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A simplified kinematic approach for the shear strength of fibre-reinforced concrete deep beams

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Abstract: Deep beams with shear span-to-depth ratios $a/d \leq 2.5$ are used to resist large shear forces due to their ability to develop direct strut action. To further enhance the shear strength and crack control of such members, researchers have studied the use of fibre-reinforced concrete (FRC). However, while this solution is promising, there is a need for rational mechanical models capable of predicting the shear strength of FRC deep beams in a sufficiently simple manner. This paper proposes such a model based on first principles: kinematics, equilibrium and constitutive relationships. The proposed model simplifies an earlier two-parameter kinematic theory (2PKT) for the complete shear behavior of FRC deep beams, to predict the shear strength and components of shear resistance in a straightforward manner. The new simplified method (S2PKT) is validated by comparing the predicted results to 22 tests from the literature, as well as to FEM and 2PKT predictions. It is shown that the proposed simplified kinematic approach predicts well the shear strength with an average experimental-to-predicted shear strength ratio of 1.12 and a coefficient of variation of 12.9%. Furthermore, the model is used to discuss the effect of shear span-to-depth ratio and fibre volumetric ratio on the shear strength of FRC deep beams.

Keywords: deep beams, shear, fibre-reinforced concrete, kinematic model

1 Introduction

Deep beams with small shear span-to-depth ratios (a/d < 2.5) can carry high shear forces by means of strut action, and thus are widely used as transfer girders, cap beams, pile caps and other heavily loaded members. Researchers have investigated the possibility of further enhancing the shear strength of such members with the use of steel fibres [1-3], which bridge the critical shear cracks and act as an additional shear-carrying mechanism. However, while fibre-reinforced concrete (FRC) deep beams exhibit improved shear-carrying capacity and greater post-peak ductility than reinforced concrete, exactly quantifying these beneficial effects remains a challenging problem. This is in large part due to the difficulty in extending the strut-and-tie method [4] (Fig. 1a) for reinforced concrete (RC) deep beams to FRC members, as the this method neglects the tension in the concrete while enhanced behavior in tension is the main advantage of FRC.

An alternative modeling approach for FRC deep beams that considers the effect of steel fibres based on compatibility of deformations, constitutive relationships and equilibrium was recently developed by Mihaylov et al. [5]. This approach is an extension of a two-parameter kinematic theory (2PKT) for RC deep beams [6], see Fig. 1b. It has been demonstrated that this kinematics-based approach is capable of modeling the complete shear response of FRC deep beams, including their post-peak behavior [5], see Fig. 1c. As evident from Fig. 1c, the kinematics-based approach accounts for five shear-carrying mechanisms in FRC deep beams; critical loading zone V_{CLZ}, aggregate interlock V_{ci}, transverse shear reinforcement V_s, dowel action of the longitudinal reinforcement V_d, and steel fibres V_f. While it provides excellent shear strength predictions and complete pre-peak, peak and post-peak load-deformation behavior of FRC deep beams, this approach requires an iterative computation procedure and does not lend itself to simple calculations.

The main aim of this paper is to simplify this approach to compute only the peak resistance of FRC deep beams in a straightforward manner with negligible compromise on the accuracy of the predictions. The main assumption used to derive the simplified model can be inferred from inspecting Fig. 1c and results from other similar analyses: the peak resistance of FRC deep beams coincides with that of the critical loading zone (CLZ). Hence, it is assumed that the failure of the beam is triggered by the crushing of the CLZ. This is a reasonable assumption to make for FRC deep beams, as the CLZs of such D-region members typically carry a significant portion of the shear, and the crushing of which is brittle in nature leading to a sudden drop in shear capacity. Similar simplification [7] has been made for the kinematic model for complete shear behavior of FRC coupling beams [8].

In addition to the derivation of the simplified kinematic model (hereafter abbreviated as S2PKT), this paper presents comparisons with 22 tests as well as with nonlinear finite element simulations. Furthermore, the predictions of the simplified method for shear strength of FRC deep beams are compared with those of the kinematic model for complete shear behavior.

2 Kinematics of deep beams

Figure 2 summarizes the compatibility equations of the two-parameter kinematic theory for reinforced concrete deep beams which have been derived earlier [6]. It is assumed that the shear failure in short shear spans occurs along a straight crack that extends from the support to the load along the diagonal of the shear span (critical diagonal crack). The important deformations in the crack include the average crack width w, crack slip s, and the strain in the stirrups ε_v halfway along the crack. As evident from Eqs. 5–7 in Fig. 2c, these deformations are expressed as functions of degrees of freedom (DOFs) $\varepsilon_{t,avg}$ and Δ_c.

The DOFs of the kinematic model are illustrated in Fig. 2a. The first DOF is the average strain $\varepsilon_{t,avg}$ along the cracked length of the bottom reinforcement, which is assumed equal to the strain in the section with maximum bending moment as characteristic of direct strut action in deep beams. This strain is associated with a flexural deformation pattern (see top diagram in Fig. 2a) where the critical crack opens but does not undergo sliding displacements. In contrast, the second DOF Δ_c, which is associated with a shear deformation pattern (see bottom diagram in Fig. 2a), results in both opening and sliding displacements. More specifically, Δ_c is the vertical displacement between the crack faces associated with crushing of the concrete in the CLZ at the top of the crack. The size of the CLZ is dependent on a key geometric property, namely the effective width of the loading plate l_{b1e} which can be determined with Eq. 1 in Fig. 2b.

Other geometric properties that are necessary to describe the kinematic model are the angle of the critical diagonal crack α_1, the cracked length along the bottom longitudinal reinforcement l_t, and the length along the bottom reinforcement l_k where the reinforcement works in double curvature. These geometric properties are expressed with Eqs. 2–4 in Fig. 2b. The angle of the crack coincides with the angle of the diagonal of the shear span α (from center of support plate to center of loading plate), except in the transition from deep to slender beams where α_1 is fixed at 35°. Angle α_1 is used to evaluate the cracked length l_t, which is in turn necessary for the evaluation of the deflection of the shear span Δ expressed with Eq. 8 in Fig. 2c.

The complete deformation pattern of the beam is obtained as a linear combination of the two basic patterns governed by DOFs $\varepsilon_{t,avg}$ and Δ_c. To determine the values of the DOFs at peak capacity, the kinematic equations are combined with equilibrium conditions and constitutive relationships for the mechanisms of shear resistance across the critical crack.

3 Mechanisms of shear resistance

As mentioned earlier, FRC deep beams carry shear through five mechanisms of shear resistance: 1) diagonal compression in the critical loading zone V_{CLZ}, 2) aggregate interlock across the critical diagonal crack V_{ci}, 3) tension in the transverse shear reinforcement V_s, 4) dowel action of the bottom longitudinal reinforcement V_d and 5) shear carried by the steel fibres V_f (see Fig. 3). The first four mechanisms have been derived elsewhere [6, 9] and are expressed with Eqs. 9-12 in Fig. 3. The shear contribution of steel fibres V_f is discussed in more detail below.

The steel fibres enhance the shear behavior of FRC deep beams mainly in two ways: 1) they act as an additional shear-carrying mechanism by transferring tension across the critical diagonal crack and 2) they improve the post-peak ductility of the CLZ in compression [10] due to the confinement effect of the fibres. However, as the second effect mainly impacts the post-peak behavior of the beam and has an insignificant impact on its peak resistance (strength), it is neglected here for the sake of simplicity. Therefore, Eq. 9 in Fig. 3 for the shear carried in the CLZ is adopted directly from the 2PKT for RC deep beams. This expression was derived based on the assumption that the CLZ is at failure under principal compressive stresses inclined at angle α (Fig. 2) [6].

The tensile behavior of steel fibres across cracks has been studied by a number of researchers [11-14], particularly for the case of pure opening of the crack without slip displacements (Mode I fracture). It is understood that the relationship between normal stress across the crack and opening of the crack can be represented by the superimposition of two stresses (Fig. 4): tension transferred directly between the two crack faces (tension softening of concrete), and tension carried by the steel fibres which are anchored on each side of the crack. The tension softening of concrete is neglected here as the kinematic model for deep beams assumes a fully-cracked member. The tensile stress transferred by straight fibres can be computed by a variable engagement model proposed by Voo and Foster [11]:

$$\sigma_f = \frac{0.396\sqrt{f_c'}\rho_f l_f}{d_f}\frac{\tan^{-1}\left(\frac{w}{d_f/3.5}\right)}{\pi}\left(1-\frac{2w}{l_f}\right)^2 \qquad (13)$$

where $0.396\sqrt{f_c'}$ is the bond stress between the concrete and the fibres, ρ_f is the volumetric ratio of steel fibres, l_f is the length of the fibres, d_f is the diameter of the fibres, and w is the crack width. However, it must be noted that Eq. 13 only considers the frictional bond behavior of steel fibres and does not take into account the mechanical anchorage effect of hooked-end steel fibres. Thus, for the shear carried by the hooked-end fibres, it is recommended to use the simplified diverse embedment model proposed by Lee et al. [14] which explicitly considers both effects, while for straight fibres the simpler model given in Eq. 13 can be used.

To use Eq. 13 to model the critical diagonal crack in FRC deep beams, it is necessary to consider that the crack does not undergo a pure opening, but a combination of opening and slip displacements (mixed Mode I and II fracture). Therefore, based on experimental observations that the vertical displacements in the crack w_v dominate the crack kinematics near shear failure [6], it is proposed to replace w in Eq. 13 with w_v, and to assume that the tensile force in the fibres is aligned with w_v. The vertical crack displacement halfway along the critical crack is derived directly from the kinematic model by applying small-displacement kinematics:

$$w_v = 0.5\varepsilon_{t,avg} l_k \cot\alpha_1 + \Delta_c \tag{14}$$

where the two terms of this equation are associated with the two DOFs of the model. Thus, with an expressed fibre stress $\sigma_f(w_v)$, the shear contribution of the fibres across the critical crack is:

$$V_f = \sigma_f(w_v) bd / \sin\alpha_1 \tag{15}$$

where b is the width of the section, d is the effective depth of the section, α_1 is the angle of the critical crack, and $d/\sin\alpha_1$ is the length of the crack.

As all shear-carrying mechanisms are expressed in terms of the two DOFs of the kinematic model, it is necessary to compute the values $\varepsilon_{t,avg}$ and Δ_c at peak resistance in order to predict the shear strength of FRC deep beams.

4 Shear strength prediction

Degree of freedom Δ_c at peak resistance is derived by following the same assumption used in the derivation of V_{CLZ}, that the failure of FRC deep beams is triggered by the crushing of the CLZ. As the CLZ is at crushing, the strain along its bottom inclined face is assumed equal to -0.0035, and the strain along its top horizontal face is neglected. By also defining the geometry of the CLZ in terms of the effective width of the loading plate l_{b1e} (Eq. 1) and the angle of the critical crack in the vicinity of the loading plate (Eq. 2), Δ_c has been expressed as [6]:

$$\Delta_c = 0.0035 \times 3 l_{b1e} \cot\alpha \qquad (16)$$

To predict the second DOF $\varepsilon_{t,avg}$, it is necessary to consider the equilibrium between the internal and external forces acting on the deep beam. This is performed by equating the summation of all the shear resistance mechanisms expressed in terms of one unknown $\varepsilon_{t,avg}$ (Eq. 9-12 and 15), with the shear demand expressed from the tension in the longitudinal reinforcement, also given in terms of $\varepsilon_{t,avg}$. Assuming that the longitudinal reinforcement remains linear elastic and the tension stiffening effect of the concrete around the reinforcement is negligible near failure, the force in the longitudinal reinforcement is:

$$T = E_s A_s \varepsilon_{t,avg} \qquad (17)$$

where E_s is the Young's modulus of steel and A_s is the area of the bottom longitudinal reinforcement.

This force is used to evaluate the shear demand V by using the moment equilibrium of the shear span:

$$V = Tz/a \tag{18}$$

where $z \approx 0.9d$ is the approximate lever arm of the longitudinal internal forces at the section with the maximum moment, and a is the shear span.

The solution procedure of finding $\varepsilon_{t,avg}$ that satisfies equilibrium between the shear capacity and the shear demand at failure is summarized in the flowchart in Fig. 5. Although the solution procedure is iterative, the convergence is very fast as demonstrated in the step-by-step calculation given in the following section.

5 Example of S2PKT calculations

The S2PKT calculation procedure will be illustrated by providing the steps taken in computing the shear strength of specimen B7 by Mansur and Ong [2], with reference to the flowchart given in Fig. 5. Specimen B7 had an a/d ratio of 1.23 and a fibre volumetric ratio $\rho_f = 0.5\%$. The effective depth of the member was $d = 463$ mm, the flexural reinforcement ratio was $\rho_l = 1.93\%$, and the stirrup ratio was $\rho_v = 0.47\%$. The complete set of material and geometric properties of specimen B7 are given in the database of shear critical FRC deep beams in Table 1.

The first step of the procedure is to compute the geometry of the kinematic model. For a shear force-to-applied point load ratio $V/P = 1$ (symmetrical three-point bending), the effective length of the loading plate l_{b1e} (Eq. 1) is 80 mm, and hence, the angle of the critical diagonal crack $\alpha_1 = \alpha$ is $41.3°$ (Eq. 2). The cracked length along the bottom reinforcement l_t (Eq. 3), which is needed to express the deflection of the beam, is 528 mm. The dowel length of the bottom longitudinal reinforcement l_k (Eq. 4) is 63 mm.

The next step of S2PKT calculations is to compute DOF Δ_c at peak capacity. As mentioned before, the proposed simplified kinematic method assumes that the peak capacity of an FRC deep beam coincides with that of the CLZ. Therefore, from Eq. 16, DOF Δ_c at the crushing of the CLZ is 0.96 mm.

The solution procedure now requires an iteration, which commences with an initial estimate of the shear strength V – for example $0.1bdf_c' \approx 140$ kN. At this load, the force in the longitudinal reinforcement T from Eq. 18 is 192 kN. Then, the second DOF $\varepsilon_{t,avg}$ is computed from Eq. 17 as 0.0012. Once both DOFs are known, it allows the computation of the deformations along the critical diagonal crack; crack width w (Eq. 5) and average strain in the stirrups ε_v (Eq. 7), which are respectively 0.78 mm and 0.0055. From Eq. 9, the shear carried by the CLZ V_{CLZ} is calculated as 75 kN. This value will stay constant during the iterations as it is governed by DOF Δ_c, which remains unchanged. The shear carried by aggregate interlock V_{ci} (Eq. 10), which depends on the crack width, is 43 kN. The shear strength component V_s depends on ε_v, and is calculated from Eq. 11 to be 62 kN. Note that as ε_v exceeds the yield strain of the steel, the stirrups are yielding at this level of loading. Furthermore, applying Eq. 12 for the dowel action of the flexural reinforcement which depends directly on DOF $\varepsilon_{t,avg}$, shear component V_d is evaluated at 27 kN. The final shear-strength component V_f (Eq. 15), which depends on the vertical displacement of the critical crack w_v (Eq. 14), is calculated to be 15 kN. Thus, at the end of the first iteration, the calculated shear capacity is $\sum V_i = 75 + 43 + 62 + 27 + 15 = 222$ kN, and the difference between the assumed shear strength and the calculated shear capacity is $|V - \sum V_i| = |140 - 222| = 82$ kN. As the two shear forces differ significantly, the iterations continue with the calculated shear capacity $\sum V_i$ as the new estimate of shear strength. After only 3 additional iterations, the error $|V - \sum V_i|$ decreases monotonically to only 1 kN, and therefore the solution can be considered converged. This fast convergence is not limited to the selected example only, but it is typical of the proposed solution procedure. Finally, at equilibrium between the internal and external forces acting on the deep beam, the predicted shear strength for this specimen is 208 kN.

The described solution procedure of the kinematic model equations performed for specimen B7 is illustrated graphically in Fig. 6. The plot shows the variation of the components of shear resistance V_d, V_s, V_{ci}, V_{CLZ} and V_f, the shear capacity $\sum V_i$ (thick black line) and the shear demand (dashed line), with increasing average tensile strain in the bottom longitudinal reinforcement $\varepsilon_{t,avg}$. As evident from the plot, the shear capacity decreases slightly with increasing $\varepsilon_{t,avg}$. This is due to the diminishing aggregate interlock as the critical diagonal crack widens, and the loss of dowel action as the dowels are subjected to increasing axial tension. Component V_{CLZ}, which provides the largest contribution to the shear capacity, remains unchanged as it is governed by DOF Δ_c only, and not DOF $\varepsilon_{t,avg}$. The iterative calculations performed earlier, which started with an estimated value of the shear strength of 140 kN, are illustrated with the spiraling set of arrows. As evident from the plot, the arrows spiral towards the solution where the shear capacity and demand curves intersect, and thus the shear forces are in equilibrium. For this beam, the obtained shear strength experimental-to-predicted ratio is V_{exp}/V_{pred} = 220 kN/208 kN = 1.06.

6 Comparisons with tests and FEM simulations

Similar analyses were performed for 22 tests from three experimental studies reported by Cho and Kim [3], Mansur and Ong [2], and Mansur and Alwist [1]. Table 1 lists the properties of the test specimens, the measured shear strengths V_{exp} and the predictions of the proposed simplified kinematics-based approach V_{S2PKT}. The shear span-to-depth ratio a/d of the beams vary from 0.31 to 1.85, the effective depth d from 168 mm to 624 mm, the compressive strength of the FRC f_c' from 25.7 MPa to 86.1 MPa, the longitudinal reinforcement ratio ρ_l from 0.81% to 2.82%, the transverse reinforcement ratio ρ_v from 0% to 1.26% and the fibre volumetric ratio ρ_f from 0% to 1.5%. The tests contained both straight and hooked-end fibres, with the length of fibres l_f varying from 30 mm to 36 mm and the diameter of fibres d_f from 0.4 mm to 0.6 mm. Table 1 also includes the shear strength predictions from the 2PKT method for complete shear response of FRC beams V_{2PKT}, as well as predictions V_{FEM} obtained with nonlinear finite element program VecTor2 based on the Disturbed Stress Field Model (DSFM [15]). The DSFM is a smeared rotating crack model that originates from the Modified Compression Field Theory [16] for reinforced concrete elements subjected to shear. To model the behavior of steel fibres across cracks, the DSFM incorporates the diverse embedment model [12, 14]. The finite element analyses were performed as part of this study following the modeling procedure described elsewhere for short FRC coupling beams [8]. It should be noted that this modeling approach is significantly more complex than the 2PKT as it requires considerable time for modeling and computations, as well as significant expertise to use properly.

For all 22 tests in Table 1, the proposed simplified kinematic approach produces an average shear strength experimental-to-predicted ratio of 1.12 with a coefficient of variation (COV) of 12.9%. For the same tests, the respective values obtained from the nonlinear FEM analyses are an average of 1.04 and a COV of 13.4 %. It is therefore evident that, as compared to the far more complex nonlinear FEM model, the proposed simplified kinematic approach produces slightly more conservative shear strength predictions with less scatter, while being rational and simple enough to be calculated by hand or spreadsheet. Figure 7a and 7b display the variation of the experimental-to-predicted ratios for all 22 tests plotted as functions of a/d and ρ_f, respectively. The largest scatter in the predictions is observed for the beams by Cho and Kim [3] without shear reinforcement and without fibres. It should be noted that these specimens are the smallest in the database with an effective depth of only 168 mm, meaning that the relative size of aggregates and voids in a concrete section is larger, and thus the shear strength of these specimens is more sensitive to random variations in the path of the shear cracks. As shown in a previous study [9], the addition of shear reinforcement and/or fibres tends to control the cracks better and reduce this sensitivity.

To further illustrate the advantages of the proposed simplified kinematic approach, comparisons were made with the shear strength predictions obtained with the kinematic model for complete shear behavior of FRC deep beams [5]. Figure 8a and 8b plot the ratio of the shear strengths calculated with the complete and simplified kinematic approaches as functions of a/d and ρ_f, respectively. It is evident from these plots that the proposed simplified kinematic approach is able to nearly replicate the shear strength predictions of the kinematic model for complete shear behavior of FRC deep beams, for the entire range of a/d and ρ_f values without any apparent bias. As it can be expected from a simplified model, it results in slightly more conservative predictions than the complete model from which it is derived.

7 Effect of test variables

Validated in this manner, the simplified kinematic approach is used to systematically study the effect of span-to-depth ratio a/d and the volumetric ratio of fibres ρ_f on the shear strength of FRC deep beams. For the purpose of comparison, the tests conducted by Mansur and Ong [2] were selected.

7.1 Effect of shear span-to-depth ratio a/d

Figure 9 shows the effect of a/d on the measured (shown as black dots) and predicted shear strengths (shown as a thick black line) for six FRC deep beams (beams B1, B2, B3, B4, B5 and B9 in Table 1). The experimental points show a decrease in shear strength as a/d increases from 0.31 to 1.85, and the simplified kinematic approach captures well this trend, albeit with slightly conservative predictions. The predicted components of shear resistance show that the global trend of decreasing shear strength with increasing a/d is mainly due to the diminishing of components V_{ci} and V_{CLZ}. As the shear span a increases for a constant d, the flexural strains $\varepsilon_{t,avg}$ also increase resulting in a wider critical diagonal crack and diminishing aggregate interlock resistance V_{ci}. Also, at smaller a/d the angle of the critical diagonal crack is steeper, resulting in a larger CLZ and higher V_{CLZ}, while at higher a/d the critical diagonal crack becomes flatter, resulting in a smaller and weaker critical loading zone. A flatter and longer critical diagonal crack crosses more stirrups and fibres, and therefore results in larger shear components V_s and V_f. The contribution of the dowel action of the longitudinal reinforcement to the shear resistance V_d also decreases with increasing a/d and diminishes completely once the longitudinal reinforcement yields in tension.

It can be seen that the shear strength predictions made by the simplified kinematic approach tend to become more conservative at very small shear span-to-depth ratios. This is because of the inherently conservative simplification to neglect the enhancing effect of the steel fibres on the shear resistance of the critical loading zone V_{CLZ}. As a/d decreases, the relative contribution of V_{CLZ} increases, and so does the effect of the adopted simplifying assumption.

Also shown in Fig. 9 are the predicted shear strengths of the six beams by nonlinear FEM analyses (shown as hollow triangles) and by the kinematic model for complete shear behavior of

FRC deep beams (shown as hollow dots). It is evident from the plot that the proposed simplified kinematic approach is able to capture the effect of a/d equally well as the aforementioned more complex methods.

7.2 Effect of fibre volumetric ratio ρ_f

Fig. 10 shows that the shear strength of FRC deep beams increases with increasing amount of steel fibres. The four test specimens used for comparison (beams B4, B6, B7 and B8 in Table 1) had a shear span-to-depth ratio of 1.23, compressive strength of the FRC of about 33 MPa, longitudinal reinforcement ratio of 1.93%, and transverse reinforcement ratio of 0.47%. As the fibre volumetric ratio increases from 0 to 1.5%, the measured shear strength increases by a factor of 1.27. The predicted shear strengths from the proposed simplified kinematic approach agree well with the observed trend in the experimental points, providing a slightly conservative estimate of the shear capacity. The nonlinear FEM analyses and the kinematic model for complete shear behavior of FRC deep beams provide equally adequate estimates of the shear strength, however in the case of the former, the predictions are slightly unconservative for the beams with fibre volumetric ratios of 0 and 0.5%. As can be seen from the predicted shear strength components, all shear resistance mechanisms remain nearly constant, except for the contribution of the fibres which increases linearly with ρ_f.

8 Concluding remarks

This paper presented a simplified kinematic approach for evaluating the shear strength of FRC deep beams. The proposed model identifies that the peak resistance of FRC deep beams occurs simultaneously with that of the critical loading zone, and uses that to simplify an earlier two-parameter kinematic model (2PKT) for complete shear behavior of FRC deep beams.

This approach was validated with 22 tests from the literature and its accuracy was compared to that of nonlinear FEM models and the original 2PKT. The simplified kinematic approach (S2PKT) predicts the shear strength of the test specimens with an average shear strength experimental-to-predicted ratio of 1.12 and a COV of only 12.9%. It was shown that the S2PKT, which uses only two degrees of freedom to describe the deformation patterns of FRC deep beams, predicts the shear strength with similar accuracy and less scatter than the complex FEM models with thousands of DOFs. Furthermore, it was demonstrated that the proposed model replicates the shear strength predictions of the 2PKT for the entire range of a/d and ρ_f values without any apparent bias. Hence, the S2PKT offers a quick and rational method of computing the shear strength of FRC deep beams without the need to perform a full load-displacement analysis. In addition to that, the proposed simplified approach captures well the decreasing and increasing trends of shear strength variation of FRC deep beams with increasing a/d and ρ_f, and explains them in a rational manner with the help of five components of shear resistance.

Notation

a	=	shear span
a_g	=	maximum size of coarse aggregate
A_s	=	area of longitudinal bars on flexural tension side
b	=	width of the section
d	=	effective depth of section
d_b	=	diameter of bottom longitudinal bars
d_f	=	diameter of fibres
E_s	=	Young's modulus of steel
f_c'	=	concrete cylinder strength
f_y	=	yield strength of longitudinal bars
f_{yv}	=	yield strength of stirrups
h	=	total depth of section
k	=	crack shape factor
l_0	=	length of heavily cracked zone at bottom of critical crack
l_{b1}	=	width of loading plate parallel to longitudinal axis of member
l_{b1e}	=	effective width of loading plate parallel to longitudinal axis of member
l_{b2}	=	width of support plate parallel to longitudinal axis of member
l_f	=	length of fibres
l_k	=	length of dowels provided by bottom longitudinal reinforcement
l_t	=	cracked length along bottom reinforcement
n_b	=	number of bottom longitudinal bars
P	=	applied concentrated load
s	=	crack slip
s_{cr}	=	distance between radial cracks along bottom edge of member
T	=	tensile force in bottom reinforcement
V	=	shear force
V_{ci}	=	shear resisted by aggregate interlock
V_{CLZ}	=	shear resisted by CLZ

V_d	=	shear resisted by dowel action
V_f	=	shear resisted by steel fibres
V_s	=	shear resisted by stirrups
w	=	crack width
w_v	=	vertical crack displacement
z	=	lever arm of longitudinal internal forces at section with maximum moment
α	=	angle of the diagonal of the shear span
α_1	=	angle of critical diagonal crack
ε_v	=	transverse web strain
$\varepsilon_{t,avg}$	=	average strain along bottom longitudinal reinforcement
ε_{yv}	=	yield strain of stirrups
σ_f	=	tensile stress transferred by fibres
$\sigma_{v,avg}$	=	average stress in stirrups along the critical crack
Δ	=	total deflection
Δ_c	=	shear distortion of critical loading zone
Δ_t	=	deflection due to elongation of bottom longitudinal reinforcement
ρ_f	=	volumetric ratio of fibres
ρ_l	=	ratio of bottom longitudinal reinforcement
ρ_v	=	ratio of transverse reinforcement

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Table 1 *Database of shear critical FRC deep beams*

No.	Ref.	Beam Name	a/d	b (mm)	d (mm)	h (mm)	a (mm)	l_{b1} (mm)	V/P	f_c' (MPa)	a_g (mm)	ρ_l (%)	f_y (MPa)	ρ_v (%)	f_{yv} (MPa)	ρ_f (%)	l_f (mm)	d_f (mm)	Fibre type	f_{yf} (MPa)	V_{exp} (kN)	V_{S2PKT} (kN)	V_{exp}/V_{S2PKT}	V_{2PKT} (kN)	V_{exp}/V_{2PKT}	V_{FEM} (kN)	V_{exp}/V_{FEM}
1		F30-0.0-13	1.43	120	168	200	240	**30**	1	34.4	13	1.32	399	0	623	0	-	-	H	1100	73.8	58	1.28	58	1.27	71	1.04
2		F30-0.5-13	1.43	120	168	200	240	**30**	1	25.7	13	1.32	399	0	623	0.5	36	0.6	H	1100	60.9	62	0.98	65	0.94	72	0.85
3		F60-0.0-13	1.43	120	168	200	240	**30**	1	54.3	13	1.32	399	0	623	0	-	-	H	1100	65.1	76	0.86	78	0.84	83	0.79
4		F70-0.0-19	1.43	120	168	200	240	**30**	1	65.3	13	2.82	456	0	623	0	-	-	H	1100	117.6	104	1.13	106	1.11	123	0.96
5		F70-0.5-19	1.43	120	168	200	240	**30**	1	70.5	13	2.82	456	0	623	0.5	36	0.6	H	1100	178.8	129	1.39	141	1.26	170	1.05
6	3	F70-1.0-19	1.43	120	168	200	240	**30**	1	67.3	13	2.82	456	0	623	1	36	0.6	H	1100	169.5	147	1.16	164	1.03	167	1.02
7		F70-1.5-19	1.43	120	168	200	240	**30**	1	67.3	13	2.82	456	0	623	1.5	36	0.6	H	1100	186.7	169	1.10	195	0.96	176	1.06
8		F80-0.0-16	1.43	120	168	200	240	**30**	1	74.1	13	2.00	442	0	623	0	-	-	H	1100	146.1	99	1.48	102	1.44	115	1.28
9		F80-0.5-16	1.43	120	168	200	240	**30**	1	82.4	13	2.00	442	0	623	0.5	36	0.6	H	1100	157.9	132	1.20	150	1.05	129	1.22
10		F80-0.0-19	1.43	120	168	200	240	**30**	1	85.2	13	2.82	343	0	623	0	-	-	H	1100	108.4	112	0.97	115	0.94	132	0.82
11		F80-0.5-19	1.43	120	168	200	240	**30**	1	86.1	13	2.82	343	0	623	0.5	36	0.6	H	1100	153.5	138	1.11	158	0.97	137	1.12
12		B1	0.31	90	**463**	500	145	**80**	1	35.7	10	1.93	440	0.42	375	1	30	**0.56**	S	-	375	326	1.15	348	1.08	323	1.16
13		B2	0.62	90	**463**	500	285	**80**	1	35.7	10	1.93	440	0.49	375	1	30	**0.56**	S	-	360	291	1.24	311	1.16	285	1.26
14		B3	0.93	90	**463**	500	430	**80**	1	35.5	10	1.93	440	0.48	375	1	30	**0.56**	S	-	291	252	1.16	267	1.09	262	1.11
15		B4	1.23	90	**463**	500	570	**80**	1	31.1	10	1.93	440	0.47	375	1	30	**0.56**	S	-	228	212	1.08	221	1.03	229	1.00
16	2	B5	1.85	90	**463**	500	855	**80**	1	31.5	10	1.93	440	0.49	375	1	30	**0.56**	S	-	183	184	0.99	190	0.97	185	0.99
17		B6	1.23	90	**463**	500	570	**80**	1	34.4	10	1.93	440	0.47	375	0	-	-	-	-	205	194	1.06	195	1.05	223	0.92
18		B7	1.23	90	**463**	500	570	**80**	1	33.8	10	1.93	440	0.47	375	0.5	30	**0.56**	S	-	220	208	1.06	213	1.04	229	0.96
19		B8	1.23	90	**463**	500	570	**80**	1	33.2	10	1.93	440	0.47	375	1.5	30	**0.56**	S	-	260	229	1.14	240	1.09	249	1.05
20		B9	1.51	90	**463**	500	700	**80**	1	29.5	10	1.93	440	0.48	375	1	30	**0.56**	S	-	224	198	1.13	203	1.10	226	0.99
21		B10	1.51	90	**463**	500	700	**80**	1	30.1	10	1.93	440	1.26	375	1	30	**0.56**	S	-	290	322	0.90	333	0.87	226	1.28
22	1	WO-1/1	0.58	80	**624**	650	360	100	1	40	10	0.81	418	0.43	304	1	30	0.4	H	-	345	316	1.09	351	0.98	334	1.03
																						Avg.	1.12		1.06		1.04
																						COV	12.9%		12.9%		13.4%

Notation: a = shear span; b = section width; d = effective depth; h = full depth; l_{b1} = width of loading plate parallel to beam axis; V/P = shear-force-to-applied-point-load ratio; f_c' = concrete cylinder strength; a_g = maximum size of coarse aggregates; ρ_l = ratio of flexural reinforcement; f_y = yield strength of flexural reinforcement; ρ_v = stirrup ratio; f_y = yield strength of stirrups; ρ_f = fibre volume ratio; l_f = fibre length; d_f = fibre diameter; S = straight fibres; H = hooked fibres; f_{yf} = yield strength of fibres; V_{exp} = measured shear strength; V_{S2PKT} = predicted shear strength from simplified kinematic approach; V_{2PKT} = predicted shear strength from 2PKT for complete shear behavior; V_{FEM} = predicted shear strength from nonlinear FEM analysis; COV = coefficient of variation.
Note: The bold numbers in the database are assumed values as the original values were not provided by the authors of the publication.

Figure captions

No.	**Caption**
Figure 1	Models for deep reinforced concrete beams; (a) Strut-and-tie model [4], (b**)** Two-parameter kinematic theory [6] and **(**c) Predicted response of an FRC deep beam without stirrups (Vs=0) based on the 2PKT [5]
Figure 2	2PKT for deep beams [6]
Figure 3	Fig. 3. Mechanisms of shear resistance in deep beams [5], [8]; f'_c = concrete compressive strength; k = crack shape factor; b = section width; a_g = maximum diameter of coarse aggregates; $\sigma_{v,avg}$ = average stress in the stirrups along the critical crack; ρ_v = stirrup ratio; α_1 = angle of the critical diagonal crack; E_s = modulus of elasticity of stirrups; ε_{yv} = yield strain of stirrups; f_{yv} = yield strength of stirrups; n_b = number of bottom longitudinal bars; d_b = diameter of longitudinal bars; l_k = length of bar-dowels; f_y = yield strength of longitudinal bars; ε_y = yield strain of longitudinal bars
Figure 4	Tension behavior of FRC (adapted from Voo and Foster [11])
Figure 5	Flow chart of the S2PKT solution procedure
Figure 6	Solution of S2PKT equations (test B7 by Mansur and Ong [2])
Figure 7	Variation of experimental-to-predicted strength ratios for 22 tests with; (a) span-to-depth ratio and (b) fibre volumetric ratio (Avg. = 1.12 and COV = 12.9% for 22 tests)
Figure 8	Variation of 2PKT complete behavior-to-simplified method strength prediction ratios for 22 tests with; (a) span-to-depth ratio and (b) fibre volumetric ratio (Avg. = 1.07 and COV = 5.70%)
Figure 9	Effect of a/d ratio (tests by Mansur and Ong [2])
Figure 10	Effect of fibre volumetric ratio (tests by Mansur and Ong [2])