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Modelling Cascading Failure of Bus-Subway Coupled Networks

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Abstract

Robustness studies on integrated urban public transport networks have attracted growing attention in recent years due to the significant influence of robustness on the overall performance of multimodal networks. In this paper, topological properties and robustness of a bus-subway coupled network in Beijing, composed of both bus and subway networks as well as their interactions, are analyzed. Three new models depicting cascading failure processes of the coupled network are proposed, based on an existing binary influence modelling approach. Simulation results show that the proposed models are more accurate than the existing method in reflecting actual passenger flow redistribution in the cascading failure process. Moreover, it was found that the larger the network size (measured by the number of nodes and edges), the more robust the coupled network is. The traffic load influence between nodes also plays a vital role in the robustness of the network. The proposed models and derived results can be utilized to improve the robustness of urban integrated public transport systems in traffic planning.

***Keywords*:** complex networks; robustness; coupled networks; cascading failure;

1. Introduction

Complex network theories have made a significant contribution to the understanding of complex networks [1-3]. A number of natural and artificial systems can be described as complex networks, such as food web, social networks, transport networks, and communication networks. In a complex network, nodes are the basic component of the system, whereas edges represent links between nodes. Research of complex networks has achieved rapid development after the discovery of small-world network effects and scale-free characteristics [3], and robustness of complex networks has become an essential research direction. Ref. [4, 5] investigated the difference in robustness between ER and scale-free networks, influenced by random failure and targeted attacks respectively. The studies show that scale-free networks are more vulnerable to targeted attacks but with high robustness to random crashes. The high robustness comes from extreme heterogeneity of node degree distribution of networks. In a scale-free network, the majority of the nodes have a small degree, with only a minority being featured with a large degree. Meanwhile, Holme et al. [6] studied the response of networks to certain attacks on nodes and edges. They found that removing essential nodes or edges would lead to changes in network structures and considerable decreases in robustness.

Many urban public transport networks have been investigated based on complex network theories, and robustness has been considered as an important measure of performance of networks [7,8,9]. The existing studies mainly focus on cascading failure modeling and single-mode transport networks, manifested by the following research. Wu et al. [10] analyzed the robustness of the bus system in Beijing under random failure and intentional attacks, Derrible et Kennedy [11] compared the robustness of subway systems among 33 cities worldwide, and Zhang et al [12] studied the robustness and topological characteristics of the Shanghai subway network. Alongside, Rodríguez-Núñez et al. [13] introduced a methodology and examined the criticality and robustness of the Madrid subway system, while Ferber et al. [14] adopted different attack strategies on 14 cities' bus transport networks and examined the influence of removing critical nodes on the structures and robustness of the networks. Moreover, Schafe et al. [15] proposed a proactive measure to enhance the robustness of heterogeneously traffic loaded networks against cascading failure, based on load-dependent weights. Zhang et al. [16,17,18] studied cascading failure of a weighted public transit network with a two-layer (including a physical and a logical layers) structure. They proposed a cascading-failure-based mesoscopic reliability model that takes congestion effects and user equilibrium evacuation into consideration. All the above-described studies improve the understanding of robustness of different urban public transport systems from the perspective of transportation science.

However, from passengers' perspectives, public transport networks, including buses, trams, light rail trains, subways, etc. are linked together forming a multimodal integrated transport environment, in which travelers can transfer between stops and stations of different networks. Research has also been conducted for the analysis of robustness of integrated networks. The representative work includes the followings. Cats et al. [19] studied the robustness of an integrated public transport system consisting of subway lines, trunk bus lines, and light rail train lines. Dynamic and stochastic robustness measures were developed with the consideration of interactions among traffic flows of the different lines (networks) in the system. Berche et al. [20] examined the robustness of the urban public transport systems across 14 cities around the world under both random and intentional attacks. In each city, the stations of different networks were regarded as homogeneous nodes in the integrated system. Moreover, Ferber et al. [21] merged urban bus, subway, and tram networks into one system based on the L space model, and compared the robustness of the systems between London and Paris. Jin et al. [22] investigated ways to enhance the robustness of a subway network through the integration of localized bus services. In the study, a two-stage stochastic programming model was developed, aimed at evaluating the inherent robustness of the subway network and improving the localized bus-subway integration strategy.

The studies on the robustness of integrated multimodal transport systems have provided great values in understanding interactions among different networks and identifying strategies to improve robustness. Nevertheless, there are certain limitations regarding the existing research. ???. In an integrated multimodal network, any disruption or event occurring on a station in one network (e.g., a bus network) will cause passengers to transfer to a station in another network (e.g., a nearby station in a subway network). Since the traffic flow volume varies in different networks, the spreading strength of disruption or events from one network to another is different. Thus, it is more accurate to describe multimodal transport networks as multi-layer complex networks, in order to reflect the uneven transfers and interactions between different networks. The importance of adopting multi-layer networks has been demonstrated by the research [23], in which the bus-subway coupled network in Beijing was represented as a two-layer network. To investigate the cascading failure process of the two-layer network, the authors developed a binary influence modelling approach which takes into account the uneven transfers and interactions between bus and subway systems. The experimental results showed the effectiveness of the approach in revealing the mechanism of the cascading failure process in the coupled system.

However, in the approach [23], once a node j in a network (e.g., a station in the subway system) fails, the whole potentially redistributed passengers on station j will be redirected to a neighboring node i in the other network (e.g., a station in the bus system). This leads to the load influence (i.e., the total potentially redistributed passenger volume) of node j all being re-imposed on node i. It's not true in real transport circumstances. In most cases, the passengers on station j will travel to the different neighbor stations of j, including station i, since travelers have different destinations. Therefore, the influence of node j on node i should only be a part of the total influence of node j on all its (i.e. node $j's$) neighbor nodes. To address the problems, in this paper, we extend the existing modelling approach by introducing three new strategies to more accurately reflect the load influence between nodes of different layers in the coupled network.

The fundamental contributions of this work lie in the following areas. (1) Three new models characterizing cascading failure processes of a coupled two-layer network (i.e., the bus-subway network in this study) are proposed, and the results are analyzed and compared between these models and the existing approach. (2) Variables including network sizes and traffic load influence have been identified as important factors affecting the robustness of the coupled network. The larger the coupled network size (i.e., the total number of nodes and edges), the more robust the network is. The load influence between nodes of different layers also plays an essential role in the cascading failure process. (3) One of the proposed models, which considers both the network size and load influence, was found to have the best performance in estimating the final relative cascading size (i.e., the fraction of the finally failed nodes to all nodes in the coupled network). This model can be utilized to improve the robustness of coupled networks in traffic planning.

The rest of this paper is organized as follows. In section 2, a two-layer coupled public transport network is presented, and the existing modelling approach [23] as well as the new models are described. In Section 3, simulation of the cascading failure process on the Beijing bus-subway coupled network is performed, and the results are analyzed. Finally, in Section 4, major conclusions are drawn and the application of the proposed models is discussed.

2. Methods and Models

2.1 Public transport network (PTN) representations

There are two common network representations for a public transport system: Space P and Space L. In the Space L scenario, a node is a station, and an edge is a route link between two nodes. If two nodes are adjacent at least on one route, there will be an edge between the two nodes. In the Space P scenario, a node is a station too, an edge links any two nodes on one route. This leads to all nodes on a route forming a complete network, i.e., a subgraph of the whole network. In this study, the Space L scenario is adopted to describe the topology of bus or subway systems.

Even with a comprehensive public transport system of buses or subways, in most cases, passengers cannot yet accomplish an entire trip from origins to destinations traveling only by bus or by subway. It's still necessary to consider a travel plan that combines both types of modes (given that both types of systems are established in a city). For rational travelers, transfers between buses and subways are constrained by an acceptable walking distance from a bus (subway) station to a subway (bus) station. Based on the above consideration, a public transport network (PTN) consisting of both bus and subway systems is modeled as a two-layer unweighted and undirected network. The two layers represent the bus and subway systems respectively, and they are independently represented by the Space L scenario. The interlayer links between the two systems are determined by a radius parameter γ. In this paper, $\gamma = 0.5$km for Beijing, according to China Urban Street Traffic Planning Standard (GB50220-1995) and China Subway Design Standard (GB50157-2013). Any bus stations, no further than γ away from a certain subway station, are connected to the subway station, and vice versa.

The data of the bus and subway systems in Beijing were collected in November 2019 from the website of the Beijing public transport company [24, 25]. The bus system consists of 1135 bus routes with 7797 stations, while the subway network is comprised of 24 subway lines accommodating 333 stations. According to the transfer restriction ($\gamma = 0.5$km), 311 interlayer (transfer) links between bus and subway stations in the coupled network are derived.

2.2 Cascading failure modelling

PTN is a typical real-world complex network, facing the challenge of cascading failure. In case of function failure occurring on one station (node), the passenger flow passing through the failed station will be redistributed to its neighbor stations. If the passenger flow on a neighbor station exceeds the capacity of the station, the neighbor station fails too and its passenger flow will be redistributed again, and so on. As the redistribution process gradually expands outward from the original failure station (node), it could cause, in the worst condition, all stations in the PTN to fail.

In this study, we first adopt the existing binary influence modelling approach [23], and refer it as M1 model. It should be noted that the passenger flow influence between two nodes in different layers varies from that between two nodes in the same layer. For instance, the passenger flow redirected from a subway station to a bus station is normally greater than that from a bus to a subway stations, since the per hour volume of passenger flow on a subway station is generally larger than that on a bus station. M1 model introduced an influence parameter (i.e., coupling strength) to reflect the difference of the influence between nodes. A node can be found in only two states: 0 and 1. State 0 is a normal state, while state 1 is a fail state in which the node is broken down. Each node has a given capacity (δ). If the traffic load on a node is larger than its capacity, the node fails (state = 1); otherwise, the node works as normal (state = 0). Initially, all the nodes are assumed to be in normal states. When a small number of nodes breaks down, it will cause traffic flow redistribution on related nodes, potentially triggering further cascading failure in a larger scale of the network. The cascading failure of M1 model can be detailed as follows.

(1) *Initialization*. Set $t = 0$ and the states of all the nodes as 0.

(2) *State changing*. For each node with state = 0, if the traffic load on the node is greater than its capacity δ, the state of the node will change to 1. Otherwise, the state remains 0.

(3) *Stationary state*. Once the state of a node turns to 1, the state remains stationary and will never change back during the cascading failure process.

(4) *End*. The cascading process will continue until no more state changes happen in the network.

In the coupled network, the state changing of a node i is described as

$$x_i^{\mathrm{M1}} = \begin{cases} 0 \left(if \colon \dfrac{\sum_{j \in \Gamma_i} \rho_{ij}^{\mathrm{M1}} x_j^{\mathrm{M1}}}{k_i} < \delta \right) \\ 1 \left(if \colon \dfrac{\sum_{j \in \Gamma_i} \rho_{ij}^{\mathrm{M1}} x_j^{\mathrm{M1}}}{k_i} \geq \delta \right) \end{cases} \qquad (0 < \delta < 1) \qquad (1)$$

where, x_i^{M1} is the state of node i in M1 model, j is a neighbor node of i, Γ_i is the neighborhood (node set) of i, k_i is the degree (i.e., the number of adjacent nodes) of i. Variable ρ_{ij}^{M1} is the coupling strength, reflecting the extent to which a node j affect its neighbor node i, and representing the redistribution of passenger flow from node j to node i. This variable is defined as

$$\rho_{ij}^{\mathrm{M1}} = \begin{cases} 1 & (i,j \in N_{bus}\ or\ i,j \in N_{subway}) \\ h & (i \in N_{bus}, j \in N_{subway}) \\ \frac{1}{h} & (i \in N_{subway}, j \in N_{bus}) \end{cases} \quad (2)$$

where, N_{bus} and N_{subway} represent the node (station) set of the bus and subway layers, respectively. Variable h is a transfer factor, defined as the ratio between the passenger volume transferring from subway to bus stations and that from bus to subway stations. This variable reveals the extent of the asymmetry of the transfer volume on the interlinks between the bus and subway layers. Yang et al. [23] found that the cascading size (i.e., the number of nodes that fail) grows with coupling strength, and when the capacity $\delta \leq 0.3$, the cascading size will spread to the whole network.

It was noted that M1 model takes $\rho_{ij}^{\mathrm{M1}} x_j^{\mathrm{M1}}$ as the influence of node j on node i. This implies that in the coupled network, the whole potentially redistributed passengers on station j will be redirected to station i. As described in Section 1, this is not realistic in actual public transport situations. In most cases, the passengers on station j will travel to the different neighbor stations of j, including station i, since travelers have different destinations. Therefore, the influence of node j on node i should only be a part of the total influence of node j on all the neighbor nodes of j. To address the problems, three new models are proposed based on M1 model, in order to more accurately characterize the influence between nodes in the coupled network.

(1) an *equalized redistribution model* (M2). It's assumed that the passenger volume on node j is evenly redistributed onto the neighbor nodes of j. Accordingly, the coupling strength from j to i is defined as

$$\rho_{ij}^{\mathrm{M2}} = \frac{\rho_{ij}^{\mathrm{M1}}}{k_j}. \quad (3)$$

(2) a *degree preferential redistribution model* (M3). It's assumed that the passenger volume on node j is preferentially redistributed onto the neighbor nodes of j, according to the degree of the neighbor nodes. The coupling strength from j to i is defined as

$$\rho_{ij}^{\mathrm{M3}} = \frac{\rho_{ij}^{\mathrm{M1}} k_i}{\sum_{v \in \Gamma_j} k_v}. \quad (4)$$

(3) a *degree and M1 coupling strength preferential redistribution model* (M4). It's assumed that the passengers on node j is preferentially redistributed onto the neighbor nodes of j, according to the degree and coupling strength of the neighbor nodes. The coupling strength from j to i is defined as

$$\rho_{ij}^{\mathrm{M4}} = \frac{\rho_{ij}^{\mathrm{M1}} k_i}{\sum_{v \in \Gamma_j} \rho_{iv}^{\mathrm{M1}} k_v}. \quad (5)$$

The state changing rule for models M2, M3, and M4 has a similar form to that for M1, i.e.,

$$x_i^{(**)} = \begin{cases} 0 \left(if\colon \sum_{j \in \Gamma_i} \rho_{ij}^{(**)} x_j^{(**)} < \delta \right) \\ 1 \left(if\colon \sum_{j \in \Gamma_i} \rho_{ij}^{(**)} x_j^{(**)} \geq \delta \right) \end{cases} \quad (0 < \delta < 1), \quad (6)$$

where $(**)$ represents one of the models M2, M3, and M4.

It was noticed that, compared to M1, more and more real features of the passenger redistribution process are accommodated into the models M2, M3 and M4, with models M2 and M3 considering the topology of the network while model M4 depicting both the topology and load influence between nodes.

3. Results

To understand the structural characteristics of the Beijing bus-subway coupled network, four variables including degrees, clustering coefficient (i.e., ?), node betweenness (i.e., ?), and shortest path length (i.e., ?) are computed based on the data described in Section 2, and the distribution of the variable values are presented in Fig. 1. The coupled network consists of 8130 nodes and 12650 edges in total. Fig. 1 (a) shows the degree distribution with the maximum degree $k_{max} = 18$ and the average degree $\overline{k}$=3.11. The $p(k)$ reaches its peak at $k = 2$, which means there are about 50% nodes in the coupled network. Fig. 1 (b) describes the cumulative distribution of clustering coefficient with the average clustering coefficient $\overline{c}$ = 0.128. According to Fig. 1 (c), the longest shortest path length is $l_{max} = 165$, and the probability distribution of shortest path length falls a Poisson distribution with the distribution parameter (λ=110). In Fig. 1 (d) the probability distribution of node betweenness with the log-binning horizontal axis is displayed. These results reveal that although each subway station couples with several bus stations, the topological-trip-distance of passengers is still unchanged, and we could find that the introduction of the subway network to the bus network did not change the randomly connected manner.

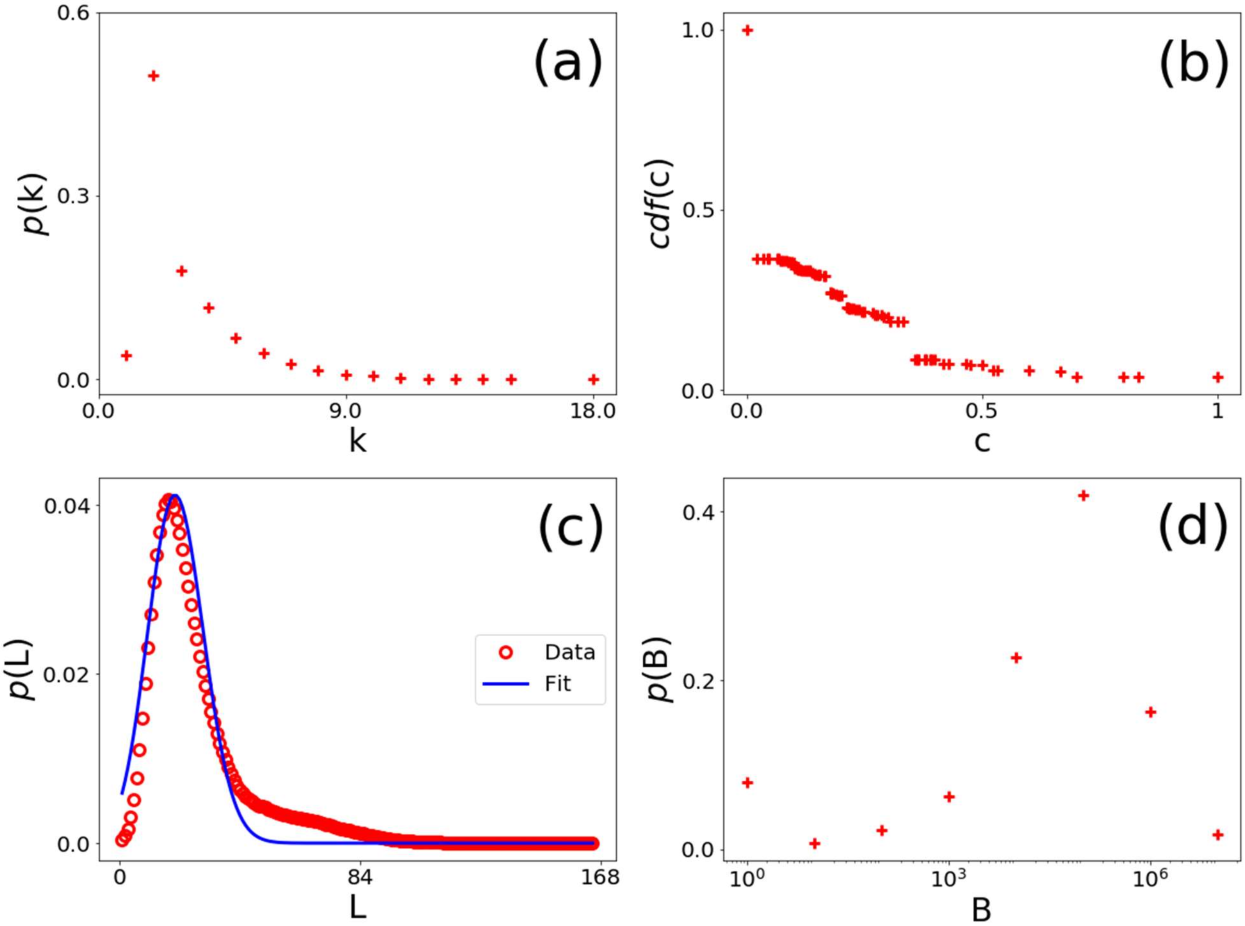

Fig. 1. Topological characteristics of the Beijing bus-subway coupled network. (a) probability distribution of degrees, (b) cumulative distribution of cluster coefficient, (c) probability distribution of shortest path length, (d) probability distribution of betweenness (log-binning).

Simulation of the cascading failure process on the Beijing bus-subway coupled network is performed, based on the above four models M1, M2, M3, and M4 respectively. In this process, the capacity δ is set as 0.5, and the influence parameter h as an integer chosen from the range [1, 50]. Since the per hour passenger volume on a subway station is usually larger than that on a bus station, the simulation is focused on the initial failure of subway stations. To measure the failure process, we define p as the fraction of the initial failure subway nodes to all nodes in the subway network, and define the relative cascading size as the fraction of the final bus and subway failure nodes to all nodes in the coupled network. Fig. 2 and Fig. 3 present the simulation results, with each data point corresponding to an average over ten times of independent simulation.

Fig. 2(a)-2(d) show the relative cascade size as a function of p and h ($h = 1,2,3,4,50$) for the four models respectively. For each model, as the initial failure p increases, the relative cascading size rises until reaching the extremum at $p = 1.0$. This is straightforward, as the larger the initial failure triggered by random or intentional events on the subway system, the worse the final damage of the whole bus-subway system. Nevertheless, it was observed that the rate of the rise in the cascading size slows down as p increases. Similarly, for each model, as h increases, the relative cascading size grows too. But the growth rate decreases for M1 and M2, fluctuates for M3, and remains almost unchanged for M4.

In the existing study [23], the same model M1 was applied to the same bus-subway network in Beijing, but the data of the coupled network were collected in 2013, 6 years older than the year (i.e. 2019) when the data in the current study were gathered. When the results obtained in [23] are compared with the ones (in Fig. 2 (a)) derived from M1 using the current data, these two groups of results show consistency in terms of the general changing trend of the cascading size as p or h changes. However, there is a large difference in the absolute extremum of the cascading size for each given value of h. For example, when $h = 50$, the extremum is about 0.35 (Fig. 2 (a)), which is less than half of the corresponding extremum value (i.e., 0.735 for $h = 50$) obtained in [23]. Actually, in Beijing, the bus system increases from 611 routes (3845 stations) in 2013 [23] to 1135 routes (7797 stations) in 2019, while the subway increases from 8 lines (144 stations) in 2015 [23] to 24 lines (7797 stations) in 2019. It's an encouraging result that, with the growth of the bus-subway coupled system, its total robustness increases more than doubled. The derived results also imply that the larger the size of the coupled network, the more the failure tolerance (robustness) is.

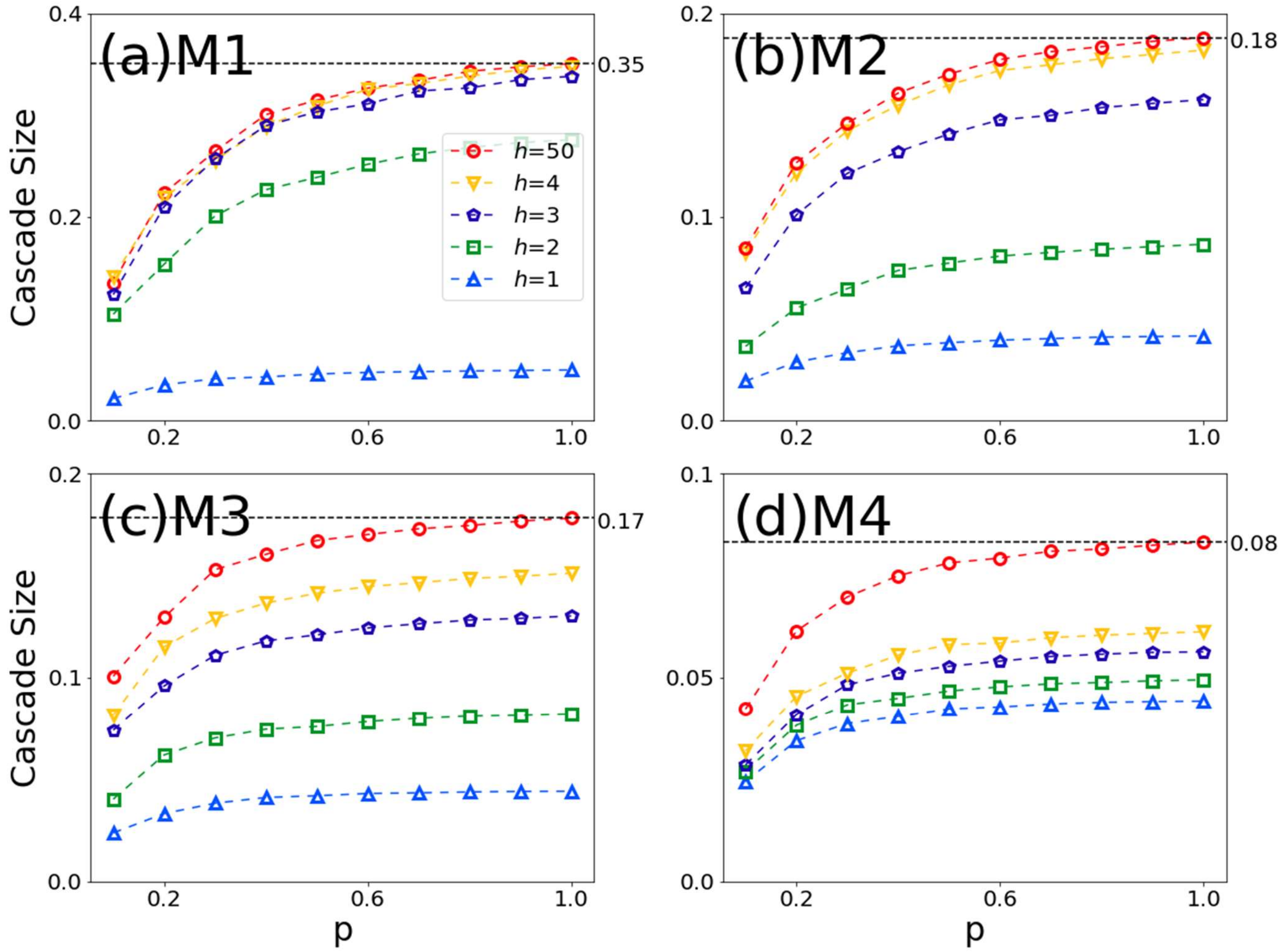

Fig. 2. Relative cascading failure size as a function of initial failure nodes.

When all the four models are compared with each other, differences were also noticed. Fig. 3 illustrate the differences among the models for $h = 1$ and $h = 50$ respectively. When the transfer volume on the interlinks between the bus and subway networks is almost symmetric ($h = 1$) (Fig. 3(a)), the final relative cascading size for each given value of p is similar (i.e., the extremum being in the range of 0.041-0.049) across the models. Particularly, M2 and M3 have almost the same cascading performance (i.e., both having 0.044). When the transfer volume is asymmetric remarkably ($h = 50$) (Fig. 3(b)), the cascading size of M2 and M3 is almost the same still (the extremum as 0.17 and 0.18 respectively), but about the half of that (0.35) for M1. The cascading size (0.08) of M4 is about one fifth of M1. This suggests that the final relative cascading size is overestimated by M1 model. Because M1 model [23] focused on revealing the mechanism of the cascading process in the bus-subway coupled system, some real traffic transfer details were not taken into account. In this paper, we propose M2, M3 and M4 models by introducing more passenger redistribution features on the bus-subway transfer stations. The aim is to improve M1 model and fill in the gap between the existing modeling approach and the real bus-subway coupled system.

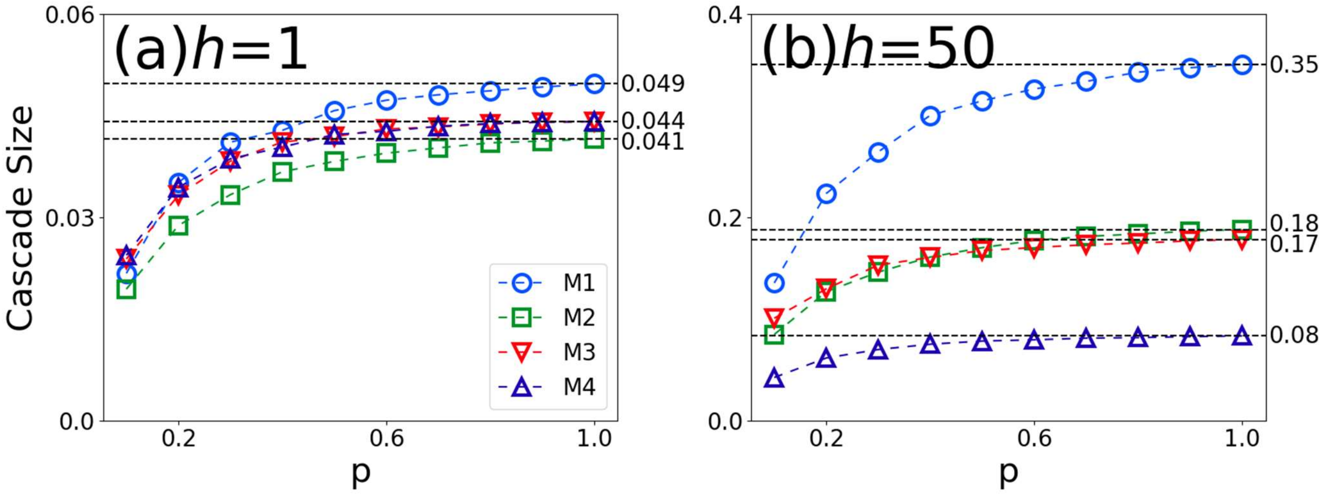

Fig. 3. Relative cascading failure size as a function of initial failure nodes.

4. Conclusions

In this paper, the cascading failure process of the Beijing bus-subway coupled network is analyzed, and three cascading failure models are proposed. The new models overcome the weak point of the existing modelling approach (M1 model), by characterizing the mechanism of the passenger flow redistribution in a way that is more consistent with the real transport situations. Simulation results show that the new models predict the final relative cascading size more realistically, whereas M1 model overestimates the total damage. Moreover, it was found that, the larger the coupled network size (i.e. the number of nodes and edges), the more robust the network is. The load influence between nodes of different layers also plays an important role in cascading failure processes. Accordingly, the model (M4) that considers both the network size and load influence shows the best performance in estimating the final cascading size, and the model can be utilized to improve the robustness of coupled networks in traffic planning.

The developed models and the results arisen from this study can be applied to the analysis of cascading failure of other combined public transport systems (apart from the bus-subway coupled network). They also provide practical suggestions on how to estimate the influence of cascading failure triggered by a tiny number of initial failure nodes as well as on how to engineer a more robust networked public transport system.

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