSP on Roll-on/Roll-of (RoRo) ships.

Ladany and Mehrez (1984) make the first contribution to the TSPPD with LIFO constraints. The motivation for their study is a real-world delivery problem in which *reshuffling* of goods inside a container causes costs and time losses. They are the first to deal with the problem of reshuffling in optimal routing design and are able to solve instances exactly with up to 3 requests. Later, Pacheco (1997, in (Iori and Martello, 2010)) develops a heuristic method to solve the TSPPD with LIFO constraints, while Carrabs et al. (2007a) develop a VNS. Carrabs et al. (2007b) develop an additive branch-and-bound method to solve the same problem exactly for instances with up to 43 vertices. In the same paper, a branch-and-bound algorithm is applied to the TSPPD with FIFO loading. Cordeau et al. (2010a) tackle the TSPPD with FIFO loading with a branch-and-cut method and are able to solve instances with up to 43 vertices. Arbib et al. (2009) present a linear programming formulation of the TSPPD with LIFO loading. The problem is solved with up to 21 vertices using CPLEX 9.0. Cordeau et al. (2010b) develop a branch-and-cut method to solve the TSPPD with LIFO for instances with up to 25 requests. Li et al. (2011) build upon and improve the VNS of Carrabs et al. (2007a) to solve the problem heuristically.

Levitin and Abezgaouz (2003) consider the routing of an Automated Guided Vehicle (AGV) which is used for carrying multiple pallets between workstations. Each additional pallet is placed on top of the pallets that are already carried by the AGV. To avoid rearranging the pallets at the workstations, a LIFO policy is assumed. They develop an exact algorithm to solve the problem with up to 100 vertices.

Côté et al. (2012b) consider the TSPPD with multiple stacks with LIFO loading. A LNS is proposed to solve the problem heuristically. Côté et al. (2012a) propose a branch-and-cut algorithm for the same problem and are able to solve instances with up to 43 vertices.

Øvstebø et al. (2011) examine a similar problem on Roll-on/Roll-of (RoRo) ships that transport cargo on wheels. The ship contains several decks and each deck may be divided into several lanes in which the cargo may be placed. The lanes may be compared to stacks in a truck. Sequence-based loading, stability constraints as well as time windows are considered, of which the former is modeled as a soft constraint. A penalty cost is incurred if the constraint is violated. This situation corresponds to reality because although reshuffling cargo represents an inconvenience, this may be allowed in RoRo setting if supplementary cargo can be carried (Øvstebø et al., 2011). Two types of stability measures concerning weight distribution are considered. The first one refers to the torque from the cargo on the ship that makes the ship lean side-ways which should be within limits at all times. The second stability measure refers to the distance of the ship's bottom deck to the center of gravity of the ship which should be less than some specified ceiling at all times. The aim of the problem is to maximize the revenue from cargo carried from optional nodes minus a penalty for cargo not carried from mandatory nodes, a penalty for violating the sequence-based loading constraint, travel cost, and cost of ship usage.

2.4.8 Double traveling salesman problem with pickups and deliveries with multiple stacks

The Double Traveling Salesman Problem with Multiple Stacks (DTSPMS) is proposed by Petersen and Madsen (2009). Pickup and delivery of goods are performed in two separate networks. All pickups are made before any delivery can take place. The goods cannot be repacked. This means that goods can not be moved to another container or shuffled inside a single container. Vertical stacking is not allowed. The goods can be placed in several rows (horizontal stacks). In each row the LIFO principle needs to be obeyed. It is assumed that each order consists of a single item. The problem is based on a real-world application in which in a first phase a container is loaded onto a truck to perform pickup operations and returned by that truck to a depot or terminal. In a second phase, the container is loaded onto a train, ship, plane or another truck and transported to another depot or terminal. In the depots or terminals, there are no facilities to repack the items. In the final phase, the container is again transferred to a truck which performs the delivery operations (Petersen and Madsen, 2009). A solution for the DTSPMS consists of a pickup and a delivery tour with a corresponding feasible packing plan for the items in the container. The total combined distance of the pickup and delivery tour is minimized. In Figure 2.3 an example of a simple DTSPMS with four items and two stacks is displayed. Items are picked up in the pickup tour (a) and delivered in the delivery tour (b). A possible feasible packing plan can be found in the last picture (c). The loading of the items in the stack is done from bottom to top and the unloading from top to bottom. In the loading plan it can be seen that the LIFO constraints in both stacks are satisfied. All DTSPMS models take into account one-dimensional packing constraints and LIFO loading in each stack. Several exact solution methods have been developed as shown in Table 2.3. A formulation of the DTSPMS is presented by Petersen and Madsen (2009). To our knowledge, none of the papers tackling the DTSPMS include time windows.

Lusby et al. (2010) propose an exact algorithm to solve the DTSPMS for instances with up to 18 requests. They first generate a set of pickup tours and a set of delivery tours. In a second step, combinations of delivery and pickup tours are matched in the TSP matching problem which verifies whether the combinations generate a feasible packing plan. Only the best delivery and pickup tours in terms of length are considered. Petersen et al. (2010) propose several different modeling approaches for an exact solution of the DTSPMS. First, a branch-and-cut approach is used on the mathematical programming formulation of the problem introduced in Petersen and Madsen (2009) which is called the 'precedence' model. Next, a variation of the precedence model is proposed and solved with a branch-and-cut approach. Finally, two new different mathematical formulations (the flow model and the TSP with Infeasible Paths (TSPIP)) are developed. To solve the flow model, again a branch-and-cut approach is used. For the TSPIP a decomposition approach is used to solve the problem. The solution of the TSPIP with a decomposition approach turned out to be the most successful approach in which the problem is solved exactly for instances with up to 25 requests. Lusby and Larsen (2011) improve the exact method developed by Lusby et al. (2010) by including an additional preprocessing technique: the longest common subsequence between the pickup and the delivery tour. This preprocessing technique significantly decreases the number of matching problems that need to be solved. This makes it possible to consider more matching problems in the same amount of time and dramatically improves the efficiency of the solution method. Instances with up to 28 requests are solved. Alba et al. (2013) develop a branch-and-cut algorithm to solve the DTSPMS exactly for instances with up to 25 requests. Felipe et al. (2011) improve the previously developed VNS in Felipe et al. (2009) by allowing intermediate infeasible solutions. Carrabs et al. (2013) consider the double TSP with two stacks. They develop a branch-and-bound algorithm to solve this problem exactly for instances with up to 29 requests. Iori and Riera-Ledesma (2015) generalize the DTSPMS by considering multiple routes. The resulting problem is the Double Vehicle Routing Problem with Multiple Stacks (DVRPMS). A heterogeneous vehicle fleet is considered, where the number of stacks as well as the capacity of the stacks may be

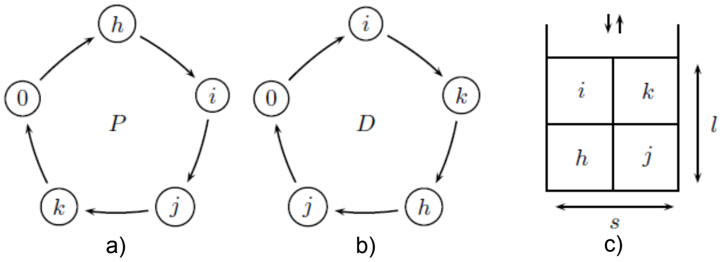

Figure 2.3: A simple DTSPMS example with a pickup tour (a), a delivery tour (b) and a loading plan (c) (figure from (Alba et al., 2013))

different.

2.4.9 VRP with pickups and deliveries with additional loading constraints

We found nine articles in the scientific literature that consider pickup and delivery problems with multiple vehicles combined with loading constraints. Six of them consider one-dimensional loading. Time windows as well as a heterogeneous vehicle fleet are sometimes included as shown in Table 2.3. Two papers propose an exact solution method (Cherkesly et al., 2015a, 2016). Fagerholt et al. (2013) present a formulation for the VRPPD with time windows, complete-shipment constraints and connectivity constraints. Cherkesly et al. (2015a) present a formulation for the VRPPD with time windows and LIFO loading. The VRPPD with multiple vehicles is a generalization of the TSPPD. As a consequence, all applications (AGVs, dial-a-ride-problems, less-than-truckload transportation ...) of the TSPPD may be considered by the VRPPD with the additional possibility of using more than a single vehicle, which is often encountered in real-life (Braekers et al., 2014).

Xu et al. (2003) present a practical pickup and delivery problem in which they consider multiple time windows, heterogeneous vehicles, compatibility constraints between items and vehicle types, separation constraints, driver's work rules and LIFO loading. The problem is solved with a hybrid approach in which heuristics are integrated in a column generation framework. Cheang et al. (2012) consider the multiple vehicle pickup and delivery problem with LIFO loading and distance constraints. A homogeneous fleet is assumed. A two-stage method is proposed to solve the problem. In the first stage the number of vehicles required is minimized using a SA and an ejection pool approach. The second stage minimizes total travel distance using a

VNS and a probabilistic TS.

Fagerholt et al. (2013) present a VRPPD with time windows and loading constraints to solve a real-life ship routing and scheduling problem that arises in tramp shipping. Complete-shipment constraints, connectivity constraints and a heterogeneous vehicle fleet are taken into account. The objective function maximizes the revenue from the optional spot cargoes minus the variable sailing and port costs through the scheduling period. A TS heuristic is proposed to solve the problem.

Cherkesly et al. (2015a) consider the VRPPD with time windows and LIFO loading. They develop three branch-price-and-cut algorithms to solve the problem exactly for instances with up to 75 requests. Cherkesly et al. (2015b) develop a population based metaheuristic to solve larger instances of the same problem heuristically. In both papers the number of vehicles is first minimized before minimizing the total traveled distance. A variant of the previous problem is the VRPPD with time windows and multiple stacks, which is defined by Cherkesly et al. (2016). They develop two branch-price-and-cut algorithms to solve the problem with one, two and three stacks to optimality for instances with up to 75 requests.

Zachariadis et al. (2013a) consider the Pickup and Delivery Routing Problem with Time Windows and Pallet loading (PDRP-TWP) which is discussed in Section 2.4.5.

Malapert et al. (2008) propose a framework to handle the two-dimensional VRPPD with multiple vehicles and sequence-based loading. Items have to be packed orthogonal to the sides of the loading surface and the orientation of the items is fixed. A constraint programming model is formulated and a variant of the bottom-left-fill is applied but turned out not to be efficient to solve the problem. Zachariadis et al. (2016) introduce and solve the VRP with simultaneous pickups and deliveries and two-dimensional loading constraints. A local search method is used for optimizing the routes and a two-dimensional packing heuristic is designed for the packing feasibility check.

2.4.10 Benchmark instances

In Table 2.4, an overview of benchmark instances on routing problems with loading constraints is provided. A distinction is made between different types of problems. For each benchmark instance, the references of papers that use the instances, the number of vertices, the number of instances and the link to the website are provided. These instances are included in order to give a complete overview of the literature concerning routing and loading. They are however not used for the analyses that are performed in this thesis.

2.5 Discussion and research opportunities

The above review of the literature on vehicle routing problems with loading constraints shows that although *classic* VRPs have received a lot of research attention, they often do not reflect the real problems faced by distributors. An important flaw of *classic* VRPs is their ignorance of several real-life loading constraints. An overview of loading constraints, mainly based on the classification of Bortfeldt and Wäscher (2013), is provided. Recently, a number of papers have addressed the integration of loading constraints in vehicle routing problems. These papers may be placed in the following categories based on the type of routing problem and the loading characteristics: Two-Dimensional Loading CVRP (2L-CVRP), Three-Dimensional Loading CVRP (3L-CVRP), multi-pile VRP, multi-compartments VRP, Pallet Packing VRP (PPVRP), Minimum Multiple Trip VRP (MMTVRP) with incompatible commodities, Traveling Salesman Problem with Pickups and Deliveries (TSPPD) with LIFO/FIFO constraints, Double TSP with Pickups and Deliveries with Multiple Stacks (DTSPMS) and Vehicle Routing Problem with Pickups and Deliveries (VRPPD) with additional loading constraints. The latter three categories consider pickup and delivery problems in which items may be picked up and delivered at customer places. For each category the relevant loading constraints that are incorporated into the models are described and the available formulations are discussed. Only a limited number of papers present a problem formulation. This phenomenon might be explained due to the fact that including loading constraints in a routing problem, increases the complexity of the problem formulation. The addition of a three-dimensional loading constraint does not imply adding a single extra row to the formulation but affects the formulation as a whole. Additionally, due to the complexity of the problem mostly heuristic methods are developed which do not often require a problem formulation.

The complexity of the problem not only depends on the complexity of the routing and loading constraints separately, but is also influenced by the combination of both types of constraints. For example, sequence-based loading becomes much more complex in a three-dimensional loading problem than in a one-dimensional problem. The type of transportation request (pickup and delivery of items, or only a single type of request) influences in return the complexity of the sequence-based loading constraint. From the literature survey it is observed that, in most models, the loading constraints are handled as a subproblem of the routing model (e.g. Gendreau et al., 2006; Doerner et al., 2007; Tarantilis et al., 2009; Bortfeldt, 2012; Fuellerer et al., 2010; Ruan et al., 2013). First, solutions of the routing problem are computed, and afterwards, a feasibility check of the loading constraints is performed. Since loading constraints

Table 2.4: Benchmark instances

	Ref	V	I	Website
2L-CVRP				
Iori et al. (2007)	82 78 170 148 103 63 94 173 58	15-255	180	http://www.or.deis.unibo.it/research.html
Martinez and Amaya (2013)		6-30	67	http://ftpprof.uniandes.edu.co/ pylo/inst/VRPM-TW-CL/instances.htm
3L CVRP				
Gendreau et al. (2006)	152 141 121 79 20 167 176 151 123 22 31	15-100	27	http://www.or.deis.unibo.it/research.html
Moura and Oliveira (2009)	124 22	25	46	www.fe.up.pt/ esicup
Bortfeldt and Homberger (2013)		100-1000	120	http://www.fernuni-hagen.de/evis/service/downloads.shtml
Ceschia et al. (2013)		11-129	13	http://www.diegm.uniud.it/ceschia/index.php?page=vrclp
Multi-pile VRP				
Doerner et al. (2007)	155 121	50-100	21	http://prolog.univie.ac.at/research/VRPandBPP/
Multi-compartments VRP				
Cornillier et al. (2009)		15-50	41	http://www.fsa.ulaval.ca/personnel/renaudj/Recherche/PSRPTW/
Pallet packing VRP				
Zachariadis et al. (2012)		50-200	70 (no TW) 68 (TW)	http://users.ntua.gr/ezach/
Zachariadis et al. (2013a)		100	36	http://users.ntua.gr/ezach/
TSPPD				
Carrabs et al. (2007a)	67 44 42 107 50 49	24-750	42	http://neumann.hec.ca/chairelogistique/data
Li et al. (2011)		24-1000	96	http://www.tigerqin.com/publicatoins/tsppdl
DTSPMS				
Petersen and Madsen (2009)	74 116 133 2 73 29	12-66	60	Website no longer available
Felipe et al. (2009)	73	132	20	http://www.mat.ucm.es/ gregoriotd/dtspmsEn.htm
VRPPD				
Cheang et al. (2012)		24-750	126	http://www.computational-logistics.org/orlib/topic/MTSPPDL/

Ref = other papers using this dataset
V = number of vertices
I = number of instances

are often complex, a considerable amount of time may be saved by only checking the *best* solutions of the routing model. There exist some exceptions to this method of incorporating loading constraints in VRP models, such as the sequential approach of Moura and Oliveira (2009) in which the container loading and the vehicle routes are planned simultaneously. Another example is the Packing First - Routing Second heuristic of Bortfeldt and Homberger (2013) in which first a feasible packing scheme for each particular customer is computed after which the routes are constructed, followed by an optimization of the overall packing plan of all customers belonging to a single route.

As the combination of vehicle routing problems with loading constraints is a fairly recent domain of research, a number of opportunities for future research can be identified. An interesting path of research could incorporate weight distribution constraints into VRPs. In the scientific literature, an even weight distribution of the cargo inside the vehicle is often achieved by placing the center of gravity of the load as close as possible to the midpoint of the container. Closely related to balancing cargo weight inside the vehicle is balancing it over the axles of the vehicle. Axle weight limits pose a challenge to transportation companies as they incur high fines in the event of non-compliance. Since weight distribution varies with every pickup or delivery, this should be monitored not just at the point of departure but throughout the journey. Axle weight constraints have not yet been considered in a VRP despite of its practical relevance. Therefore, in the next chapters of this dissertation, the CVRP with sequence-based pallet loading and axle weight constraints is considered. This is a special case of 2L-CVRP in which all items are pallets of equal size which may be placed in two horizontal rows in the vehicle. The problem may also be seen as a special case of multi-pile VRP in which the horizontal rows represent piles. In the CVRP with sequence-based pallet loading and axle weight constraints however, pallets are alternately packed in the left and right row, while this is not a requirement in the multi-pile VRP.

Another line of future research could focus on pickup and delivery problems with loading constraints. Except for a single paper (Malapert et al., 2008), the current literature concerning PDPs only takes one dimension into account.

Next, few solutions methods for PDPs with loading constraints and multiple vehicles have so far been developed. Future research could analyze PDPs with multiple vehicles and multiple dimensions. As for the multi-compartments VRP, one might focus on scheduling over multiple periods or over multiple trips in a single tour where contamination from load residuals may be considered. A compartment carrying a specific product might not be available after emptying for another product before cleaning.

Furthermore, it appears that few exact methods have been devised to solve VRPs with loading constraints. Hence future research could focus on creating exact methods to solve VRPs with loading constraints to which heuristic solutions may be compared.

A final observation is that other *rich* constraints are rarely incorporated into the current VRP models with loading constraints. Even time windows are not often included in the current models. Including time windows or other additional constraints such as the use of a heterogeneous fleet, maximum route length and duration or drivers' regulations in current VRP models with loading constraints would go some considerable way towards making these more realistic.

Chapter 3

CVRP with sequence-based pallet loading and axle weight constraints: Problem formulation and description

3.1 Introduction

This chapter [1] focuses on the formulation of the VRP with axle weight constraints (Figure 3.1). Contacts with logistics service providers pointed out that they are faced with loading problems in their decisions on route scheduling. Current commercial route scheduling software does not take into account most of these loading constraints, which makes the routes often not feasible in practice. This gives rise to last-minute and often non-optimal changes. The development of vehicle routing models that incorporate loading constraints is therefore vital for an efficient scheduling of routes. The focus of this chapter and the following chapters is on the combination of a VRP with the loading of pallets inside a vehicle since this is a problem setting often encountered by distributors. Pallets may be placed in two rows inside the vehicle but cannot be stacked on top of each other because of their weight, fragility or customer preferences. Sequence-based loading is assumed which ensures that when arriving at a customer, no items belonging to customers served later on the route

[1] This chapter is based on Pollaris et al. (2016a).

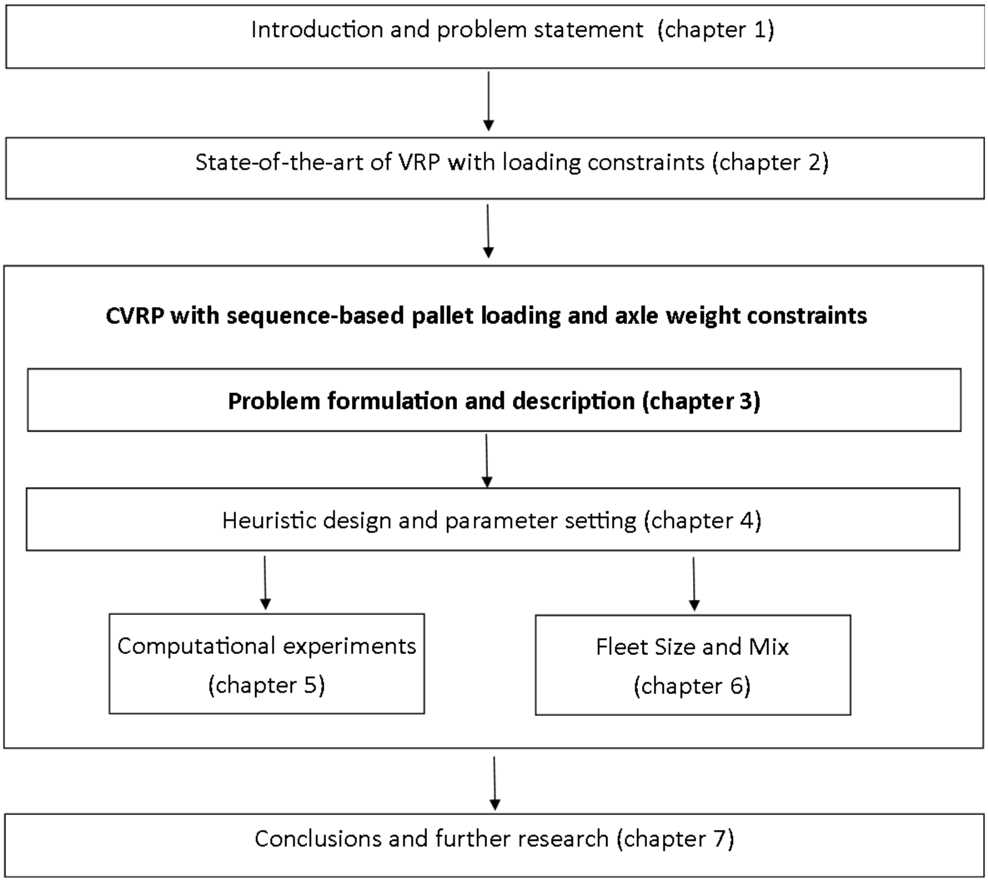

Figure 3.1: Outline of the thesis

block the removal of the items of the current customer. Furthermore, the capacity of a truck is not only expressed in total weight and number of pallets but also consists of a maximum weight on the axles of the truck.

In this chapter, two problem formulations for the CVRP with sequence-based pallet loading and axle weight constraints are developed. Section 3.2 describes the problem characteristics. The calculation of the weight on the axles is described in Section 3.3. In Section 3.4, the impact of the incorporation of axle weight constraints in a routing problem is illustrated with an example. A mixed integer linear programming formulation for the CVRP with sequence-based pallet loading and axle weight constraints is presented in Section 3.5. In Section 3.6, a set partitioning formulation for the problem is presented. In the final section (Section 3.7), conclusions and future research opportunities are discussed.

3.2 Problem Description

The problem of interest in this chapter is a CVRP with sequence-based pallet loading and axle weight constraints. To the best of the author's knowledge, it is the first time that axle weight constraints are incorporated into a VRP. The problem is based on a real-world distribution problem. Following assumptions are made:

(a) The objective is to minimize total distance traveled.

(b) No time windows are considered.

(c) The vehicle fleet is homogeneous.

(d) Each customer has to be visited exactly once.

(e) The demand of the customers consists of europallets (80x120 cm) and is heterogeneous.

(f) Pallets may be placed inside a truck in two horizontal rows and are packed dense in the truck. This means there cannot be a gap between two consecutive pallets inside the truck. Pallets are alternately packed in the left and right row. This implies that the pallets of a single customer cannot be aligned in a single row. Moreover, the pallets of the last customer are placed at the deepest portion of the loading area.

(g) All pallets of a single customer have the same weight and the weight is uniformly distributed inside each pallet i.e. the center of gravity of a pallet lies in its geometric midpoint.

(h) The truck can only be unloaded at the rear side.

(i) Sequence-based loading is imposed.

(j) Vertical stacking is not allowed.

Assumptions a, b, c and g are the least realistic for real problems. In reality, the objective of a transportation company is to minimize total costs instead of minimizing total distance traveled. Furthermore, companies are faced with time windows within which a delivery must take place. Companies also usually dispose over a heterogeneous instead of a homogeneous fleet of vehicles. Finally, in most distribution problems, pallets of a single customer do not necessarily have the same weight.

On the other hand, assumptions d, f and i are common in real problems. Customers have to be visited exactly once because split deliveries are generally not allowed. Dense packing is often used in practice since the stability of the load is much higher when the pallets are packed dense than when gaps are allowed between pallets. Therefore, less time needs to be spent on securing the cargo. Sequence-based loading is often assumed in real problems to avoid moving pallets of other customers when arriving at a customer.

Assumptions e, h and j are realistic for a number of real problems, but may also be restrictive for a number of real problems. The demand of the customers may consist of non-palletized goods. In this case, dense packing may also not always be possible. Most trucks are loaded and unloaded through the rear side, however there also exist trucks that may be unloaded through the long side, and/or via the top side. Finally, vertical stacking of pallets may sometimes be allowed when non-fragile pallets are considered.

The axle weight is the weight that is placed on the axles of the truck. A truck with five axles is illustrated in Figure 3.2. The first axle, also called the *steering axle*, and the second axle, called the *driving axle*, both belong to the tractor. The axles of the semi-trailer are assumed to be *tridem axles*. Tridem axles are three successive axles with a distance between the middle of the first axle and the middle of the second axle and between the middle of the second axle and the middle of the third axle of less than 1.8 and more than 1 meter. When item j is placed onto a vehicle, the weight of the item is divided over the axles of the tractor and the axles of the semi-trailer. a_j^F represents the weight of the items of customer j placed on the coupling of the truck (which is the link between the tractor and the semi-trailer), the *first* axle point. The weight on the coupling is carried by the axles of the tractor. a_j^R represents the weight of the items of customer j on the axles of the semi-trailer, the *rear* axles of the truck. As a truck delivers items to several customers on a single route, the weight on the axles of the truck changes. A load that is placed at the rear of the vehicle (behind the axles of the semi-trailer), has a negative weight on the axles of the tractor. For this reason, it is possible that by unloading this item a violation of the weight limits of the axles of the tractor is induced. It is therefore important that axle weights are considered during the whole trip of the vehicle and not only when the vehicle departs from the depot. To our knowledge, Lim et al. (2013) and Alonso et al. (2017) are the only authors that address axle weight constraints in a container loading problem. Lim et al. (2013) develop a heuristic method to tackle the single container loading problem with axle weight constraints. Alonso et al. (2017) develop integer linear programming models to tackle multi-container loading problems with axle weight constraints in which items are first packed on pallets and afterwards, pallets are placed onto trucks.

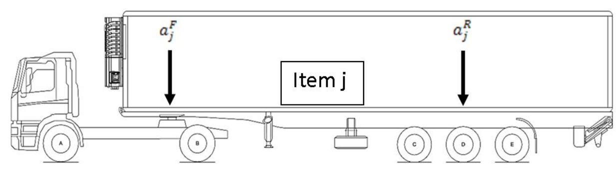

Figure 3.2: Tractor (with two axles) and semi-trailer (with tridem axles) (figure adapted from TruckScience)

Information on the vehicle fleet (measurements, capacity, mass, axle weight limits) is derived from information from a Belgian logistics service provider. In this chapter, a homogeneous fleet of 30-foot trucks is considered. In total, 22 pallets may be placed inside a truck in two horizontal rows. The 30-foot trucks consist of a tractor, a semi-trailer and a container. The length, width and height of the inside dimensions of the container are respectively 9.12 meters, 2.44 meters and 2.44 meters. The mass of the empty tractor is 6.82 tonnes of which 4.88 tonnes is carried by the steering axle and 1.97 tonnes is supported by the driving axle. The tare weight of the container is 3 tonnes of which 2 tonnes is supported by the coupling and 1 ton is support by the axles of the semi-trailer. The mass of the empty trailer is 2 tonnes which is carried by the axles of the trailer. The maximum weight on the coupling of the tractor is 13.6 tonnes. This is subtracted by the weight of the container carried by the coupling (2 tonnes), which leads to a maximum weight of the load that may be placed on the coupling of 11.6 tonnes. 80% of the weight on the coupling is supported by the driving axles of the tractor, while the remaining 20% is carried by the steering axle. The maximum weight capacity of the tridem axles of the semi-trailer is 24 tonnes. This is subtracted by the weight of the semi-trailer (2 tonnes) and the weight of the empty container carried by the axles of the semi-trailer (1 tonnes) which gives a total of 21 tonnes. This is the maximum weight of the load that may be carried by the axles of the semi-trailer. The maximum weight of the vehicle is 44 tonnes. This is subtracted by the empty weight of the tractor (6.82 tonnes), the tare weight of the container (3 tonnes) and the weight of the semi-trailer (2 tonnes), which results in a maximum weight of the load of 32.2 tonnes.

3.3 Axle weight calculation

In this section, the calculation of the weight on the axles of a truck, in which pallets are placed in two horizontal rows, is described. The calculation depends on the center of gravity of the pallets inside the truck and on vehicle-specific parameters. In Subsection 3.3.1, the formulas for the calculation of the axles weights are presented. In Subsection 3.3.2, the determination of the center of gravity is described.

3.3.1 Calculation of the weight on the axles of a truck

The calculation of the weight of the pallets on the axles is based on the static equilibrium condition derived from Newton's first law of motion. The law may be stated as follows:

If the resultant force acting on a particle is zero, the particle will remain at rest (if originally at rest) or will move with constant speed in a straight line (if originally in motion).

This implies that if the axles are at equilibrium, the net force acting upon the axles should be zero Newton. Therefore, if all forces are added together as vectors, then the resultant force (the vector sum) should be zero Newton. Figure 3.3 presents a truck with a single pallet of customer j.

The mass of the pallet of customer j is denoted by m_j. Parameter CG_j represents the distance from the front of the container to the center of gravity of the pallet of customer j. Parameter c denotes the distance from the front of the container to the coupling. Parameter d represents the distance between the coupling and the central axle of the semi-trailer. Figure 3.4 presents a free-body diagram on which the forces that are applied by the pallet on the coupling (denoted by P_1) and on the axles of the semi-trailer (denoted by P_2) are displayed.

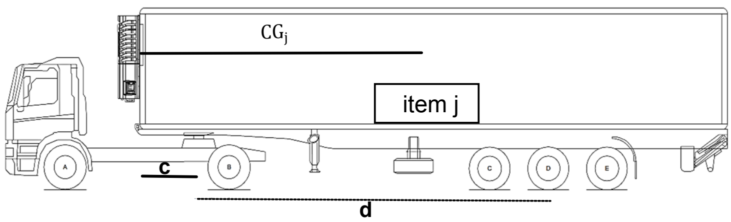

Figure 3.3: Tractor (with two axles) and semi-trailer (with tridem axles) (figure adapted from TruckScience)

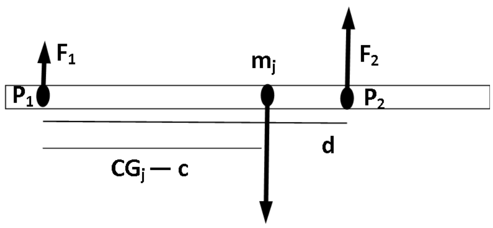

Figure 3.4: Free body diagram

According to the equilibrium condition, following equation holds in P_1:

$$d \cdot F_2 - (CG_j - c) \cdot m_j \cdot a = 0 \tag{3.1}$$

With gravitational constant $a = 9.81 \frac{m}{s^2}$.

From this equation, the following formula may be derived for F_2:

$$F_2 = \frac{(CG_j - c) \cdot m_j \cdot a}{d} \tag{3.2}$$

Since $F = m \cdot a$ (second law of Newton), the mass from the pallet from customer j that is placed on the axles of the trailer, P_2, may be calculated as follows:

$$m_{j2} = \frac{(CG_j - c) \cdot m_j}{d} \tag{3.3}$$

An analogous reasoning leads to the following formula for the mass of the pallet from customer j that is placed on the coupling, P_1:

$$m_{j1} = \frac{(d - (CG_j - c)) \cdot m_j}{d} \tag{3.4}$$

This formula may be rearranged to the following:

$$m_{j1} = m_j - \frac{(CG_j - c) \cdot m_j}{d} = m_j - m_{j2} \tag{3.5}$$

Note that in physics, weight refers to force (F) and is expressed in Newton. However, in this thesis, weight is used to refer to mass (expressed in tonnes) since this is common practice in the VRP literature. The general formula for the weight of the pallets of customer j when traveling from customer i to customer j on the coupling point or the axles of the tractor (a_{ij}^F) and on the axles the semi-trailer (a_{ij}^R) is presented in equations (3.6) and (3.7).

$$a_{ij}^R = \frac{(CG_j - c)}{d} m_j \tag{3.6}$$

$$a_{ij}^F = m_j - a_{ij}^R \tag{3.7}$$

The weight of the pallets is divided over the axles of the semi-trailer and the axles of the tractor. The distribution of the weight over the axles depends on the distance between the pallet and the axles. In the first part of equation (3.6) the percentage of the weight that is assigned to the axles of the semi-trailer is computed by dividing the distance between the coupling and the center of gravity of the item by the distance between the coupling and the central axle of the semi-trailer. In the second part of equation (3.6), this percentage is multiplied by the weight of the item to compute the weight that is carried by the axles of the semi-trailer. The larger the distance between the item and the coupling, the higher the percentage of weight that is distributed to the axles of the semi-trailer will be. The weight on the coupling is computed in equation (3.7) by subtracting the weight on the axles of the semi-trailer from the weight of the item.

Based on real-world information, parameters c and d respectively have a value of 1 meter and 5.5 meters in a 30-foot truck. In the next paragraphs, calculations are expressed in *pallet places* instead of meters. A pallet place is a length unit and equals the length of a single pallet inside the container. The europallets (1.20 m x 0.8 m) are placed in two rows with a width of 1.20 m, which makes the length of each pallet inside the container (= a single pallet place) equal to 0.8 m. The value of c becomes 1.25 ($= \frac{1m}{0.8m/palletplace}$) pallet places and d has a value of 6.875 ($= \frac{5.5m}{0.8m/palletplace}$) pallet places.

While c, d and m_j are parameters which are known beforehand, determining the value of CG_j is less straightforward since the center of gravity depends on the location of item j in the truck. In the next subsection, the determination of the center of gravity is discussed.

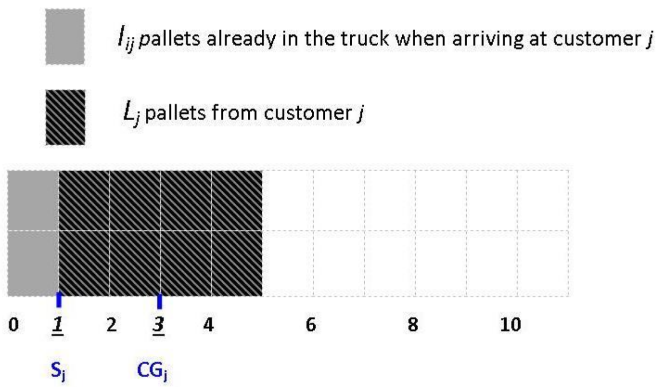

Figure 3.5: Top view of a container with indication of starting point (S_j) and center of gravity (CG_j)

3.3.2 Calculation of the center of gravity

The calculation of the center of gravity is illustrated in Figure 3.5 in which the grey shaded pallets represent pallets that are already in the truck when arriving at customer j and the black shaded pallets represent the pallets of customer j. Since it is assumed that the center of gravity lies in the geometric midpoint of the pallets of customer j, the center of gravity of the eight pallets of customer j in Figure 3.5 equals 3. The starting point (S_j) is the point at which the first pallet of customer j (when coming from customer i) is placed inside the vehicle, which is at point 1 in Figure 3.5. The number of pallets that are already in the truck when traveling from customer i to customer j is denoted by l_{ij} and L_j represents the number of pallets of customer j.

In the following paragraphs, a formula is presented to calculate the center of gravity of the pallets of a customer. The author is not aware of other sources that use a similar approach. The calculation of the center of gravity is composed of two parts. The first part determines the starting point (S_j) at which the first pallet of customer j will be placed. This depends on l_{ij}. If the number of pallets in the truck is even, $S_j = \frac{l_{ij}}{2}$. When the number of pallets already in the truck is odd, $S_j = \frac{l_{ij}}{2} - 0.5$. In Figure 3.5, $l_{ij} = 2$ which results in $S_j = 1$. The second part of the calculation of the center of gravity determines the distance between the center of gravity of the pallets of customer j and S_j. This depends on the value of l_{ij}, L_j and the capacity of the vehicle in terms of pallets, L.

When l_{ij} is even, the second part of the equation for the center of gravity equals E_j. E_j is the center of gravity of the pallets of customer j when the truck is empty upon arrival ($l_{ij} = 0$). E_j corresponds to equation (3.8) or (3.9) depending on whether L_j is respectively even or odd. In equation (3.9), the center of gravity of every pallet separately is added up and divided by the number of pallets of customer j (L_j).

If L_j is even:

$$E_j = \frac{L_j}{4} \tag{3.8}$$

If L_j is odd:

$$E_j = \frac{(\frac{L_j}{2} + Max[0, (L_j - 2)] + Max[0, (L_j - 4)] + ... + Max[0, (L_j - (L - 2))])}{L_j} \tag{3.9}$$

When l_{ij} is odd, the second part of the equation for the center of gravity equals O_j. O_j is the center of gravity of the pallets of customer j when a single pallet is placed in the truck upon arrival ($l_{ij} = 1$). O_j equals equations (3.10) or (3.11) depending on whether L_j is respectively even or odd. The calculation of O_j is similar to the calculation of E_j. In equation (3.11), the last two terms equal the center of gravity of the first pallet of customer j that is placed inside the truck.

If L_j is even:

$$O_j = \frac{L_j}{4} + 0.5 \tag{3.10}$$

If L_j is odd:

$$O_j = \frac{(L_j + Max[0, (L_j - 2)] + Max[0, (L_j - 4)] + ... + Max[0, (L_j - (L - 2))] - 0.5)}{L_j} \tag{3.11}$$

Since the number of pallets of customer j (L_j) and the vehicle capacity in terms of pallets (L) are known in advance, O_j and E_j can be treated as parameters or constants in the mathematical model in Section 3.5. To integrate equations (3.8), (3.9), (3.10) and (3.11) and the calculation of S_j into a single formula to define the center of gravity, parameter P_j and variable C_{ij} are created. $P_j = 1$ if L_j is even and $P_j = -1$ when L_j is odd. Variable C_{ij} is defined to keep track of the variable l_{ij}. When a vehicle travels from i to j, $C_{ij} = 1$ when l_{ij} is even and $C_{ij} = -1$ when l_{ij} is odd. When a vehicle does not travel from i to j, $C_{ij} = 0$.

The integrated formula of CG_j is displayed in equation (3.12). Note that this is a linear equation.

$$\begin{aligned} CG_j =& \frac{\sum_{i \in V} l_{ij}}{2} - \frac{1}{4} \cdot (1 - \sum_{i \in V} C_{ij}) + O_j \cdot \frac{1}{2} \cdot (1 - \sum_{i \in V} C_{ij}) \\ &+ E_j \cdot \frac{1}{2} \cdot (1 + \sum_{i \in V} C_{ij}) \forall j \in V \end{aligned} \tag{3.12}$$

The formula consists of two parts.

$$\frac{\sum_{i \in V} l_{ij}}{2} - \frac{1}{4} \cdot (1 - \sum_{i \in V} C_{ij}) \tag{3.12.1}$$

$$O_j \cdot \frac{1}{2} \cdot (1 - \sum_{i \in V} C_{ij}) + E_j \cdot \frac{1}{2} \cdot (1 + \sum_{i \in V} C_{ij}) \tag{3.12.2}$$

In equation (3.12.1), the starting point S_j at which the first pallet of customer j will be placed is determined. When l_{ij} is even, $C_{ij} = 1$ and the second term will become 0. Only the first term remains. When l_{ij} is odd, $C_{ij} = -1$ and the second term becomes -0.5. Equation (3.12.2) calculates the distance between the center of gravity of the pallets of customer j and the front of the container, when no pallets or a single pallet are inside the truck. This distance equals the distance between the center of gravity of the pallets and the starting point S_j. When l_{ij} is even, $C_{ij} = 1$ and the first term becomes zero while the second term becomes E_j. When l_{ij} is odd, $C_{ij} = -1$ and the first term becomes O_j while the second term turns to zero.

3.4 Illustrative example

In this section, the impact of incorporating axle weight constraints in a routing model with sequence-based loading is illustrated with an example. In Figure 3.6, a depot with four customers is presented. Each customer has a demand of five europallets. The total mass of the pallets of customer 1, 2, 3 and 4 is respectively 12 tonnes, 2 tonnes, 2 tonnes and 12 tonnes. The distance matrix of the customer nodes and the depot may be found in Table 3.1. The shortest route between the depot and customer locations is computed with and without taking axle weight constraints into account.

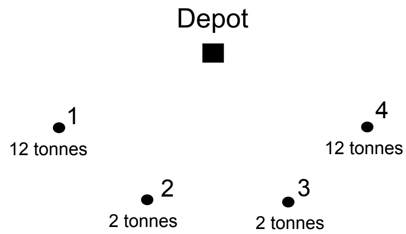

Figure 3.6: Graphical representation of a depot with four customers

Table 3.1: Distance matrix illustrative example

	Depot	Customer 1	Customer 2	Customer 3	Customer 4
Depot	0.00	3.16	2.24	2.24	3.16
Customer 1	3.16	0.00	2.24	4.12	6.00
Customer 2	2.24	2.24	0.00	2.00	4.12
Customer 3	2.24	4.12	2.00	0.00	2.24
Customer 4	3.16	6.00	4.12	2.24	0.00

The objective is to minimize total distance traveled. An optimal vehicle route when axle weight constraints are not considered is graphically represented in Figure 3.7(a). The vehicle starts in the depot, visits customer nodes 1, 2, 3, 4 and returns to the depot. The loading scheme of the container may be found in Figure 3.8(a). In Table 3.2, the total mass of the load, the weight of the load on the coupling and on the axles of the semi-trailer when the truck arrives at each customer node is given. The weight on the coupling is calculated by taking the sum of the weights on the coupling of all pallets that are inside the truck. For each customer, the weight of the pallets on the coupling is calculated with the formula presented in equation (3.6). Similarly, the weight on the axles of the semi-trailer is calculated by taking the sum of the weights on the semi-trailer of all pallets that are inside the truck. The weight of the pallets on the axles of the semi-trailer is calculated for each customer with the formula presented in equation (3.7).

The total mass of the load is well below the maximum weight capacity (32.2 tonnes) of the vehicle. The cargo weight on the coupling is greater than the weight limit (11.6 tonnes) when the vehicle departs from the depot until it arrives at the last customer. This means that the axle weight limits on the axles of the tractor are

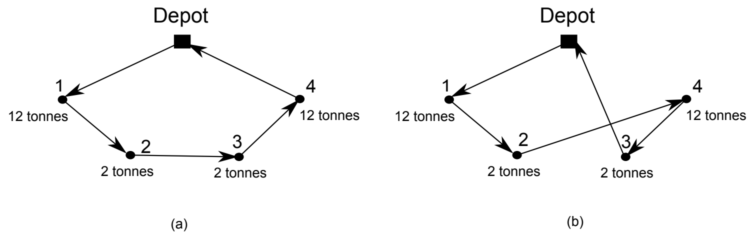

Figure 3.7: Graphical representation of an optimal vehicle route (a) without axle weight constraints, (b) with axle weight constraints

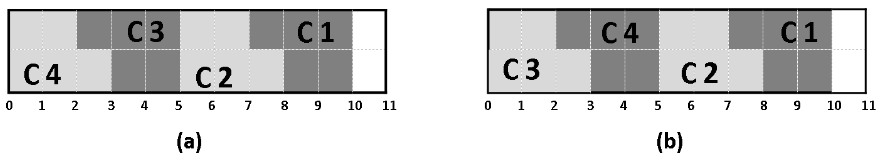

Figure 3.8: Loading scheme of a container (in top view) of the optimal route (a) without axle weight constraints, (b) with axle weight constraints. The load of respectively customer 1, 2 , 3 and 4 is indicated by C1, C2, C3 and C4

violated on the vehicle route. The highest axle weight violation takes place between customer 1 and customer 2. When the vehicle departs from customer 1, the weight on the coupling is 18% higher than the limit. Therefore the solution is not feasible for the distribution company. In this symmetric VRP without axle weight constraints, visiting sequence 4-3-2-1 (the reverse order of customers from the first solution) is also an optimal route with the same total distance traveled as visiting sequence 1-2-3-4. This sequence generates the same axle weight violations as visiting sequence 4-3-2-1.

Table 3.2: Results of the illustrative example without axle weight constraints

Customer	Total mass (kg)	Weight coupling (kg)	Weight axles semi-trailer (kg)
1	28,000	12,727*	15,273
2	16,000	13,731*	2,269
3	14,000	13,200*	800
4	12,000	11,913*	87

* >= 11.6 tonnes

In Figure 3.7(b), the optimal vehicle route when axle weight constraints are considered is graphically represented. The vehicle starts in the depot, visits customer nodes 1, 2, 4, 3 and returns to the depot. In Table 3.3, the total mass of the load and the weight of the load on the coupling and on the axles of the semi-trailer when the truck arrives at each customer node is given. Total distance traveled is 14, which is an increase of 9 percent compared to the optimal solution without axle weight constraints. The maximum weight on the coupling is 10.2 tonnes which does not exceed the weight limit on the coupling (11.6 tonnes). The maximum load on the axles of the semi-trailer is 18.8 tonnes which is below the weight limit on the axles of the semi-trailer (21 tonnes). In Figure 3.8(b), the loading scheme of the container is presented. Note that although the change only consists of swapping two customers (customer 3 and 4) on the route, all axle weight violations have disappeared. This is because the heavy pallets of customer 4 are no longer at the front of the vehicle and are therefore not only carried by the coupling, but also partially by the axles of the semi-trailer. To conclude, considering axle weight constraints in route scheduling may lead to a higher distance traveled, but ensures a feasible weight distribution of the load inside the vehicle.

Table 3.3: Results of illustrative example with axle weight constraints

Customer	Total mass (kg)	Weight coupling (kg)	Weight axles semi-trailer (kg)
1	28,000	9,236	18,764
2	16,000	10,240	5,760
4	14,000	9,709	4,291
3	2,000	1,985	15

3.5 Problem formulation: MILP

In this section, a Mixed Integer Linear Programming (MILP) formulation of a CVRP with sequence-based pallet loading and axle weight constraints is presented. Note that the considered problem is a delivery problem, as illustrated in Section 3.3. The calculation for the center of gravity is however less complex when formulated as a pickup problem. Therefore, the problem in this section is formulated as a pickup problem. Since we consider a symmetric problem (symmetric distance matrix) and do not assume time windows, the optimal visiting sequences of the pickup problem can be reversed to determine the optimal visiting sequences of the delivery problem. Note that the drawback of this formulation is that if the problem is extended with time windows or other realistic features such as time-dependent routing, the pickup routes cannot be simply reversed to find the delivery routes.

To formulate the problem the following notation is used:

$V = \{0, 1, ..., n+1\}$ set of vertices with customers (node $1, \ldots, n$) and depot (node 0, $n+1$) (indices i, j, k)

$E =$ set of edges (i, j) where $i, j \in V$ and $i \neq j$

c_{ij} = travel distance on edge(i, j)

L_j = number of pallets demanded by customer j

L = maximum number of pallets per vehicle

Q_j = total mass of the pallets of customer j

Q = maximum mass capacity of each vehicle

E_j = center of gravity of the pallets of customer j when the container is empty upon arrival at customer j

O_j = center of gravity of the pallets of customer j when 1 pallet is in the container upon arrival at customer j

A^F = maximum weight on the coupling

A^R = maximum weight on the axles of the semi-trailer

W^T = mass of the empty truck

W^{TD} = weight of the empty truck on the driving axle

W^{TR} = weight of the empty truck on the axles of the semi-trailer

h = fraction of the weight on the coupling that is carried by the driving axle

c = distance between the front of the container and the coupling

d = distance between the coupling and the center of the axles of the semi-trailer

$$P_j = \begin{cases} 1 & \text{if } L_j \text{ even} \\ -1 & \text{if } L_j \text{ odd} \end{cases}$$

The decision variables are defined as:

$$x_{ij} = \begin{cases} 1 & \text{if a vehicle travels from } i \text{ to } j \text{ with } (i,j) \in \text{E} \\ 0 & \text{otherwise} \end{cases}$$

$CG_j =$ center of gravity of the pallets of customer j

$$l_{ij} = \begin{cases} \text{total number of pallets on this link} & \text{if a vehicle travels from } i \text{ to } j \\ 0 & \text{otherwise} \end{cases}$$

$$q_{ij} = \begin{cases} \text{total cargo mass on this link} & \text{if a vehicle travels from } i \text{ to } j \\ 0 & \text{otherwise} \end{cases}$$

$$a_{ij}^F = \begin{cases} \text{total cargo weight on the coupling on this link} & \text{if a vehicle travels from } i \text{ to } j \\ 0 & \text{otherwise} \end{cases}$$

$$a_{ij}^R = \begin{cases} \text{total cargo weight on the axles of the semi-trailer on this link} & \text{if a vehicle travels from } i \text{ to } j \\ 0 & \text{otherwise} \end{cases}$$

$$C_{ij} = \begin{cases} 1 & \text{if } l_{ij} \text{ even and a vehicle travels from } i \text{ to } j \\ -1 & \text{if } l_{ij} \text{ odd and a vehicle travels from } i \text{ to } j \\ 0 & \text{otherwise} \end{cases}$$

The problem is formulated as follows:

$$\min \sum_{(i,j)\in E} c_{ij} x_{ij} \tag{3.13}$$

Subject to

$$\sum_{i\in V} x_{ij} = 1 \qquad \forall j \in V \backslash \{0, n+1\} \tag{3.14}$$

$$\sum_{j\in V} x_{ij} = 1 \qquad \forall i \in V \backslash \{0, n+1\} \tag{3.15}$$

$$x_{n+1,j} = 0 \qquad \forall j \in V \tag{3.16}$$

$$x_{j,0} = 0 \qquad \forall j \in V \tag{3.17}$$

$$l_{0j} = 0 \qquad \forall j \in V \tag{3.18}$$

$$l_{ij} \leq L x_{ij} \qquad \forall (i,j) \in E \tag{3.19}$$

$$\sum_{i\in V} l_{ij} + L_j = \sum_{k\in V} l_{jk} \qquad \forall j \in V \backslash \{0, n+1\} \tag{3.20}$$

$$q_{0j} = 0 \qquad \forall j \in V \tag{3.21}$$

$$q_{ij} \leq Qx_{ij} \qquad \forall (i,j) \in E \tag{3.22}$$

$$\sum_{i \in V} q_{ij} + Q_j = \sum_{k \in V} q_{jk} \qquad \forall j \in V \backslash \{0, n+1\} \tag{3.23}$$

$$C_{0j} = x_{oj} \qquad \forall j \in V \tag{3.24}$$

$$C_{ij} \leq x_{ij} \qquad \forall (i,j) \in E \tag{3.25}$$

$$C_{ij} \geq -x_{ij} \qquad \forall (i,j) \in E \tag{3.26}$$

$$\sum_{i \in V} C_{ij} P_j = \sum_{k \in V} C_{jk} \qquad \forall j \in V \backslash \{0, n+1\} \tag{3.27}$$

$$a_{0j}^F = 0 \qquad \forall j \in V \tag{3.28}$$

$$a_{0j}^R = 0 \qquad \forall j \in V \tag{3.29}$$

$$a_{ij}^F \leq A^F x_{ij} \qquad \forall (i,j) \in E \tag{3.30}$$

$$a_{ij}^R \leq A^R x_{ij} \qquad \forall (i,j) \in E \tag{3.31}$$

$$a_{ij}^F \geq -W^{TD} x_{ij} \qquad \forall (i,j) \in E \tag{3.32}$$

$$a_{ij}^R \geq -W^{TR} x_{ij} \qquad \forall (i,j) \in E \tag{3.33}$$

$$a_{ij}^F h + W^{TD} \geq 0.25(W^T + q_{ij}) \qquad \forall (i,j) \in E \tag{3.34}$$

$$\sum_{i \in V} a_{ij}^F + Q_j - \frac{(CG_j - c)Q_j}{d} = \sum_{k \in V} a_{jk}^F \qquad \forall j \in V \backslash \{0, n+1\} \tag{3.35}$$

$$\sum_{i \in V} a_{ij}^R + \frac{(CG_j - c)Q_j}{d} = \sum_{k \in V} a_{jk}^R \qquad \forall j \in V \backslash \{0, n+1\} \tag{3.36}$$

$$\begin{aligned} CG_j = & \frac{\sum_{i \in V} l_{ij}}{2} - \frac{1}{4} \cdot (1 - \sum_{i \in V} C_{ij}) + O_j \cdot \frac{1}{2} \cdot (1 - \sum_{i \in V} C_{ij}) \\ & + E_j \cdot \frac{1}{2} \cdot (1 + \sum_{i \in V} C_{ij}) \forall j \in V \end{aligned} \tag{3.37}$$

$$x_{ij} \in \{0, 1\} \tag{3.38}$$

$$l_{ij} \geq 0 \qquad \forall (i,j) \in E \tag{3.39}$$

$$q_{ij} \geq 0 \qquad \forall (i,j) \in E \tag{3.40}$$

The objective function (3.13) aims to minimize total distance traveled. Constraints (3.14) and (3.15) ensure that each customer is visited exactly once. Constraints (3.16) make sure that no route begins in the destination depot (node $n+1$), while constraints (3.17) ensure that no route arrives in the start depot (node 0). Constraints (3.18) initialize the value of l_{0j} to 0 since no pallets are inside a container when it departs from the start depot. Constraints (3.19) limit l_{ij} to the maximum number of pallets that may be placed in each vehicle. Constraints (3.20) keep track of l_{ij} by adding up the number of pallets in the vehicle when arriving at customer j (l_{ij}) with the number of pallets of customer j (L_j). Constraints (3.21) initialize the values of q_{0j} to 0 since a container is empty when it departs from the start depot. Constraints (3.22) limit q_{ij} to the maximum mass capacity (Q) of the vehicle. Constraints (3.23) keep track of q_{ij} by adding up the weight of the load in the container when arriving at customer j (l_{ij}) with the pallet weight of customer j (Q_j). In constraints (3.24), the value of the variable C_{0j} is set to 0 if x_{0j}=0 and set to 1 if x_{0j}=1. Since a container is empty when it departs from the start depot, it has an even number of pallets (0 pallets). Constraints (3.25) and (3.26) guarantee that C_{ij} can only have a non-zero value when a vehicle travels from i to j. Constraints (3.27) keep track of C_{ij} by multiplying the value of C_{ij} when arriving at customer j with parameter P_j. Constraints (3.28) and (3.29) initialize the values of the weight on the coupling (a^F_{ij}) and the weight on the rear axles (a^R_{ij}) to zero. Constraints (3.30), (3.31), (3.32) and (3.33) ensure that a^F_{ij} and a^R_{ij} only have a non-zero value when a vehicle travels from i to j. Constraints (3.30) and (3.31) also specify the upper bounds of respectively a^F_{ij} and a^R_{ij}. The values of the upper bounds A^F and A^R depend on the vehicle characteristics and are specified by legislation. The lower bound of a^F_{ij} may also be fixed in legislation. Belgian legislation (KB 15.03.1968 art 32bis) specifies that the mass corresponding to the load on the driving axle must be at least 25 percent of the total mass of the loaded truck which is captured in constraints (3.34). On the left-hand side of constraints (3.34), the weight of the empty truck on the driving axle (W^{TD}) is added up with parameter h (percentage of the weight on the coupling that is carried by the driving axle of the tractor) multiplied by the weight of the load that is placed on the coupling (a^F_{ij}). On the right-hand side 25 percent of the total mass of the empty truck and the total weight of the load is computed. Since there are no guidelines concerning the lower bound of the weight on the axles of the semi-trailer, constraints (3.33) ensure that this should be at least equal to $-W^{TR}$ to avoid a negative axle weight on the rear axles. Constraints (3.35) keep track of a^F_{ij} by adding up the weight on the coupling when arriving at customer j (a^F_{ij}) with the weight on the coupling of the pallets of customer j. Constraints (3.36) keep track of a^R_{ij} in a similar way. Constraints (3.37) determine the center of gravity of the pallets of customer j (CG_j) as a function of C_{ij} and l_{ij}. This constraint is the same as equation (3.12) and is explained in Section 3.3. Finally, constraints (3.38) to (3.40) define the domain of the decision variables.

3.6 Problem formulation: set partitioning formulation

In this section, a Set Partitioning (SP) formulation for the CVRP with sequence-based pallet loading and axle weight constraints is presented. This formulation is introduced because the MILP is only able to solve instances with up to 20 customers, as will be discussed in Chapter 5. The main drawback of a SP model is the exponential number of variables when the instance size increases because each feasible route results in an additional variable. In the current work however, the number of customers in a route is limited because the maximum number of europallets inside a vehicle is limited (in a 30-foot truck a maximum of 22 pallets may be placed). The number of feasible routes is therefore smaller than for traditional CVRP problems. For this reason, a SP formulation is a promising option for the CVRP with sequence-based pallet loading and axle weight constraints.

Balinski and Quandt (1964) originally proposed a SP formulation for solving a CVRP. First all feasible routes are enumerated. A route is feasible if the vehicle capacity in terms of number of pallets and total weight is not exceeded and if the axle weight limits are not violated. Next, a set partitioning model is solved for all feasible routes. Let the index set of all feasible routes be $\mathcal{R} = \{1, 2, ..., R\}$. Let c_r be the length of route r. Define the parameter a_{ir} for each customer $i \in V$ and for each route $r \in \mathcal{R}$:

$$a_{ir} = \begin{cases} 1 & \text{if customer } i \text{ is served in route } r \\ 0 & \text{otherwise} \end{cases}$$

Define the decision variables for each route $r \in \mathcal{R}$ as:

$$y_r = \begin{cases} 1 & \text{if route } r \text{ is selected} \\ 0 & \text{otherwise} \end{cases}$$

The SP model is defined as follows:

$$\min \sum_{r \in \mathcal{R}} c_r y_r \tag{3.41}$$

Subject to

$$\sum_{r \in \mathcal{R}} a_{ir} y_r = 1 \qquad \forall i \in V \backslash \{0, n+1\} \tag{3.42}$$

$$y_r \in \{0, 1\} \qquad \forall r \in \mathcal{R} \tag{3.43}$$

$$\tag{3.44}$$

The objective function (3.41) aims to minimize total distance traveled. Constraint (3.42) ensures that each customer i must be present in exactly a single route. Constraint (3.42) ensures that y_r is a binary variable.

The set of feasible routes $\mathcal{R}$ and the corresponding travel distance matrix c_r of each route r are constructed as follows. In the first step, all routes with a single customer are considered and checked for feasibility. In the next step, all routes with two customers are considered, etc. Each combination of customers that may be visited feasibly on a single route, is inserted into the set $\mathcal{R}$ and the corresponding travel distance is inserted in the travel distance matrix c_r. When a combination of customers may lead to multiple feasible routes, the travel distance of the shortest feasible route is inserted in the travel distance matrix. The process is terminated either when the step number (i.e. the number of customers that are considered in a route) equals the number of customers in the network or when in a step every possible route exceeds the vehicle capacity in terms of number of pallets or total weight.

The feasibility check consists of two stages. First, the capacity of the vehicle in terms of number of pallets and total weight is checked. If the sum of the pallets or the pallet weight of the customers in the route exceed vehicle capacity, the combination of these customers is no longer considered, also not in future steps. In the second stage, for each combination all visiting sequences are listed and tested in terms of axle weight violations. If at least a single visiting sequence leads to a feasible route without axle weight violations, the combination is feasible. Note that it is possible that for a combination of customers no feasible route can be created without an axle weight violation, while in the next step when another customer is added to this combination, a feasible route may be created.

3.7 Conclusions

Axle weight limits have become an increasingly important issue for transportation companies. Transporters are faced with high fines when violating these limits, while commercial scheduling programs do not incorporate these constraints. Although research has been done on Vehicle Routing Problems (VRP) combined with loading constraints, axle weight constraints have not yet been integrated in a VRP.

This chapter therefore introduces the CVRP with sequence-based pallet loading and axle weight constraints. The calculation of the weight on the axles of a truck in which pallets are placed in two horizontal rows is described. The calculation depends on the center of gravity of the pallets inside the truck and on vehicle-specific parameters. An illustrative example shows that the integration of axle weight constraints in a vehicle routing problem may lead to a higher distance traveled, but ensures a feasible weight distribution of the load inside the vehicle. Two problem formulations for the CVRP with sequence-based pallet loading and axle weight constraints are presented.

The first formulation is a Mixed Integer Linear Programming formulation. In the second formulation, the problem is formulated as a set partitioning model. Both formulations are tested on small-size instance sets in Chapter 5. Since the CVRP is NP-hard, finding an exact solution within a reasonable time limit for large-size instances will be difficult. Therefore, in Chapter 4, a heuristic method is presented to solve large-size instances.

Chapter 4

CVRP with sequence-based pallet loading and axle weight constraints: Heuristic design and parameter setting

4.1 Introduction

In the previous chapter, the capacitated vehicle routing problem with sequence-based pallet loading and axle weight constraints has been presented. Two problem formulations have been proposed to solve the problem to optimality for small-size networks. In order to solve realistic-size instances for the CVRP with sequence-based pallet loading and axle weight constraints, an Iterated Local Search (ILS) metaheuristic is developed. In this chapter [1], the design of the ILS is presented in detail and a sensitivity analysis of the parameters of the ILS is performed (Fig 4.1). In the next chapter, the computational results of the ILS and the problem formulations will be discussed. The remainder of this chapter is organized as follows. In Section 4.2 the generation of the problem instances that are used to test the ILS on are described. Section 4.3 describes the ILS. In Section 4.4, the parameter tuning of the ILS is investigated. Section 4.5 provides a sensitivity analysis of the parameters of the algorithm. Section 4.6 presents the conclusions of this chapter.

[1] This chapter and the next chapter are based on Pollaris et al. (2017).

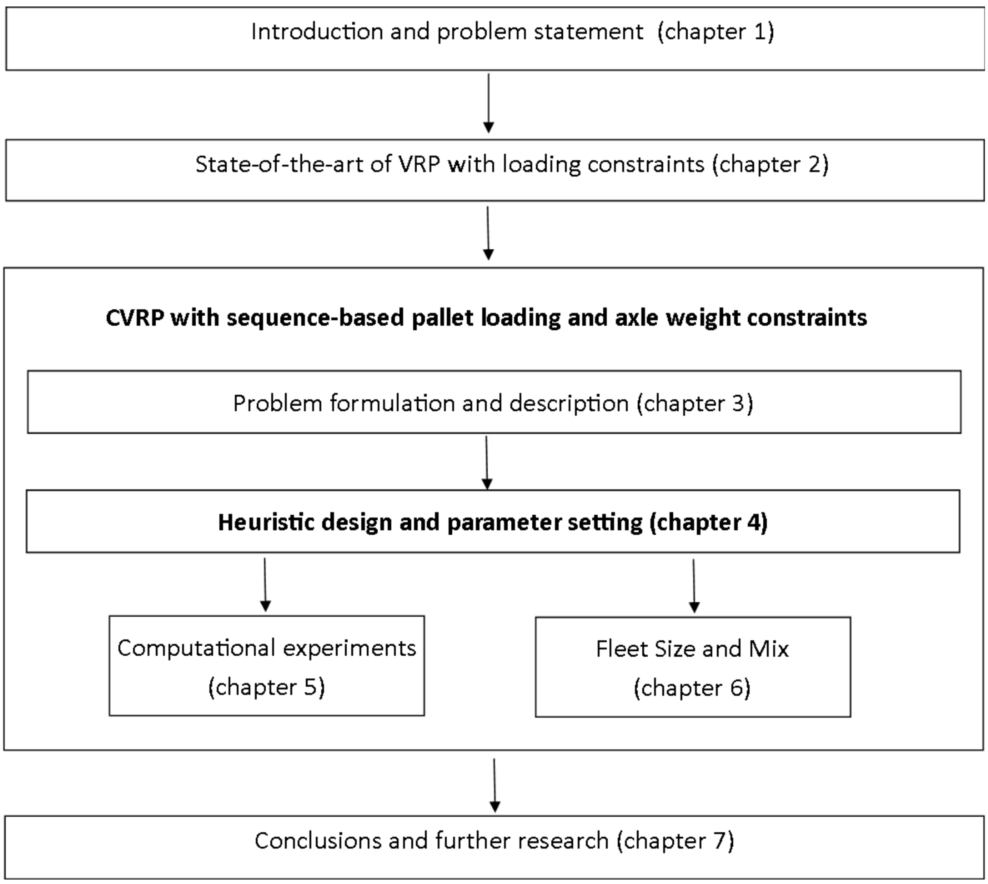

Figure 4.1: Outline of the thesis

4.2 Generation of problem instances

To analyze the performance of the ILS, 192 instances are created with networks consisting of 10, 15, 20, 50, 75 and 100 customers. For each network size, 32 instances are considered with randomly generated coordinates (x, y) with $x \in [0, 10]$ and $y \in [0, 10]$. The position of the depot is fixed to position (5,5). Travel distances are computed by taking the Euclidean distance between the coordinates of each node pair. Four different problem classes are created by varying the values for the number of pallets of each customer (L_i) and the total mass of the pallets of each customer (Q_i). In Table 4.1, the problem classes are presented. The number of pallets may have a low variation (4 - 7 pallets per customer) or a high variation (1 - 15 pallets per customer). With respect to the weight of the pallets, axle weight constraints do not play a role when only light pallets (100 - 500 kg) are considered. Therefore a distinction is made between customer demands of only heavy pallets (1000 - 1500 kg) and a fifty-fifty percent mix between customer demands with light pallets (100 - 500 kg) and customer demands with heavy pallets. The number of pallets and total weight for each customer are generated randomly in the intervals specified in Table 4.1, depending on the problem class. For each network size, eight instances are created in each problem class.

Table 4.1: Problem classes based on parameters Q_i and L_i

	Heavy pallets	Mix between light and heavy pallets
Low variation	Problem class 1	Problem class 3
High variation	Problem class 2	Problem class 4

4.3 Design of a solution method based on the ILS framework

The proposed solution method is based on an Iterated Local Search (ILS) framework which is proven to be a highly effective heuristic for routing problems (Lourenço et al., 2010). The ILS consists of four procedures (Generate initial solution, Local Search, Perturbation, Acceptance). The general structure is presented in Algorithm 1. First, an initial solution (s^o) is constructed. Second, this solution is improved using local search until a local optimum is reached. The local search is performed by a Variable Neighborhood Descent (VND) method. Third, the following steps are performed iteratively. In order to escape from the local optimum, a new starting point for the local search is generated by perturbing the current solution (s). This solution is improved using local search. Then, the acceptance criterion determines with which solution the process continues. The ILS stops after a number of α consecutive non-improving iterations. A non-improving iteration ($non\_improving\_it$) is an iteration in which no new best solution was found. For more information regarding the general ILS framework, the reader is referred to Lourenço et al. (2010). Note that since the local search is performed by a VND, the algorithm may also be called Iterated Variable Neighborhood Descent, as used in Chen et al. (2010).

The framework of ILS is chosen because iteratively building upon an embedded heuristic (in this case a VND) has been proven to lead to far better solutions than random restarts of that heuristic (Lourenço et al., 2010). This is confirmed during the construction of our metaheuristic, since the perturbation phase has proven to work very well. Because of this, ILS was chosen instead of a multi-start heuristic. Furthermore, ILS is simple, easy to implement, robust and highly effective.

To improve the efficiency of the algorithm, a pool of feasible and infeasible routes is constructed. Each time a route is proven to be (in)feasible in terms of loading, this route is stored in the appropriate pool. This avoids duplicate loading feasibility checks of a single route. In the following subsections, the implementation of the ILS is described.

Algorithm 1 Steps of the ILS

Initialization
1: $s^o \leftarrow$ Generate initial solution
2: $s, s^b \leftarrow$ Local search on s^o
3: **repeat**
$s \leftarrow$ Perturbation on s
$s \leftarrow$ Local search on s
$s, s^b \leftarrow$ Acceptance criterion
4: **until** $non\_improving\_it > \alpha$

4.3.1 Initial solution

Routes are constructed by inserting nodes one by one. To obtain a feasible initial solution, special attention is given to the insertion of *difficult* nodes. Nodes are considered *difficult* if they cannot be inserted feasibly in the front of a truck because the mass of the pallets exceeds the capacity of the axles of the tractor. These axles typically have the lowest axle weight capacity. The load of those nodes should therefore be placed more towards the end of the truck. Since sequence-based loading is assumed and no gaps are allowed between the front of the truck and the load due to the dense packing constraint, these nodes can only be feasibly inserted after the insertion of one or several other (*non-difficult*) nodes in a route. A classical insertion heuristic for the VRP (such as regret-2 insertion) does not take this into account. Therefore, in order to obtain a feasible initial solution, following methodology is used to ensure that all *difficult* nodes are feasibly inserted.

For each *difficult* node, a list is constructed with all options consisting of nodes or combinations of two nodes that would lead to a feasible packing scheme when the load of these (and only these) nodes precede the load of the *difficult* node. To decide for each *difficult* node which option is chosen, a binary constraint satisfaction problem (BCSP) is solved. The following notation is used:

Ω = set of *difficult* nodes (index j)

Φ = set of *non-difficult* nodes (index i)

Ψ_j = set of options for *difficult* node j (index a)

$$k_{ai}^{j} = \begin{cases} 1 & \text{if } \textit{non-difficult} \text{ node } i \text{ belongs to option } a \text{ of } \textit{difficult} \text{ node } j \\ 0 & \text{otherwise} \end{cases}$$

The decision variables are defined as:

$$x_{aj} = \begin{cases} 1 & \text{if option } a \text{ is chosen for } \textit{difficult} \text{ node } j \\ 0 & \text{otherwise} \end{cases}$$

The constraints are as follows:

$$\sum_{a \in \Psi_j} x_{aj} = 1 \qquad \forall j \in \Omega \tag{4.1}$$

$$\sum_{j \in \Omega} \sum_{a \in \Psi_j} k_{ai}^{j} \cdot x_{aj} \leq 1 \qquad \forall i \in \Phi \tag{4.2}$$

No objective function is specified since the only goal is to find a solution that meets all constraints. Constraint (4.1) ensures that for every *difficult* node, a single option is chosen. Constraint (4.2) makes sure that each *non-difficult* node i can only be inserted once. The BCSP is solved with CPLEX 12.6 with the default parameters. Preliminary tests have shown that a solution can often already be obtained when allowing only a single non-difficult node to precede each difficult node. Additionally, for some instances, considering both a single and a combination of two *non-difficult* nodes, considerably increases computation time. Therefore, the BCSP is first solved with options consisting of a single *non-difficult* node only. When the BCSP is not able to not find a feasible solution, a combination of two nodes is allowed as well.

When the BCSP finds a feasible solution, each *difficult* node is inserted into a route along with the node(s) from the corresponding option that was selected by the BCSP. Therefore, as many routes as *difficult* nodes are created. The remaining *non-difficult* nodes are inserted with a regret-2 insertion heuristic (Røpke and Pisinger, 2006). The regret value of a node is defined as the absolute difference in objective value between the cheapest insertion of a node and the second cheapest insertion of that node into another route. In each iteration, insertions in the existing routes are considered as well as in an additional empty route. The node with the highest regret value is inserted in its best insertion position. This procedure continues until all nodes are feasibly inserted into a route.

The need for the BCSP in the generation of the initial solution is demonstrated by generating an initial solution with three different insertion methods for the 192 instances which are described in Section 4.2. In the first method, an initial solution is generated by using a regret-2 insertion heuristic without giving special attention to difficult nodes. In the second method, a BCSP with options existing out of a single non-difficult node is solved, followed by a regret-2 heuristic. The third method solves a BCSP with options existing out of a single node or combinations of two nodes followed by a regret-2 heuristic. Appendix A reports for each instance whether a feasible solution is found for each insertion method. The regret-2 insertion heuristic was able to find a feasible solution in 132 out of 192 instances. The instances for which no feasible solution was found are instances containing customers with large demands in terms of number of pallets. In 187 out of 192 instances, a feasible solution was found when considering only a single non-difficult node in each option. For the remaining five instances a feasible initial solution was found when a single node and a combination of two non-difficult nodes was allowed in each option. These instances contain besides demands with a high number of pallets also only demands with heavy pallets. Furthermore, the number of customers in the network in these instances is small and ranges from 15 to 20.

4.3.2 Local Search

The local search is performed by a VND in which four neighborhoods are used. The exchange operator (Waters, 1987) swaps two nodes which can be either from the same route or from different routes. The 2-opt operator (Croes, 1958) removes two arcs of a single route and generates two new arcs in such a way that the section between the removed arcs is reversed. Only arc-pairs which are separated with at least four customers are considered in this neighborhood to avoid scanning the same moves as the exchange operator. As an example, consider the move from route A (0-1-2-3-0) to route B (0-3-2-1-0). This move may be performed by a 2-opt operator by removing the first and the last arc and reversing the section between the arcs and by an exchange operator by swapping nodes 3 and 1.

The cross-exchange operator (Taillard et al., 1997) interchanges two segments of different routes while preserving the orientation of the segments and the routes. At least one of two segments needs to be of size greater than one to avoid scanning the same moves as in the exchange neighborhood. As an example, consider the move from solution 1 containing routes (0-1-2-0) and (0-3-0) to solution 2 containing routes (0-3-2-0) and (0-1-0). This move may be performed by an exchange operator (swap nodes 1 and 3) and by a cross-exchange operator (interchange segment -1- and -3-). There is no upper bound on the size of the segments.

Algorithm 2 Local search

```
Neighborhoods = {exchange, 2-opt, cross-exchange, relocate}
s = initial solution
s' := s
stop := 0
repeat
    for i := 1 to 4 do
        next_neighborhood = 0
        repeat
            s'' ← Local search with Neighborhoods[i] on s'
            if s' = s'' then
                next_neighborhood := 1
            else
                s' := s''
            end if
        until next_neighborhood = 1
    end for
    if s = s' then
        stop := 1
    else
        s := s'
    end if
until stop = 1
```

Finally, the relocate operator (Waters, 1987) removes a node from its route and reinserts it in another place in its original route or in another route. This move may reduce or increase the number of routes in the solution, since relocation to an empty route is also considered. For each neighborhood, all possible moves are identified after which a best improvement strategy is applied. An overview of the local search procedure is presented in Algorithm 2.

Relocate, exchange and two-opt are classical local search operators often used in the CVRP literature. In the state-of-the-art algorithms for the 2L-CVRP of Wei et al. (2015), Leung et al. (2013) and Zachariadis et al. (2013b) these operators are used as well. The cross-exchange operator is implemented in order to allow for the exploration of solutions which are substantially different from the neighborhoods explored by the other three operators since this operator allows to exchange segments of customers between different routes. This operator is also frequently used in the VRP literature (e.g. Strodl et al., 2010; Tricoire et al., 2011; Bräysy, 2003).

The sequence of the neighborhoods is fixed. Since relocate is also used in the perturbation phase, this operator is placed last in the local search procedure to prevent that changes made during perturbation can easily be undone. When a local optimum is reached for a neighborhood, the local search proceeds to the next neighborhood. When a local optimum is reached for the last neighborhood, the local search procedure is repeated until no further improvement is found in any of the neighborhoods.

4.3.3 Perturbation

In the perturbation phase, the relocate operator is considered once for each customer, using a randomized objective function. The effect of relocating a customer to another position is randomized by adding a noise factor to the insertion cost. The insertion cost is calculated as the sum of the length of the arcs that are created when inserting a customer in a new position minus the length of the arcs that are removed. Similar to Røpke and Pisinger (2006), the noise value is calculated as a random number in the interval $[-\eta * maxD, \eta * maxD]$ where $\eta \in]0, +\infty[$ is a parameter to control the amount of noise and $maxD$ is the maximum distance between two nodes in the network.

When the randomized insertion cost is positive or zero, the next insertion position for the customer is considered. When the randomized insertion cost is negative, the move is immediately implemented and the perturbation process continues with the next customer. Codenotti et al. (1996) apply a similar method in the perturbation phase of the ILS, but instead of adding noise directly to the insertion costs, the coordinates of the cities are changed which also results in changes in the insertion cost matrix.

The general framework of the perturbation procedure is presented in Algorithm 3. Customers are considered in a random order sequence. For each customer, a first improvement strategy is used because the goal of the perturbation phase is merely to change the current solution. It is therefore not necessary to choose the move with the largest improvement. The insertion positions of a customer are considered in random order, i.e., the first route that is considered for insertion is chosen randomly and in each route, the first position that is considered is also chosen randomly.

Initially, the value for η is determined by the value of parameter η^0. If the perturbation does not change the solution s, η is increased with the value of parameter η_{incr} and the perturbation is repeated. After δ consecutive non-improving iterations (*non_improving_it*) of the ILS, a heavy perturbation is applied. This means that η increases with the parameter value of η_{heavy} to increase the level of diversification. When an improvement is found after the local search procedure, the number of consecutive non-improving iterations becomes 0 (as may be seen in Algorithm 4) and the value for η is set to the value of η^0.

Algorithm 3 Perturbation

```
if non_improving_it > δ then
    η := η^0 + η_heavy
else
    η := η^0
end if
repeat
    s' ← Relocate for each customer on s with noise η
    η := η + η_incr
until s ≠ s'
s := s'
```

4.3.4 Acceptance Criterion

An overview of the acceptance procedure is presented in Algorithm 4. The acceptance criterion that is used in this ILS algorithm is based on record-to-record travel, introduced by Dueck (1993). The solution s obtained after local search is always accepted to become the new incumbent solution s of the next ILS iteration if the objective value is smaller than the objective value of the current best solution s^b. When the objective value of s is higher than the objective value of s^b and no heavy perturbation will be applied in the next iteration of the ILS (i.e. *non_improving_it* $<\delta$), the solution is still accepted if the worsening is smaller than a certain threshold value. This threshold value corresponds to a fraction β of the objective value of s^b. In case a heavy perturbation will be applied in the next ILS iteration, a worsening is never accepted in order not to deviate too far from the current best solution. In case s is not accepted to become the new incumbent solution, the search continues from s^b. The acceptance of worse solutions appears to have a positive impact on solution quality, as shown in the sensitivity analysis in Section 4.5.

Algorithm 4 Acceptance criterion

if $objectivevalue[s] < objectivevalue[s^b]$ **then**
  $s^b := s$
  $non\_improving\_it := 0$
else
  $non\_improving\_it := non\_improving\_it + 1$
  if $(objectivevalue[s] > objectivevalue[s^b] \cdot (1 + \beta))$ or $(non\_improving\_it > \delta)$ **then**
    $s := s^b$
  end if
end if

4.4 Parameter tuning

This section presents the parameter tuning of the algorithm described in the previous section. Parameter tuning is important because the value of the parameters may have a substantial impact on the efficacy of a heuristic algorithm (Hoos, 2012). Pellegrini and Birattari (2011) compared the performance of five metaheuristics (tabu search, simulated annealing, genetic algorithm, iterated local search and ant colony optimization) with and without automated parameter tuning on a VRP with stochastic demands. The parameters from the non-tuned algorithms were randomly drawn within a given range while the parameters from the tuned versions were obtained through an automatic configuration process based on the F-Race algorithm (Birattari et al., 2002). For every metaheuristic, the tuned version achieves significantly better results than the corresponding non-tuned version. While traditionally parameter values have been set manually using expertise and experimentation, recently several automated tuning methods have been proposed (Rasku et al., 2014). Rasku et al. (2014) compare the performance of seven state-of-the-art algorithm configuration methods on different routing metaheuristics. Their findings confirm the results of Pellegrini and Birattari (2011) that the performance of the routing algorithm can be clearly improved by using parameter tuning. The results also reveal that there is no single best tuning method for routing algorithms, but that the Iterated F-Race algorithm seems to be the most robust. Iterated F-race also performed very well on the ILS metaheuristic (Rasku et al., 2014) and other metaheuristics (Balaprakash et al., 2007; Birattari et al., 2002). Therefore, Iterated F-race will be used for the parameter tuning of the ILS in this chapter.

The Iterated Local Search algorithm makes use of a set of six parameters (α, δ, η^0, η_{heavy}, η_{incr}, β). The number of consecutive non-improving iterations after which the ILS is stopped is denoted by α. δ represents the number of non-improving iterations of the ILS after which a heavy perturbation is applied. η^0 controls the initial amount of noise in the perturbation phase. η_{heavy} represents the increase in noise during a heavy perturbation. The increment in the value of noise when the solution is not changed during the perturbation phase is denoted by η_{incr}. Finally, β is the factor which determines the threshold value in the acceptance criterion.

For the tuning of the parameters, 20 test instances with sizes ranging from 20 to 75 customers are used. The instances are generated in a similar way as the instances that are described in Section 4.2. The value for parameter α is determined based on the results of a single run of the test instances, in which no substantial improvement was found after more than 220 consecutive non-improving iterations. The values of the other parameters were determined for this run based on preliminary tests. Since this value was the result of a single run, α is set to 250, to incorporate a margin of 10 %. The parameter space for the remaining five parameters of the ILS algorithm is denoted by $X = \{\delta, \eta^0, \eta_{heavy}, \eta_{incr}, \beta\}$. Each parameter $X_d \in X$ may take different values within a specified range $[\underline{x}_d, \overline{x}_d]$. A configuration of the algorithm $\theta = \{x_1, x_2, x_3, x_4, x_5\}$ is a unique assignment of values to these parameters (López-Ibáñez et al., 2016). The tuning problem is stated by Birattari (2009) as the problem of finding the configuration θ that provides the lowest expected cost on a set of problem instances. In order to find this configuration, the irace package provided by López-Ibáñez et al. (2016) is used. The irace package is designed for automatic algorithm configuration and implements the iterated racing procedure, which is an extension of the Iterated F-race procedure (López-Ibáñez et al., 2016). Irace is also successfully used by other authors for the tuning of heuristic algorithms in similar applications to ours such as by Ceschia et al. (2013) and François et al. (2016). The iterated racing procedure is presented in Algorithm 5.

The input of the iterated racing procedure consists of an instance set I, parameter space X, a cost function C and a tuning budget B. The cost function returns the cost of configuration θ on instance i. The tuning budget refers to the number of calls to the ILS that irace will perform. Based on the number of parameters that

Algorithm 5 Iterated Racing (López-Ibáñez et al., 2016)

Require: $I = \{I_1, I_2, ...\}$,

1: parameter space: X,
2: cost measure: $C(\theta, i) \in \mathbb{R}$,
3: tuning budget: B
4: $\Theta_1 \sim SampleUniform(X)$
5: $\Theta^{elite} := Race(\Theta_1, B_1)$
6: $j := 2$
7: **while** $B_{used} \leq B$ **do**
8: $\quad \Theta^{new} \sim Sample(X, \Theta^{elite})$
9: $\quad \Theta_j := \Theta^{new} \cup \Theta^{elite}$
10: $\quad \Theta^{elite} := Race(\Theta_j, B_j)$
11: $\quad j := j + 1$
12: **end while**

are tuned (N^{param}), an estimation of the number of iterations N^{iter} is made with $N^{iter} = \lfloor 2 + \log_2 N^{param} \rfloor$. Since five parameters are tuned in our algorithm, the value for $N^{iter} = 4$. Note that N^{iter} is an estimation of the number of iterations. In case, after N^{iter} iterations, there is still enough budget to perform a new race, the algorithm continues. The tuning budget B_j for iteration j depends on the tuning budget B, the tuning budget that is already used in previous iterations B_{used}, N^{iter} and the iteration number j:

$$B_j = \frac{B - B_{used}}{N^{iter} - j + 1} \tag{4.3}$$

In the first step of the first iteration ($j = 1$) of the iterated racing procedure, candidate configurations Θ_1 are sampled according to a uniform distribution in the parameter space X (line 4 in Algorithm 5). In the second step, a racing procedure is used to select elite candidate configurations Θ_{elite} from the configurations sampled in the previous step (line 5). The racing procedure consists of several steps. In each step of the race, the candidate configurations are evaluated on a set of instances by means of a cost measure C. The order in which the instances are considered is randomized. For the evaluation of the candidate configurations, the rank-based Friedman test is used. If a candidate configuration performs statistically worse than at least one other configuration, this configuration is discarded in the next step. The racing procedure is terminated when a minimum number of surviving configurations is reached ($N_j^{surv} \leq N^{min}$) or when the number of surviving configurations N_j^{surv}

exceeds the remaining tuning budget B_j for race j.

At the end of the race, the surviving configurations are ranked according to the mean cost and are assigned to a rank value r_z. The set of the elite configurations Θ_{elite} is composed of the $N_j^{elite} = min(N_j^{surv}, N^{min})$ configurations with the lowest rank. For the generation of a new candidate configuration for the next race, a parent configuration θ_z is sampled from the set of elite configurations Θ_{elite} with a probability p_z proportional to its rank r_z. A higher ranked elite configuration has a higher probability of being selected as a parent:

$$p_z = \frac{N_{j-1}^{elite} - r_z + 1}{N_{j-1}^{elite} \cdot (N_{j-1}^{elite} + 1)/2} \tag{4.4}$$

A new value is sampled for each parameter X_d within the given range according to a normal distribution $\mathcal{N}(x_d^z, \sigma_d^j)$ (line 8). For the integer parameter δ, the sampled value is rounded to the nearest integer. The mean of the distribution x_d^z is the value of parameter X_d in elite configuration θ_z. The standard deviation σ_d^j decreases at each iteration j and depends on the value of the standard deviation in the previous iteration (σ_d^{j-1}), the number of newly sampled configurations (N_j^{new}) and the number of parameters to be tuned (N^{param}). The parameter σ_d^1 is set to $(\underline{x}_d - \overline{x}_d)/2$.

$$\sigma_d^j = \sigma_d^{j-1} \cdot \left(\frac{1}{N_j^{new}}\right)^{1/N^{param}} \tag{4.5}$$

The new set of candidate configurations consists of N_{j-1}^{elite} elite configurations from the previous iteration Θ_{elite} and the N_j^{new} newly sampled configurations (line 9). These configurations are used in a new racing procedure to select elite candidate configurations Θ_{elite} (line 10).

This process (lines 8 to 11) is repeated until the number of experiments B_{used} exceeds the maximum number of experiments specified in the tuning budget B. The number of candidate configurations N_j in iteration j depends on the tuning budget B_j, the iteration number j and parameter μ:

$$N_j = \frac{B_j}{\mu + min(5, j)} \tag{4.6}$$

Parameter μ has a default value of 5 in irace. This value may be changed in order to influence the ratio between budget and number of configurations. At each iteration, the number of candidate configurations Θ_j decreases to allow for more evaluations per candidate configuration in later iterations. After five iterations, the number of candidate configurations remains constant at value Θ_5 in order to avoid having too few configurations in a single race. For more information regarding the tuning process, the reader is referred to López-Ibáñez et al. (2016).

Irace is run with default parameter values on the ILS heuristic. A tuning budget B of 5000 runs is specified. The parameters tuned by irace may be found in Table 4.1, along with the type, range and tuned value. δ is integer (i), while η^0, η_{heavy}, η_{incr}, β are real parameters (r). The ranges for the parameters are intuitively determined. The upper bound of δ is equal to 250, which is the value for α. For the other parameters, the upper bound was increased when the tuned value was close to an upper bound. The lower bound of β is set to zero to test the case in which a deterioration of the solution value is not accepted as new incumbent solution. The lower bound of η_{heavy} is also set to zero to test the case without a heavy perturbation. The lower bound of η^0 is greater than 0 since there must be a non-zero value for noise in the perturbation phase to escape from the local optimum. For the real parameters (η^0, η_{incr}, η_{heavy}, η_{incr}, β), two decimal places are considered.

Name	Description	Type	Range	Tuned value
δ	# non_improving_it heavy perturbation	i	(1, 250)	196
η^0	initial value η	r	(0.01, 0.80)	0.33
η_{heavy}	increase η heavy perturbation	r	(0.0, 1.0)	0.14
η_{incr}	increase η when solution is not changed during perturbation	r	(0.0, 0.50)	0.22
β	threshold value	r	(0.0, 0.50)	0.10

Table 4.2: Parameter list

Both the introduction of a heavy perturbation as the acceptance of a worse solution based on record-to-record appear to have a positive influence on solution quality since η_{heavy} and β have non-zero tuned values. In the next section, this influence is further analyzed by means of a sensitivity analysis.

4.5 Sensitivity analysis

In this section, the performance of the algorithm in terms of solution quality is tested with respect to different parameter values. The purpose of this analysis is to verify whether the parameter tuning produced logical results as well as to test the importance of the parameter values for the efficacy of the ILS algorithm. For each parameter, different values are tested on four instance sets while keeping other parameters at their tuned value. The *first* instance set consists of the 20 instances used for the tuning of the algorithm with instance sizes ranging from 20 to 75 customers. To test wether the tuned parameter configuration also performs well on other instances, three additional instance sets are considered. The *second* instance set consists of 32 instances of size 20 and is used to test the effect of the parameters of the ILS on small-size instances. The *third* and *fourth* instance sets consist of 12 instances of size 50 and 12 instances of size 75, respectively, to evaluate the parameter setting on realistic-size instances. Note that these instance sets are smaller than the second instance set because the CPU time for each run is considerably longer. The size of the instance sets is however sufficient for the purpose of the sensitivity analysis. The instances in the four instance sets only differ in network size. Other characteristics such as pallet weight and number of pallets per customer are assigned in a similar way to all instance sets. The instances in instance set 2, 3 and 4 are also used in Section 4.3.1. The performance of each parameter setting is measured with five independent runs. The best and average increase in objective compared to the lowest objective value found over all experiments for that instance are plotted. Note that for each instance set a different scale is used on the vertical axis of the graph because of the large difference in increase in objective value between the different instance sets. Instances with a high number of customers tend to have a higher increase in objective value than instances with a low number of customers. For all graphs of the same instance set however, the same scale on the vertical axis is maintained for the sensitivity analyses of the average and best objective value of all parameters.

4.5.1 Sensitivity analysis of η^0

The impact of η^0, the initial value for noise, on the solution quality of instance sets 1, 2, 3 and 4 are plotted in Figures 4.2, 4.3, 4.4 and 4.5 respectively. The following values are considered for η^0: 0.01, 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.80 and 1.0. The tuned configuration in which η^0 has a value of 0.33 is also included. As expected, in the first instance set, the tuned value ($\eta^0 = 0.33$) renders the best results. For the instances of set 2, the increase in objective value drops significantly when η^0 increases from 0.01 to 0.30. After this point, an increase in η^0 does not have an influence on solution quality. For the instances of set 3, we see that when η^0 increases beyond 0.40, the solution quality deteriorates. For the instances of set 4, we have a similar trend, but here the solution quality already deteriorates when η^0 increases beyond 0.30. Because of the difference in influence of η^0 on the small-size instances in instance set 2 and the instances of size 50 and 75 in instance sets 3 and 4, it may be concluded that there is an interaction effect between the size of the network and the impact of the initial value of noise.

The variation in solution quality for the instances of sets 3 and 4 with respect to changes in the initial value of noise is explored by means of boxplots which are presented in Figures 4.6 and 4.7. In each boxplot the first, second (median) and third quartile as well as the minimum and maximum increase in objective value over all instances in the instance set are indicated. Note that a different scale is used on the vertical axis since the maximum increase in objective value highly exceeds the average increase in objective value over all instances in the instance set reported in the previous graphs. The current scale however is maintained for all the analyses with boxplots. Figure 4.6 shows that the variation in solution quality for the instances of set 3 decreases when η^0 increases to 0.40. For the instances of set 4, the variation decreases when η^0 increases to 0.30, as displayed in Figure 4.7. For this instance set, the variation increases rapidly when η^0 increases beyond 0.30, while for the instances of set 3, this increase only starts from a value of 0.6 and is much smaller. This confirms the above finding that there is an interaction effect between network size and the impact of η^0 on solution quality.

4.5.2 Sensitivity analysis of β

For the sensitivity analysis of β, the factor that determines the threshold value in the acceptance criterion, values between 0.0 and 0.5 are considered in steps of 0.10. To have an idea of the effect of a very high value of β, the value 1.0 is also considered. In this setting, a solution with a distance traveled twice as high as the distance of the best known solution will still be accepted as the incumbent solution in the next iteration of the ILS. Figure 4.8 shows as expected that for the tuning instances (instance set 1), the lowest objective value increment (average and best) may be found at the tuned value $\beta = 0.1$, although the difference with the other values for β is very small. For the instances of size 20 of instance set 2, depicted in Figure 4.9, the solution quality improves when a worsening solution is accepted ($\beta > 0.0$). Based on the figure it appears that the tuned value 0.10 leads to the best solution quality although the difference with higher values of β is again very small. In Figure 4.10, a similar trend may be observed for the instances of set 3. The solution quality clearly benefits from accepting worse solutions and in addition, the tuned value ($\beta = 0.1$) renders the best

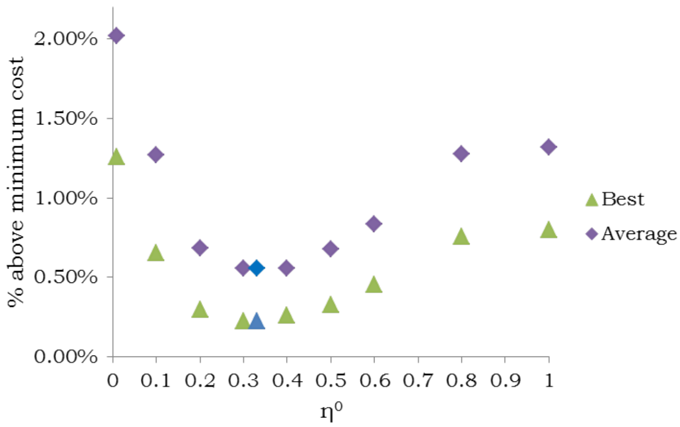

Figure 4.2: Sensitivity of η^0 on instance set 1 (tuning instances)

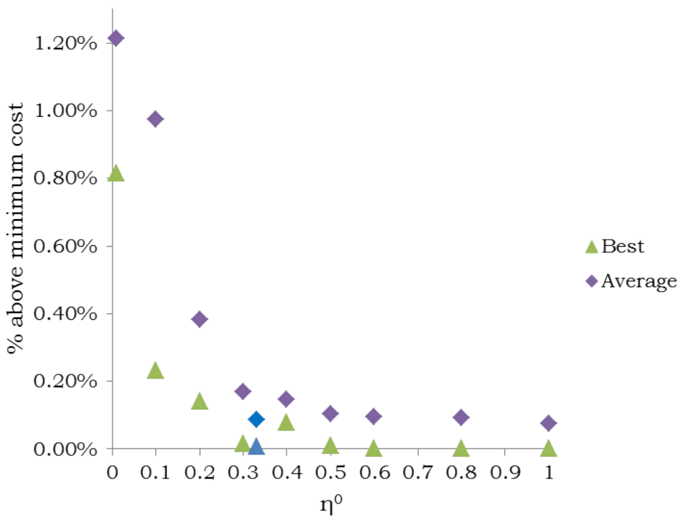

Figure 4.3: Sensitivity of η^0 on instance set 2 (20 customers)

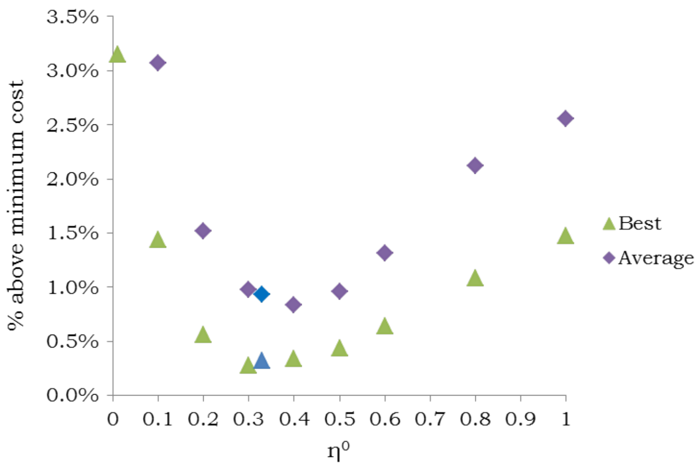

Figure 4.4: Sensitivity of η^0 on instance set 3 (50 customers)

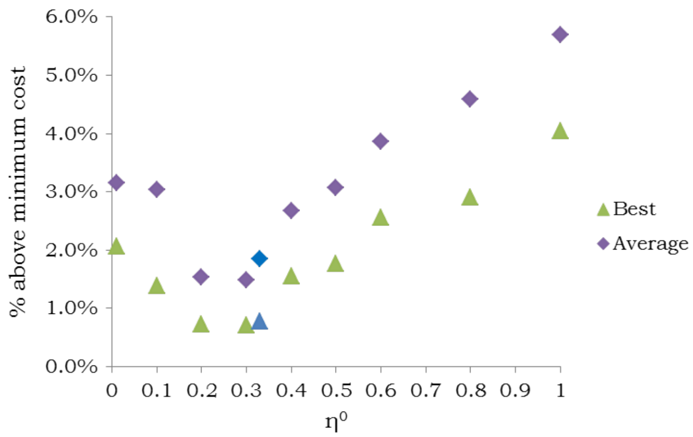

Figure 4.5: Sensitivity of η^0 on instance set 4 (75 customers)

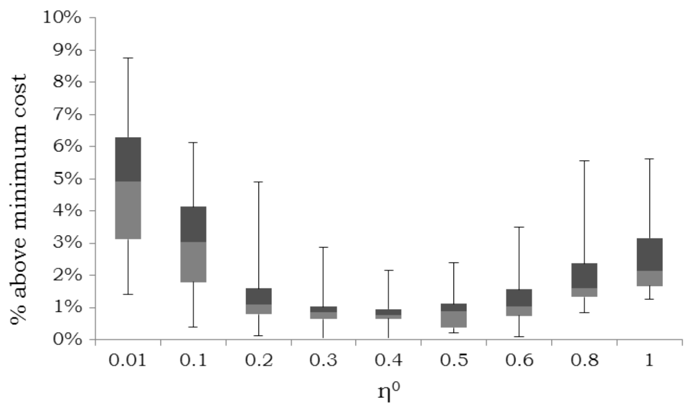

Figure 4.6: Boxplot of the objective value increase in function of η^0 in instance set 3 (50 customers)

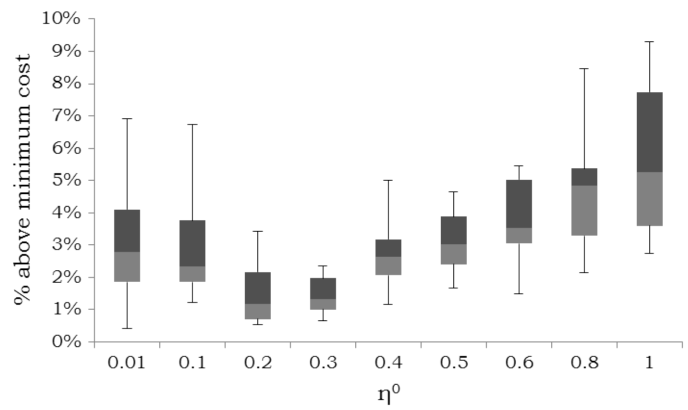

Figure 4.7: Boxplot of the objective value increase in function of η^0 in instance set 4 (75 customers)

solution quality although the difference with higher values of β is small. For the instances in set 4, illustrated in Figure 4.11, we see a small improvement in solution quality when $\beta = 0.10$ in comparison with $\beta = 0.0$. However, when β further increases to 0.20 and higher, there is a deterioration in solution quality compared to the case in which the threshold value is zero.

Figure 4.12 presents the variation in solution quality for the instances of set 3 with respect to changes in the threshold value. The boxplot of the objective value increase of the tuned value of β clearly lies lower than the boxplot of the configuration with a threshold value of $\beta = 0.0$, except for the minimum values which are equal for both configurations. When β increases to a value higher than 0.1, no clear trend can be distinguished in the graph. The variation of the objective value increase of the instances in set 4 for the different values for β are presented in Figure 4.13. The median of the configuration with a threshold value of 0.0 is lower than the median of the other configurations. This is different from the findings from instance set 3. Based on this observation and on the small improvement in average solution quality for instance set 4 when β increases from 0.0 to 0.10, it can be concluded that incorporating a record-to-record procedure seems to have a smaller effect on solution quality for instances with 75 customers than for instances with 50 customers or fewer.

4.5.3 Sensitivity analysis of δ and η_{heavy}

The impact of δ, the number of non-improving iterations of the ILS after which a heavy perturbation is applied, on solution quality is measured simultaneously with the impact of η_{heavy}, the increase in η during a heavy perturbation. To this end, ten combinations are created by varying the values for δ and η_{heavy}. The first combination represents the situation in which no heavy perturbation is applied. In Table 4.3, the remaining nine combinations are presented. δ may have a low (50), medium (125) or high (200) value. Similarly, η_{heavy} is assigned to a low (0.10), medium (0.30) and high (0.60) level. The tuned values of δ and η_{heavy} are 196 and 0.14 which correspond most to combination 4 with a high value of δ combined with a low value of η_{heavy}. In Figure 4.14, the average solution quality is plotted for each of the combinations for the instances of the first instance set. Combinations 2, 3 and 4 with a low value for η_{heavy} have the best solution quality although the difference with the other combinations is small. The solution quality of the combinations with a high value for η_{heavy} is slightly worse than the solution quality of combination 1 in which no heavy perturbation is applied. No clear trend may be distinguished for the value of δ.

Figure 4.15 shows that for the instances of size 20 in instance set 2 there are on

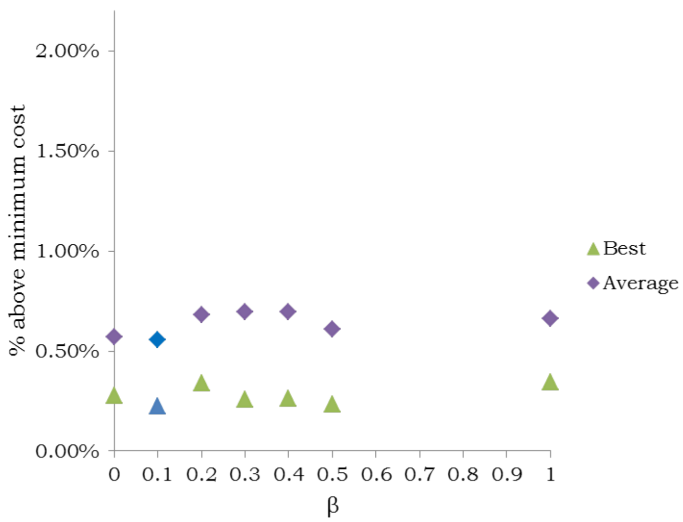

Figure 4.8: Sensitivity of β on instance set 1 (tuning instances)

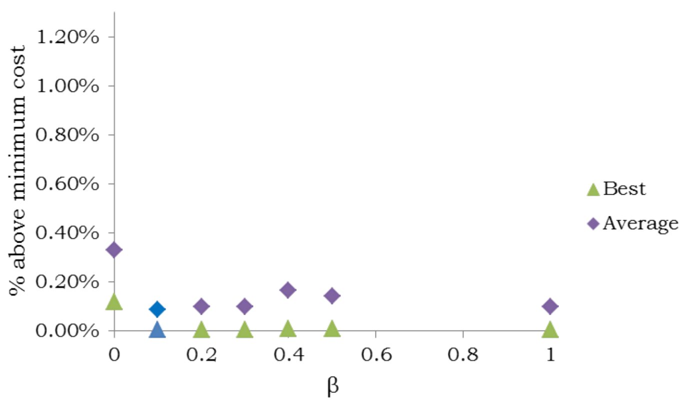

Figure 4.9: Sensitivity of β on instance set 2 (20 customers)

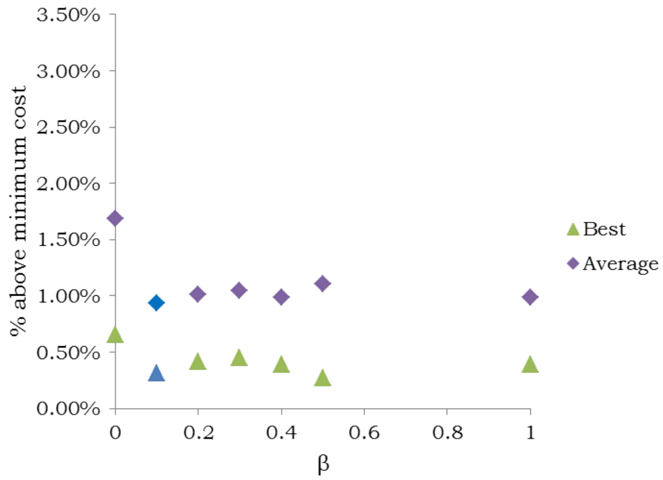

Figure 4.10: Sensitivity of β on instance set 3 (50 customers)

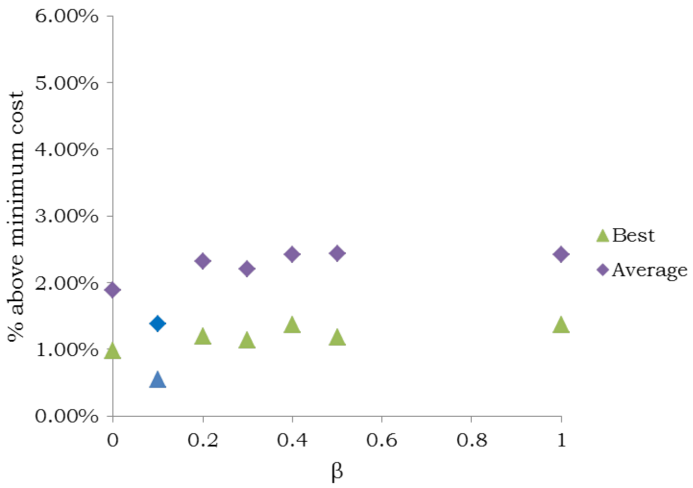

Figure 4.11: Sensitivity of β on instance set 4 (75 customers)

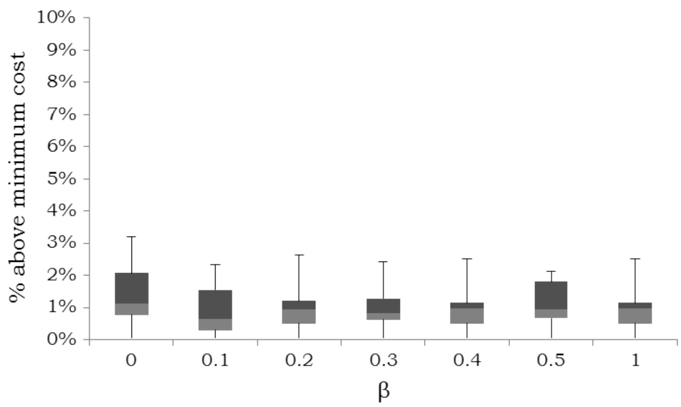

Figure 4.12: Boxplot of the objective value increase in function of β in instance set 3 (50 customers)

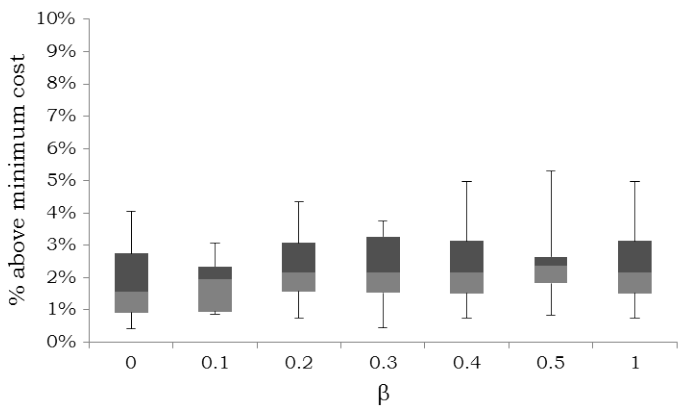

Figure 4.13: Boxplot of the objective value increase in function of β in instance set 4 (75 customers)

average only small differences in solution quality between the combinations. Figure 4.16 plots the average solution quality of the instances of instance set 3 with respect to the different combinations. A similar trend as for the first instance set may be distinguished in which the combinations with a low value of η_{heavy} have a higher solution quality than the other combinations. Furthermore, when a moderate or high value of η_{heavy} is considered, the value of δ also seems to influence the objective value. A low value of δ appears to have a negative effect on solution quality. Note that, surprisingly, combination 1 in which no heavy perturbation is applied has the best solution quality although the difference with the combinations with a low value of heavy noise is negligible. The introduction of heavy noise therefore does not appear to have a positive impact on solution quality for the instances of size 50 in instance set 3. The average solution quality for the instances of size 75 from instance set 4 for the different combinations is presented in Figure 4.17. The combinations with a low value of heavy noise, clearly outperform the other combinations including the first combination without heavy noise. Based on this figure, one can also distinguish a trend for the value of δ. For each value of η_{heavy}, the solution quality seems to increase when δ increases.

Table 4.3: Combinations heavy noise based on the values of δ and η_{heavy}

	$\delta = 50$	$\delta = 125$	$\delta = 200$
$\eta_{heavy} = 0.1$	combination 2	combination 3	combination 4
$\eta_{heavy} = 0.3$	combination 5	combination 6	combination 7
$\eta_{heavy} = 0.6$	combination 8	combination 9	combination 10

4.5.4 Sensitivity analysis of η_{incr}

As discussed in Section 4.3, in case the solution is not changed during the perturbation phase, the value of noise is incremented with η_{incr} and the perturbation is repeated. However in all runs of the instances of size 50 and 75, the solution changes during every perturbation. Therefore the value of η_{incr} does not have an impact on the solution quality of the instances in set 3 and 4. In 5 % of the instances of size 20 a single or several noise increments are performed. To measure the impact on even smaller size instances, a fifth and sixth instance set are considered, each containing

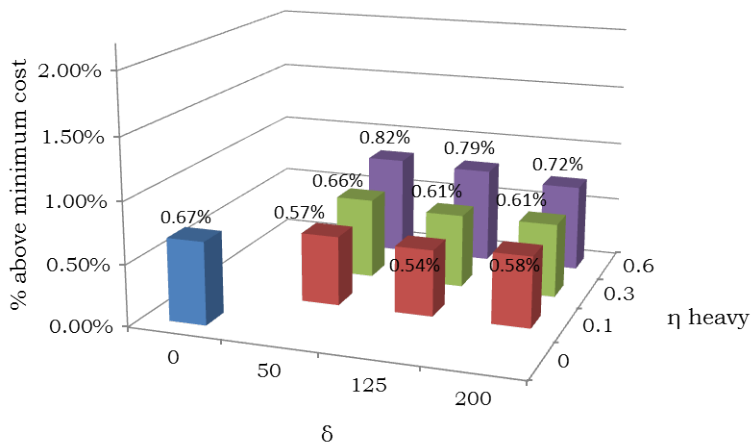

Figure 4.14: Sensitivity of η_{heavy} and δ on instance set 1 (tuning instances)

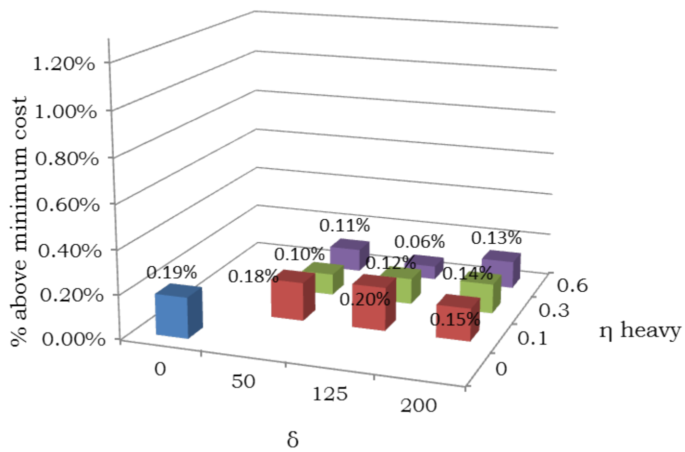

Figure 4.15: Sensitivity of η_{heavy} and δ on instance set 2 (20 customers)

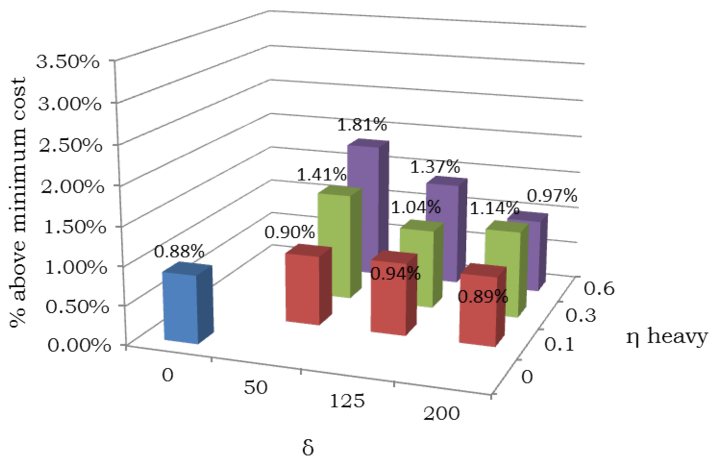

Figure 4.16: Sensitivity of η_{heavy} and δ on instance set 3 (50 customers)

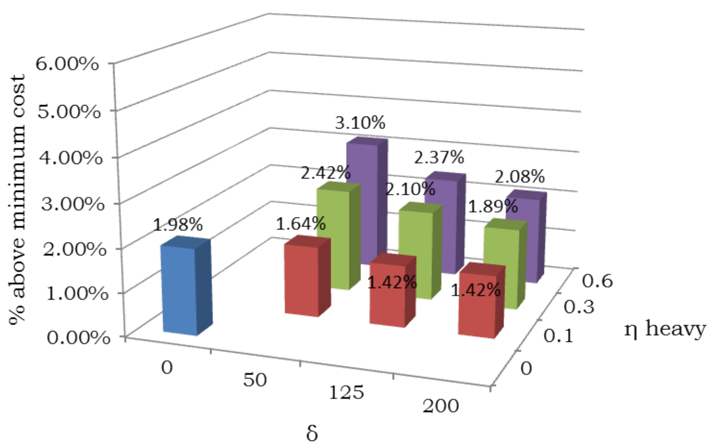

Figure 4.17: Sensitivity of η_{heavy} and δ on instance set 4 (75 customers)

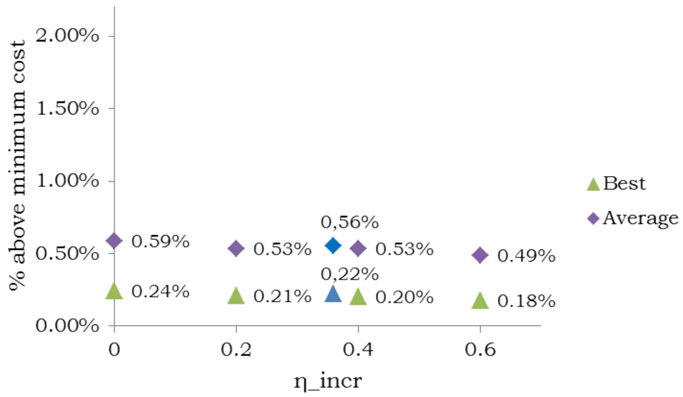

Figure 4.18: Sensitivity of η_{incr} on instance set 1 (tuning instances)

32 instances of size 10 and 15. In 70 % of the instances of size 10 and in 28 % of the instances of size 15 a single or several noise increments are performed. The reason for this may be that for very small-size instances, it is more difficult to find a feasible neighboring solution. The likelihood of finding another solution in the perturbation phase is therefore much smaller for instances of size 10 compared to instances of size 20 or 50. Because η_{incr} is primarily used on instances of size 10, instance set 5 is considered in the sensitivity analysis of this parameter. The impact of η_{incr} on the solution quality of instance sets 1, 2 and 5 are plotted in Figures 4.18, 4.20 and 4.19, respectively. Instance sets 3 and 4 are not considered because η_{incr} is never used in the instances contained in these sets. The following values are considered for η_{incr}: 0.0, 0.20, 0.40 and 0.60. The tuned configuration in which η_{incr} has a value of 0.22 is also included. The value for η_{incr} only has a very small impact on the solution quality of the instances of the tuning set. This may be explained by the fact that the tuning set contains instances ranging from 20 to 75 customers. For the instances of size 10, an increase in solution quality is detected when the value for η_{incr} increases. For the second instance set with 20 customers, the impact on solution quality is negligible. This is expected since on average in only 5 % of the instances, a noise increment is performed.

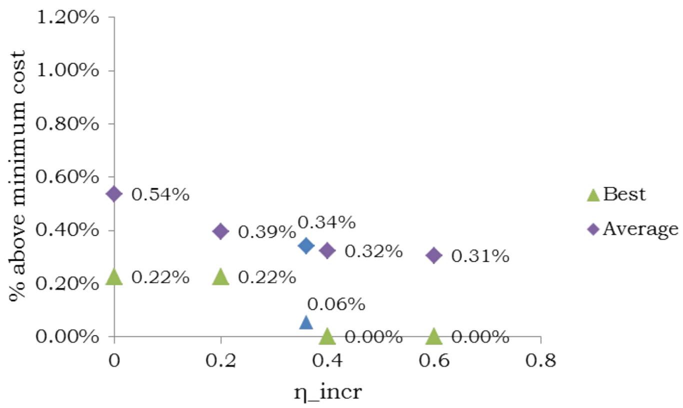

Figure 4.19: Sensitivity of η_{incr} on instance set 5 (10 customers)

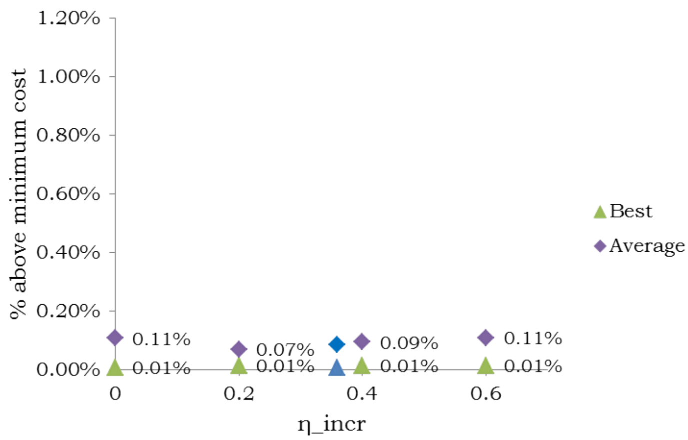

Figure 4.20: Sensitivity of η_{incr} on instance set 2 (20 customers)

4.5.5 Sensitivity analysis of α

The impact of the number of consecutive non-improving iterations after which the ILS is stopped (α) on solution quality and CPU time is illustrated in Figures 4.21, 4.22, 4.23 and 4.24 for respectively the instances of the first, second, third and fourth instance set. Values between 50 and 500 with a step of 50 are considered for α. As one can expect, if α increases, CPU time and solution quality also increase. The largest gains in solution quality are obtained when α is small. For instance sets 1 and 2, an increment in α beyond the tuned value 250 does not lead to a significant increase in solution quality. For the instances of instance set 3, solution quality does not increase much beyond $\alpha = 300$. For the instances of set 4, the solution quality does not increase beyond $\alpha = 350$.

4.5.6 Contribution of local search operators

In this section, the contribution to solution quality of the local search algorithm and the different local search operators is analyzed. Five variants of the ILS are analyzed on the instance sets that were used for the sensitivity analysis of the parameters. In the first variant, only an initial solution is generated. No local search is performed. In the other variants, each time a single local search operator is removed from the search.

Five independent runs of the variants of the ILS are performed on each instance. Table 4.4 gives an overview of the results. When the local search is removed, the average gap with the original algorithm is very large, ranging from 69.51% to 98.24% as can be expected. Individually, the local search operators also have a contribution to solution quality although much smaller. The influence of the local search operators is larger on realistic size instances with 50 and 75 customers from instance sets 3 and 4 than on the first two instance sets. The relocate operator seems to have the largest influence on the results for the realistic-size instances. Note that interaction effects between local search operators are ignored in this analysis.

4.5.7 Conclusions sensitivity analysis

To conclude, it appears that there is an interaction between the size of the network and the effect of the parameters on the solution quality of the metaheuristic. As a result, the acceptance of worse solutions based on record-to-record travel, the heavy noise perturbation and the noise increment do not have an added value for instances of all sizes. The analysis points out however that in these cases the solution quality

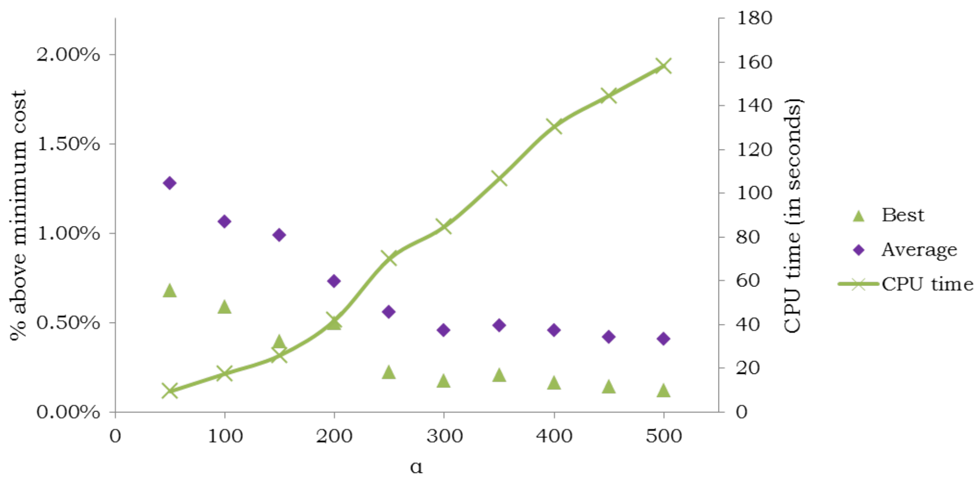

Figure 4.21: Sensitivity of α on instance set 1 (tuning instances)

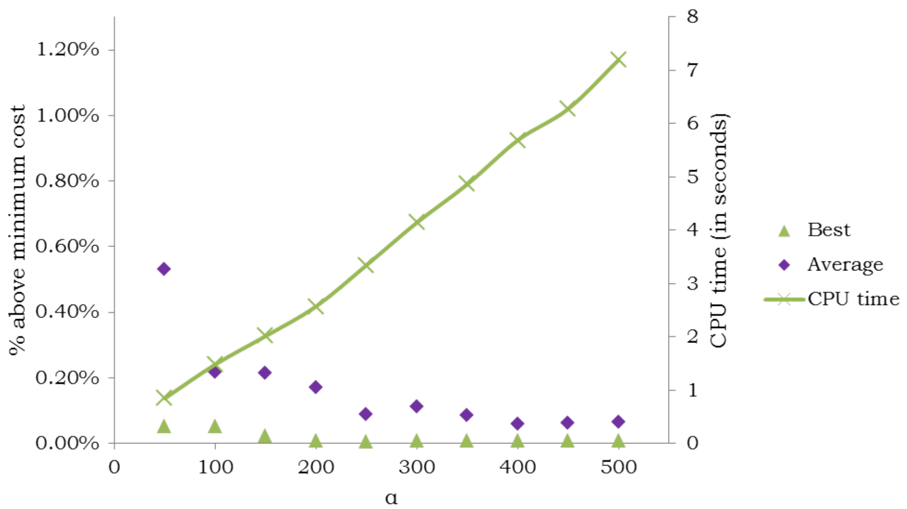

Figure 4.22: Sensitivity of α on instance set 2 (20 customers)

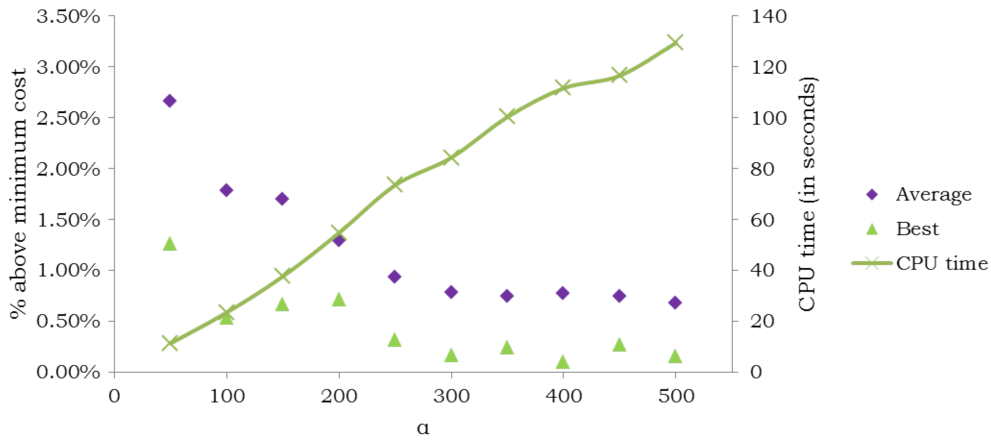

Figure 4.23: Sensitivity of α on instance set 3 (50 customers)

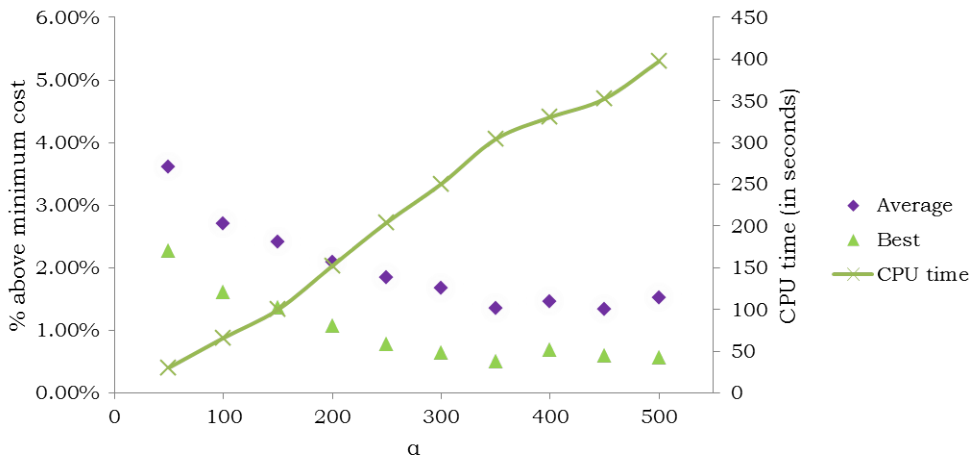

Figure 4.24: Sensitivity of α on instance set 4 (75 customers)

Table 4.4: Contribution of the local search operators

	Average gap with original algorithm			
	Instance set 1 (tuning)	Instance set 2 (20 customers)	Instance set 3 (50 customers)	Instance set 4 (75 customers)
No local search (only initial solution)	69.51%	69.60%	96.50%	98.24%
No exchange	0.03%	0.00%	0.25%	0.53%
No 2-opt	0.01%	0.00%	0.20%	0.52%
No cross-exchange	-0.05%	-0.01%	0.10%	0.46%
No relocate	-0.01%	0.02%	0.27%	0.68%

is also not negatively influenced. The algorithm needs to perform well on instances of all sizes and these mechanisms all have proven to contribute to the solution quality for at least a subset of instances.

The initial value of noise η^0 has the largest impact on solution quality. For all instance sets, the solution quality greatly improves when η^0 increases to a value of 0.30. For the tuning instances and the instances of size 20, the solution quality does not change when η^0 further increases. For the instances of size 50 and 75 however, the solution quality reaches a maximum when η^0 reaches 0.30 and 0.40 respectively. The solution quality decreases when η^0 further increases. The value of β has a considerable impact on instances with 20 or 50 customers. For these instances, the solution quality greatly improves when β has a non-zero value. The tuned value $\beta = 0.10$ renders the best solution quality, although the difference with higher values of β is very small. For the instances of size 75, the tuned value also renders the best solution quality, where higher values of β produce worse solutions than the configuration where $\beta = 0.0$. The impact of δ and η_{heavy} on solution quality is investigated simultaneously. The impact of these parameters on small-size instances of set 2 is rather small, while for the instances of sets 1, 3 and 4, the configurations with a low value of η_{heavy} produced a higher solution quality than the configurations with a moderate and high value of η_{heavy}. Furthermore, in these instance sets a low value of δ appears to have a negative effect on solution quality, especially when combined with a moderate or a high value for η_{heavy}. The value of η_{incr} is only relevant in very small-size instances. The reason for this is that an increment is only performed when the perturbation does not change the solution, which does not occur in the instances with 50 or 75 customers and occurs only rarely in the instances of size 20. The analysis shows that for instances of size 10, a value for η_{incr} of 0.4 or 0.6 has the highest solution quality.

As a result, the sensitivity analysis shows that the tuned setting for the parameters tuned by irace (δ, η^0, η_{heavy}, η_{incr}, β) renders a good solution quality for all instance sizes. With regards to the value of α, it may be concluded that on instance sets 1 and 2 the increase in α beyond the tuned value of 250 does not yield a quality increase. For the instances of set 3 and 4, the solution quality does not increase much beyond $\alpha = 300$ and $\alpha = 350$, respectively. Based on these results, α is set to 300.

The relevance of the algorithmic components of the ILS has been demonstrated in the foregoing analyses. The contribution of the acceptance of worse solutions based on record-to-record travel, the heavy noise perturbation and the noise increment is analyzed in the sensitivity analysis of the parameters. Record-to-record travel has a positive influence on solution quality since for all instance sets, a threshold value (β) of 0.10 leads to a higher solution quality than a threshold value equal to zero. A high value of noise after a number of consecutive non-improving iterations also has a positive effect on solution quality, although this effect appears to depend on the instance size. A noise increment after a perturbation that did not change the solution is only relevant in very small-size instances. The contribution of this component can only be demonstrated in instances of size 10. For larger instances, this component has no influence on solution quality. For the contribution of the local search operators, it appears that the relocate operator has the largest impact on solution quality for realistic size instances. The impact on solution quality of the local search operators individually does not seem to be very large. Interaction effects between local search operators have however not been considered. Furthermore, there is an interaction effect between the size of the network and the contribution of the local search operators.

Note that although the analysis shows that all algorithmic components contribute to the solution quality for at least a subset of instances, it may be interesting to look at the possibility of using a simplified heuristic method consisting of less local search operators while maintaining or even increasing the efficiency of the solution method. Christiaens and Vanden Berghe (2016) obtain high-quality results on benchmark instances of the CVRP with a heuristic method consisting of a single destroy operator and a single repair operator. This heuristic outperformed more complicated state-of-the-art algorithms for the CVRP on known benchmark instances.

4.6 Conclusions

In this chapter, a metaheuristic algorithm for the capacitated vehicle routing problem with sequence-based pallet loading and axle weight restrictions is proposed. The design and analysis of an Iterated Local Search (ILS) algorithm is presented. The parameters of the metaheuristic are tuned with an automatic algorithm configuration software which implements an iterated racing procedure, which is an extension of the Iterated F-race procedure (López-Ibáñez et al., 2016). Furthermore, a sensitivity analysis is performed to analyze the impact of the values of the parameters of the ILS and to test the contribution of the algorithmic components of the ILS on solutions quality.

Although the results show that the tuned configuration renders a good solution quality for all instance sizes, the sensitivity analysis also points out that the impact of the parameter values depends on the size of the network. Furthermore, the contribution of the algorithmic components seems to interact with the instance size.

In the current research, the aim is to develop an algorithm which performs well on instances of all sizes. However, future research may focus on making a distinction during the tuning process of the algorithm between networks of different sizes. The differences from the current algorithm to algorithms designed for a specific network size may be analyzed. In the next chapter, the performance of the ILS is analyzed by comparing the results to the optimal solutions. Furthermore, the effect of introducing axle weight constraints in a CVRP on travel distance is examined.

Chapter 5

CVRP with sequence-based pallet loading and axle weight constraints: Computational experiments

5.1 Introduction

In this chapter [1], computational experiments on the CVRP with sequence-based pallet loading and axle weight constraints are presented (Figure 5.1). In the previous chapters, three solution methods for the CVRP with sequence-based pallet loading and axle weight constraints are presented. The Mixed Integer Linear Programming model (MILP) of the problem and the Set Partitioning (SP) model designed in Chapter 3 are used to obtain an optimal solution for small-size instances (10 to 20 customers) and instances with 50 customers, while the ILS method developed in Chapter 4 is used to obtain a heuristic solution for instances with up to 100 customers.

This chapter is organized as follows. Section 5.2 describes the test setting and the generation of the instances. In Section 5.3, the computational experiments are discussed. First, the results of the MILP model, the SP model and the ILS are compared on instances containing 10 to 50 customers. Next, the effect of axle weight constraints in a CVRP is analyzed on instances with 10 to 100 customers. In Section 5.4, con-

[1] This chapter and the previous chapter are based on Pollaris et al. (2017).

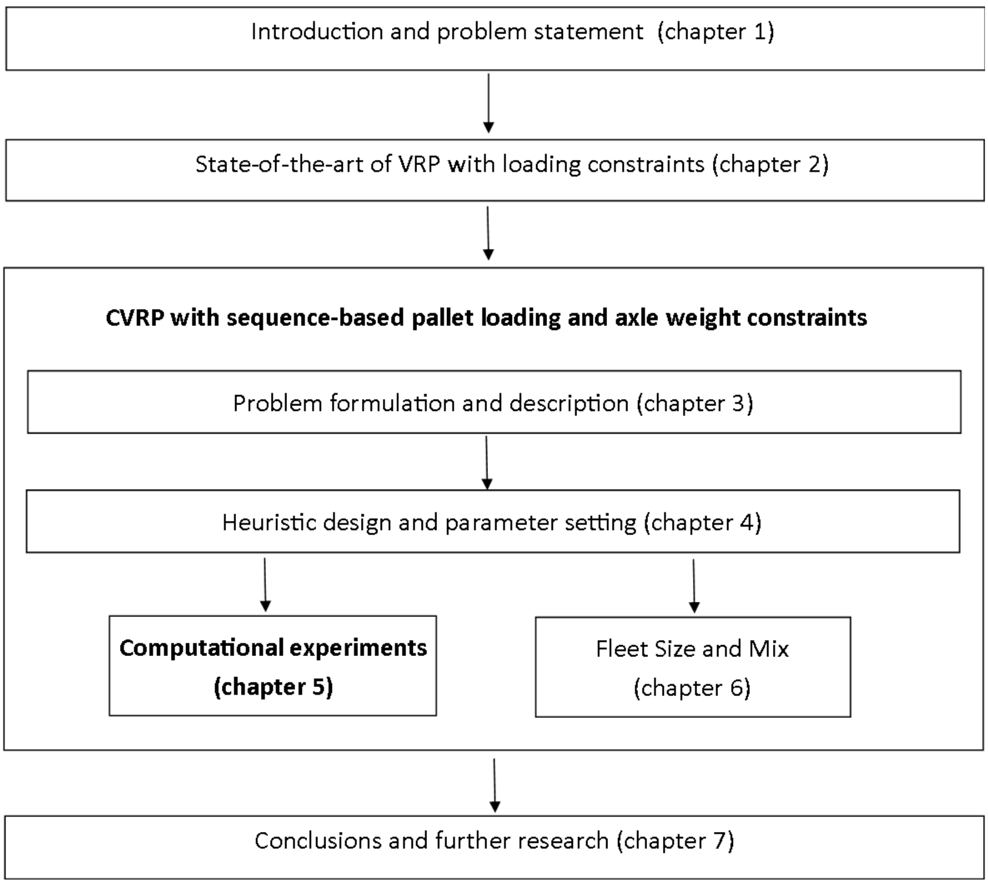

Figure 5.1: Outline of the thesis

clusions and future research opportunities are discussed.

5.2 Test Setting

For the computational tests, the 192 instances that are described in Section 4.2 are used. It was not possible to transform current benchmark instances in such a way that the optimal solution is not affected. In existing benchmark instances of the CVRP, demand and capacity are expressed in a (single) number of fictive units. For example, vehicle capacity = 100, demand customer 1 = 23, demand customer 2 = 56. In our problem, demand and capacity are expressed both in terms of total weight (in kg) and in number of pallets. Transforming a classic CVRP instance to an instance for our problem would firstly require to transform the fictive units to weight units (in kilogram). A transformation to number of pallets is not possible since this needs to be an integer value. Secondly, a number of pallets should be attributed to each customer in such a way that the optimal solution is not affected. This is necessary in order to compare the objective value to other solutions methods used for the benchmark instance. For most benchmark instances, the number of customers in the vehicle routes of the final solutions is at least 10 (although usually even more). The capacity of a 30-foot truck is 22 pallets. This leads to customer demands consisting of on average 2 pallets, which is very low. Furthermore, in the optimal solutions in the benchmark instances, the capacity utilization of the vehicles is very high (90% or higher). After transformation of the instances, this leads to certain pallets weighing more than 2000 kg, which is not feasible.

Also the benchmark instances mentioned in Table 2.4 which were designed for specific types of VRPs with loading constraints are not suited for our problem. In the benchmark instances for the 2L-CVRP, customer demand is expressed in length and width of the cargo and vehicle capacity is expressed in length and width of the loading area, which cannot be transformed to total weight and number of pallets. In the benchmark instances of the multi-pile VRP, the demand consist of different types of items (long and small chipboards).

An unlimited number of vehicles is considered. Characteristics of the vehicle fleet (measurements, capacity, mass, axle weight limits) are based on information from a Belgian logistics service provider. The vehicle type that is considered is a 30-foot truck that consists of a two-axle tractor (steering axle and driving axle) and a semi-trailer with tridem axles. In total, 22 pallets may be placed inside the truck. The total weight capacity of the truck consists of 32.2 tonnes. No more than 11.6 tonnes may be placed on the coupling, while no more than 21 tonnes may be placed on the tridem axles of the semi-trailer. The distance from the front of the container to the coupling (parameter c in equations (3.6) and (3.7)) equals 1 meter. The distance between the coupling and the central axle of the semi-trailer (parameter d in equations (3.6) and (3.7)) equals 5.5 meters. For more information regarding the vehicle characteristics, the reader is referred to the problem description in Chapter 3.

The experiments of the MILP are performed with AIMMS 3.13 (using CPLEX 12.5) on a 2.5 GHz Intel Core i5 laptop with 4 GB RAM. The set partitioning model is implemented in Python 2.7 and uses CPLEX 12.6 on a Xeon E5-2680v3 CPU at 2.5 GHz with 64 GB of RAM. The Iterated Local Search algorithm is implemented in Python 2.7 on a Xeon E5-2680v3 CPU at 2.5 GHz with 64 GB of RAM.

Because the experiments of the MILP are performed on a different computer than the experiments of the SP and ILS, hardware benchmarking is used in order to compare the speed of the algorithms. The computation times of the MILP that are reported in this chapter are recalculated to align with the result if the MILP would have been run on a Xeon E5-2680v3 CPU at 2.5 GHz. For information on the CPU speed of both computers, www.cpubenchmark.net is consulted. On this website, the values for the CPU speed are determined from thousands of performance tests benchmark results.

The parameters of the ILS algorithm are set to their tuned values, as described in Section 4.4. The value of α, the number of consecutive non-improving iterations of the ILS is set to 300 as a result of the sensitivity analysis described in Section 4.5.

5.3 Experimental results

In this section, the experimental results of the CVRP with sequence-based pallet loading and axle weight constraints are discussed. In Subsection 5.3.1, the results of the MILP model, the SP model and the ILS are compared on the instances of size 10 to 50. In Subsection 5.3.2, the effect of axle weight constraints in a CVRP is analyzed.

5.3.1 Comparison of MILP, SP and ILS

In this section, the results of the MILP model, the SP model and the ILS are compared for the CVRP with sequence-based pallet loading with and without axle weight constraints. A maximum computation time of 30 hours is considered for the exact approaches. For the instances of size 75 and 100, no optimal solutions were obtained.

Results of both CVRPs (with and without axle weight constraints) for each problem class on networks of 10, 15, 20 and 50 customers are presented in Tables 5.1, 5.2, 5.3 and 5.4. The computation times of the MILP, SP and ILS are reported (t(s)). The computation time of the SP model includes the time for the generation of the routes and the time to solve the SP with CPLEX. The optimal solutions of the instances of size 50 in 5.4 are only obtained by the SP model and not by the MILP formulation. Because of the stochastic character of the ILS, twenty independent runs are performed on each instance. *Opt. gap* Z^{avg}(%) presents the optimality gap of the average objective value out of twenty runs. *Opt. gap* Z^{best}(%) presents the optimality gap of the best solution out of twenty runs. Both gaps are reported in Tables 5.1, 5.2, 5.3 and 5.4. In Section 5.3.1.1 the computations times of the models are discussed while in Section 5.3.1.2 the objective value of the ILS is compared to the optimal value.

5.3.1.1 Computation times

For the networks of 10 customers, the computation times of the MILP model with axle weight constraints are presented for the case in which the constraints concerning the axle weight limits (equations (3.30) to (3.34)) are defined as lazy constraints as well as for the case in which these constraints are defined as regular constraints. Lazy constraints are initially not part of the active model. The model is solved without the lazy constraints and each solution is checked to see if any of the constraints in the lazy pool is violated. If a lazy constraint is violated, this constraint is added to the active model. On average, the computation time reduces with 45% when axle weight constraints are defined as lazy constraints. For that reason also the instances with 15 and 20 customers are solved with the MILP including lazy constraints.

Tables 5.1, 5.2 and 5.3 show average computation times for the SP model of 0.5 seconds, 2 seconds and 57 seconds for the instances of size 10, 15 and 20 respectively for the model with axle weight constraints. The computation time increases as the number of customers in the network increases. An explanation for the speed of the SP formulation may be that the number of feasible routes is limited because of the vehicle capacity of 22 pallets. The demand of the customers in the instances in problem classes 1 and 3 lies between 4 and 7 pallets which implies that a single vehicle can not visit more than five customers. The number of pallets per customer in the instances in problem classes 2 and 4 lies between 1 and 15. In instances in which several customers have a small number of pallets, the number of customers in a route may be larger than 5, which leads to a higher number of possible routes. For instances in which the majority of the customers have a large number of pallets, the number of possible routes will decrease. This explains the large variation in computation time for the instances of size 20 in problem classes 2 and 4 for the SP model. The computation times are higher in the model with axle weight constraints than in the model without axle weight constraints although the number of feasible routes is smaller. The reason for this is that the generation of the routes takes longer in the model with axle weight constraints because of the axle weight feasibility check.

The computation time for the MILP formulation for the CVRP with axle weight constraints (with lazy pool constraints) is on average 13 seconds, 1,918 seconds and 5,586 seconds for the instances of size 10, 15 and 20 respectively. Note that the MILP does not find an optimal solution for all instances within 30 hours of computation time. For the model with axle weight constraints (with lazy pool constraints) a solution is found in all instances with 10 customers, in 31 out of 32 instances with 15 customers and in 27 out of 32 instances with 20 customers. For the CVRP without axle weight constraints a solution is found for all instances.

Note that although a different version of CPLEX is used for the SP and the MILP as indicated in the test setting, it is clear that the SP model is much faster than the MILP model. Because the SP outperforms the MILP in terms of computational efficiency, the instances of size 50 are only solved by the SP and not by the MILP. For the instances of size 50, the SP formulation obtains an optimal solution for 27 out of 32 instances. The results for these instances are reported in Table 5.4. For the remaining 5 instances of size 50, no optimal solution was found within 30 hours of computation time. These instances are therefore not included in Table 5.4. The computation time for the generation of the routes as well to solve the SP problem for the instances with 50 customers for the CVRP without axle weight constrains is on average 2.1 hours. For the model with axle weight constraints, the average computation time for the instances of size 50 is 4.2 hours.

The computation time of the ILS for the CVRP without axle weight constraints is on average 0.8 seconds, 2 seconds, 5 seconds and 107 seconds for the instances of size 10, 15, 20 and 50. For the CVRP with axle weight constraints the computations times are on average 0.6 seconds, 2 seconds, 3 seconds and 84 seconds for the instances of size 10, 15, 20 and 50. Note that for small-size instances of 10 to 20 customers, the difference with the computation times of the SP is on average very small, while for the instances of size 50, the ILS is clearly much faster than the SP formulation.

5.3.1.2 Objective value

A summary of the comparison between the objective value obtained by the ILS and the optimal solution for each problem size is presented in Table 5.5 for the CVRP without axle weight constraints and in Table 5.6 for the CVRP with axle weight constraints. In the second column of both tables, the number of instances for which the optimal solution is known is reported. In the third and fourth column, the average optimality gap of the best solution out of twenty runs (*Opt. gap* $Z^{best}(\%)$) and the number of instances in which the best solution of the ILS does not equal the optimal solution ($Z^{best} \neq$ Z*) is reported. In the last columns, the average optimality gap of the average solution quality out of twenty runs (*Opt. gap* $Z^{avg}(\%)$) is reported as well as the number of instances in which the average solution of the ILS does not equal the optimum solution ($Z^{avg} \neq$ Z*).

For the CVRP without axle weight constraints, the ILS is able to find the optimum solution in each run for 92 out of 96 instances for the instances of size 10, 15 and 20. For the remaining 4 instances, the optimum solution is found in at least one run of the ILS. For the instances of size 50, the ILS finds the optimum solution in at least one run for 24 out of 27 instances. For the remaining 3 instances, the optimality gaps are very small. The average optimality gap of the best solution found by the ILS for the instances of size 50 is 0.04 %. The average optimality gap of all runs is 0.28 %.

For the CVRP with axle weight constraints, the ILS finds the optimal solutions in the majority of the instances. In case the optimal solution is not found, the optimality gap is very small. For the instances with networks of 10, 15 and 20 customers, the average optimality gaps are respectively 0.34 %, 0.35 % and 0.18 %. In 66 out of 96 instances, the optimal solution is found in all runs of the ILS. For 92 instances, the optimal solution is found in at least a single run. For 19 out of 27 instances of size 50 the optimal solution is found in at least a single run of the ILS. The average optimality gap for all runs for the instances of size 50 is 0.84 %, while the average optimality gap of the best run is only 0.08 %. The results show that the ILS is able to find good quality solutions for both problem types (CVRP with and without axle weight constraints).

5.3.2 Effect of axle weight constraints

In this section, the effect of the integration of axle weight constraints in a CVRP is analyzed by comparing the results of the ILS for the CVRP with and without axle weight constraints. Tables 5.7, 5.8 and 5.9 provide the results of the ILS on the instances with networks of realistic sizes with 50, 75 and 100 customers. For both problem types (CVRP with and without axle weight constraints), the average CPU time (t (s)), the average objective value (Z^{avg}) and the best objective value (Z^{best}) out of 20 runs is reported. For the problem without axle weight constraints, the number of axle weight violations ($\#\ V$) and maximum violation ($Max\ V$) are also shown. The number of violations represents the number of arcs traveled by a vehicle in which the coupling is overloaded. The total number of arcs traveled with a loaded vehicle is equal to the number of customers in the network. The maximum violation is expressed as a percentage of the weight capacity of the coupling (11.6 t). In all instances, the largest violation that occurs is a violation of the weight limit on the coupling (and thus on the axles of the tractor). Violations of the weight limit on the axles of the semi-trailer occur less frequently and are in all instances smaller than the violations on the axles of the tractor. This may be explained by the higher weight capacity of the axles of the semi-trailer (21 t) compared to the weight capacity of the coupling (11.6 t). For the problem with axle weight constraints, the increase in average objective value compared to the average objective value in the problem

Table 5.1: Comparison MILP, SP and ILS on networks of 10 customers

Instance	Model without axle weight constraints					Model with axle weight constraints					
	MILP t(s)	SP t(s)	ILS t(s)	Opt. gap Z^{avg}(%)	Opt. gap Z^{best}(%)	MILP t(s)	MILP t_{lazy}(s)	SP t(s)	ILS t(s)	Opt. gap Z^{avg}(%)	Opt. gap Z^{best}(%)
Problem class 1											
1	1	0.5	1.0	0.00	0.00	203	115	0.5	1.0	0.88	0.00
2	1	0.5	1.0	0.00	0.00	2	2	0.5	0.5	0.00	0.00
3	5	0.5	1.0	0.00	0.00	6	10	0.5	0.5	1.77	1.77
4	2	0.5	1.0	0.00	0.00	79	58	0.5	0.5	0.00	0.00
5	3	0.5	1.0	0.00	0.00	5	2	0.5	0.5	0.00	0.00
6	3	0.5	1.0	0.00	0.00	29	7	0.5	0.5	0.00	0.00
7	2	0.5	1.0	0.00	0.00	6	4	0.5	0.5	0.00	0.00
8	1	0.5	0.5	0.00	0.00	7	2	0.5	0.5	0.68	0.00
Problem class 2											
1	6	0.5	1.0	0.00	0.00	63	25	0.5	0.5	4.97	0.00
2	1	0.5	0.5	0.00	0.00	24	10	0.5	0.5	0.00	0.00
3	4	0.5	1.0	0.00	0.00	15	9	0.5	0.5	2.46	0.00
4	2	0.5	0.5	0.00	0.00	5	4	0.5	0.5	0.00	0.00
5	35	0.5	1.0	0.00	0.00	31	19	1	0.5	0.19	0.00
6	1	0.5	0.5	0.00	0.00	9	5	0.5	0.5	0.00	0.00
7	1	0.5	0.5	0.00	0.00	21	2	0.5	0.5	0.00	0.00
8	0.5	0.5	1.0	0.00	0.00	2	2	0.5	0.5	0.00	0.00
Problem class 3											
1	1	0.5	1.0	0.00	0.00	0.5	0.5	0.5	0.5	0.00	0.00
2	4	0.5	0.5	0.00	0.00	12	8	0.5	0.5	0.00	0.00
3	3	0.5	0.5	0.00	0.00	4	2	0.5	0.5	0.00	0.00
4	2	0.5	1.0	0.00	0.00	11	4	0.5	1.0	0.00	0.00
5	0.5	0.5	1.0	0.00	0.00	4	2	0.5	1.0	0.00	0.00
6	2	0.5	1.0	0.00	0.00	4	2	0.5	0.5	0.00	0.00
7	2	0.5	1.0	0.00	0.00	5	2	0.5	1.0	0.00	0.00
8	2	0.5	1.0	0.00	0.00	6	2	0.5	1.0	0.00	0.00
Problem class 4											
1	2	0.5	0.5	0.00	0.00	2	1	0.5	0.5	0.00	0.00
2	2	0.5	0.5	0.00	0.00	24	6	0.5	0.5	0.00	0.00
3	0.5	0.5	1.0	0.00	0.00	1	0.5	0.5	0.5	0.00	0.00
4	0.5	0.5	0.5	0.00	0.00	1	1	0.5	0.5	0.00	0.00
5	6	0.5	0.5	0.00	0.00	7	6	0.5	0.5	0.00	0.00
6	2	0.5	1.0	0.00	0.00	24	19	1	0.5	0.00	0.00
7	0.5	0.5	0.5	0.00	0.00	2	1	0.5	0.5	0.00	0.00
8	1	0.5	0.5	0.00	0.00	2	1	0.5	0.5	0.00	0.00
Average	3	0.5	0.8	0.00	0.00	19	10	0.5	0.6	0.34	0.06

t_{lazy} = computation time when axle weight limits are defined as lazy constraints

Table 5.2: Comparison MILP, SP and ILS on networks of 15 customers

Instance	Model without axle weight constraints					Model with axle weight constraints				
	MILP t(s)	SP t(s)	ILS t(s)	Opt. gap Z^{avg}(%)	Opt. gap Z^{best}(%)	MILP t(s)	SP t(s)	ILS t(s)	Opt. gap Z^{avg}(%)	Opt. gap Z^{best}(%)
Problem class 1										
1	24	0.5	3	0.00	0.00		2	2	0.60	0.00
2	4	0.5	2	0.00	0.00	20066	1	1	0.00	0.00
3	102	0.5	2	0.32	0.00	391	0.5	1	0.64	0.00
4	79	0.5	3	0.00	0.00	1017	1	2	0.00	0.00
5	15	0.5	2	1.59	0.00	78	1	2	1.54	0.00
6	45	0.5	2	0.00	0.00	3846	0.5	1	0.29	0.00
7	117	0.5	2	0.00	0.00	1400	0.5	2	0.00	0.00
8	10	0.5	2	0.00	0.00	650	2	1	0.00	0.00
Problem class 2										
1	8	1	2	0.00	0.00	187	4	1	3.85	0.00
2	28	0.5	2	0.00	0.00	31	0.5	1	0.00	0.00
3	79	0.5	2	0.00	0.00	539	2	1	1.88	0.00
4	13	0.5	2	0.00	0.00	23	1	1	0.00	0.00
5	12	0.5	2	0.00	0.00	45	0.5	1	0.40	0.40
6	76	9	3	0.00	0.00	16418	29	2	0.15	0.00
7	1568	0.5	2	0.00	0.00	1326	1	2	0.00	0.00
8	64	0.5	2	0.00	0.00	157	1	1	0.15	0.00
Problem class 3										
1	227	0.5	3	0.00	0.00	590	1	2	0.36	0.00
2	67	0.5	2	0.00	0.00	257	1	2	0.00	0.00
3	24	0.5	2	0.00	0.00	215	2	2	0.00	0.00
4	12	0.5	2	0.00	0.00	19	2	2	0.00	0.00
5	16	0.5	2	0.00	0.00	42	1	1	0.35	0.00
6	72	0.5	3	0.00	0.00	173	1	2	0.00	0.00
7	47	0.5	1	0.00	0.00	57	1	2	0.00	0.00
8	32	0.5	2	0.00	0.00	309	1	2	0.00	0.00
Problem class 4										
1	5	0.5	1	0.00	0.00		0.5	2	0.43	0.00
2	54	0.5	2	0.00	0.00	102	1	2	0.00	0.00
3	45	0.5	2	0.00	0.00	68	1	2	0.00	0.00
4	44	0.5	2	0.00	0.00	19	0.5	1	0.00	0.00
5	37	0.5	2	0.15	0.00	45	0.5	2	0.46	0.00
6	26	0.5	2	0.00	0.00	19	2	2	0.13	0.00
7	167	0.5	2	0.00	0.00	107	1	2	0.00	0.00
8	31	0.5	2	0.00	0.00	65	0.5	1	0.00	0.00
Average	99	0.8	2	0.06	0.00	1,609	2	2	0.35	0.01

Table 5.3: Results of the CVRP with sequence-based pallet loading with and without axle weight constraints on instances with 20 customers

Instance	Model without axle weight constraints					Model with axle weight constraints				
	MILP t(s)	SP t(s)	ILS t(s)	Opt. gap Z^{avg}(%)	Opt. gap Z^{best}(%)	MILP t(s)	SP t(s)	ILS t(s)	Opt. gap Z^{avg}(%)	Opt. gap Z^{best}(%)
Problem class 1										
1	24	1	4	0.00	0.00	280	3	3	0.00	0.00
2	670	5	5	0.00	0.00	654	12	4	0.00	0.00
3	594	3	6	0.00	0.00		8	5	0.00	0.00
4	253	3	5	0.00	0.00	1338	7	4	0.00	0.00
5	4806	1	5	0.00	0.00		3	4	0.56	0.00
6	17	2	4	0.00	0.00	17621	5	2	0.00	0.00
7	20	2	5	0.00	0.00	45	3	3	0.00	0.00
8	1759	3	7	0.00	0.00		5	4	2.15	1.34
Problem class 2										
1	591	0.5	3	0.00	0.00	13317	2	3	0.10	0.00
2	608	0.5	4	0.00	0.00	1310	2	3	0.00	0.00
3	75	0.5	3	0.00	0.00	27433	2	2	0.32	0.00
4	250	0.5	4	0.00	0.00	907	1	2	0.00	0.00
5	228	0.5	4	0.00	0.00		1	2	0.19	0.00
6	1153	0.5	3	0.00	0.00	3591	1	1	0.27	0.27
7	52	0.5	4	0.00	0.00	1060	2	3	0.00	0.00
8	507	196	6	0.00	0.00	3308	338	3	1.31	0.00
Problem class 3										
1	173	2	5	0.00	0.00	729	5	4	0.00	0.00
2	543	4	5	0.00	0.00	1262	8	3	0.00	0.00
3	925	1	5	0.00	0.00	3536	2	3	0.00	0.00
4	4434	1	6	0.00	0.00	1917	3	4	0.00	0.00
5	812	4	7	0.00	0.00	5634	9	5	0.00	0.00
6	841	1	4	0.00	0.00	1178	3	3	0.00	0.00
7	166	4	7	0.00	0.00	3307	12	5	0.00	0.00
8	2075	2	9	0.00	0.00	16662	6	5	0.00	0.00
Problem class 4										
1	2490	229	6	0.00	0.00	12813	635	5	0.00	0.00
2	283	237	6	0.00	0.00		723	5	0.16	0.00
3	802	2	5	0.45	0.00	1150	6	4	0.56	0.00
4	73	0.5	4	0.00	0.00	296	2	2	0.00	0.00
5	175	0.5	5	0.00	0.00	1074	0.5	3	0.00	0.00
6	105	0.5	4	0.00	0.00	536	2	4	0.08	0.00
7	121	1	4	0.00	0.00	1493	2	3	0.00	0.00
8	30	1	4	0.00	0.00	67	6	4	0.00	0.00
Average	802	22	5	0.01	0.00	4,538	57	3	0.18	0.05

Table 5.4: Comparison SP and ILS on networks of 50 customers

Instance	Model without axle weight constraints				Model with axle weight constraints			
	SP t(s)	ILS t(s)	Opt. gap Z^{avg}(%)	Opt. gap Z^{best}(%)	SP t(s)	ILS t(s)	Opt. gap Z^{avg}(%)	Opt. gap Z^{best}(%)
Problem class 1								
1	3088	114	0.15	0.00	3863	91	0.38	0.00
2	22439	161	0.15	0.00	8423	88	2.21	0.66
3	5714	135	0.29	0.00	4728	93	0.32	0.00
4	20498	120	0.07	0.00	11375	60	1.78	0.00
5	2503	76	0.00	0.00	1879	72	0.76	0.00
6	3425	103	0.03	0.00	4137	58	0.39	0.00
7	15997	139	1.32	0.72	10763	74	1.42	0.00
8	3115	148	1.87	0.00	1170	106	0.60	0.00
Problem class 2								
2	12103	98	0.15	0.00	36853	66	0.98	0.00
4	8506	110	0.19	0.00	39903	72	1.04	0.11
5	629	85	0.10	0.00	1352	60	1.05	0.00
8	16384	71	0.07	0.00	65521	56	1.47	0.26
Problem class 3								
1	2742	146	0.19	0.00	2686	99	0.46	0.17
2	3478	127	0.09	0.00	3457	108	0.71	0.00
3	6895	107	0.07	0.00	10217	144	1.30	0.00
4	19602	131	0.12	0.08	13930	137	0.71	0.25
5	2056	118	0.03	0.00	2935	121	0.50	0.00
6	6096	120	0.29	0.00	6284	111	0.89	0.00
7	5364	145	0.54	0.17	7770	103	1.09	0.18
8	5234	88	0.11	0.00	3685	107	0.81	0.00
Problem class 4								
2	6545	93	0.24	0.00	23907	48	0.05	0.00
3	5085	84	0.75	0.00	12944	59	0.65	0.00
4	17803	86	0.00	0.00	93099	76	0.49	0.00
5	756	81	0.56	0.00	1267	84	0.74	0.00
6	9168	81	0.14	0.00	39379	75	0.46	0.16
7	360	71	0.10	0.00	547	66	0.24	0.00
8	138	47	0.00	0.00	415	46	1.09	0.35
Average	7619	107	0.28	0.04	15277	84	0.84	0.08

Table 5.5: Validation ILS on CVRP without axle weight constraints

	# instances	Opt. gap Z^{best}(%)	$Z^{best} \neq$ Z*	Opt. gap Z^{avg}(%)	$Z^{avg} \neq$ Z*
10 customers	32	0.00	0	0.00	0
15 customers	32	0.00	0	0.06	3
20 customers	32	0.00	0	0.01	1
50 customers	27	0.04	3	0.28	24

Table 5.6: Validation ILS on CVRP with axle weight constraints

	# instances	Opt. gap Z^{best}(%)	$Z^{best} \neq$ Z*	Opt. gap Z^{avg}(%)	$Z^{avg} \neq$ Z*
10 customers	32	0.06	1	0.34	6
15 customers	32	0.01	1	0.35	14
20 customers	32	0.05	2	0.18	10
50 customers	27	0.08	8	0.84	27

without axle weight constraints (Z^{avg} *incr* (%)) is reported, as well as the increase in best objective value compared to the best objective value in the problem without axle weight constraints (Z^{best} *incr* (%)).

For all instances with 50 to 100 customers, the solution of the ILS for the CVRP without axle weight constraints generates axle weight violations. The number of arcs in which there is an axle weight violation for networks of 50, 75 and 100 customers is equal to 14, 19 and 27, on average, respectively. This means that in more than 25 % of the arcs traveled with a loaded vehicle, there is an axle weight violation. The extent of the violations is also considerable, with, on average, a maximum violation of 13 %, which would lead to a high fine in practice. Results show that these violations may be avoided with a relatively small increase in objective value. On average the increase in average objective value in the model with axle weight constraints compared to the average objective value in the model without axle weight constraints is 2.50 %, 2.49 % and 3.35 % for the networks of respectively 50, 75 and 100 customers. The average increase in best objective value compared to the best objective value in the model without axle weight constraints is 1.84 %, 1.58 %, 2.29 % for the networks with respectively 50, 75 and 100 customers. The CPU time for the instances of size 50, 75 and 100 is on average respectively 83, 260 and 333 seconds for the model with axle

Table 5.7: Results of the CVRP with sequence based pallet loading with and without axle weight constraints on networks of 50 customers

Instance	Model without axle weight					Model with axle weight				
	Z^{avg}	Z^{best}	t (s)	# V	Max V (%)	Z^{avg}	Z^{avg} incr (%)	Z^{best}	Z^{best} incr (%)	t (s)
Problem class 1										
1	152.5	152.2	114	9	12	154.7	1.44	154.1	1.25	91
2	134.1	133.8	161	22	16	144.4	7.68	142.2	6.28	88
3	145.7	145.3	135	13	17	148.2	1.72	147.7	1.65	93
4	151.0	150.8	120	15	13	154.8	2.52	152.1	0.86	60
5	166.1	166.1	76	15	17	169.4	1.99	168.2	1.26	72
6	153.2	153.2	103	12	16	156.8	2.35	156.2	1.96	58
7	143.1	142.3	139	16	12	148.3	3.63	146.3	2.81	74
8	144.9	142.3	148	22	16	148.4	2.42	147.5	3.65	106
Problem class 2										
1	170.3	170.3	88	19	16	178.8	4.99	177.5	4.23	79
2	200.7	200.5	98	24	18	205.8	2.54	203.8	1.65	66
3	189.2	188.4	84	14	12	196.9	4.07	192.8	2.34	77
4	183.4	183.0	110	12	18	189.0	3.05	187.2	2.30	72
5	191.0	190.9	85	18	15	196.5	2.88	194.5	1.89	60
6	168.4	168.4	74	11	14	169.6	0.71	168.4	0.00	53
7	184.2	184.2	74	26	17	195.5	6.13	190.1	3.20	63
8	192.7	192.5	71	14	16	202.4	5.03	200.0	3.90	56
Problem class 3										
1	144.1	143.9	146	8	3	144.9	0.56	144.5	0.42	99
2	157.2	156.9	127	6	5	158.4	0.76	157.3	0.25	108
3	143.8	143.8	107	8	5	146.1	1.60	144.3	0.35	144
4	149.2	149.0	131	9	6	150.2	0.67	149.5	0.34	137
5	158.4	158.3	118	10	7	160.6	1.39	159.8	0.95	121
6	143.7	143.3	120	6	4	145.5	1.25	144.3	0.70	111
7	147.3	146.5	145	9	6	148.2	0.61	146.8	0.20	103
8	130.1	130.0	88	9	14	131.2	0.85	130.1	0.08	107
Problem class 4										
1	182.1	181.4	105	19	19	185.4	1.81	184.6	1.76	95
2	210.0	210.0	93	11	10	210.8	0.38	210.7	0.33	48
3	198.7	197.9	84	13	8	200.3	0.81	199.0	0.56	59
4	193.7	193.7	86	16	10	200.5	3.51	199.5	2.99	76
5	199.4	199.3	81	19	17	207.0	3.81	205.5	3.11	84
6	212.8	212.5	81	8	5	213.4	0.28	212.8	0.14	75
7	199.5	199.4	71	15	8	203.4	1.95	202.9	1.76	66
8	222.0	222.0	47	22	17	236.5	6.53	234.8	5.77	46
Average			103	14			2.50		1.84	83

V = number of violations
Max V = maximum violation

Table 5.8: Results of the CVRP with sequence based pallet loading with and without axle weight constraints on networks of 75 customers

Instance	Model without axle weight					Model with axle weight					
	Z^{avg}	Z^{best}	t (s)	# V	Max V (%)	Z^{avg}	Z^{avg} incr (%)	Z^{best}	Z^{best} incr (%)	t (s)
Problem class 1										
1	201.8	201.1	551	20	14	205.9	2.03	203.8	1.34	208
2	208.0	206.7	679	24	16	216.0	3.85	214.0	3.53	214
3	208.1	207.8	340	24	18	216.5	4.04	214.2	3.08	215
4	212.7	211.5	450	17	15	219.2	3.06	216.2	2.22	221
5	200.1	199.9	484	17	15	205.0	2.45	202.2	1.15	206
6	199.9	199.1	418	22	14	207.2	3.65	204.7	2.81	209
7	213.8	213.2	458	24	18	224.4	4.96	220.9	3.61	222
8	207.3	206.4	629	23	16	214.4	3.42	211.6	2.52	215
Problem class 2										
1	273.8	273.8	226	30	18	286.9	4.78	283.8	3.65	287
2	261.0	260.8	344	21	17	265.7	1.80	262.7	0.73	266
3	299.6	298.9	315	32	18	309.9	3.44	304.7	1.94	311
4	276.0	275.1	364	25	16	285.8	3.55	282.5	2.69	288
5	323.9	323.7	437	27	15	331.8	2.44	329.9	1.92	330
6	263.2	262.5	506	22	18	266.0	1.06	262.8	0.11	267
7	315.3	314.8	393	27	20	327.3	3.81	322.7	2.51	327
8	326.1	325.8	447	34	18	335.1	2.76	333.1	2.24	336
Problem class 3										
1	221.2	220.6	616	14	13	225.9	2.12	221.9	0.59	227
2	195.3	194.5	366	11	6	197.7	1.23	195.2	0.36	197
3	201.5	201.4	444	13	4	206.6	2.53	202.9	0.74	204
4	204.6	203.7	527	10	5	211.2	3.23	206.2	1.23	216
5	204.9	204.3	409	13	9	207.4	1.22	205.6	0.64	210
6	229.7	229.4	349	12	9	231.8	0.91	230.1	0.31	233
7	202.8	202.0	330	11	10	204.4	0.79	202.0	0.00	204
8	211.0	209.4	668	12	9	215.2	1.99	212.4	1.43	215
Problem class 4										
1	337.7	337.3	433	20	14	342.3	1.36	340.1	0.83	343
2	326.9	326.7	333	23	12	335.2	2.54	331.7	1.53	333
3	328.1	327.0	369	13	9	333.7	1.71	331.3	1.31	335
4	324.4	322.1	425	16	14	331.0	2.03	326.9	1.49	329
5	271.5	270.5	317	15	11	274.0	0.92	271.8	0.48	275
6	287.0	286.4	351	16	8	290.2	1.11	288.3	0.66	290
7	288.0	287.8	342	24	15	297.6	3.33	295.0	2.50	299
8	269.0	268.6	326	9	9	272.8	1.41	269.7	0.41	270
Average			426	19			2.49		1.58	260

V = number of violations
Max V = maximum violation

Table 5.9: Results of the CVRP with sequence based pallet loading with and without axle weight constraints on networks of 100 customers

Instance	Model without axle weight					Model with axle weight				
	Z^{avg}	Z^{best}	t (s)	# V	Max V (%)	Z^{avg}	Z^{avg} incr (%)	Z^{best}	Z^{best} incr (%)	t (s)
Problem class 1										
1	272.4	271.1	1172	29	16	281.3	3.27	278.4	2.69	282
2	281.4	277.9	1349	33	16	292.6	3.98	289.8	4.28	290
3	256.5	253.7	1290	32	18	268.8	4.80	264.7	4.34	268
4	274.9	273.4	1560	31	14	291.6	6.07	283.9	3.84	293
5	260.6	260.0	1215	31	16	270.1	3.65	266.2	2.38	268
6	275.3	274.4	1343	36	17	289.6	5.19	282.2	2.84	287
7	275.2	274.2	1344	32	15	290.6	5.60	287.3	4.78	292
8	271.9	271.2	1277	25	16	283.8	4.38	277.5	2.32	284
Problem class 2										
1	388.3	387.6	1326	42	21	400.9	3.24	396.5	2.30	400
2	362.4	360.7	1230	27	16	368.7	1.74	364.3	1.00	370
3	355.9	354.8	1219	41	19	368.6	3.57	364.3	2.68	366
4	383.3	382.3	1448	41	20	396.4	3.42	389.2	1.80	399
5	364.8	364.0	1373	40	17	375.5	2.93	372.1	2.23	375
6	332.6	331.6	1377	37	19	344.0	3.43	337.8	1.87	344
7	382.2	381.5	1284	27	19	395.2	3.40	389.9	2.20	397
8	383.6	383.0	1424	24	16	393.1	2.48	387.6	1.20	391
Problem class 3										
1	249.8	248.6	1375	14	11	256.1	2.52	253.0	1.77	255
2	266.8	265.0	1260	13	7	274.9	3.04	269.2	1.58	274
3	272.9	270.8	1185	13	8	279.2	2.31	274.5	1.37	280
4	253.5	252.5	1467	17	6	262.3	3.47	259.0	2.57	259
5	266.3	265.6	1096	12	11	271.3	1.88	267.8	0.83	272
6	277.4	275.6	1449	12	7	285.3	2.85	280.4	1.74	284
7	260.6	259.1	1493	18	12	266.5	2.26	264.0	1.89	267
8	266.0	265.4	1033	15	7	269.9	1.47	266.3	0.34	272
Problem class 4										
1	392.9	392.0	1403	29	17	400.5	1.93	397.1	1.30	400
2	411.7	410.5	816	21	11	422.5	2.62	418.3	1.90	424
3	431.0	430.0	1030	31	12	443.2	2.83	437.8	1.81	441
4	356.3	355.5	1300	25	7	367.9	3.26	361.8	1.77	368
5	354.2	353.3	990	29	13	367.1	3.64	361.1	2.21	372
6	364.2	362.5	1207	25	15	378.3	3.87	373.3	2.98	381
7	392.3	390.4	1261	29	16	407.1	3.77	403.1	3.25	407
8	375.1	374.7	985	31	16	391.2	4.29	386.4	3.12	389
Average			1268	27			3.35		2.29	333

V = number of violations

Max V = maximum violation

weight constraints. For the model without axle weight constraints, the CPU time for the instances of size 50, 75 and 100 is on average respectively 103, 426 and 1268 seconds.

Table 5.10 presents a comparison of the results per problem class and number of customers in the network. In this table, the small-size instances with 10 to 20 customers are also considered. Detailed results of the ILS on the small-size networks of 10, 15 and 20 customers are available in Appendix B. The increase in average and best costs in problem classes 1 and 2 are for the small-size instances on average higher than for the instances of size 50 to 100. For problem classes 3 and 4, there is not much difference in average and best increase in objective value between the small-size and large-size instances.

For all instance sizes, the number of violations and the increase in objective value are larger in problem classes 1 and 2 (where only heavy pallets are considered) than in problem classes 3 and 4 (where a fifty-fifty percent mix of heavy and light pallets are considered). For the instances with a mix between light and heavy pallets, an increase in number of violations may be detected when we move from a low variation (problem class 3) to a high variation in number of pallets (problem class 4). Likewise, for the instances with only heavy pallets, the number of violations increases when the variation in number of pallets increases. The highest number of violations may therefore be found in problem class 2, while the instances in problem class 3 have on average the lowest number of violations.

The positive effect of mixing light pallets with heavy pallets on the increase in objective value can be explained by the fact that this allows for more flexibility in the packing process. If lighter pallets are packed first in the truck, the weight of the heavy pallets will mostly be carried by the axles of the semi-trailer, which have a higher weight capacity. Heavy pallets are therefore better transported together with light pallets even though the total weight capacity of the vehicle is sufficient to transport only heavy pallets. A possible explanation for the negative effect on increase in objective value and number of violations of a higher variation in number of pallets per customer may be that a variation between 1 and 15 pallets per order leads to on average half of the orders which have more than 8 pallets which is much less flexible than orders between 4 and 7 pallets per customer. An order of 15 pallets with a pallet weight of 1.4 tonnes, leads to a total weight of the order of 21 tonnes, which is less flexible to position on a truck than several smaller orders with a high pallet weight. The probability of an axle weight violation and the extent of this violation is much larger when a high variation of number of pallets is considered.

Figures 5.2 and 5.3 present for each problem class an overview of the variation

Table 5.10: Summary of the increase in objective value and number of violations for each problem class

n	# V	Z^{avg} incr (%)	Z^{best} incr (%)
Problem class 1			
10	3	4.16	3.95
15	5	6.48	6.33
20	4	1.99	1.81
50	15	2.97	2.47
75	21	3.43	2.53
100	21	4.62	3.43
Problem class 2			
10	4	6.63	5.63
15	6	6.75	5.95
20	8	4.94	4.69
50	17	3.68	2.44
75	27	2.96	1.97
100	35	3.03	1.91
Problem class 3			
10	1	0.95	0.95
15	2	0.85	0.76
20	2	0.00	0.00
50	8	0.96	0.41
75	12	1.75	0.66
100	14	2.47	1.51
Problem class 4			
10	2	1.62	1.62
15	4	2.56	2.44
20	5	2.02	1.98
50	15	2.39	2.05
75	17	1.80	1.15
100	28	3.28	2.29
Average	12	3.01	2.46

n = network size

V = number of violations

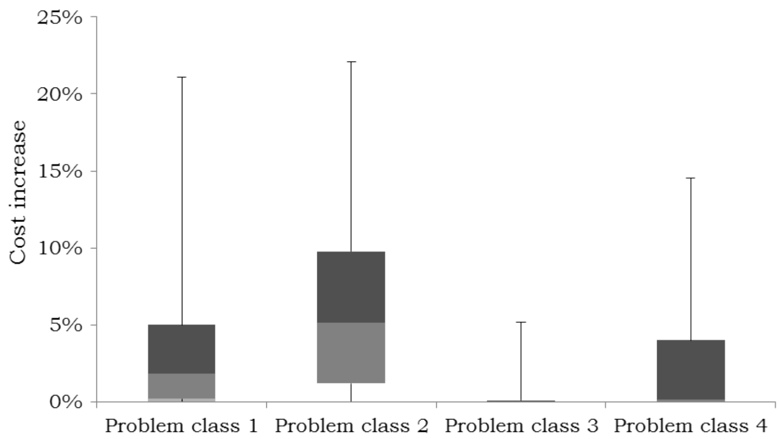

Figure 5.2: Boxplot of the increase in objective value after the integration of axle weight constraints per problem class of small-size instances with 10, 15 and 20 customers

of the average increase in objective value of the model with axle weight constraints compared to the model without axle weight constraints for respectively small-size (10 to 20 customers) and large-size (50 to 100 customers) instances. Note that a different scale is used on the vertical axis of both graphs because the variation of the increase in objective value is much larger for the small-size instances than for large-size instances. In particular this holds true for the first two problem classes.

For the small-size instances, the 25$^{\text{th}}$, 50$^{\text{th}}$ and 75$^{\text{th}}$ percentiles of the increase in objective value in problem class 2 are considerably higher than the corresponding percentiles in the first problem class. This is not the case for the large-size instances. The 25$^{\text{th}}$ and 50$^{\text{th}}$ percentiles of the increase in objective value of the large-size instances in problem class 1 and problem class 2 are similar. In problem class 1, 50 % of the large-size instances have an increase in objective value lower than 3.7 % while in problem class 2 the median is situated at 3.3 %. The 75$^{\text{th}}$ percentile of the average increase in objective value of the large-size instances in problem classes 1 and 2 is situated at 4.5 % and 3.7 % respectively.

For both instance sets, problem class 3 has on average the lowest increase in objective value. For the small-size instances, 75 % of the instances of this problem class have an increase lower than 0.1 %. For the large-size instances, the increase in

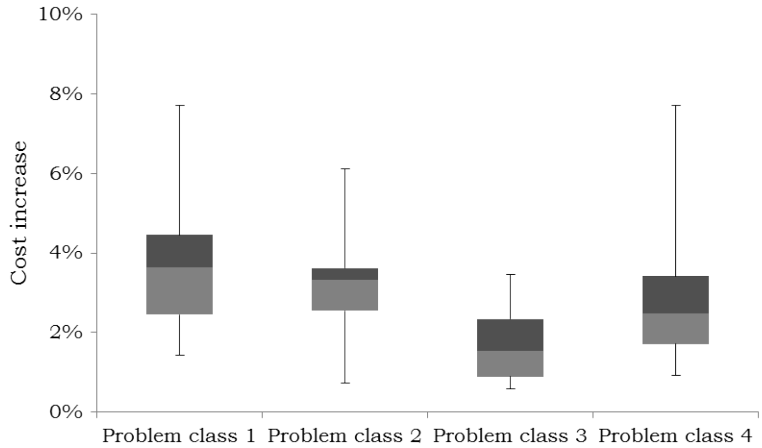

Figure 5.3: Boxplot of the increase in objective value after the integration of axle weight constraints per problem class of large-size instances with 50, 75 and 100 customers

objective value in problem class 3 is for 75 % of the instances lower than 2.5 %. The 75^{th} percentile of the instances of problem class 4 are for both instance sets higher with on average 3.9 % for the small-size instances and 3.5 % for the large-size instances. Furthermore, the variation in increase in objective value between the instances is for both instance sets larger in problem class 4 compared to problem class 3.

Based on the foregoing analysis, following conclusions may be composed with regards to the impact of the weight of the demand and the variation in number of pallets on the integration of axle weight constraints in the route scheduling. Mixing heavy and light pallets leads to fewer violations in the model without axle weight constraints and a smaller increase in objective value for all instance sizes. Furthermore, a large variation in number of pallets (1 to 15 pallets per customer) leads to a larger increase in objective value than a small variation in number of pallets (4 to 7 pallets) for the instances with a mix between light and heavy pallets (problem classes 3 and 4) for both small-size and large-size instances. In small-size instances with only heavy pallets (problem classes 1 and 2), the variation in number of pallets has a similar influence on the increase in objective value. In large-size instances with only heavy pallets (problem classes 1 and 2), the variation in number of pallets does not appear to have an influence on the increase in objective value.

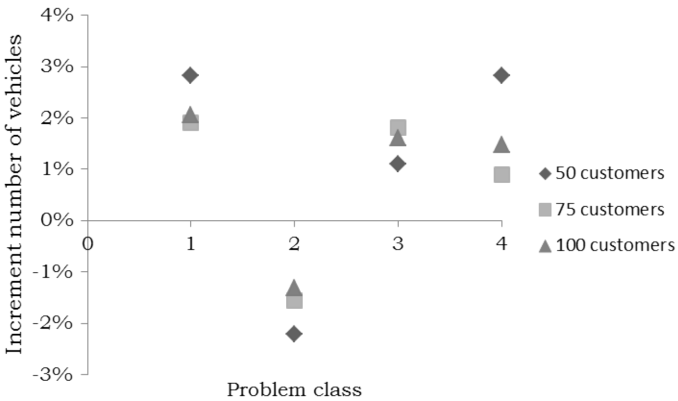

Figure 5.4: Increase in vehicles used per problem class of large-size instances with 50, 75 and 100 customers

Figure 5.4 presents the increase in number of vehicles in the model with axle weight constraints compared to the model without axle weight constraints for the large-size instances. Note that the minimization of the number of vehicles in the solution is not included in the objective function. For problem classes 1, 3 and 4 the number of vehicles increases with on average 2.3 %, 1.5 % and 1.7 % respectively, while in problem class 2, the number of vehicles decreases with on average 1.7 %. The reason for this may be that this problem class contains the most difficult loads, since only heavy pallets are considered and customer demands may contain a large number of pallets. These difficult loads need to be combined with lighter loads to obtain a feasible packing plan. Therefore it is more likely that in a solution with axle weight constraints, customers are combined in a single route even when they are situated far from each other and therefore are served separately in the solution without axle weight constraints.

5.4 Conclusions and future research

This chapter presents computational experiments of the capacitated vehicle routing problem with sequence-based pallet loading and axle weight constraints. The performance of the MILP model and the set partitioning (SP) formulation for the CVRP with sequence-based pallet loading and axle weight constraints presented in Chapter 3 is illustrated on instances of 10 to 50 customers. Within 30 hours of computation time, the MILP model is able to produce optimal solutions for most instances up to 20 customers. The SP model solves all instances up to 20 customers and is considerably faster than the MILP model. The majority of the instances with 50 customers are also solved by the SP model. The Iterated Local Search (ILS) algorithm, developed in Chapter 4, has proven to produce high-quality solutions, with very small optimality gaps on instances with up to 50 customers. Furthermore, the ILS is used to analyze the effect of introducing axle weight constraints in a CVRP on the objective value in instances with networks consisting of 10 to 100 customers. Results show that integrating axle weight constraints does not lead to a large increase in objective value, while not including axle weight constraints may induce major axle weight violations. In several instances axle weight violations can even be avoided without an increase in objective value. The effect of including axle weight constraints on the objective value depends on the number of pallets per customer and the weight of the pallets. When only light pallets are packed, axle weight limits do not play a role in the packing process. The effect of integrating axle weight limits on the objective value is higher when only heavy (1000 - 1500 kg) pallets are considered compared to a fifty-fifty percent mix of heavy and light pallets.

Since research on vehicle routing problems with axle weight constraints is very scarce, many research opportunities still exist with regards to solution methods and problem extensions. The exact methods based on the MILP and the SP formulations that have been developed are only able to solve small-size instances within a reasonable time limit. Future research could therefore focus on the development of more efficient exact methods. Since the SP model appears to outperform the MILP model in terms of computational efficiency, it may be useful to develop a new exact method based on the SP formulation. In the SP model, each feasible route is represented by a variable. Although the number of feasible routes is smaller in the CVRP with sequence-based pallet loading than in most other VRP models, it still causes the number of variables to become very large when the number of customers in the network is larger than 50. Column generation therefore presents an interesting research opportunity to solve the SP model. This method has proven to work well on linear models with many variables. With regards to the MILP formulation, a branch-and-cut method could be used to improve the efficiency of the model. In the literature concerning the VRP, good results have already been obtained with branch-and-cut algorithms (e.g. Iori et al., 2007; Tricoire et al., 2011; Cordeau et al., 2010b; Alba et al., 2013; Côté et al., 2012a). Furthermore, the development of matheuristics presents a promising research direction. A matheuristic combines mathematical programming with a heuristic solution method. In most matheuristics in the field of VRPs with loading constraints, the loading constraints are handled as a subproblem of the routing model (e.g. Doerner et al., 2007; Tricoire et al., 2011; Fuellerer et al., 2009). The packing subproblem is solved through mathematical programming models to optimality, while the routing model is solved heuristically.

Although the ILS metaheuristic combined with record-to-record travel provides good results for the CVRP with sequence-based pallet loading and axle weight constraints, it may be interesting to compare its performance to a simplified heuristic method consisting of less local search operators while maintaining or even increasing the efficiency of the solution method.

Future research could furthermore integrate other realistic features in the current problem such as time windows, time-dependent travel times, legal driving hours and the use of a heterogeneous vehicle fleet. Additionally, other loading constraints may be added to the current model. Another line of future research could be to integrate axle weight constraints in other types of VRPs such as three-dimensional loading VRP, multi-compartment VRP and pickup and delivery problems. In the next chapter, the effect of axle weight constraints in a CVRP with a heterogeneous fleet is analyzed.

Chapter 6

CVRP with sequence-based pallet loading and axle weight constraints: Fleet size and mix

6.1 Introduction

In Chapters 3, 4 and 5, the CVRP with sequence-based pallet loading and axle weight constraints is introduced and analyzed. As in most of the VRP literature, a homogeneous fleet of vehicles is assumed. This assumption is however not realistic in many real-life applications (Hoff et al., 2010). Most transportation companies dispose of a heterogeneous vehicle fleet to meet customer demands. Furthermore, the objective that is used in the previous chapters is the minimization of total distance traveled. Although this objective is traditionally used in VRP literature, it does not correspond to the objective faced by companies. In reality, companies aim to minimize total transport cost when scheduling their routes.

In order to examine the impact of axle weight constraints for a realistic vehicle fleet, this chapter introduces a heterogeneous vehicle fleet for the CVRP with sequence-based pallet loading and axle weight constraints (Figure 6.1). The vehicles in the fleet may have different capacities as well as costs. The goal of this chapter is fourfold. The first is to introduce the Fleet Size and Mix CVRP with sequence-based pallet loading and axle weight constraints. The second goal is to show that the ILS metaheuristic, developed in Chapter 4, may be used to tackle this problem. The third goal is to evaluate the impact of the vehicle fleet on the integration of axle weight constraints in a VRP. To this end, three heavy-duty vehicle fleet compositions are compared: a homogeneous fleet of 30-foot trucks, a homogeneous fleet of 45-foot trucks and a heterogeneous fleet with 30-foot and 45-foot trucks. We expect that the impact of integrating axle weight constraints is larger when a fleet of 45-foot trucks is considered because more pallets may be placed inside a 45-foot truck, while the weight capacity of the axles does not increase. Therefore, the probability of an axle weight violation in 45-foot trucks is larger than in 30-foot trucks. The fourth goal is to analyze the impact of a more realistic objective function which takes into account total transport costs on the effect of axle weight constraints in a VRP.

This chapter is organized as follows. Section 6.2 provides a problem description. In Section 6.3, related literature is discussed. In Section 6.4, the test setting is presented. Section 6.5 discusses experimental results. Section 6.6 presents conclusions and opportunities for further research.

6.2 Problem description

This chapter integrates a heterogeneous vehicle fleet in the CVRP with sequence-based pallet loading and axle weight constraints. Since a vehicle fleet is usually heterogeneous in real-life, the extension of the VRP to heterogeneous vehicles is highly relevant (Bräysy et al., 2009). Besides a heterogeneous vehicle fleet instead of a homogeneous fleet, the problem characteristics of the VRP presented in Chapter 3 remain unchanged. The demand of the customers consists of europallets (80x120 cm) and is delivered from a depot to customer locations. Pallets are packed dense in a truck in two horizontal rows. It is assumed that all pallets of a single customer have the same weight and that the weight is uniformly distributed inside each pallet, i.e., the center of gravity of a pallet lies in its geometric midpoint. The container can only be unloaded at the rear side. To avoid moving pallets of other customers when arriving at a customer, sequence-based loading is imposed. Vertical stacking is not allowed. The vehicle types in the fleet are different in terms of tare weight and measurements. Consequently, the capacity in terms of number of pallets and payload is different as well as the weight capacity of the axles.

Two scenarios with a different objective function are considered. In the first scenario, the objective is the minimization of total distance traveled, which was also

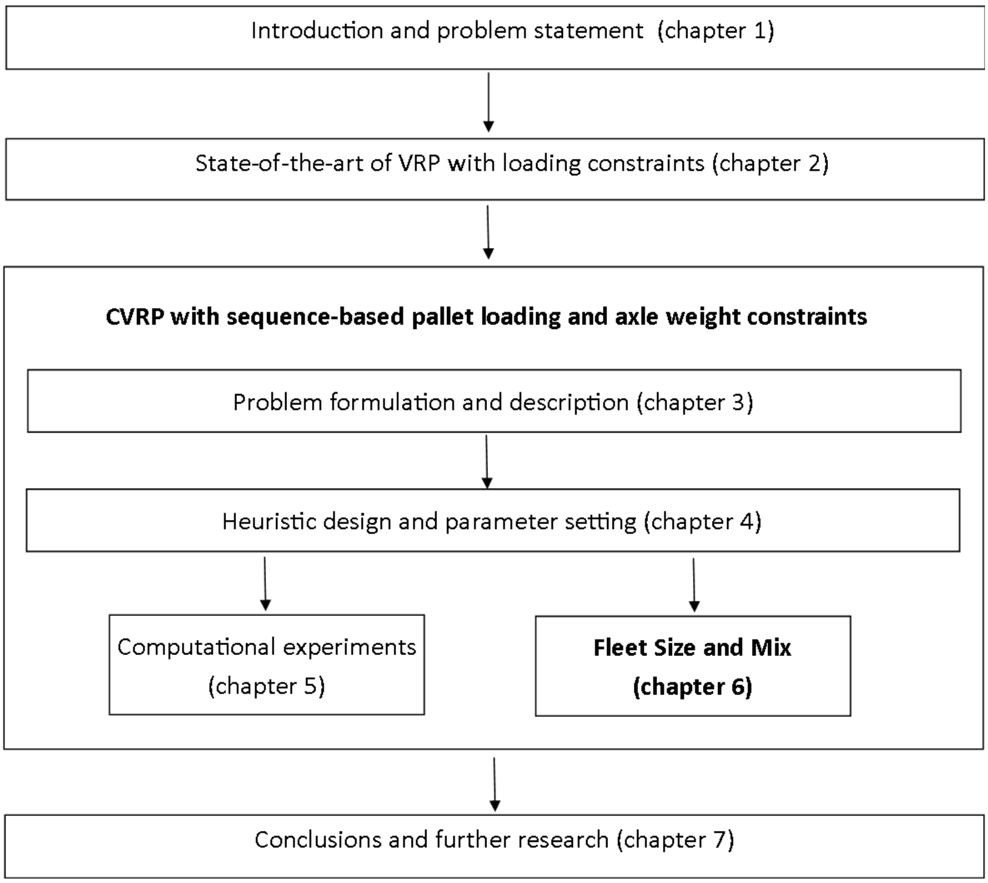

Figure 6.1: Outline of the thesis

the objective of the original problem described in Chapter 3. In the second scenario, the objective function minimizes total transport costs. A distinction is made between hour costs and kilometer costs based on Blauwens et al. (2012). The first objective is traditionally used in the VRP literature, while the second objective corresponds more to the objective of transportation companies in real-life. Both objectives are considered to examine whether the objective function influences the results of the analysis.

We expect that the impact of the integration of axle weight constraints in the scheduling of routes will be smaller in the second scenario. In the second objective function, fuel costs are taken into account which depend on the gross weight of the vehicle. Routes visiting customers with the heaviest pallets first on the route will therefore be favored over routes in which these customers are visited last in order to minimize fuel costs. Since sequence-based loading is assumed, this implies that heavy pallets will be placed more towards the rear of the vehicle, carried by the axles of the trailer. Since the axles of the trailer have a larger weight capacity, this leads to a smaller probability of an axle weight violation.

To formulate the objective function of the second scenario, the following notation is used:

$V = \{0, 1, ..., n+1\}$ set of vertices with customers (node 1, $\ldots$, n) and depot (node 0, $n+1$) (indices i, j)

$E =$ set of edges (i, j) where $i, j \in V$ and $i \neq j$

$c_{ij} =$ distance between nodes i and j (km)

$v =$ average speed (km/h)

$f =$ fuel price (€/l)

$kc =$ kilometer cost coefficient (excluding fuel costs) (€/km)

$hc =$ hour cost coefficient (€/h)

Variables x_{ij} and fe_{ij} are defined as follows:

$$x_{ij} = \begin{cases} 1 & \text{if a vehicle travels from } i \text{ to } j \text{ with } (i,j) \in \mathrm{E} \\ 0 & \text{otherwise} \end{cases}$$

$fe_{ij} =$ fuel efficiency on edge (i,j) considering the gross weight of the truck on (i,j) (km/l)

The objective function may be formulated as follows:

$$\min \sum_{(i,j)\in E} (kc + \frac{hc}{v} + \frac{1}{fe_{ij}} \cdot f) \cdot c_{ij} x_{ij} \tag{6.1}$$

The objective function 6.1 aims to minimize total transport costs. The calculation of the transport costs is based on Blauwens et al. (2012). The cost components of the hour coefficient (hc) and kilometer coefficient (kc) are presented in Table 6.1 and Table 6.2, respectively. These figures are based on data sampled from professional hauliers in Europe in 2004 (Blauwens et al., 2012). Note that the assumption is made that these cost figures are equal for the different vehicle types. Contacts with a logistics service provider confirm that purchasing costs as well as maintenance costs are comparable for heavy-duty vehicles with the same axle configuration (tractor and semi-trailer with tridem axles). The hour coefficient is equal to € 24.18 and consists of fixed costs that are charged per hour. The hour coefficient is divided by the average speed of a truck to calculate the cost per kilometer. The kilometer coefficient in Blauwens et al. (2012) is equal to € 0.08.

Table 6.1: Hour costs of a tractor and semi-trailer (in €) (Blauwens et al., 2012)

Interest and depreciation (fixed rate)	3.34
Insurance	1.79
Road tax, Euro-vignette, contributions, dues	0.67
Driver's wages (inc. all charges and premiums)	16.37
Others (buildings, management, administration)	2.01
Hour coefficient (hc)	24.18

Fuel consumption depends on the gross weight of the truck. The objective functions is therefore not linear. This cost component is calculated separately and is not included in the kilometer coefficient. The effect of weight on the fuel efficiency of

Table 6.2: Kilometer costs of a tractor and semi-trailer without fuel costs (in €) (Blauwens et al., 2012)

Interest and depreciation (variable)	0.04
Tires	0.01
Maintenance, repairs, fines	0.03
Kilometer coefficient (kc)	0.08

heavy-duty vehicles can be characterized by a linear function $fe = \pi + \tau \cdot L$ (Kopfer et al., 2014; Xiao et al., 2012). This formula presents the fuel efficiency in function of the weight of the payload L. For the values of π and τ, a study on the fuel efficiency of heavy-duty vehicles of the UK department of Transport (2007) is consulted. In this study, the fuel efficiency of several vehicle types is tested including a truck consisting of a tractor with two axles and semi-trailer with tridem axles. The tests were performed on a typical distribution route combining motorways, dual-carriageways as well as single-carriageways. During the first trip, the truck is empty. To measure the effect of increasing payload, in total seven trips are performed, each time with an increment in payload. Based on a regression analysis with a very high regression fit ($R^2 = 0.99$), following values are determined: $\pi = 4.5915$, $\tau = -0.0591$. Note that the fuel efficiency in this study is expressed in miles per gallon and that this has been recalculated to kilometers per liter in this chapter. The weight of the payload is expressed in tonnes. The tare weight of the truck is 15.6 tonnes. Based on this study, the following formula is constructed for the calculation of the fuel efficiency in our problem setting expressed in kilometers per liter:

$$\text{Fuel efficiency } = 4.5915 - 0.0591 \cdot (\text{payload + tare weight truck - 15.6}) \tag{6.2}$$

Note that only weight is included in the calculation of the fuel efficiency, while other factors such as road gradient, speed and acceleration are not considered. It is assumed that these factors are constant for the different vehicle types since the vehicles in the heterogeneous fleet are assumed to be heavy-duty vehicles. In case a mix between low-duty, medium-duty and heavy-duty vehicles with different values for average speed and acceleration are considered in the vehicle fleet, fuel efficiency models that include these factors may be used. For an overview of fuel consumption models in literature, the reader is referred to Kopfer et al. (2014) and Demir et al. (2011).

6.3 Related literature

The literature concerning heterogeneous vehicle fleet routing is divided into two major problem classes (Koç et al., 2014). The first class is the Fleet Size and Mix Vehicle Routing Problem (FSM) and considers an unlimited vehicle fleet. The problem is introduced by Golden et al. (1984). The objective is to minimize total distribution cost and determine the optimal fleet size and mix. The second problem class is the Heterogeneous Vehicle Routing problem (HVRP) and considers a limited vehicle fleet. The goal of the HVRP is to minimize total distribution costs, given an available fleet. This problem is introduced by Taillard (1999).

Based on the type of costs that are taken into account (vehicle dependent variable costs and/or fixed costs), following five problem variants have been proposed (Baldacci et al., 2008; Koç et al., 2014): 1) the Heterogeneous VRP with Vehicle Dependent Routing Costs, denoted by HVRPD; 2) the Heterogeneous VRP with Fixed Costs and Vehicle Dependent Routing Costs, denoted by HVRPFD; 3) the Fleet Size and Mix VRP with Fixed Costs, denoted by FSMF; 4) the Fleet Size and Mix VRP with Vehicle Dependent Routing Costs, denoted by FSMD; 5) the Fleet Size and Mix VRP with Fixed Costs and Vehicle Dependent Routing Costs, denoted by FSMFD.

Golden et al. (1984) introduce the FSM with fixed costs (FSMF). They develop several heuristics including a savings based heuristic as well as techniques to generate a lower bound. Twenty test problems are created containing 12 to 75 nodes with different combinations of capacity and fixed costs for the vehicle types considered in the fleet. The FSM with fixed costs and vehicle dependent routing cost (FSMFD) is introduced by Ferland and Michelon (1988). They propose three heuristic algorithms and two branch-and-bound procedures. Taillard (1999) introduces the FSM and HVRP with vehicle dependent routing costs (FSMD, HVRPD). The problems are tackled with a heuristic column generation method. A vehicle dependent routing cost component is added to eight instances of Golden et al. (1984) of size 50 and larger. The HVRP with vehicle dependent routing costs and fixed costs (HVRPFD) is introduced by Li et al. (2007). The authors propose a record-to-record travel algorithm which is tested on the instances of Golden et al. (1984) with the vehicle dependent routing costs components from Taillard (1999). Furthermore, they generate a new set of five large-scale vehicle routing problems with 200 to 360 customers.

A common extension of the FSM and the HVRP is the addition of time windows. Liu and Shen (1999) consider the FSMF with time windows. They develop several heuristic solution techniques to tackle the problem. The heuristics are tested on the data sets for VRPTW of Solomon (1987) to which costs for the different types of vehicles are added. For each original Solomon instance, three different sets of vehicle costs (large, medium, small) are considered. In total, 168 problem instances are created. Dullaert et al. (2002) develop three insertion-based heuristics for the same problem, which significantly outperform the heuristics proposed by Liu and Shen (1999). For some instances, solution improvements of more than 50% are attained. Paraskevopoulos et al. (2008) consider the HVRPFD with time windows. The authors propose a two-phase multi-start metaheuristic based on Tabu Search. The instances of Liu and Shen (1999) for the FSM with time windows are used, with the obtained best fleet size and mix by Liu and Shen (1999) to be the given fixed fleet.

Recently, Pasha et al. (2016) introduced the Multi-Period FSMFD. In this variant of the FSM, customer demands may vary over a set of periods. The objective is to find the best fleet composition to fulfill the customer demands and to find the best routing plan with that fleet for each of the different periods in the scheduling horizon. The authors develop a tabu search method to solve the problem. Other extensions of the FSM and HVRP include the integration of multiple depots (e.g. Salhi and Sari, 1997; Bettinelli et al., 2011), pollution-routing (e.g. Kwon et al., 2013; Koç et al., 2014), backhauls (e.g. Tütüncü, 2010; Salhi et al., 2013), split deliveries (e.g. Tavakkoli-Moghaddam et al., 2007; Belfiore and Yoshizaki, 2013) and container loading (Leung et al., 2013). Since the introduction of the FSM and HVRP many solution techniques, mainly heuristics, have been developed. For an overview of the literature, the reader is referred to Koç et al. (2016), Soonpracha et al. (2014) and Baldacci et al. (2008).

Based on the literature review, the problem that is studied in this chapter is the Fleet Size and Mix VRP. Since the tare weight depends on the vehicle type, the fuel costs will be (partially) vehicle dependent. The problem may therefore be classified as a Fleet Size and Mix VRP with Vehicle Dependent Routing Costs (FSMD).

6.4 Test Setting

For the computational tests of the FSM with sequence-based pallet loading and axle weight constraints, two different types of vehicles are considered. The first is a 30-foot truck, which is also used in the experiments in Chapter 5. For more information regarding the vehicle characteristics of the 30-foot truck, the reader is referred to Section 5.2. The second vehicle type is a 45-foot truck. As for the 30-foot truck, the characteristics of the 45-foot truck (measurements, capacity, mass, axle weight limits) are based on information from a Belgian logistics service provider. The 45-

foot truck consist of a tractor with two axles, a semi-trailer with tridem axles and a container. The length, width and height of the inside dimensions of the container are respectively 13.49 meters, 2.44 meters and 2.44 meters. The tare weight of the container and the semi-trailer is respectively 4.3 tonnes and 3.55 tonnes. The 45-foot truck has a similar axle configuration as the 30-foot truck. The tractor that is used in the 45-foot truck is the same as for the 30-foot truck. Furthermore, the semi-trailer also has tridem axles, but the distance between the axles is different. In the 45-foot truck, the distance from the front of the container to the coupling (parameter c in equations (3.6) and (3.7)) is 1.96 meters. The distance between the coupling and the central axle of the semi-trailer (parameter d in equations (3.6) and (3.7)) is 7.6 meters. In total, 32 pallets may be placed inside the 45-foot truck in two horizontal rows. The total weight capacity of the truck consists of 29.35 tonnes. No more than 10.75 tonnes may be placed on the coupling, while no more than 18.84 tonnes may be placed on the tridem axles of the semi-trailer. Note that the total weight capacity as well as the weight capacity of the axles is smaller for the 45-foot truck than for the 30-foot truck. The reason for this is that the tare weight of a 45-foot truck (14.65 tonnes) is higher than the tare weight of a 30-foot truck (11.8 tonnes), while the limits on the gross weight (44 tonnes) and on the axles of the loaded truck remain unchanged. In Table 6.3, the main characteristics of the 30-foot and 45-foot truck are summarized. An unlimited number of both 30-foot and 45-foot vehicles is considered. The price of fuel (f in equation 6.1) is fixed to € 0.70 per liter. For the average speed of a truck (v in equation 6.1) a value of 50 km/h is adopted (Van den Driest et al., 2011).

Table 6.3: Comparison of the characteristics of the 30-foot truck and 45-foot truck

	30-foot	45-foot
Tare weight	11.8 t	14.65 t
Capacity in terms of pallets	22	32
Weight capacity	32.2 t	29.35 t
Max weight coupling	11.6 t	10.75 t
Max weight axles trailer	21 t	18.84 t

The Iterated Local Search metaheuristic developed in Chapter 4 is used to solve the FSM with sequence-based pallet loading and axle weight constraints. The changes on the ILS algorithm to tackle this problem are limited. For the first scenario the objective remains the minimization of total distance traveled. The only changes to the algorithm are situated in the loading feasibility check because the different types of vehicles do not have the same characteristics in terms of capacity. In case a given route does not lead to a feasible packing plan for the first vehicle type, the feasibility is checked for the other vehicle type. Since an unlimited number of vehicles is considered and the objective value does not depend on the vehicle type, it does not matter which vehicle is used when both vehicle types are feasible. For the second scenario, the objective is the minimization of total transport costs. For this scenario, vehicle dependent routing costs are considered since the tare weight of the truck is included in the calculation of the fuel consumption. In case both vehicle types lead to a feasible packing plan for a given route, the lightest vehicle is chosen since this leads to the lowest transport costs.

The computational tests are performed on a new instance set consisting of 48 instances with networks of 50, 75 and 100 customers. The instances are generated in a similar way as the instances in Chapter 5. The main difference lies in the generation of the coordinates. Where the coordinates for the instance sets used in Chapter 5 were randomly drawn in a 10 x 10 coordinate matrix, the coordinates for the instances of the current set are randomly drawn in a 250 x 250 matrix. The reason for the increase in size of the coordinate matrix is that in this analysis, the distance units represent kilometers and a 10 x 10 matrix would not lead to realistic route lengths. The position of the depot is fixed to (125, 125). Distances are computed by taking the Euclidean distance between the coordinates of each node pair. The problem classes that are defined in Chapter 5 for the generation of the number of pallets and the weight of the pallets are presented in Table 6.4. Preliminary analysis points out that when only heavy pallets (1000 - 1500 kg) are considered, a feasible loading plan can not be achieved in a 45-foot truck. When more than 11 pallets of 1000 kg are packed dense in the truck, the weight on the coupling exceeds the limit of 10.75 tonnes. For this reason, problem classes 1 and 2 in which only heavy pallets are considered, are not included in this instance set. Problem classes 3 and 4 in which a fifty-fifty percent mix between customer demands with light pallets (100 - 500 kg) and customer demands with heavy pallets are considered, are included in the analysis. The number of pallets has a low variation (between 4 and 7 pallets per customer) in problem class 3 and a high variation (between 1 and 15 pallets per customer) in problem class 4. The number of pallets and total weight for each customer are generated randomly in the above mentioned intervals, depending on the problem class. For each network size, eight instances are created in each problem class, leading to in total 48 test instances.

The experiments are run on a Xeon E5-2680v3 CPU at 2.5 GHz with 64 GB of RAM. The parameters of the ILS algorithm are set to their tuned values, as described in Section 4.4. The value of α, the number of consecutive non-improving iterations

Table 6.4: Problem classes based on variation in number of pallets and pallet weight

	Heavy pallets	Mix between light and heavy pallets
Low variation	Problem class 1	Problem class 3
High variation	Problem class 2	Problem class 4

of the ILS is set to 300 as a result of the sensitivity analysis described in Section 4.5. Because of the stochastic character of the ILS algorithm, ten independent runs of the algorithm are performed. The average results for each instance are reported in Section 6.5.

6.5 Experimental results

In this section, the effect of a heterogeneous vehicle fleet on the integration of axle weight constraints in a CVRP is analyzed. The Fleet Size and Mix CVRP with sequence-based pallet loading and axle weight constraints with a 30-foot and a 45-foot truck is compared to a homogeneous vehicle fleet CVRP with 30-foot trucks and a homogeneous fleet CVRP with 45-foot trucks. Section 6.5.1 describes results for the three fleet compositions with the objective to minimize total distance (scenario 1). The computation times of the ILS may be found in Appendix C. In Section 6.5.2 results with the objective to minimize total transport costs are described (scenario 2). The computation times of the ILS for this scenario are presented in Appendix D.

6.5.1 Results for distance minimization

In this section, the results of the first scenario are discussed. Table 6.5 gives an overview of the percentage of routes in the final solutions of the FSM with and without axle weight constraints under the first scenario that may be performed by 30-foot trucks, 45-foot trucks and by both trucks. Detailed results of the number of vehicles of each vehicle type in the final solutions for each instance may be found in Appendix C. The majority of the routes can only be performed by 45-foot trucks, although there is a difference between the FSM with and without axle weight constraints. In the model without axle weight constraints 93.43 % of the routes in the final solution can only be performed by 45-foot trucks and only 0.34 % can only be performed by 30-foot trucks. The remaining 6.23 % may be performed by both trucks. This indicates that in the largest part of the routes more than 22 pallets (the capacity

Table 6.5: Routes performed by 30-foot trucks and 45-foot trucks in the final solutions of the FSM in percentage - Scenario 1

	30 ft truck	45 ft truck	30 ft or 45 ft truck
FSM without axle weight constraints	0.34 %	93.43 %	6.23 %
FSM with axle weight constraints	14.44 %	85.10 %	0.46 %

of a 30-foot truck) are transported. The small percentage of routes that can only be performed by 30-foot trucks indicates that the total weight of the load that is transported on the routes is rarely higher than 29.35 tonnes (the weight capacity of a 45-foot truck). In the model with axle weight constraints the percentage of routes that can only be performed by 45-foot trucks is 85.10 %, which is lower than in the model without axle weight constraints. The number of routes that can only be performed by 30-foot trucks is 14.44 %, which is in turn considerably higher than in the model without axle weight constraints. The number of routes that may be performed by both trucks is reduced to 0.46 %. The explanation for the shift of 45-foot trucks to 30-foot trucks in the model with axle weight constraints is twofold. First, because the tare weight of a 45-foot truck is higher than the tare weight of a 30-foot truck, the maximum weight of the load that may be applied on the axles of a 45-foot truck is lower. Second, because the distance between the front of the truck and the coupling (parameter c in equations (3.6) and (3.7)) and between the coupling and the central axle of the semi-trailer (parameter d in equations (3.6) and (3.7)) is larger in 45-foot trucks, more weight will be placed on the coupling. While for 30-foot trucks already most violations occur on the coupling in the model without axle weight constraints, the possibility of a violation on the coupling will be even larger for 45-foot trucks.

Table 6.6 presents the relative decrease in total distance traveled in the solution of the CVRP with sequence-based pallet loading and axle weight constraints with a homogeneous vehicle fleet consisting of 45-foot trucks and with a heterogeneous vehicle fleet with 30-foot and 45-foot trucks compared to the distance traveled in the solution of the CVRP with a homogeneous fleet with 30-foot trucks. For each instance size (50, 75 and 100 customers), the decrease in total distance with respect to a 30-foot fleet is provided. For all instances, there is a decrease in distance when considering a 45-foot fleet instead of a 30-foot fleet. The average decrease is 19.15 %, 20.80 % and 21.14 % for the instances of size 50, 75 and 100, respectively. An explanation for the decrease is that the capacity of a 45-foot truck in terms of number of pallets is almost 50 % higher than the capacity of a 30-foot truck. As expected,

Table 6.6: Decrease of the total distance traveled in percentage with respect to the homogeneous fleet CVRP with 30-foot trucks - Scenario 1

	50 customers		75 customers		100 customers	
Instance	45-foot	FSM	45-foot	FSM	45-foot	FSM
Problem class 3						
1	18.84	20.05	18.75	19.79	21.07	20.02
2	19.23	20.40	20.28	20.59	22.30	23.41
3	18.17	18.85	19.59	19.64	21.59	21.21
4	19.81	19.47	19.20	19.49	22.62	22.19
5	18.91	20.05	21.75	20.26	22.33	21.19
6	17.10	18.54	20.80	21.26	20.42	21.29
7	17.96	18.19	19.97	20.62	22.05	21.93
8	20.23	20.66	19.87	20.88	21.04	20.28
Problem class 4						
1	20.53	20.73	19.46	20.58	18.24	20.33
2	17.92	18.34	18.46	22.26	21.88	22.58
3	21.28	21.79	21.49	22.22	20.52	20.31
4	18.84	19.31	20.42	22.75	19.99	19.99
5	21.33	21.40	22.13	23.52	21.62	22.93
6	18.28	19.73	26.60	26.93	21.98	22.93
7	21.24	21.65	23.00	24.13	20.13	21.51
8	16.81	19.29	21.02	21.82	20.48	22.09
Average	19.15	19.90	20.80	21.67	21.14	21.51

the distance is lowest for the FSM although the difference with a homogeneous fleet of 45-foot trucks is quite small. This may be explained by the fact that on average 85.56 % of the routes in the solution of the FSM may be performed by 45-foot trucks.

Table 6.7 presents the number of routes in the solution from the CVRP with sequence-based pallet loading and axle weight constraints for a vehicle fleet consisting of 30-foot trucks, a fleet consisting of 45-foot trucks and a heterogeneous vehicle fleet with 30-foot and 45-foot trucks. For all instance sizes, the number of routes is lowest when only 45-foot trucks are considered. This implies that although the weight capacity of a 45-foot truck is smaller than the weight capacity of a 30-foot truck, in the given instances more customers may be visited with a 45-foot truck because of its

Table 6.7: Number of routes in the solution - Scenario 1

	50 customers			75 customers			100 customers		
Instance	30-foot	45-foot	mixed	30-foot	45-foot	mixed	30-foot	45-foot	mixed
Problem class 3									
1	13.0	9.0	9.5	20.0	14.1	14.5	27.6	19.3	19.8
2	13.0	9.3	9.3	19.0	13.5	13.9	26.9	19.4	19.4
3	13.0	9.0	9.6	20.1	14.3	14.4	27.3	19.3	19.9
4	14.0	9.8	9.8	19.9	14.0	14.0	28.0	19.3	20.1
5	14.0	9.2	10.0	20.1	14.4	14.9	26.7	18.9	19.4
6	14.0	10.0	10.6	20.1	14.7	14.9	26.6	18.9	19.1
7	13.9	10.0	10.0	20.0	14.2	14.4	26.9	18.8	18.9
8	13.4	9.1	10.0	19.8	14.0	14.1	27.2	19.2	20.2
Problem class 4									
1	21.0	14.5	15.0	27.8	20.1	20.9	39.4	28.6	29.3
2	19.9	13.7	14.1	30.1	20.2	21.0	37.4	26.5	27.8
3	21.0	14.2	15.2	31.8	22.2	22.6	40.1	28.6	29.2
4	17.0	12.0	12.1	32.0	22.3	22.8	37.0	26.3	26.7
5	17.6	12.0	13.0	34.2	24.2	25.2	40.8	29.6	30.1
6	21.0	15.0	16.0	30.8	22.0	23.1	46.3	32.3	33.7
7	20.6	14.0	14.4	29.7	20.4	21.2	41.3	29.6	31.1
8	21.0	15.0	16.4	30.9	22.0	22.4	40.7	29.0	29.6
Average	16.7	11.6	12.2	25.4	17.9	18.4	33.8	24.0	24.6

larger capacity in terms of number of pallets. Because mostly 45-foot trucks are used in the FSM solutions, there is only a small difference in number of routes between the fleet with 45-foot trucks and the heterogeneous fleet consisting of 45-foot trucks and 30-foot trucks.

Table 6.8 presents the increase in distance traveled of the CVRP with axle weight constraints and sequence-based pallet loading compared to the equivalent CVRP without axle weight constraints. For each instance size (50, 75 and 100 customers), the increase in distance is provided for the model with a homogeneous fleet with 30-foot trucks, a homogeneous fleet with 45-foot trucks and a heterogeneous fleet with 30-foot and 45-foot trucks. Results show that the effect of axle weight constraints on total distance traveled is highest when a homogeneous vehicle fleet of 45-foot trucks is considered with an average increase in distance of 6.72 %. The increase for a homo-

Table 6.8: Increase in total distance in percentage due to the integration of axle weight constraints - Scenario 1

	50 customers			75 customers			100 customers		
Instance	30-foot	45-foot	FSM	30-foot	45-foot	FSM	30-foot	45-foot	FSM
Problem class 3									
1	1.06	3.34	1.61	1.12	6.08	4.39	2.34	6.32	7.44
2	0.23	5.38	3.86	2.83	5.46	4.79	3.07	6.10	4.10
3	0.37	4.80	3.86	1.79	5.09	4.45	2.13	5.74	5.71
4	1.67	3.55	4.08	2.18	4.82	4.32	2.84	6.26	6.34
5	0.66	4.63	3.18	1.53	3.82	5.45	2.69	6.03	7.08
6	1.63	5.09	3.17	3.02	6.95	5.91	2.27	7.74	6.07
7	0.46	3.21	2.88	2.18	3.94	2.92	2.72	6.39	5.63
8	0.31	2.73	1.95	2.36	4.67	3.28	3.66	5.63	6.13
Problem class 4									
1	1.76	3.20	2.76	2.69	7.24	5.66	2.21	9.88	6.73
2	4.93	7.12	7.41	3.46	9.22	3.76	1.93	8.84	7.24
3	1.99	6.31	5.63	2.38	8.79	7.79	4.35	8.19	8.21
4	1.43	4.70	4.14	2.78	10.64	7.23	4.15	9.69	8.76
5	2.85	8.22	8.13	2.10	9.64	7.47	5.54	8.89	6.41
6	1.37	9.12	7.37	2.73	6.91	6.14	3.80	9.75	7.72
7	1.95	6.56	5.33	3.04	7.16	5.26	2.25	9.74	7.41
8	0.92	11.65	8.18	2.94	6.72	5.27	2.94	10.63	8.08
Average	1.48	5.60	4.60	2.45	6.70	5.26	3.06	7.86	6.82

geneous fleet of 30-foot trucks is considerably lower with an average of 2.33 %. The larger effect on 45-foot trucks is due to the fact that axle weight violations are more likely to occur on 45-foot trucks due to a higher tare weight and because more weight is applied on the coupling, as discussed above. As may be expected, the increase in distance due to the integration of axle weight constraints in the FSM is lower than when only 45-foot trucks are considered, but considerably higher than when only 30-foot trucks are included with an average of 5.56 %. It may therefore be concluded that the vehicle fleet strongly influences the impact of the integration of axle weight constraints on the objective value when the objective is to minimize total distance. When the fleet consists of 45-foot trucks, it is more important to consider axle weight constraints during route scheduling, than when only 30-foot trucks are considered.

Table 6.9: Routes performed by 30-foot trucks and 45-foot trucks in the final solutions of the FSM in percentage - Scenario 2

	30 ft truck	45 ft truck
FSM without axle weight constraints	16.13 %	83.87 %
FSM with axle weight constraints	16.69 %	83.31 %

6.5.2 Results for total transport cost minimization

In this section, the results of the second scenario are discussed. The objective function is the minimization of total transport costs, as presented in equation 6.1. Table 6.9 presents the percentage of routes in the final solutions of the FSM that are performed by 30-foot trucks and by 45-foot trucks with and without axle weight constraints. Detailed results of the number of vehicles of each vehicle type in the final solutions for each instance may be found in Appendix D. For the model without axle weight constraints on average 83.87 % of the routes in the final solutions are performed by 45-foot trucks, while the remaining 16.13 % are performed by 30-foot trucks. If a route leads to a feasible packing plan for both truck types, the route is assigned to a 30-foot truck because the tare weight of a 30-foot truck is lower than the tare weight of a 30-foot truck and weight is a determining factor of fuel costs. For this reason, the percentage of 45-foot trucks in the problem without axle weight constraints is lower than in the first scenario in which fuel costs are not considered. The average fleet composition of the problem with axle weight constraints is almost identical to the fleet composition of the problem without axle weight constraints with on average 83.31 % of the routes that are performed by 45-foot trucks and 16.69 % of the routes that are performed by 30-foot trucks. This indicates that for the given instance set, axle weight constraints may be integrated without an impact on the ideal composition of the vehicle fleet in the second scenario.

Table 6.10 presents the decrease in solution cost in percentage from the CVRP with sequence-based pallet loading and axle weight constraints with a homogeneous vehicle fleet consisting of 45-foot trucks and with a heterogeneous vehicle fleet with 30-foot and 45-foot trucks compared to the CVRP with a homogeneous fleet with 30-foot trucks. For each instance size (50, 75 and 100 customers), the cost decrease with respect to a 30-foot fleet is shown. For all instances, there is a cost decrease when considering a 45-foot fleet compared to a 30-foot fleet with an average decrease of 17.76

%, 20.07 % and 20.42 % for the instances of size 50, 75 and 100 respectively. Note that the decreases in objective function value are smaller than in the first scenario. The reason for this may be that in the current scenario, weight is a determining factor of fuel consumption and therefore has an impact on solution cost. Since the tare weight of a 30-foot truck is smaller than the tare weight of a 45-foot truck and the payload of 45-foot trucks is on average larger, the cost advantage of 45-foot trucks in comparison to 30-foot trucks is smaller in this scenario. As in the first scenario, the objective value is lowest for the FSM with 30-foot and 45-foot trucks although the difference with a homogeneous fleet of 45-foot trucks is still quite small. The average decrease in cost of the FSM compared to the CVRP with 30-foot trucks is 18.83 %, 20.75 % and 20.68 % for the instances of size 50, 75 and 100 respectively. These are smaller than the decrease in objective value in the first scenario, which may be explained by the fact that the majority of the trucks in the FSM solutions are 45-foot trucks which have on average a higher fuel consumption than 30-foot trucks.

The number of routes in the solution of the CVRP with sequence-based pallet loading and axle weight constraints for a vehicle fleet consisting of 30-foot trucks, a fleet consisting of 45-foot trucks and a heterogeneous vehicle fleet with 30-foot and 45-foot trucks are presented in Table 6.11. The results strongly resemble the results in Table 6.7 which indicates that the number of routes in the solutions are not affected by the different objective function. This is also reflected in the fleet composition since the percentage of 45-foot and 30-foot trucks in both scenarios are very similar for the problem with axle weight constraints.

Table 6.12 presents the cost increase of the CVRP with axle weight constraints and sequence-based pallet loading compared to the equivalent CVRP without axle weight constraints. For each instance size (50, 75 and 100 customers), the cost increase is provided for the model with a homogeneous fleet with 30-foot trucks, a homogeneous fleet with 45-foot trucks and a heterogeneous fleet with 30-foot and 45-foot trucks. As in the first scenario, the increase in objective value of the integration of axle weight constraints is highest when a homogeneous vehicle fleet of 45-foot trucks is considered with an average cost increase of 2.99 %. The cost increase of the integration of axle weight constraints is on average 2.28 % for the FSM and 0.54 % when a homogeneous fleet of 30-foot trucks is considered. Note that for all fleet compositions the increase in objective value is much lower than in the first scenario. An explanation for this is that because fuel consumption is considered in the objective function, there will be a tendency to visit customers with heavy pallets early on the route. Therefore heavy pallets are placed towards the rear of the vehicle. Since the weight capacity of the axles of the semi-trailer is larger than the weight capacity of the axles of the

Table 6.10: Decrease in solution cost in percentage with respect to the homogeneous fleet CVRP with 30-foot trucks - Scenario 2

	50 customers		75 customers		100 customers	
Instance	45-foot	FSM	45-foot	FSM	45-foot	FSM
Problem class 3						
1	18.68	17.87	18.69	18.91	20.79	20.23
2	18.84	19.55	17.39	17.55	20.56	20.10
3	16.05	17.58	18.92	19.20	20.41	20.11
4	18.31	18.45	17.50	18.29	21.59	21.33
5	17.68	18.43	20.37	19.79	20.65	20.20
6	16.07	17.64	19.33	19.53	19.38	18.18
7	15.98	17.00	19.60	19.65	21.00	20.01
8	19.19	19.43	19.03	19.05	19.26	19.31
Problem class 4						
1	18.55	19.53	20.43	19.87	17.97	19.19
2	15.52	17.21	17.97	21.40	22.07	22.12
3	19.45	20.91	20.97	21.55	20.23	20.80
4	16.96	18.01	20.20	22.01	18.62	19.72
5	20.53	21.82	21.03	22.54	21.49	22.33
6	18.22	19.29	27.06	27.27	20.78	23.79
7	19.02	19.91	22.25	23.83	20.55	21.47
8	15.15	18.66	20.31	21.55	21.34	21.97
Average	17.76	18.83	20.07	20.75	20.42	20.68

Table 6.11: Number of routes in the solution - Scenario 2

	50 customers			75 customers			100 customers		
Instance	30-foot	45-foot	mixed	30-foot	45-foot	mixed	30-foot	45-foot	mixed
Problem class 3									
1	13.0	9.1	9.7	19.9	14.2	14.5	27.9	19.5	20.2
2	13.1	9.1	9.6	19.0	13.7	14.0	26.8	19.8	19.6
3	13.1	9.8	9.6	20.1	14.3	14.7	27.7	19.2	20.4
4	14.0	9.7	10.0	19.9	14.0	14.0	28.2	19.5	20.0
5	14.2	9.9	10.0	20.2	14.4	14.9	26.8	18.9	19.3
6	14.1	10.2	10.5	20.1	14.7	14.9	26.4	19.1	19.1
7	13.9	10.0	10.0	20.0	14.1	14.3	26.4	19.0	18.8
8	14.0	10.0	10.0	19.9	14.1	14.3	27.0	19.5	20.3
Problem class 4									
1	21.0	15.1	15.0	28.0	20.3	21.3	39.2	29.0	29.7
2	20.0	14.0	14.4	30.0	20.3	21.0	37.2	26.6	28.1
3	21.0	14.8	15.6	32.3	22.6	23.0	39.9	28.6	29.0
4	17.0	12.0	12.3	32.0	22.0	23.5	37.0	26.4	26.8
5	17.9	12.6	13.0	34.7	23.9	25.0	41.2	29.5	30.2
6	21.8	15.9	16.0	31.1	22.0	23.2	46.2	32.5	34.1
7	20.5	14.5	14.8	29.9	20.4	21.2	41.5	29.4	31.3
8	21.2	15.8	16.5	31.0	22.1	22.5	40.7	28.9	29.8
Average	16.9	12.0	12.3	25.5	17.9	18.5	33.8	24.1	24.8

Table 6.12: Cost increase in percentage of the integration of axle weight constraints - Scenario 2

	50 customers			75 customers			100 customers		
Instance	30-foot	45-foot	FSM	30-foot	45-foot	FSM	30-foot	45-foot	FSM
Problem class 3									
1	-0.17	-0.63	0.69	0.42	2.44	2.49	0.12	1.68	1.52
2	-0.04	3.39	2.06	-0.31	1.77	1.93	-0.27	2.85	3.51
3	-0.57	3.33	1.58	0.07	2.05	1.85	-1.84	1.78	2.09
4	0.39	2.17	2.60	0.53	2.04	0.88	-0.93	0.49	1.75
5	-0.26	2.06	1.81	-1.35	-0.60	-0.12	-0.17	1.46	2.08
6	0.02	3.03	1.32	-0.07	2.05	1.22	-1.14	2.28	4.07
7	0.11	2.86	1.99	1.28	0.97	0.97	-0.61	1.25	3.56
8	-0.18	0.39	0.54	0.25	1.49	0.59	0.46	1.34	1.70
Problem class 4									
1	1.33	2.43	1.52	0.40	1.88	1.99	-0.29	3.69	2.21
2	4.44	6.79	5.67	0.99	4.56	-0.04	0.25	2.50	2.68
3	1.06	6.63	4.39	1.76	5.38	4.84	1.01	2.18	1.47
4	0.49	4.30	3.39	1.19	6.06	3.59	-0.11	3.49	1.77
5	1.04	5.20	3.02	1.65	6.08	3.99	3.35	2.52	1.87
6	0.98	6.18	4.83	3.33	2.75	3.46	1.85	4.49	1.17
7	-0.06	3.19	2.64	1.13	2.18	1.09	0.31	3.09	2.56
8	0.54	9.54	5.43	2.62	4.12	1.90	0.80	2.01	1.11
Average	0.57	3.81	2.72	0.87	2.83	1.91	0.17	2.32	2.20

tractor, the number of violations and the extent of the violation is therefore smaller. Consequently, the difference in solution cost between the models with and without axle weight constraints is also smaller. Note that for some instances an average cost decrease is reported. This indicates that for these instances, the solution of the heuristic solution method for the problem without axle weight constraints deviates from the optimal solution. Since the ILS has proven to be an effective solution method for the CVRP with sequence-based pallet loading and axle weight constraints with small optimality gaps in Chapter 5, it does not affect the conclusions of this research.

6.6 Conclusions and future research

In real-life applications, the vehicle fleet of a transportation company is generally not homogeneous but consists of several types of vehicles. Vehicles in the fleet may differ in terms of capacity, costs and other factors such as speed and acceleration. This chapter considers the integration of a heterogeneous fleet in the CVRP with sequence-based pallet loading and axle weight constraints. Since an unlimited number of vehicles in the heterogeneous fleet is considered, the resulting problem is defined as the Fleet Size and Mix CVRP with sequence-based pallet loading and axle weight constraints.

From a scientific point of view, it is the first time that axle weight constraints are considered in the Fleet Size and Mix CVRP. Furthermore, it is the first time that the effect of axle weight constraints for different fleet compositions is compared. A slightly modified version of the ILS, developed in Chapter 4 for the CVRP with sequence-based pallet loading and axle weight constraints, is used to solve the problem. To measure the impact of the vehicle fleet on the integration of axle weight constraints in a VRP, a heterogeneous fleet with 30-foot and 45-foot trucks is compared to a homogeneous fleet with 30-foot trucks and a homogeneous fleet with 45-foot trucks. Furthermore, two scenarios are analyzed for which the objective function differs: in the first scenario the objective is to minimize total distance while in the second scenario the objective is the minimization of total transport costs. Instances with only heavy pallets (1000 - 1500 kg) can not be feasibly solved when a homogeneous fleet of 45-foot trucks is considered. Therefore, only instances with a fifty-fifty percent mix between heavy and light pallets (100 - 500 kg) are used for the analysis this chapter. The experimental results indicate that the effect of axle weight constraints on the objective value is highest when a homogeneous fleet of 45-foot trucks is considered. The reason for this is that axle weight violations are more likely to occur in 45-foot trucks since the capacity in terms of number of pallets is higher while the maximum weight of the load on the axles is lower. Furthermore, the results indicate that although the ideal composition of the fleet in the FSM mainly consists of 45-foot trucks in our test instances, the effect of the integration of axle weight constraints on the objective value is smaller than when a homogeneous fleet of 45-foot trucks is considered. As expected, the overall objective value is lowest for the FSM with 30-foot and 45-foot trucks. Based on these results, it may be concluded that the type of vehicle as well as the combination of vehicle types in the fleet is of importance when calculating the impact of axle weight constraints on the objective function value. Besides, it may be interesting to look at total transport costs instead of only distance in the objective function since the impact of the integration of axle weight constraints on the objective value is different for both objective functions. When total transport costs are considered, the increase in objective value due to the integration of axle weight constraints is considerably smaller than when the objective is to minimize distance traveled in our test instances.

From a managerial point of view, it may be useful to consider axle weight constraints in long-term planning decisions concerning the acquisition of new vehicles. This may lead to a more diverse vehicle fleet with, for instance, vehicles with a smaller capacity in terms of number of pallets and a similar capacity in terms of total weight (e.g. 30-foot trucks instead of 45-foot trucks). Furthermore, the results indicate that when the vehicle fleet of a company consists of 45-foot trucks, it is even more important to consider axle weight constraints during route scheduling then when smaller vehicles are considered because the impact of axle weight constraints on the route scheduling is larger.

Future research could introduce the Heterogeneous VRP with sequence-based pallet loading and axle weight constraints in which a limited vehicle fleet is assumed. Another line of future research would be to analyze the impact of axle weight constraints when other vehicle types are considered such as medium-duty trucks with different axle configurations. Furthermore, the effect of axle weight constraints on the FSM with sequence-based pallet loading and axle weight constraints with a fleet consisting of medium-duty and heavy-duty trucks may be analyzed. For this problem, fuel consumption models that do not only consider weight but also vehicle specific parameters such as average speed and acceleration may be used.

Chapter 7

Conclusions and further research

This thesis has studied the integration of loading constraints in vehicle routing models. Special attention was paid to the consideration of axle weight limits in route scheduling. Chapter 2 discusses the state-of-the-art of vehicle routing problems with loading constraints and identifies research gaps. In Chapters 3 to 6, the Capacitated Vehicle Routing Problem (CVRP) with sequence-based pallet loading and axle weight constraints is analyzed. Chapter 3 presents the calculation of the axle weights and two problem formulations. In Chapter 4, an Iterated Local Search (ILS) method is developed to solve the problem heuristically. Computational experiments of the problem formulations and the ILS are presented in Chapter 5. Chapter 6 considers the integration of a heterogeneous vehicle fleet in the CVRP with sequence-based pallet loading and axle weight constraints. Finally, in this chapter, general conclusions are formulated and further research opportunities are discussed (Figure 7.1).

7.1 Final conclusions

Since its introduction almost 60 years ago, the vehicle routing problem is a well-studied research topic in the Operational Research community. Most studies however do not reflect the real problems faced by transportation companies. The classic capacitated vehicle routing problem consists of the delivery of items from a depot to a set of geographically scattered customers with a homogeneous vehicle fleet while minimizing total distance traveled. It does not consider several real-life characteristics of route

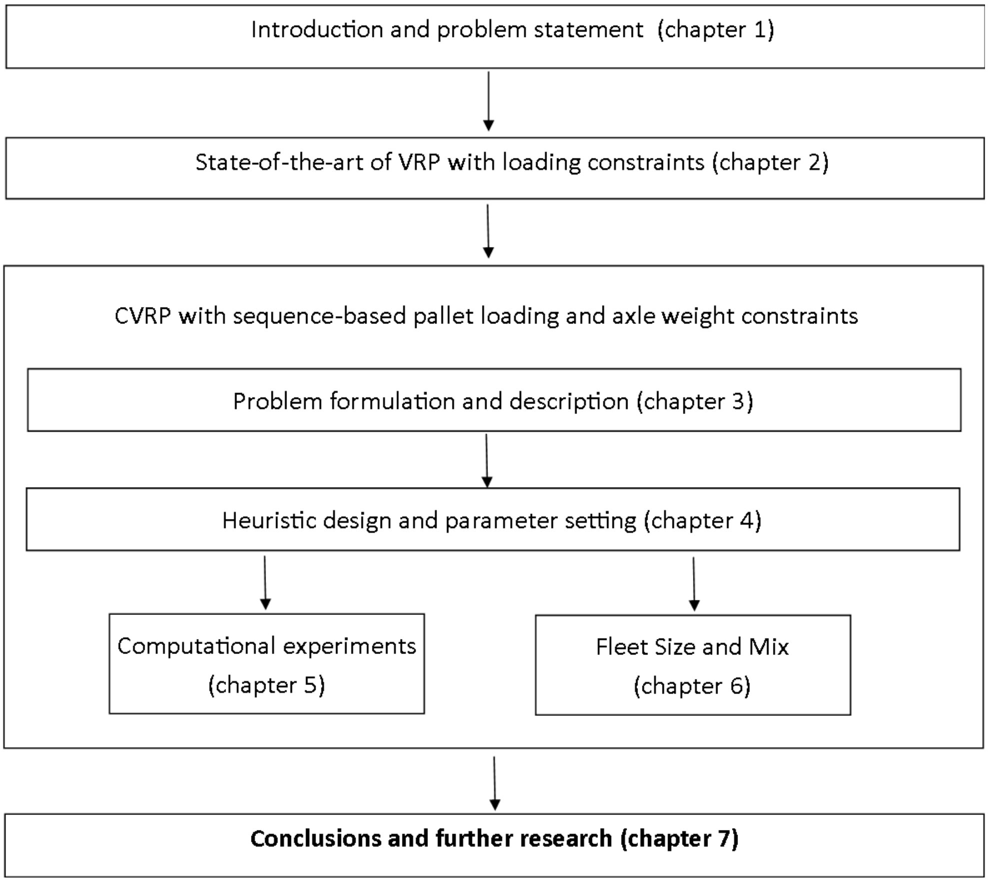

Figure 7.1: Outline of the thesis

scheduling such as loading constraints, a heterogeneous vehicle fleet, legal driving hours and time-dependent travel times. Therefore, in recent years, there is a scientific shift towards *rich* vehicle routing problems which incorporate some of these real-life constraints. In this thesis, the integration of axle weight limits and a heterogeneous vehicle fleet in vehicle routing problems is studied.

Recently, a number of papers have addressed the integration of loading constraints in vehicle routing problems. In the first part of the thesis, the existing literature is reviewed. Based on the type of routing problem and loading characteristics, the existing literature is divided in the following categories: Two-Dimensional Loading CVRP, Three-Dimensional Loading CVRP, multi-pile VRP, multi-compartments VRP, Pallet Packing VRP, Minimum Multiple Trip VRP with incompatible commodities, Traveling Salesman Problem with Pickups and Deliveries with LIFO/FIFO constraints, Double TSP with Pickups and Deliveries with Multiple Stacks and Vehicle Routing Problem with Pickups and Deliveries with additional loading constraints. The latter three categories consider pickup and delivery problems in which items may be picked up and delivered at customer places. From the literature review it is observed that only few problem formulations are developed for VRPs with loading constraints, which may be explained by the fact that the integration of loading constraints in a routing problem greatly increases the complexity of the formulation. Consequently, mainly heuristic methods are developed to tackle these problems. It is also observed that the complexity of the problem is influenced by the combination of routing and loading constraints. When sequence-based loading is added to a three-dimensional loading problem, the problem becomes more complex than when it is considered in a one-dimensional loading problem. In addition, the type of transportation request (pickup and delivery of items, or only a single type of request) influences the complexity of the loading constraints. Furthermore, the review points out that in most models, loading constraints are handled as a subproblem of the routing model. First, solutions of the routing problem are computed, and afterwards, a loading feasibility check is performed for the best solutions.

The second part of the thesis focuses on the integration of axle weight constraints in a VRP. Axle weight limits impose a real challenge for transportation companies because they are faced with high fines in the event of non-compliance. Furthermore, violations are a threat to traffic safety and may cause serious damage to the road surface. Current commercial route scheduling programs do not incorporate these limits which leads to violations or last-minute changes. Despite of the practical relevance of axle weight constraints, this has not yet been studied in combination with VRP. In this thesis, the CVRP with sequence-based pallet loading and axle weight constraints is introduced. The distribution of pallets and sequence-based loading is considered, which is a problem setting often encountered in real-life problems. Sequence-based loading ensures that when arriving at a customer, no pallets of customers to be served later on the route block the removal of the pallets of the current customer. Furthermore, pallets are packed dense inside the truck. This means that the pallets of the last customer are placed at the deepest portion of the loading area and that there is no gap between two consecutive pallets inside the truck. This pallet configuration increases the stability of the load and is therefore often used in practice.

Two problem formulations for the CVRP with sequence-based pallet loading and axle weight constraints are tested on small-size instance sets. Additionally, an Iterated Local Search (ILS) algorithm is developed to solve the problem heuristically for realistic-size instances with networks consisting of up to 100 customers.

The analysis shows that the integration of axle weight constraints does not lead to a large increase in distance traveled, while not including axle weight constraints may induce major axle weight violations. In several instances, axle weight violations can even be avoided without a cost increase. Furthermore, the results indicate that the effect of including axle weight constraints may depend on demand characteristics. When only light pallets (100 - 500 kg) are packed, axle weight limits do not play a role in the loading process. The effect of integrating axle weight limits on routing costs is higher when only heavy pallets (1000 - 1500 kg) are considered compared to a fifty-fifty percent mix of heavy and light pallets. This may be explained by the fact that the number of violations and the magnitude of the violation when axle weight constraints are not included in the route scheduling are larger when only heavy pallets are considered. Heavy pallets are therefore better transported together with light pallets even though the total weight capacity of the vehicle is sufficient to transport only heavy pallets. Furthermore, the number of violations and the extent of the violations are larger when a high variation (between 1 and 15 pallets) in number of pallets per customer is considered compared to a small variation (between 4 and 7 pallets). The reason for this may be that a high variation in number of pallets leads to on average half of the orders consisting of more than 8 pallets, which is less flexible than orders between 4 and 7 pallets.

The vehicle fleet of transportation companies generally consists of different types of vehicles. Therefore, in the final part of the thesis, the Fleet Size and Mix CVRP with sequence-based pallet loading and axle weight constraints is introduced. Two types of heavy-duty vehicles are considered: a 30-foot truck and a 45-foot truck. The trucks differ in tare weight and measurements. The capacity in terms of number of pallets and total weight is therefore different as well as the maximum axle loads. Two scenarios are defined for which the objective function differs. In the first scenario the objective is to minimize total distance. In the second scenario, the objective is the minimization of total transport costs. Total transport costs include interest and depreciation costs, taxes, insurance, driver wage, maintenance and fuel costs. Fuel consumption depends on the gross weight of the truck. Since the tare weight of the vehicles in the fleet are different, fuel costs are (partially) vehicle dependent. The analysis points out that the impact of axle weight constraints on the objective value in our test instances is smaller when the objective is to minimize total transport costs compared to the minimization of total distance traveled, which is the traditional objective in the VRP literature.

The impact of the vehicle fleet on the integration of axle weight constraints in a VRP is analyzed by comparing three heavy-duty vehicle fleet compositions: a homogeneous fleet of 30-foot trucks, a homogeneous fleet of 45-foot trucks and a heterogeneous fleet with 30-foot and 45-foot trucks. The cost increase for integrating axle weight constraints is considerably higher when a homogeneous fleet of 45-foot trucks is considered compared to a homogeneous fleet of 30-foot trucks. This may be explained by the fact that the capacity in terms of number of pallets is higher for 45-foot trucks while the maximum weight of the load on the axles is lower which increases the probability of an axle weight violation. It may be concluded that the vehicle fleet strongly influences the impact of the integration of axle weight constraints on the solution costs.

7.2 Managerial implications

When axle weight constraints are ignored during the planning process, transporters face high fines due to axle weight violations or costs due to last minute changes in planning. The findings of this thesis may encourage transportation companies to consider axle weight constraints during their route scheduling. The analysis points out that ignoring axle weight constraints during route scheduling may lead to considerable axle weight violations, while including axle weight constraints does not lead to a large cost increase in our test instances.

Furthermore, since the ideal composition of the vehicle fleet may change when axle weight constraints are integrated in route scheduling, it may be useful for transportation companies to consider axle weight constraints already when taking purchasing decisions. In the FSM, an unlimited vehicle fleet is considered for all vehicle types which may be helpful to determine the optimal fleet size and mix and may therefore support long-term planning decisions of transportation companies with regards to the acquisition of new vehicles. This may enable them to analyze the impact of an investment in trucks with varying capacities and tare weight on solution cost. For instance, it may be used to assess the benefits of an investment in a truck that is specifically designed to have a low weight in order to maximize the net weight capacity.

Finally, transportation companies may be encouraged to consider all relevant costs and not only total distance traveled if they want to evaluate the impact of the integration of axle weight constraints in their own route scheduling. The analysis points out that the impact of axle weight constraints is larger when the objective is to minimize total distance traveled, which is the traditional objective in the VRP literature, compared to the more realistic objective of the minimization of total transport costs. The reason for this is that because total transport cost includes fuel consumption there will be a tendency to visit customers with heavy pallets early on the route. Therefore heavy pallets are placed towards the rear of the vehicle. Since the weight capacity of the axles of the semi-trailer is larger than the weight capacity of the axles of the tractor, the number of violations and the extent of the violation is therefore already smaller in the model without axle weight constraints.

7.3 Further research

The combination of vehicle routing problems with *loading constraints* is a fairly recent domain of research. Therefore a number of opportunities for future research may be identified. An interesting future research direction would be to focus on the pickup and delivery problem with multiple vehicles and multiple dimensions since currently little research has been done on this topic. Future research may also consider contamination issues in the multi-compartments VRP by focussing on scheduling over multiple periods or over multiple trips in a single tour. Finally, the literature review shows that current VRP models with loading constraints rarely incorporate other *rich* constraints. The inclusion of *rich* characteristics such as the use of a heterogeneous fleet, time-dependent routing or drivers' regulations in current VRP models with loading constraints would go some considerable way towards making these more realistic.

A number of research opportunities exist on the development of solution methods for vehicle routing problems with *axle weight constraints*. The exact methods that are developed to solve the problem are only able to solve small-size instances within a reasonable time limit. A promising research direction is to use column generation to solve the Set Partitioning model. Column generation has proven to work very well on linear models with a large number of variables and a small number of constraints, which is the case for the SP formulation. Additionally, the MILP formulation could be strengthened in a branch-and-cut algorithm. Furthermore, future research may look into the development of matheuristics to solve the routing problem heuristically and the packing subproblem exactly. Finally, it would be interesting to compare the performance of the ILS metaheuristic to the performance a simplified heuristic with less local search operators.

Another line of future research could focus on extending the current model. In practice, transportation companies are often faced with time windows (hard or soft) within which a delivery must take place. Furthermore, they need to comply to legislation concerning legal driving hours and working time of drivers. These features could be considered in the current model in order to make it more realistic. Following this, the impact of axle weight constraints may be analyzed on real-life scheduling data. Furthermore, the impact of axle weight constraints may be analyzed when the delivery of non-palletized goods is considered. In this case, depending on the possibility of vertical stacking, a two-dimensional or three-dimensional bin packing problem needs to be solved. Stability constraints will also be an important challenge since dense packing will not always be possible when considering goods of different sizes. Cargo-securing measures should therefore be considered to prevent the load from moving inside the truck.

In this thesis, the Fleet Size and Mix CVRP with sequence-based pallet loading and axle weight constraints is analyzed with a heterogeneous fleet consisting of heavy-duty vehicles. Future research could analyze the impact of axle weight constraints when other vehicle types are considered such as medium-duty trucks with different axle configurations. Additionally, the analysis could include a fleet consisting of both medium-duty and heavy-duty vehicles. In this case, the vehicles in the fleet will differ in terms of average speed and acceleration, which may lead to the use of different fuel consumption models that take into account these vehicle specific parameters. Finally, future research could focus on the Heterogeneous VRP with sequence-based pallet loading and axle weight constraints in which a limited vehicle fleet is assumed. This model could analyze the effect of the integration of axle weight constraints on an operational scheduling level for day-to-day route scheduling.

Appendix A

Results insertion methods

This appendix shows the results of three insertion methods on instances with 10, 15, 20, 50, 75 and 100 customers. For each instance size, 32 instances are considered. The first insertion method is a regret-2 heuristic. In the second method, the BCSP formulated in Section 4.2.1 is solved for options consisting of a single non-difficult node ($\sum_{i\in\Phi} k^j_{ai} = 1\ \forall j \in \Omega, a \in \Psi_j$), followed by a regret-2 heuristic. The third method solves the BCSP formulated in Section 4.2.1 with options existing of a single node or combinations of two nodes ($\sum_{i\in\Phi} k^j_{ai} <= 2 \quad \forall j \in \Omega, a \in \Psi_j$) followed by a regret-2 heuristic. In case the insertion method leads to a feasible initial solution, this is reported by a cross (x).

Table A.1: Results of three insertion methods on networks of 10 customers

	Insertion method		
Instance	Regret-2	BCSP - 1 node	BCSP - 2 nodes
Problem class 1			
1	x	x	x
2	x	x	x
3	x	x	x
4	x	x	x
5	x	x	x
6	x	x	x
7	x	x	x
8	x	x	x
Problem class 2			
1	x	x	x
2		x	x
3	x	x	x
4	x	x	x
5		x	x
6	x	x	x
7		x	x
8	x	x	x
Problem class 3			
1	x	x	x
2	x	x	x
3	x	x	x
4	x	x	x
5	x	x	x
6	x	x	x
7	x	x	x
8	x	x	x
Problem class 4			
1	x	x	x
2	x	x	x
3	x	x	x
4	x	x	x
5	x	x	x
6	x	x	x
7		x	x
8		x	x
Total	27	32	32

Table A.2: Results of three insertion methods on networks of 15 customers

	Insertion method		
Instance	Regret-2	BCSP - 1 node	BCSP - 2 nodes
Problem class 1			
1	x	x	x
2	x	x	x
3	x	x	x
4	x	x	x
5	x	x	x
6	x	x	x
7	x	x	x
8	x	x	x
Problem class 2			
1		x	x
2		x	x
3			x
4	x	x	x
5			x
6			x
7	x	x	x
8	x	x	x
Problem class 3			
1	x	x	x
2	x	x	x
3	x	x	x
4	x	x	x
5	x	x	x
6	x	x	x
7	x	x	x
8	x	x	x
Problem class 4			
1		x	x
2		x	x
3	x	x	x
4	x	x	x
5	x	x	x
6		x	x
7	x	x	x
8	x	x	x
Total	24	29	32

Table A.3: Results of three insertion methods on networks of 20 customers

	Insertion method		
Instance	Regret-2	BCSP - 1 node	BCSP - 2 nodes
Problem class 1			
1	x	x	x
2	x	x	x
3	x	x	x
4	x	x	x
5	x	x	x
6	x	x	x
7	x	x	x
8	x	x	x
Problem class 2			
1	x	x	x
2	x	x	x
3			x
4			x
5		x	x
6		x	x
7		x	x
8	x	x	x
Problem class 3			
1	x	x	x
2	x	x	x
3	x	x	x
4	x	x	x
5	x	x	x
6	x	x	x
7	x	x	x
8	x	x	x
Problem class 4			
1	x	x	x
2	x	x	x
3	x	x	x
4		x	x
5	x	x	x
6		x	x
7	x	x	x
8		x	x
Total	24	30	32

Table A.4: Results of three insertion methods on networks of 50 customers

	Insertion method		
Instance	Regret-2	BCSP - 1 node	BCSP - 2 nodes
Problem class 1			
1	x	x	x
2	x	x	x
3	x	x	x
4	x	x	x
5	x	x	x
6	x	x	x
7	x	x	x
8	x	x	x
Problem class 2			
1		x	x
2		x	x
3		x	x
4		x	x
5		x	x
6		x	x
7		x	x
8		x	x
Problem class 3			
1	x	x	x
2	x	x	x
3	x	x	x
4	x	x	x
5	x	x	x
6	x	x	x
7	x	x	x
8	x	x	x
Problem class 4			
1	x	x	x
2	x	x	x
3		x	x
4		x	x
5		x	x
6	x	x	x
7		x	x
8		x	x
Total	19	32	32

Table A.5: Results of three insertion methods on networks of 75 customers

	Insertion method		
Instance	Regret-2	BCSP - 1 node	BCSP - 2 nodes
Problem class 1			
1	x	x	x
2	x	x	x
3	x	x	x
4	x	x	x
5	x	x	x
6	x	x	x
7	x	x	x
8	x	x	x
Problem class 2			
1		x	x
2	x	x	x
3		x	x
4	x	x	x
5		x	x
6		x	x
7		x	x
8		x	x
Problem class 3			
1	x	x	x
2	x	x	x
3	x	x	x
4	x	x	x
5	x	x	x
6	x	x	x
7	x	x	x
8	x	x	x
Problem class 4			
1		x	x
2		x	x
3		x	x
4		x	x
5		x	x
6	x	x	x
7		x	x
8	x	x	x
Total	20	32	32

Table A.6: Results of three insertion methods on networks of 100 customers

	Insertion method		
Instance	Regret-2	BCSP - 1 node	BCSP - 2 nodes
Problem class 1			
1	x	x	x
2	x	x	x
3	x	x	x
4	x	x	x
5	x	x	x
6	x	x	x
7	x	x	x
8	x	x	x
Problem class 2			
1		x	x
2		x	x
3		x	x
4	x	x	x
5		x	x
6		x	x
7		x	x
8		x	x
Problem class 3			
1	x	x	x
2	x	x	x
3	x	x	x
4	x	x	x
5	x	x	x
6	x	x	x
7	x	x	x
8	x	x	x
Problem class 4			
1		x	x
2		x	x
3		x	x
4	x	x	x
5		x	x
6		x	x
7		x	x
8		x	x
Total	18	32	32

Appendix B

Detailed results: ILS algorithm on small-size instances

This appendix shows the detailed results of the ILS algorithm on the CVRP with sequence-based pallet loading with and without axle weight constraint on small-size instances with 10, 15 and 20 customers. Because of the stochastic character of the algorithm, twenty independent runs of the algorithm are performed. The average solution cost (Z^{avg}) and best solution cost (Z^{best}) for each instance are reported. For the problem without axle weight constraints, the number of axle weight violations (# V) and maximum violation in percentage (Max V) are also shown. For the problem with axle weight constraints, the increase in average cost compared to the average cost in the problem without axle weight constraints ($Z^{avg} incr(\%)$) is reported, as well as the increase in best cost compared to the best cost in the problem without axle weight constraints ($Z^{best} incr$ (%)).

Table B.1: Results of the CVRP with sequence based pallet loading with and without axle weight constraints on networks of 10 customers

Instance	Model without axle weight					Model with axle weight				
	Z^{avg}	Z^{best}	t (s)	# V	Max V (%)	Z^{avg}	Z^{avg} incr (%)	Z^{best}	Z^{best} incr (%)	t (s)
Problem class 1										
1	38.4	38.4	1.0	4	11.7	45.6	18.75	45.2	17.71	1.0
2	38.5	38.5	1.0	2	4.0	38.5	0.00	38.5	0.00	0.5
3	39.3	39.3	1.0	5	5.6	40.2	2.29	40.2	2.29	0.5
4	41.9	41.9	1.0	4	11.9	45.7	9.07	45.7	9.07	0.5
5	51.7	51.7	1.0	2	9.9	51.7	0.00	51.7	0.00	0.5
6	43.4	43.4	1.0	2	4.0	44.2	1.84	44.2	1.84	0.5
7	45.2	45.2	1.0	2	2.1	45.2	0.00	45.2	0.00	0.5
8	44.0	44.0	0.5	2	11.1	44.6	1.36	44.3	0.68	0.5
Problem class 2										
1	41.2	41.2	1.0	5	13.9	46.5	12.86	44.3	7.52	0.5
2	44.7	44.7	0.5	4	11.7	51.3	14.77	51.3	14.77	0.5
3	56.3	56.3	1.0	2	3.6	58.2	3.37	56.8	0.89	0.5
4	50.3	50.3	0.5	5	7.1	50.7	0.80	50.7	0.80	0.5
5	49.9	49.9	1.0	6	15.7	53.9	8.02	53.8	7.82	0.5
6	49.5	49.5	0.5	3	13.8	53.3	7.68	53.3	7.68	0.5
7	64.6	64.6	0.5	2	6.2	68.2	5.57	68.2	5.57	0.5
8	40.5	40.5	1.0	3	9.4	40.5	0.00	40.5	0.00	0.5
Problem class 3										
1	37.4	37.4	1.0	0		37.4	0.00	37.4	0.00	0.5
2	37.4	37.4	0.5	3	7.0	38.3	2.41	38.3	2.41	0.5
3	41.0	41.0	0.5	0		41.0	0.00	41.0	0.00	0.5
4	43.4	43.4	1.0	0		43.4	0.00	43.4	0.00	1.0
5	38.8	38.8	1.0	2	9.5	40.8	5.15	40.8	5.15	1.0
6	41.3	41.3	1.0	1	4.4	41.3	0.00	41.3	0.00	0.5
7	44.4	44.4	1.0	2	2.1	44.4	0.00	44.4	0.00	1.0
8	46.5	46.5	1.0	0		46.5	0.00	46.5	0.00	1.0
Problem class 4										
1	57.3	57.3	0.5	3	8.7	57.3	0.00	57.3	0.00	0.5
2	47.3	47.3	0.5	5	16.9	49.3	4.23	49.3	4.23	0.5
3	46.9	46.9	1.0	0		46.9	0.00	46.9	0.00	0.5
4	53.3	53.3	0.5	0		53.3	0.00	53.3	0.00	0.5
5	44.7	44.7	0.5	0		44.7	0.00	44.7	0.00	0.5
6	50.2	50.2	1.0	2	3.6	52.2	3.98	52.2	3.98	0.5
7	57.2	57.2	0.5	3	9.2	59.9	4.72	59.9	4.72	0.5
8	50.1	50.1	0.5	0		50.1	0.00	50.1	0.00	0.5
Average			0.8	2.3			3.34		3.04	1.0

\# V = number of violations
Max V = maximum violation

Table B.2: Results of the CVRP with sequence based pallet loading with and without axle weight constraints on networks of 15 customers

Instance	Model without axle weight					Model with axle weight				
	Z^{avg}	Z^{best}	t (s)	# V	Max V (%)	Z^{avg}	Z^{avg} incr (%)	Z^{best}	Z^{best} incr (%)	t (s)
Problem class 1										
1	59.3	59.3	3.0	5	8.9	66.6	12.31	66.2	11.64	2.0
2	54.6	54.6	2.0	9	8.8	66.1	21.06	66.1	21.06	1.0
3	62.3	62.1	2.0	4	4.6	62.5	0.32	62.1	0.00	1.0
4	54.1	54.1	3.0	4	5.6	55.1	1.85	55.1	1.85	2.0
5	57.5	56.6	2.0	3	5.3	59.3	3.13	58.4	3.18	2.0
6	64.1	64.1	2.0	6	8.1	68.7	7.18	68.5	6.86	1.0
7	62.5	62.5	2.0	3	13.4	63.6	1.76	63.6	1.76	2.0
8	56.2	56.2	2.0	6	8.7	58.6	4.27	58.6	4.27	1.0
Problem class 2										
1	56.6	56.6	2.0	5	10.3	62.0	9.54	59.7	5.48	1.0
2	87.5	87.5	2.0	2	2.3	87.5	0.00	87.5	0.00	1.0
3	60.3	60.3	2.0	8	14.5	65.2	8.13	64.0	6.14	1.0
4	58.7	58.7	2.0	3	10.4	58.7	0.00	58.7	0.00	1.0
5	62.0	62.0	2.0	10	21.3	75.7	22.10	75.7	22.10	0.5
6	59.4	59.4	3.0	8	12.6	65.6	10.44	65.5	10.27	2.0
7	68.8	68.8	2.0	5	9.8	68.8	0.00	68.8	0.00	2.0
8	65.8	65.8	2.0	7	17.6	68.3	3.80	68.2	3.65	1.0
Problem class 3										
1	56.2	56.2	3.0	5	2.3	56.4	0.36	56.2	0.00	2.0
2	51.5	51.5	2.0	2	0.2	51.5	0.00	51.5	0.00	2.0
3	53.3	53.3	2.0	3	6.8	54.7	2.63	54.7	2.63	2.0
4	57.8	57.8	2.0	0		57.8	0.00	57.8	0.00	2.0
5	55.6	55.6	2.0	2	0.3	56.9	2.34	56.7	1.98	1.0
6	58.0	58.0	3.0	0		58.0	0.00	58.0	0.00	2.0
7	59.0	59.0	1.0	0		59.0	0.00	59.0	0.00	2.0
8	46.4	46.4	2.0	2	0.0	47.1	1.51	47.1	1.51	2.0
Problem class 4										
1	81.7	81.7	1.0	6	10.2	93.6	14.57	93.2	14.08	2.0
2	79.7	79.7	2.0	9	15.4	83.4	4.64	83.4	4.64	2.0
3	77.9	77.9	2.0	6	3.2	78.4	0.64	78.4	0.64	2.0
4	88.0	88.0	2.0	1	0.0	88.0	0.00	88.0	0.00	1.0
5	65.5	65.4	2.0	3	0.4	65.7	0.31	65.4	0.00	2.0
6	74.7	74.7	2.0	3	5.5	74.8	0.13	74.7	0.00	2.0
7	63.9	63.9	2.0	6	1.5	64.0	0.16	64.0	0.16	2.0
8	63.3	63.3	2.0	1		63.3	0.00	63.3	0.00	1.0
Average			2.1	4			4.16		3.87	2.0

V = number of violations
Max V = maximum violation

Table B.3: Results of the CVRP with sequence based pallet loading with and without axle weight constraints on networks of 20 customers

Instance	Model without axle weight					Model with axle weight				
	Z^{avg}	Z^{best}	t (s)	# V	Max V (%)	Z^{avg}	Z^{avg} incr (%)	Z^{best}	Z^{best} incr (%)	t (s)
Problem class 1										
1	74.0	74.0	4.0	4	20	74.2	0.27	74.2	0.27	3.0
2	73.1	73.1	5.0	0		73.1	0.00	73.1	0.00	4.0
3	72.6	72.6	6.0	5	9	73.9	1.79	73.9	1.79	5.0
4	72.0	72.0	5.0	2	0	72.0	0.00	72.0	0.00	4.0
5	70.8	70.8	5.0	4	4	72.4	2.26	72.0	1.69	4.0
6	61.7	61.7	4.0	5	10	62.1	0.65	62.1	0.65	2.0
7	69.8	69.8	5.0	3	7	69.8	0.00	69.8	0.00	3.0
8	68.5	68.5	7.0	10	13	76.0	10.95	75.4	10.07	4.0
Problem class 2										
1	91.7	91.7	3.0	8	12	96.0	4.69	95.9	4.58	3.0
2	87.8	87.8	4.0	5	12	87.8	0.00	87.8	0.00	3.0
3	113.0	113.0	3.0	11	14	125.2	10.80	124.8	10.44	2.0
4	99.5	99.5	4.0	5	12	101.4	1.91	101.4	1.91	2.0
5	94.4	94.4	4.0	12	13	104.9	11.12	104.7	10.91	2.0
6	104.6	104.6	3.0	9	12	111.6	6.69	111.6	6.69	1.0
7	90.8	90.8	4.0	7	18	93.5	2.97	93.5	2.97	3.0
8	91.4	91.4	6.0	6	11	92.6	1.31	91.4	0.00	3.0
Problem class 3										
1	67.3	67.3	5.0	0	0	67.3	0.00	67.3	0.00	4.0
2	68.5	68.5	5.0	1	5	68.5	0.00	68.5	0.00	3.0
3	78.7	78.7	5.0	4	6	78.7	0.00	78.7	0.00	3.0
4	63.1	63.1	6.0	2	1	63.1	0.00	63.1	0.00	4.0
5	68.3	68.3	7.0	1	0	68.3	0.00	68.3	0.00	5.0
6	78.4	78.4	4.0	3	2	78.4	0.00	78.4	0.00	3.0
7	63.6	63.6	7.0	2	2	63.6	0.00	63.6	0.00	5.0
8	67.0	67.0	9.0	3	0	67.0	0.00	67.0	0.00	5.0
Problem class 4										
1	80.9	80.9	6.0	5	4	81.0	0.12	81.0	0.12	5.0
2	59.5	59.5	6.0	3	2	61.5	3.36	61.4	3.19	5.0
3	89.3	88.9	5.0	3	0	89.4	0.11	88.9	0.00	4.0
4	86.6	86.6	4.0	3	5	89.3	3.12	89.3	3.12	2.0
5	100.3	100.3	5.0	3	0	100.3	0.00	100.3	0.00	3.0
6	122.7	122.7	4.0	9	14	127.7	4.07	127.6	3.99	4.0
7	92.6	92.6	4.0	7	10	97.5	5.29	97.5	5.29	3.0
8	89.8	89.8	4.0	5	17	89.9	0.11	89.9	0.11	4.0
Average			4.9	5			2.24		2.12	3.0

V = number of violations

Max V = maximum violation

Appendix C

Detailed results: minimization of total distance

Table C.1 shows the number of trucks of each vehicle type in the final solution of the FSM *with* sequence-based pallet loading and axle weight constraints, while Table C.2 presents the number of trucks of each vehicle type in the final solution of the FSM *without* axle weight constraints for the scenario of distance minimization. For all instances, the number of routes in the final solutions of the FSM that may be performed by 30-foot trucks, 45-foot trucks and by both trucks are reported.

The computation times of the ILS for the CVRP with sequence-based pallet loading and axle weight constraints for a vehicle fleet consisting of 30-foot trucks, a fleet consisting of 45-foot trucks and a heterogeneous vehicle fleet *with* 30-foot and 45-foot trucks are presented in Table C.3. Finally, Table C.4 presents the computation times of the ILS for the CVRP *without* axle weight constraints for a vehicle fleet consisting of 30-foot trucks, a fleet consisting of 45-foot trucks and a heterogeneous vehicle fleet with 30-foot and 45-foot trucks.

Table C.1: Number of trucks of each vehicle type in the final solution for the FSM with sequence-based pallet loading and axle weight constraints - Distance minimization

	50 customers			75 customers			100 customers		
Instance	30-foot	45-foot	both	30-foot	45-foot	both	30-foot	45-foot	both
Problem class 3									
1	1.0	8.5	0.0	2.7	11.8	0.0	2.5	17.3	0.0
2	0.8	8.5	0.0	2.0	11.9	0.0	2.6	16.8	0.0
3	1.9	7.7	0.0	1.0	13.0	0.4	2.8	17.1	0.0
4	1.0	8.7	0.1	1.2	12.7	0.1	2.1	18.0	0.0
5	1.6	8.4	0.0	2.1	12.6	0.2	2.1	17.2	0.1
6	2.5	8.1	0.0	3.0	11.9	0.0	2.2	16.9	0.0
7	0.5	9.0	0.5	2.0	12.4	0.0	1.1	17.7	0.1
8	0.0	9.0	1.0	0.9	13.0	0.2	3.5	16.7	0.0
Problem class 4									
1	0.8	13.8	0.4	3.0	17.9	0.0	6.6	22.7	0.0
2	1.2	12.9	0.0	4.5	16.5	0.0	4.1	23.7	0.0
3	2.1	13.1	0.0	2.0	20.6	0.0	4.0	25.2	0.0
4	2.0	10.1	0.0	3.4	19.4	0.0	2.7	24.0	0.0
5	3.0	10.0	0.0	5.3	19.9	0.0	4.6	25.5	0.0
6	3.1	12.9	0.0	4.0	19.1	0.0	6.1	27.6	0.0
7	1.5	12.9	0.0	3.8	17.4	0.0	6.8	24.3	0.0
8	4.5	11.9	0.0	2.4	20.0	0.0	3.4	26.2	0.0
Average	1.7	10.3	0.1	2.7	15.6	0.1	3.6	21.1	0.0

Table C.2: Number of trucks of each vehicle type in the final solution for the FSM without axle weight constraints - Distance minimization

	50 customers			75 customers			100 customers		
Instance	30-foot	45-foot	both	30-foot	45-foot	both	30-foot	45-foot	both
Problem class 3									
1	0.0	8.9	0.1	0.0	12.9	1.0	0.0	18.6	0.5
2	0.0	9.0	0.3	0.0	13.0	0.1	0.0	17.7	0.8
3	0.0	8.8	0.3	0.0	13.1	0.9	0.0	17.8	1.3
4	0.0	9.0	0.7	0.0	13.0	0.5	0.0	17.9	1.1
5	0.0	9.0	0.2	0.0	13.2	0.8	0.0	18.0	0.1
6	0.0	9.0	1.0	0.0	13.0	1.0	0.0	17.8	0.2
7	0.0	9.0	1.0	0.0	12.5	1.5	0.0	17.7	0.4
8	0.0	9.0	0.2	0.0	13.2	0.8	0.0	18.0	1.0
Problem class 4									
1	0.0	14.2	0.5	0.0	18.6	0.7	0.2	25.3	2.6
2	1.2	11.0	1.8	0.1	17.7	2.6	0.0	24.0	2.0
3	0.0	13.3	1.0	0.0	21.0	1.0	0.0	26.1	1.9
4	0.0	11.0	1.0	0.0	20.8	0.9	0.2	24.7	0.5
5	0.0	11.0	1.0	0.1	21.5	1.4	0.2	27.6	1.2
6	0.0	14.0	1.0	0.0	20.0	1.4	0.0	28.9	3.3
7	0.0	13.1	0.9	0.2	17.5	2.3	0.1	26.6	2.1
8	0.0	14.0	1.0	0.0	18.8	2.5	0.2	26.9	0.9
Average	0.1	10.8	0.8	0.0	16.2	1.2	0.1	22.1	1.2

Table C.3: Computation times of the ILS for the CVRP with sequence-based pallet loading and axle weight constraints for a vehicle fleet consisting of 30-foot trucks, a fleet consisting of 45-foot trucks and a heterogeneous vehicle fleet with 30-foot and 45-foot trucks - Distance minimization

	50 customers			75 customers			100 customers		
Instance	30-foot	45-foot	FSM	30-foot	45-foot	FSM	30-foot	45-foot	FSM
Problem class 3									
1	125	224	208	276	211	499	931	585	944
2	113	130	179	218	337	654	543	746	1,969
3	108	118	183	342	309	666	372	670	904
4	135	170	151	306	401	664	505	862	829
5	90	152	186	288	456	285	648	501	1,554
6	122	200	176	283	394	429	543	489	1,887
7	96	169	156	322	278	572	499	631	956
8	114	181	132	223	259	588	408	421	976
Problem class 4									
1	48	113	107	217	207	425	516	372	852
2	48	89	105	189	297	565	342	430	1,088
3	61	116	128	193	189	507	458	402	689
4	113	155	212	226	168	460	428	353	668
5	109	127	170	189	197	319	378	321	1,250
6	62	137	122	158	266	340	344	330	785
7	75	143	125	197	275	386	469	374	897
8	72	90	114	170	285	425	459	322	922
Average	93	145	153	237	283	486	490	488	1,073

Table C.4: Computation times of the ILS for the CVRP *without* axle weight constraints for a vehicle fleet consisting of 30-foot trucks, a fleet consisting of 45-foot trucks and a heterogeneous vehicle fleet with 30-foot and 45-foot trucks - Distance minimization

	50 customers			75 customers			100 customers		
Instance	30-foot	45-foot	FSM	30-foot	45-foot	FSM	30-foot	45-foot	FSM
Problem class 3									
1	148	234	557	532	444	1,713	2,348	1,534	5,796
2	103	152	284	570	609	2,797	1,850	1,427	5,216
3	122	144	259	583	365	2,441	1,951	1,413	4,725
4	236	274	462	681	510	2,068	2,393	1,435	4,492
5	150	215	421	678	599	1,952	1,825	1,116	6,026
6	168	260	414	920	619	2,324	2,018	1,167	4,590
7	140	215	530	629	447	1,554	1,948	1,137	5,055
8	139	316	453	600	531	2,037	1,820	1,093	4,325
Problem class 4									
1	55	212	234	630	348	1,760	1,068	1,015	4,325
2	55	150	269	661	348	1,246	1,737	1,107	3,192
3	70	130	371	475	421	1,717	2,388	976	3,628
4	77	182	430	324	316	1,037	1,523	1,073	3,758
5	124	167	315	635	352	1,167	833	832	3,572
6	79	185	331	504	313	1,256	1,537	992	3,084
7	137	218	367	406	371	1,306	1,417	721	3,618
8	84	142	289	369	292	955	2,134	865	3,368
Average	118	200	374	575	430	1,708	1,799	1,119	4,298

Appendix D

Detailed results: minimization of total transport cost

Table D.1 shows the number of trucks of each vehicle type in the final solution of the FSM *with* sequence-based pallet loading and axle weight constraints, while Table D.2 presents the number of trucks of each vehicle type in the final solution of the FSM *without* axle weight constraints for the scenario of total transport cost minimization. For all instances, the number of routes in the final solutions of the FSM that are performed by 30-foot trucks and 45-foot trucks are reported.

The computation times of the ILS for the CVRP with sequence-based pallet loading and axle weight constraints for a vehicle fleet consisting of 30-foot trucks, a fleet consisting of 45-foot trucks and a heterogeneous vehicle fleet *with* 30-foot and 45-foot trucks are presented in Table D.3. Finally, Table D.4 presents the computation times of the ILS for the CVRP *without* axle weight constraints for a vehicle fleet consisting of 30-foot trucks, a fleet consisting of 45-foot trucks and a heterogeneous vehicle fleet with 30-foot and 45-foot trucks.

Table D.1: Number of trucks of each vehicle type in the final solution for the FSM with sequence-based pallet loading and axle weight constraints - Transport cost minimization

	50 customers		75 customers		100 customers	
Instance	30-foot	45-foot	30-foot	45-foot	30-foot	45-foot
Problem class 3						
1	1.1	8.6	2.9	11.6	3.3	16.9
2	1.2	8.4	2.2	11.8	2.7	16.9
3	1.8	7.8	2.5	12.2	3.5	16.9
4	1.9	8.1	1.1	12.9	2.3	17.7
5	1.6	8.4	2.3	12.6	2.4	16.9
6	2.3	8.2	3.0	11.9	2.1	17.0
7	1.0	9.0	1.9	12.4	0.9	17.9
8	1.0	9.0	1.4	12.9	3.6	16.7
Problem class 4						
1	1.0	14.0	4.2	17.1	7.8	21.9
2	3.1	11.3	4.5	16.5	5.8	22.3
3	3.1	12.5	3.3	19.7	3.4	25.6
4	2.3	10.0	4.8	18.7	2.5	24.3
5	3.0	10.0	4.7	20.3	4.2	26.0
6	3.6	12.4	4.2	19.0	7.0	27.1
7	2.4	12.4	3.4	17.8	6.5	24.8
8	4.5	12.0	2.6	19.9	4.8	25.0
Average	2.2	10.1	3.1	15.5	3.9	20.9

Table D.2: Number of trucks of each vehicle type in the final solution for the FSM without axle weight constraints - Transport cost minimization

	50 customers		75 customers		100 customers	
Instance	30-foot	45-foot	30-foot	45-foot	30-foot	45-foot
Problem class 3						
1	0.8	8.7	1.0	13.0	5.9	16.7
2	0.8	9.0	1.7	12.4	2.4	17.0
3	2.0	8.2	1.4	12.9	2.9	17.3
4	1.0	8.9	1.6	13.0	3.6	17.5
5	0.9	9.2	3.7	12.3	3.2	17.2
6	1.4	8.7	3.5	12.5	3.6	16.7
7	1.0	9.0	2.6	12.1	1.5	17.4
8	1.2	8.8	2.2	12.5	2.4	17.5
Problem class 4						
1	1.1	13.8	2.0	18.2	7.3	23.4
2	2.9	11.6	5.0	17.3	6.2	22.2
3	1.6	13.5	2.7	20.5	7.2	23.9
4	0.9	11.1	4.5	18.7	6.1	22.5
5	1.7	10.9	4.4	20.3	7.1	25.0
6	2.0	13.8	3.4	19.5	7.6	27.3
7	2.5	12.5	4.1	17.0	6.0	25.0
8	3.2	12.3	4.3	18.6	7.1	24.5
Average	1.6	10.6	3.0	15.7	5.0	20.7

Table D.3: Computation times of the ILS for the CVRP with sequence-based pallet loading and axle weight constraints for a vehicle fleet consisting of 30-foot trucks, a fleet consisting of 45-foot trucks and a heterogeneous vehicle fleet with 30-foot and 45-foot trucks - Transport cost minimization

	50 customers			75 customers			100 customers		
Instance	30-foot	45-foot	FSM	30-foot	45-foot	FSM	30-foot	45-foot	FSM
Problem class 3									
1	297	499	669	410	1,562	2,760	997	3,683	6,778
2	264	291	959	588	1,207	2,802	1,144	3,622	4,433
3	333	366	1,182	316	1,277	3,154	1,129	2,834	3,497
4	253	338	810	401	1,296	3,762	1,187	4,037	6,321
5	246	233	746	468	1,578	2,534	1,137	2,277	7,447
6	284	474	693	568	2,224	2,201	1,532	3,427	4,241
7	140	331	561	445	1,458	3,008	773	3,638	3,937
8	183	321	611	415	1,353	2,658	933	2,784	4,856
Problem class 4									
1	100	145	415	148	785	1,720	312	1,240	4,455
2	100	114	385	248	790	2,461	342	2,072	4,520
3	108	148	376	104	526	1,566	319	1,520	3,490
4	213	278	626	131	417	1,764	327	1,686	5,023
5	249	251	417	140	409	1,450	229	1,306	3,141
6	89	147	388	107	618	1,231	247	872	4,758
7	175	258	530	170	798	1,776	261	1,364	4,295
8	117	111	390	113	489	1,630	229	2,001	5,246
Average	197	269	610	298	1,049	2,280	694	2,398	4,777

Table D.4: Computation times of the ILS for the CVRP *without* axle weight constraints for a vehicle fleet consisting of 30-foot trucks, a fleet consisting of 45-foot trucks and a heterogeneous vehicle fleet with 30-foot and 45-foot trucks - Transport cost minimization

	50 customers			75 customers			100 customers		
Instance	30-foot	45-foot	FSM	30-foot	45-foot	FSM	30-foot	45-foot	FSM
Problem class 3									
1	1,040	1,036	1,367	4,092	5,195	6,131	9,476	6,740	12,418
2	955	1,134	1,055	3,488	3,288	4,921	11,763	9,623	11,448
3	1,023	1,015	1,352	3,710	3,755	5,994	11,068	6,409	9,944
4	909	1,041	1,205	3,845	3,856	5,941	7,816	9,397	11,330
5	789	979	1,198	3,499	2,875	4,462	7,219	7,793	8,699
6	981	1,218	1,207	2,482	2,868	5,181	11,452	8,243	11,363
7	809	968	1,330	3,463	4,399	4,758	10,452	8,263	13,281
8	937	934	1,221	3,585	3,610	4,657	8,782	7,137	9,738
Problem class 4									
1	501	422	735	1,890	2,368	3,652	5,684	5,539	7,071
2	472	483	699	2,021	2,310	2,862	7,081	7,217	7,618
3	497	541	567	1,968	2,593	3,252	5,888	5,890	9,182
4	658	620	796	2,178	1,864	2,499	6,550	5,679	8,696
5	576	606	1,071	2,050	1,654	2,741	6,326	5,517	6,327
6	445	531	637	2,158	1,916	3,502	5,132	5,546	7,012
7	670	732	863	1,976	2,396	2,918	5,883	5,205	8,240
8	439	558	726	1,734	1,862	2,678	5,834	5,980	7,403
Average	731	801	1,002	2,759	2,926	4,134	7,900	6,886	9,361

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