Simulation Evaluation of Threshold-Based Bus Control Strategy Under the Mixed Traffic Condition

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Abstract—Implementing operational strategies is a sustainable and effective way for providing good bus serviceability, but the effects of the strategies on other traffic participants, e.g., cars, have drawn little attention. This paper thus aims to explore the effects of the mutual interference between buses and private cars, when applying a bus control strategy to a regular transit line. Three threshold-based strategies are compared, including (a) holding control (HC); (b) limited boarding control (LBC) and (c) holding combined with limited boarding control (H-LBC). A cellular automaton based model is proposed to depict the interaction between cars and buses, and the model parameters are calibrated using data collected from a real-life bus route in Beijing, China. The control strategies are evaluated by their benefits for three stakeholders involved in the system, including passengers, bus operator and car drivers. Simulation results show that a good bus control strategy does not only improve the efficiencies of bus operating and passenger travel, but also speed up the car running by alleviating traffic congestion. In turn, the car volume is an important factor when setting the optimal parameter for control strategies. Moreover, the comparison suggests that H-LBC outperforms the other two strategies in improving the service level for passengers, buses as well as cars, especially under crowded scenarios.

I. Introduction

Prompting mode sharing rates of buses is recognized as one of the most practical and sustainable ways for the development of transportation systems. However, bus operation processes suffer from the problem of bus bunching, i.e., several buses traveling together. This makes the services inefficient since larger headways between buses make people wait for more time, while a delayed bus traveling at slower average speed causes longer passenger in-bus time. Transit scholars and operators have devoted considerable efforts implementing some control strategies to improve the reliability of bus services, e.g., bus holding [1], skip-stop [2], limited boarding [3] and speed regulation [4].

Among the existing control strategies, bus holding has been viewed as a simple yet effective way to reduce bus bunching. By delaying early-arriving vehicles, buses with shorter headways (e.g., less than 10 min) can arrive at stops at the evenly-distributed time, while buses with longer headways (e.g., more than or equal to 10 min) can meet the timetable. There is rich literature on the development of the bus holding strategy models; the representative work includes the following. Eberlein et al. [5], Dessouky et al. [6] and Zhao et al. [7] extended the research from traditional static holding patterns (i.e., Osuna and Newell [8], Hickman [9]) to real-time dynamic control on a bus line; while Hernández et al. [10]

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and Schmöcker et al. [11] took the interaction among different bus lines into consideration to examine holding control effectiveness on multiline systems. Koehler [12] developed a workable optimization method with low computational complexity, which allowed for real-time use; and Bartholdi and Eisenstein [13] suggested a more self-coordinating holding method, which could deal with emergencies where large disruptions in ridership occur. Moreover, Wu et al. [14] went even further in the development of the holding strategy by considering both vehicle overtaking and distributed passenger boarding behavior, which could lead to much better headway regularity. The above-described studies have advanced the bus holding strategy models and greatly reduced bus bunching, leading to potential benefits for both passengers and operators. Nevertheless, longer travel time caused by holding buses at controlled stops could also bring inconvenience to passengers and reduce the total time savings achieved. This leads to less applicability of the holding models in practice. To deal with this vital problem, integrated control considering holding and other services has aroused attention. The state-of-the-art development of the integration can be typified by the following work. Cortés et al. [15] and Sáez et al. [16] respectively developed hybrid predictive control formulation and implemented control strategies considering both holding and skip-stop control. Their results show potential savings of 20% and 10% in the total travel time for passengers when the proposed strategies are tested. However, a real-timeskip-stop decision may give rise to extra transfer trips for passengers whose destinations are located at the skipped stops, which makes the service less attractive. To overcome this weakness, Delgado et al. [17], [18] proposed a more advanced model by combining both holding and limited boarding control strategies. In this model, passengers can alight at their destination stops but sometimes are constrained to enter a bus with a non-full load. Their experiment results demonstrated that the integrated control outperforms the holding control alone in terms of comfort and reliability for both passengers and operators.

In this paper, we will further extend the existing research on the integrated strategies of both holding and limited boarding control (H-LBC). Note that, the studied control is only applied to high-frequency bus lines, thus is headway-based. To the best of our knowledge, the existing bus holding studies are normally limited to the condition of exclusive bus lanes; little is known about the usefulness of the control methods when the bus lanes are mixed with other classes of traffic. However, in reality, bus operation is often heavily impacted by other vehicles, e.g., cars, especially for general lanes without bus exclusive right. Ignoring the impact of other participants may lead to improper bus control design. To address this issue, we will in this study make a first significant innovation by applying the integrated H-LBC strategy to a bus route that is under mixed traffic conditions. The aim is to find appropriate control settings to ameliorate the operation conditions of both buses and private cars. Three important research questions will be investigated, including (a) how the integrated control affects the three players involved in the system, i.e., passengers, bus companies and car drivers; (b) what are the differences between the control settings for dedicated bus corridors and those for mixed traffic systems; and (c) whether the integrated control is always more beneficial for mixed traffic systems than the single control (i.e., either holding or limited boarding control) is.

To find solutions to the above questions, the key issue to answer is: how to determine holding points (or limited boarding points) and how long the bus needs to wait at a given control point (or how many passengers are allowed to board on a bus). To address this issue, Eberlein et al. [5], Delgado et al. [17], [18] and Cortés et al. [15] made decisions based on a mathematical control formulation model with an explicit objective function e.g., minimizing total passenger waiting time; whereas Fu and Yang [19] and Yu and Yang [20] adopted a threshold-based control mode, where buses are held based on the deviation of their headways from the desired ones. Including the interaction between buses and cars in optimization control strategies would lead to the problem being intractable or time-consuming to solve, which makes it less useful in real-world applications. Therefore, the threshold-based control mode proposed in [19], [20] is chosen herein, and it will be combined with the integrated H-LBC control strategy.

Various methods have been adopted in the existing threshold-based control mode to determine the bus holding time at stops. The conventional threshold-based control makes decisions only based on vehicle information on the stop points, and holds a bus until the preceding headway (headway between the current bus and its preceding bus) is up to the threshold. In the latest development of the techniques, many other factors are taken into account in the control decision-making process, e.g., using real-time data gathered from Automatic Vehicle Location (AVL) or Automatic Fare Collection (AFC) system. For example, apart from the preceding bus headway, the two-headway-based control introduced by Fu and Yang [19] considers the following headway (headway between the current bus and its following bus) as well. The dynamic control proposed by Yu and Yang [20] does not only guarantee even headway distributions at the current stop, but also maintains on-time performance of the controlled bus operation at the next stop. Thus, we will in this study utilize the advantages of the existing methods [19], [20], and combine them into a new dynamic threshold-based mode. This represents the second novelty of our research. Moreover, this paper also makes the third significant contribution in terms of developing an innovative threshold-based limited boarding strategy.

To evaluate the effectiveness of the H-LBC strategy under mixed traffic conditions, simulation will be adopted to analyze bus operational control models. Some ready-made software systems, e.g., Paramics, SimTransit and BusMezzo, have been developed to simulate bus operation [19]–[21]. However, in these systems, certain operation rules for vehicles or passengers are pre-defined and not flexibly modifiable. To make up for the deficiency, we develop a novel cellular automaton (CA) simulation framework to characterize the operation of a bus route in a mixed two-lane system. The braking light (BL) CA model addressed by Knospe et al. [22] deserves much attention because it can reproduce real traffic phenomena. Recently, an improved BL model has been proposed by Tian et al. [23], and it considers the deceleration capabilities of vehicles. In our framework, the model in [23] will be adopted as the vehicle moving rules, and novel rules for lane changes will be further proposed. This makes up the fourth contribution of our work.

The remaining parts of this paper are organized as follows. Sect. II introduces the typical transit system targeted in this research, while Sect. III presents the CA simulation framework for the system. In Sect. IV, the principles and functions of the threshold-based holding and limited boarding control are described, and in Sect. V, the effectiveness of the different control modes is compared based on a real-world bus line. Finally, in Sect. VI, major conclusions are drawn, and future research is pointed out.

II. System Characteristics

The system underlying our model is a heterogeneous two-lane corridor (Fig. 1). Buses are allowed to drive only on the right lane, while cars can travel freely on both lanes. There are N_s stops in total in the system, indicated by $i = 1,2,\ldots,N_s$. Buses visit the stops in increasing order (from stop 1 to stop N_s). For a specific stop, the first visiting bus is bus m, and the next is $m-1$. Let l denote the lanes; $l = 1$ refers to the left lane and $l = 2$ refers to the right one. The planned bus headway is assumed to be a constant (H_0) in the concerned time interval. The notations are summarized in Table 1.

III. Simulation Model Description

In this section, we adopt a cellular automaton model to depict the kinetic characteristics of buses and cars. For simplicity, we consider the following assumptions:

- The buses have two doors, with one for boarding and the other for alighting. The passenger serving time for a bus at a stop is the maximum value between the boarding time and the alighting time.
- The boarding passengers obey the discipline of the first arrival first on.
- The passengers at a stop always board the first arrival bus, unless the bus reaches vehicle capacity or boarding limits.

We apply the CA model to portray the vehicle kinematics. Both the lane and the time are discrete in such a model. The distance and the velocity are in cells and cells/time step, respectively [24]. Assume that the current time step is t, the next time step is $t+1$.

At each time step, a passenger arrives at stop i with a fixed probability λ_i. A new arrival passenger at stop i will increase the number of waiting passengers: $W_i(t) = W_i(t-1)+1$. While once the passenger gets on a bus, $W_i(t) = W_i(t-1)-1$.

A. Vehicle Forward Operation

a) Vehicle Moving

The modeling of vehicle movement is mainly based on the work of Tian et al. [23].

Step 1: Acceleration. If vehicle n and its previous vehicle $n+1$ are not braking, and their distance is large enough, vehicle n may accelerate:

If $(b_{l,n+1}(t) = 0$ and $b_{l,n}(t) = 0)$ and $t_h \geq t_s$, then:

$$v_{l,n}(t+1) = \min(v_{l,n}(t) + a_k, v_{k,\max})$$

else:

$$v_{l,n}(t+1) = v_{l,n}(t). \tag{1}$$

Where $t_h = d_{l,n}/v_{l,n}(t)$ represents the time needed to reach the nearest obstacle in front (vehicle $n+1$ or a bus stop). $t_s = \min(v_{l,n}(t), t_c)$ stands for the safety gap of vehicle n which prevents a driver from reacting to the braking light of a distant predecessor that is very far away. t_c determines the range of interaction with the brake light. $d_{l,n}$ is the real spatial distance between adjacent vehicles or the gap between a bus and its next stop: $d_{l,n} = x_{l,n+1} - x_{l,n} - len_k$ if vehicle n is a car, otherwise, $d_{l,n} = \min(x_{l,n+1} - x_{l,n} - len_k, S_{e_m} - x_{l,n})$.

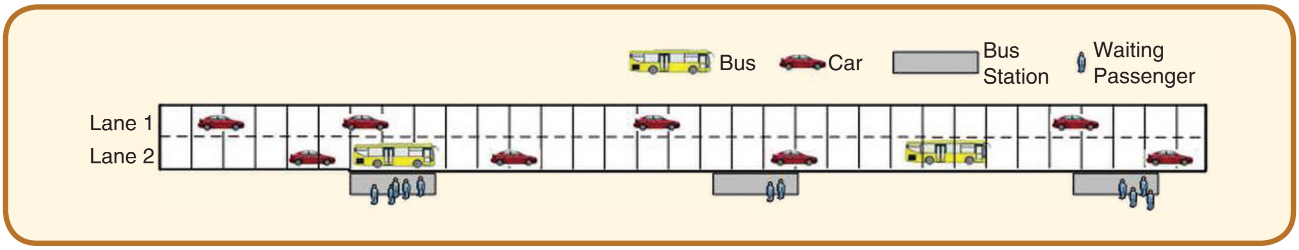

FIG 1 Schematic illustration of a bus corridor on a dual-lane urban road.

Note that $S_{e_m} - x_{l,n}$ denotes that each bus should dwell at stops, where S_{e_m} is the next stop location for bus m (vehicle n).

Step 2: Deceleration. In the BL models [22], [23], the deceleration action is limited by the effectiveness distance $d_{l,n}^{\text{eff}}$ instead of the real spatial distance $d_{l,n}$, to consider the anticipation effects of the preceding vehicle $n+1$. Set velocity restriction $(\lceil d_{l,n}^{\text{eff}}/T \rceil)$ when vehicle $n+1$ is too close, then:

$$v_{l,n}(t+1) = \min(\lceil d_{l,n}^{\text{eff}}/T \rceil, v_{l,n}(t+1)), \tag{2}$$

where, $\lceil x \rceil$ is the minimum integer larger than x; $T > 1$ is the desired time gap that vehicles hope to keep, which reflects the fact that cars tend to move more slowly to avoid unrealistic oversized deceleration in the next time step. $d_{l,n}^{\text{eff}} = d_{l,n} + \max(v_{\text{anti}} - gap_{\text{safety}}, 0)$ is the effective distance between vehicle n and $n+1$. Where $v_{\text{anti}} = \min(d_{l,n+1}, v_{l,n+1})$ is the expected velocity of vehicle

Table 1. List of notations.

Indices and Parameters:			
h^*	Holding control parameter, $h^* \in [0,1]$	S_i	Location of stop i
s^*	Limited boarding control parameter, $s^* \in [1, +\infty)$	λ_i	Passenger arrival probability for stop i at every second.
ρ	Car entry probability for one lane, $\rho \in [0,1]$	p_i	The proportion of passengers alighting at stop i
l	Index of lanes, $l \in \{1,2\}$	$T_{\max}^{\text{hold}}$	The maximum holding time of a bus at each stop
n	Index of vehicles, including buses and cars.	len_k	The length of vehicles with the vehicle type k
m	Index of buses	$v_{k,\max}$	The maximum speed of vehicles with the vehicle type k
i	Index of bus stops	a_k	Acceleration capacity of the vehicle type k
k	Vehicle type. $k=1$ represents car, $k=2$ represents bus	T	The desired time gap that vehicles hope to keep
t	Index of time	gap_{safety}	A vehicle parameter controls the effectiveness of the anticipation
N_s	Total number of bus stops	t_c	A parameter that determines the range of the interaction with the brake light
H_0	The desired bus headway	p_0	The probability reflects the delay-to-start behaviors of some vehicles locates on the downstream front of the traffic jam
C	The bus capacity	p_b	The probability considers the impacts of the decelerating vehicle in close front
κ	Additional dwell time at a bus stop, e.g., time to open and close doors	p_d	The probability for all other situations
α, β	Average alighting, boarding time per passenger respectively		
Auxiliary Variables Regarding the Vehicle Moving:			
$x_{l,n}$, $v_{l,n}$	Position, velocity of vehicle n on lane l, respectively	$p_{l,n}^{\text{rand}}$	The randomization probability of vehicle n on lane l to reflect the stochastic deceleration behavior
$d_{l,n}$, $d_{l,n}^{\text{eff}}$	Distance, effective distance between vehicle n and its preceding vehicle $n+1$ on lane l, respectively	t_s, t_h	Safety time gap, time headway between vehicle n and its preceding vehicle $n+1$, respectively
$b_{l,n}(t)$	Braking status of vehicle n on lane l at time t, $b_{l,n}(t)$=1 (or 0) means the brake light is on (or off)	v_{anti}	The expected velocity of the preceding vehicle $n+1$ in the next time step
Auxiliary Variables Regarding bus Dwelling at Stops:			
e_m	The next stop of bus m	$On_{m,i}^{\max}$	The maximum allowed number of boarding passengers for bus m at stop i
$\tau_{m,i}$	The time for passengers boarding and alighting of bus m at stop i	$On_{m,i}$, $Off_{m,i}$	The actual number of boarding, alighting passengers for bus m at stop i, respectively
$A_{m,i}$, $D_{m,i}$	Arrival, departure time of bus m at stop i respectively	$W_i(t)$	Total number of passengers waiting for bus at stop i at time t
$T_{m,i}^{\text{hold}}$	The holding time of bus m at stop i	$\psi_{m,i}$	Total number of passengers on bus m before arriving at stop i
$\overline{On}_{m,i}$	The potential number of passengers who can enter into bus m when the bus just arrives stop i.		

$n+1$ at time $t+1$; and gap_{safety} controls the effectiveness of the anticipation.

Step 3: Randomization. Due to drivers' random behavior, vehicle n may decelerate at the next time step.

If $(rand() < p_{l,n}^{\text{rand}})$, then:

$$v_{l,n}(t+1) = \min(v_{l,n}(t+1) - 1, 0), \tag{3}$$

where, $rand()$ is a random number smaller than 1; $p_{l,n}^{\text{rand}}$ is the randomization probability for vehicle n. If the vehicle is at rest, $p_{l,n}^{\text{rand}} = p_0$; if the brake light of the vehicle in front is switched on and it is found within the interaction horizon, $p_{l,n}^{\text{rand}} = p_b$; otherwise, $p_{l,n}^{\text{rand}} = p_d$. Thus

$$p_{l,n}^{\text{rand}} = \begin{cases} p_b : \text{if } b_{l,n+1}(t) = 1 \quad \text{and} \quad t_h < t_s \\ p_0 : \text{if } v_{l,n}(t) = 0 \\ p_d : \text{in all other cases} \end{cases}. \tag{4}$$

Step 4: Determining braking status and positions at the next time step.

$$b_{l,n}(t+1) = \begin{cases} 1 & : \text{ if } v_{l,n}(t+1) < v_{l,n}(t) \\ 0 & : \text{ if } v_{l,n}(t+1) > v_{l,n}(t) \\ b_{l,n}(t) & : \text{ if } v_{l,n}(t+1) = v_{l,n}(t) \end{cases} \tag{5}$$

$$x_{l,n}(t+1) = x_{l,n}(t) + v_{l,n}(t+1) \tag{6}$$

Step 5: Dwell decision making. If vehicle n is a bus (bus m) and arrives at location $x_n = S_{e_m}$, it will dwell.

b) Bus Dwelling

If the time gap between two neighbor buses is beyond a threshold, a holding or limited boarding control may be triggered. For bus m dwelling at stop i:

① $A_{m,i} < t \le A_{m,i} + \kappa + \tau_{m,i}$: The process of the door opening, passenger serving and the door closing.

If there is no limited boarding control, $On_{m,i}^{\max}$ is only constrained by vehicle capacity: $On_{m,i}^{\max} = C - \psi_{m,i} + Off_{m,i}$; otherwise, it will be set in Section 4.1. Where $Off_{m,i} = p_i \psi_{m,i}$ stands for the number of alighting passengers.

The passengers already waiting at stop i along with the subsequent newcomers can get on bus m until the number of number of boarding passengers, then the passenger serving time is $\tau_{m,i} = \max(\beta \cdot On_{m,i}, \alpha \cdot Off_{m,i})$, and the number of passengers on bus m and at stop i is updated as: $\psi_{m,i+1} = \psi_{m,i} + On_{m,i} - Off_{m,i}$ and $W_i(t) = W_i(t-1) - On_{m,i}$.

Holding or not: If limited boarding control is not triggered, we check whether or not the bus needs to be held, and determine its departure time $D_{m,i}$ and holding time $T_{m,i}^{\text{hold}}$ according to Section 4.2. If no holding control is needed, bus m will depart immediately after passengers being served, and $D_{m,i} = A_{m,i} + \kappa + \tau_{m,i}$; otherwise, it will be held for $T_{m,i}^{\text{hold}}$.

② $A_{m,i} + \kappa + \tau_{m,i} < t \le D_{m,i}$: The bus is held at the stop, and no passengers are allowed to board.

B. Car Lane-Changing Motion

Buses are only allowed to travel on Lane 2, thus the lane changing rules are just applied to cars. Here we consider symmetric lane changing rules, which include incentive criterion and safety criterion.

- **Incentive criterion**: A car has the incentive to change lanes to guarantee a non-decreasing movement.
  1) If $(b_{l,n+1}(t) = 0$ and $b_{l,n}(t) = 0)$ and $t_h \ge t_s$, a car may accelerate its movement in the next step, then
     1.1) If $v_{l,n}(t) \le \lceil d_{l,n}^{\text{eff}}/T \rceil < v_{l,n}(t) + a_k$ and $d_{\text{other},n}^{\text{eff}} > d_{l,n}^{\text{eff}}$ (the car cannot accelerate on lane l, while the condition on the other lane is better), then
        ① If $b_{\text{other},n+1}(t) = 0$ or $t_{\text{other},h} \ge t_s$ (a non-decreasing movement can be guaranteed on the other lane), the car wishes to change lane.
        ② Otherwise, it does not wish to change lane.
     1.2) If $v_{l,n}(t) > \lceil d_{l,n}^{\text{eff}}/T \rceil$ and $d_{\text{other},n}^{\text{eff}} > d_{l,n}^{\text{eff}}$ (the car would decelerate if continuing running on lane l, while the condition on the other lane is better), the car wishes to changes lane.
     1.3) Otherwise, it does not wish to change lane.
  2) In all other cases, the car cannot accelerate:
     2.1) If $v_{l,n}(t) > \lceil d_{l,n}^{\text{eff}}/T \rceil$ and $d_{\text{other},n}^{\text{eff}} > d_{l,n}^{\text{eff}}$, the car wishes to change lane.
     2.2) Otherwise, it keeps running on lane l.
- **Safety criterion**: A safe lane-changing action must be performed without threatening someone else's security.

$$d_{\text{other},n-1}^{\text{eff}} > v_{\text{other},\max}$$

Here, $d_{\text{other},n}^{\text{eff}}$ (or $d_{\text{other},n-1}^{\text{eff}}$) is the effective distance between vehicle n and its front vehicle $n+1$ (or back vehicle $n-1$) on the other lane; $t_{\text{other},h}$ is the headway between vehicle n and its front vehicle on the other lane; and $b_{\text{other},n+1}$ (or $v_{\text{other},\max}$) is the brake light status for the front vehicle (or the maximum speed of the back vehicle) on the other lane.

IV. Threshold-Based Bus Control

To limit the deviation of actual headways from the desired ones, we define an early threshold and a late threshold as the minimum and maximum time gap between neighbor buses, respectively. For the headway-based control, $h^* H_0 (0 \le h^* H_0 \le H_0)$ and $s^* H_0 (s^* H_0 \ge H_0)$ denote the early, late threshold, respectively. If the actual headway is smaller than $h^* H_0$, a bus holding action is taken; if it is larger than $s^* H_0$, a limited boarding control is needed; in all other cases, the bus needs no control actions. Therefore, $h^* (0 \le h^* \le 1)$ and $s^* (s^* \ge 1)$ can also be referred as the holding control parameter and the limited boarding control parameter, respectively. The following assumptions are made for the bus control:

- The first departed bus needs no control strategy because it can travel freely without being blocked by other buses.

- The control can be conducted at any stops except stop N_s, where no passengers wait for boarding.

A. Limited Boarding Control

When bus m arrives at stop i, if the difference between the expected departure time of bus m and that of the preceding bus $m+1$ is beyond the late threshold, i.e.,

$$[A_{m,i} + \max(\alpha \cdot Off_{m,i}, \beta \cdot \overline{On}_{m,i}) + \kappa] - D_{m+1,i} > s^* H_0, \quad (7)$$

some boarding behaviors will be restricted in late bus m. Note that, $\overline{On}_{m,i}$ is the number of potential boarding passengers at time $A_{m,i} : \overline{On}_{m,i} = \min\{C - \psi_{m,i} + p_i \psi_{m,i}, W_i(A_{m,i})\}$. In this case, the allowed maximum number of passengers is:

$$On_{m,i}^{\max} = \min\{On_{m,i}^{\text{limited}}, C - \psi_{m,i} + Off_{m,i}\}, \quad (8)$$

where

$$On_{m,i}^{\text{limited}} = \max\left\{\left\lceil \frac{\alpha \cdot Off_{m,i}}{\beta} \right\rceil, \left\lceil \frac{max(s^* H_0 + D_{m+1,i} - A_{m,i} - \kappa, 0)}{\beta} \right\rceil\right\}.$$

$\lceil (\alpha \cdot Off_{m,i})/\beta \rceil$ represents the number of boarding passengers during the alighting process; $\max(s^* H_0 + D_{m+1,i} - A_{m,i} - \kappa, 0)$ stands for the maximum passenger serving time for bus m to avoid late departure from stop i. Generally, the larger s^* is, the looser the control is. Positive infinite values of s^* refers to no limited boarding control. On the contrary, $s^* = 1$ represents the tightest control situation in which a bus takes limited boarding actions whenever its headway is larger than H_0. The situation may lead to passengers' antipathy and is thus usually not recommended, especially during peak traffic periods.

B. Holding Control

a) Calculation of Holding Time

Step 1: Determining holding action. At stop i, if the time gap between bus $m+1$ and bus m is less than $h^* H_0$, i.e.,

$$(A_{m,i} + \kappa + \tau_{m,i}) - D_{m+1,i} < h^* H_0, \quad (9)$$

then go to Step 2 to calculate $D_{m,i}$ and $T_{m,i}^{\text{hold}}$ for bus m. Otherwise, bus m is dispatched immediately, i.e., $D_{m,i} = A_{m,i} + \kappa + \tau_{m,i}$ and $T_{m,i}^{\text{hold}} = 0$. $(A_{m,i} + \kappa + \tau_{m,i})$ denotes the time that bus m has just finished the processes of the door opening, passenger serving and the door closing at stop i.

Step 2: Calculating holding time.

Step 2.1: Based on the rules proposed by Fu and Yang [19], we calculate the possible departure time $D_{m,i}$ of bus m from stop i.

If $\dfrac{\tilde{D}_{m-1,i} - D_{m+1,i}}{2} > h * H_0$,

$$D_{m,i} = D_{m+1,i} + h * H_0$$

else

$$D_{m,i} = D_{m+1,i} + \left(h * H_0 + \frac{\tilde{D}_{m-1,i} - D_{m+1,i}}{2}\right) \Big/ 2. \quad (10)$$

Step 2.2: Yu and Yang [20] stated that, if the forecasted departure time from stop $i+1$ would still be ahead of the scheduled one, the bus should be held at stop i; otherwise, it would be controlled for a short period or not controlled at all.

if $\tilde{D}_{m,i+1} - \tilde{D}_{m+1,i+1} < H_0$

$$D_{m,i} = D_{m,i}$$

else

$$D_{m,i} = \max\{A_{m,i} + \kappa + \tau_{m,i}, \quad D_{m,i} - (\tilde{D}_{m,i+1} - \tilde{D}_{m+1,i+1} - H_0)\} \quad (11)$$

Step 2.3: The holding time of bus m at stop i is

$$T_{m,i}^{\text{hold}} = \min[T_{\max}^{\text{hold}}, D_{m,i} - (A_{m,i} + \kappa + \tau_{m,i})]. \quad (12)$$

In the equation (10–12), $\tilde{D}_{m-1,i}$, $\tilde{D}_{m,i+1}$ and $\tilde{D}_{m+1,i+1}$ are the estimated departure time, which will be explained in the following subsection. Note that, $\tilde{D}_{m,i+1}$ is predicted based on the assumption that bus m is held at stop i till time $D_{m,i}$; if bus $m+1$ has departed from stop $i+1$, $\tilde{D}_{m+1,i+1} = D_{m+1,i+1}$. Moreover, $(\tilde{D}_{m-1,i} - D_{m+1,i})/2$ represents the average value of the previous headway and following headway for bus m.

The holding time calculation process indicates that the larger the value of h^* is, the tighter the control is. For example, $h^* = 0$ represents no holding control being taken, and $h^* = 1$ refers to the full control – a situation in which a bus is held whenever its headway is smaller than H_0.

b) Prediction of Bus Departure Time from Stops

We record travel time between stops of all buses, and derive the average travel speed v'_j between stops $j-1$ and j. In our simulation, v'_j is not time-dependent, and it will keep stable after simulation warm-up period. All the control actions are ignored in the prediction process.

The estimated arrival time for bus m (Suppose it is vehicle n on Lane 2, then its location is $x_{2,n}$) at the downstream stop j is:

$$\tilde{A}_{m,j} = \begin{cases} T_0 + (S_j - x_{2,n})/v'_j & ,j = e_m \\ \tilde{D}_{m,j-1} + (S_j - S_{j-1})/v'_j, & j > e_m \end{cases}, \quad (13)$$

and the estimated departure time is

$$\tilde{D}_{m,j} = \tilde{A}_{m,j} + \kappa + \max\{\alpha \cdot \tilde{Off}_{m,j}, \beta \cdot \tilde{On}_{m,j}\}. \quad (14)$$

Where, T_0 is the current time instant; $\tilde{On}_{m,j}$ and $\tilde{Off}_{m,j}$ are the estimated number of boarding and alighting passengers at stop j for bus m respectively, and they can be calculated as

$$\tilde{On}_{m,j} = \min\,[C - \tilde{\psi}_{m,j} + p_j\tilde{\psi}_{m,j}, W_j(T_0) + \lambda_j\tilde{A}_{m,j} - T_0], \quad (15)$$

$$\tilde{Off}_{m,j} = p_j\tilde{\psi}_{m,j}. \quad (16)$$

$\tilde{\psi}_{m,j}$ is the number of passengers on the bus before arriving at stop j, which can be estimated by

$$\tilde{\psi}_{m,j} = \begin{cases} \psi_{m,j}, & j = e_m \\ \tilde{\psi}_{m,j-1} - \tilde{Off}_{m,j-1} + \tilde{On}_{m,j-1}, & j > e_m \end{cases}. \quad (17)$$

V. Case Studies and Simulation Analysis

To validate the effectiveness of the H-LBC strategy, a numerical test is conducted in this section based on BRT (Bus rapid transit) Line 1 in Beijing, China. The line has 17 stops, and the real survey data for the line are listed in Table 2. The data include the average passenger arrival probability λ_i, alighting probability p_i and stop location S_i of each stop. For the estimation details of the three parameters see Huang et al. [2].

In the simulations, each cell length and time step corresponds to 1 meter and 1 s in reality, respectively. A car (or bus) occupies 6 (or 18) cells, travels with a maximum velocity of 22 (or 15) cells, and accelerates at $a_1 = 1\,\text{m/s}^2$ (or $a_2 = 2\,\text{m/s}^2$). Buses with the capacity of 180 passengers are departed every 3 min. Based on the real survey data, the additional dwell time κ, the average alighting time α and the average boarding time β are set to be 6 s, 1.5 s and 2.0 s, respectively. The maximum holding time is taken as $T_{\max}^{\text{hold}} = 90\,\text{s}$ (Cortés et al. [15]). Other parameters $p_d = 0.1$, $p_b = 0.94$, $p_0 = 0.5$, $t_c = 6\,\text{s}$, $T = 1.8\,\text{s}$ and $gap_{\text{safe}} = 10$ cells are adopted from the existing research (Tian et al. [23]).

Five indicators are used to evaluate the system performance, including the standard deviation of bus headways, the bus travel time, the passenger waiting time, the passenger in-bus time and the passenger weighted travel time. The value of the passenger waiting time is set to be double that of the in-bus time (Delgado et al. [18]), and the passenger weighted travel time is the sum of the passenger in-bus time and the doubled waiting time. Alongside the five indicators, two additional measures including the car flow rate and average car speed are also adopted for assessing car travel performance.

Randomness is guaranteed by averaging 50 replications. Each run is extended for 6 h with the first two-hour being considered as the warm-up period and thus removed from the analysis process.

A. The Impact of Control Parameters on System Performance

a) Dedicated Bus Corridor Systems

Fig. 2 presents the influence of h^* and s^* on the dedicated bus lane systems. The standard deviation of bus headways is used to capture the variability among the observed headways at all bus stops. From Fig. 2(a), we observed that the bus control is effective in regularizing headways, and the tighter control even performs better.

Secondly, good bus service reliability benefits decrease the passenger waiting time. This can be demonstrated in Fig. 2(b), where the tighter holding control with lower headway standard deviation contributes to less passenger waiting time. However, if the tighter limited boarding control is implemented, although more even-distributed bus headways are obtained, the passenger waiting time may have an increase due to the additional waiting time caused by passengers who are left behind. This is consistent with the results in Fig. 2(b) showing that the waiting time increases as s^* decreases.

Thirdly, the passenger in-bus time and bus travel time are computed as the average time taken for passengers and buses to move from their origins to destinations, respectively. The two indicators both include moving and stopping time during the travel along the entire route. As shown in Fig. 2(c) and (e), the effects of h^* and s^* on the bus travel time are similar to those on the passenger in-bus time; both types of the time go down as s^* decreases. H-LBC brings about the effects in two ways. One is to decrease the probability of the subsequent buses being blocked by the leading buses by means of mitigating bus bunches. In this case, the subsequent buses can speed up and further experience shorter running time.

Table 2. Average arrival probability λ_i, alighting rate p_i (%) and location S_i (km) for each stop.

Station	1	2	3	4	5	6	7	8	9
λ_i	0.19	0.12	0.03	0.03	0.06	0.09	0.06	0.05	0.03
p_i	0.00	0.66	2.46	3.14	1.41	1.84	1.17	1.05	1.53
S_i	0.30	2.10	2.80	3.60	4.60	5.30	5.80	6.20	6.80
	10	11	12	13	14	15	16	17	
λ_i	0.03	0.02	0.02	0.04	0.06	0.01	0.01	0.00	
p_i	5.75	3.29	15.08	21.95	14.16	3.97	10.62	100	
S_i	8.00	8.70	9.70	10.70	11.50	12.70	13.30	15.90	

The other is to increase (or decrease) the stopping time by holding buses at stops (or preventing individuals from boarding). Moreover, it is also noted that imposing strict holding control may encounter the problem that the additional holding time overwhelms the reduced moving time. For example, when $s^* \leq 1.6$ (in which the limited boarding control is strict enough, and the headway deviation is much reduced), a loose holding control ($h^* = 0.4$) action is

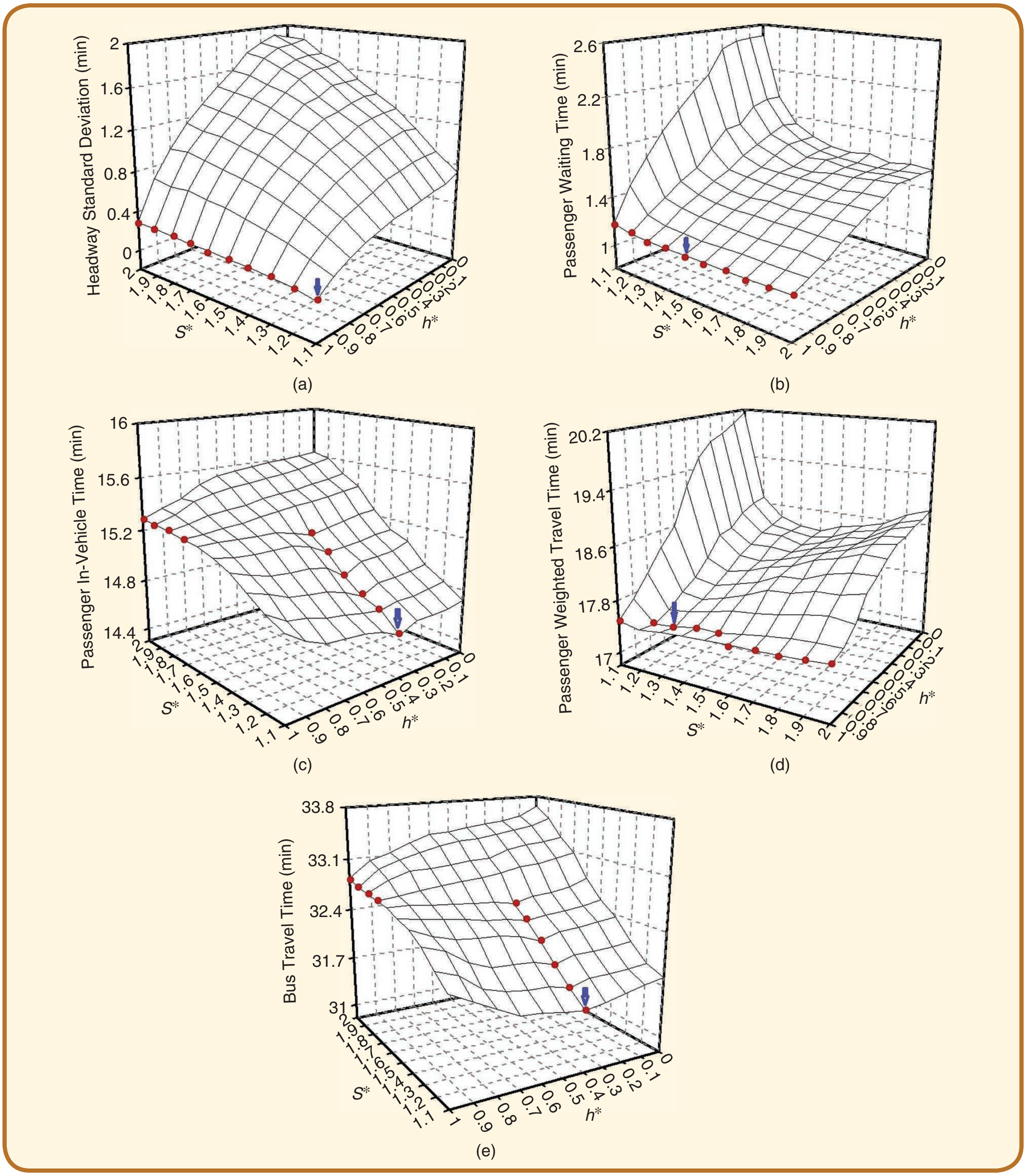

FIG 2 Performance measures for buses and passengers when implementing different degrees of H-LBC on a dedicated bus lane ($\rho = 0$). (The solid red dots indicate the minimum values for each case of s^*, of which pointed by a blue arrow is the minimum one). (a) Standard deviation of headway, (b) passenger waiting time, (c) passenger in-bus time, (d) passenger weighted travel time and (e) bus travel time.

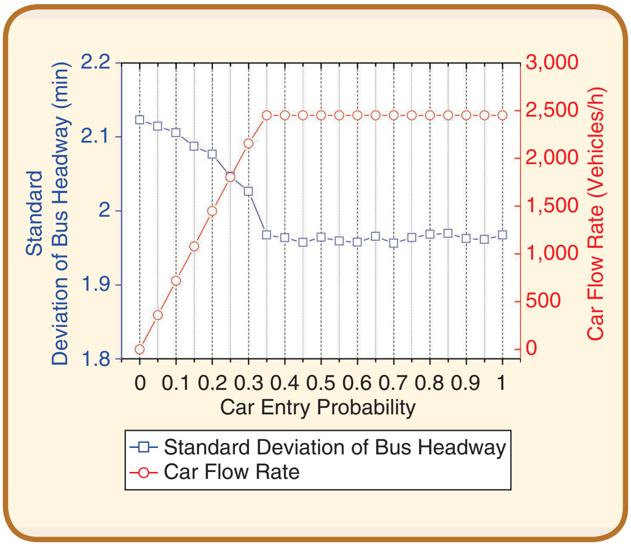

FIG 3 The effects of ρ on mixed traffic system with no control.

more beneficial in reducing the passenger in-bus time (or the bus travel time).

Fourthly, from Fig. 2 (d), we obtain that the passenger weighted travel time gets the minimum value when $h^* = 0.9$ and $s^* = 1.3$.

b) Bus Route Systems Mixed with Private Vehicles

As shown in Fig. 3, when ρ is smaller than the critical value 0.35, the mixed traffic system is uncongested as the car flow rate increases with the growth of the car entry probability; while it remains almost unchanged in a congested traffic state if ρ is larger than the critical value. The increased private cars will block the approach of buses more frequently, and further reduce the probability of bus bunching. Therefore, the standard deviation of bus headway decreases with car flow rate, which fits the results in Fig. 3. To further assess the effectiveness of H-LBC strategy under mixed traffic conditions, we examine the effects

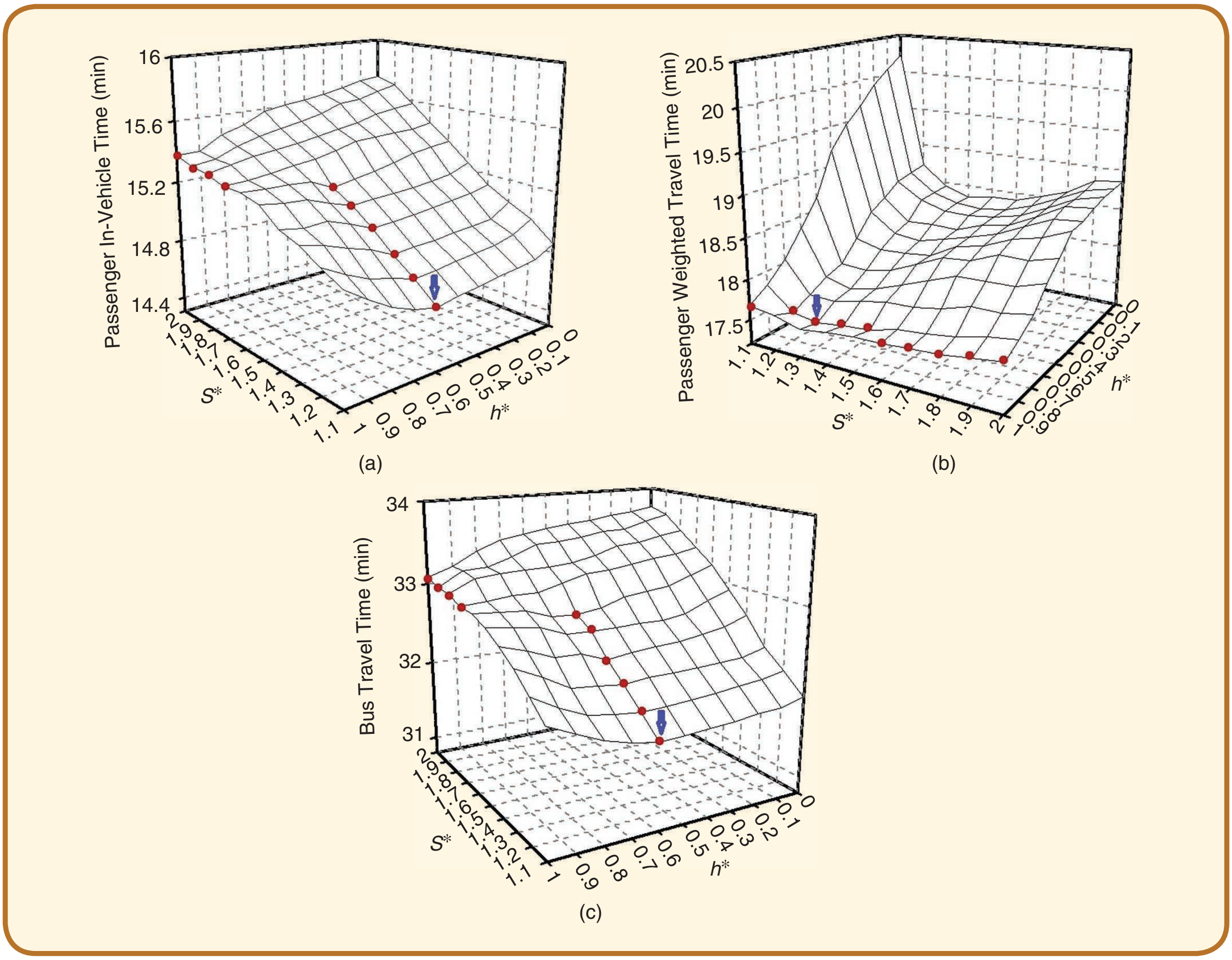

FIG 4 Performance measures for buses and passengers when implementing different degrees of H-LBC in the case of $\rho = 0.15$. (a) Passenger in-bus time, (b) passenger weighted travel time, and (c) bus travel time.

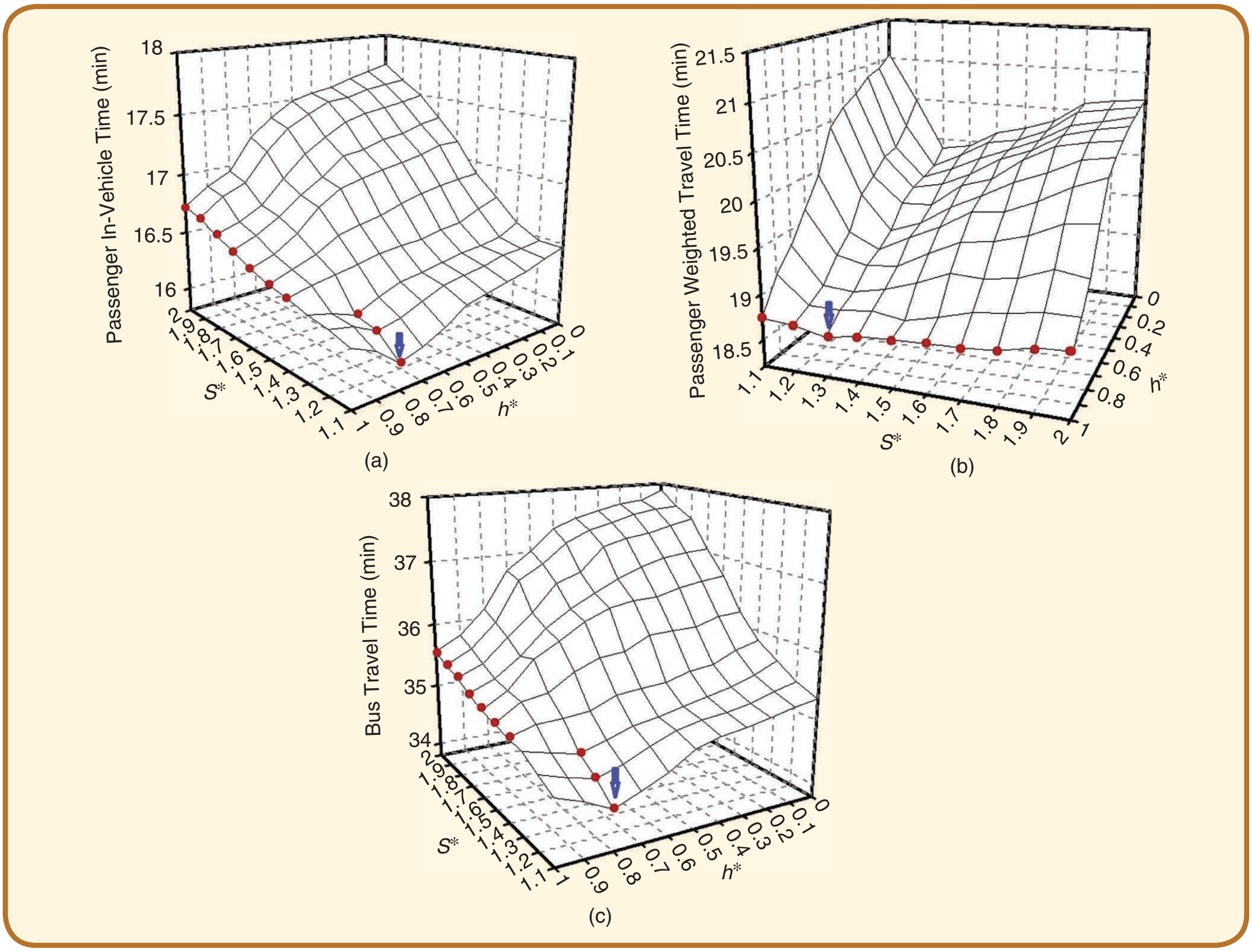

FIG 5 Performance measures for buses and passengers when implementing different degrees of H-LBC in the case of $\rho = 0.7$. (a) Passenger in-bus time, (b) passenger weighted travel time, and (c) bus travel time.

of h^* and s^* on the system with various car entry probabilities. For a given control scenario, the results show similar trends whatever the congestion level is. For brevity, only two congestion level cases are presented in the following, i.e., $\rho = 0.15$ and $\rho = 0.7$. Moreover, the simulation shows that the effects of h^* and s^* on the headway standard deviation or the passenger waiting time are independent of the car entry probability. Thus, the two indicators are not documented in this subsection.

1) The Effects on Passengers and Buses

From Fig. 4 and Fig. 5, we can conclude that to minimize the passenger in-bus time (or bus travel time), looser holding control is preferable under a less crowded traffic scenario. For example, when $s^* = 1.5$, h^* should get the value of 0.6 (or 1.0) for $\rho = 0.15$ (or $\rho = 0.7$), even though $h^* = 0.4$ is the best holding value in the case of $\rho = 0$. See Fig. 6, at congestion state, the standard deviation of travel time among buses decreases, which thus makes the travel time more predictable. In this case, fixing the problem all at once (holding the bus until it reaches the desired headway) is the best alternative because all buses will continue to travel at the same speed. Under noncongestion scenarios,

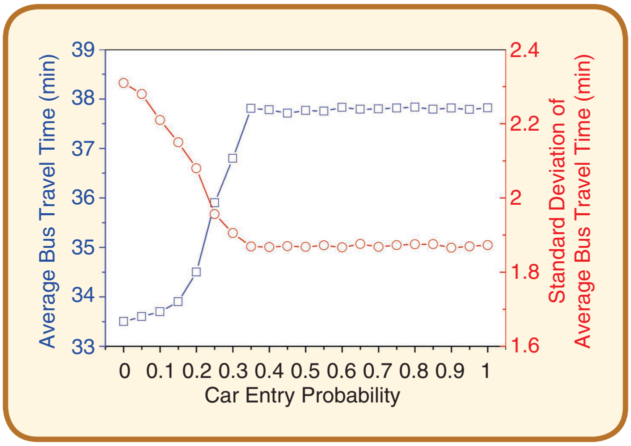

FIG 6 Average bus travel time and its standard deviation under different car entry probabilities.

buses can travel at different speeds, and trying to fix all at once is thus not the best alternative since it is an extreme action without taking into account the fact that conditions may change or be very different at downstream stops. Therefore, tighter holding control works better in a more congested scenario than in a noncongestion case.

Influenced by the passenger in-bus time, when the congestion level increases, the passenger weighted travel time is reduced under tighter holding control. For example, when $s^* = 1.3$, h^* should be 0.9 (or 1.0) for $\rho = 0.15$ (or $\rho = 0.7$) (see Fig. 4(d) and Fig. 5(d)).

2) The Effects on Private Vehicles

A bus control may also impact the operation of cars passing along the route. It is noted from Fig. 7(a) and (b) that, H-LBC has no obvious influence on the car flow rate, no matter what the congestion levels are. We also found from Fig. 7(c) and (d) that, under a low congestion level (e.g., $\rho = 0.15$), the average car speed does not vary with the different values of h^* and s^*. However, under more crowded traffic situations (e.g., $\rho = 0.7$), tighter control is beneficial to speed up the car operation.

To further explore the mechanism of control effectiveness on the car operation, we analyze typical vehicle trajectories under different services for $\rho = 0.15$ and $\rho = 0.7$ in Fig. 8 and Fig. 9, respectively. Given that there is no bus overtaking, bus bunches might occur. Buses with larger preceding headways are likely to be further delayed since they have to serve more passengers. When overtaking the slow-moving buses, certain following cars have to look for opportunities to merge into the other lane. The limited changing-lane opportunities may lead to the car crowdedness behind a bus. This phenomenon is not obvious when few cars travel along the route, as shown in Fig. 8. T cars can run freely before and after implementing a bus control. While for a crowded case, the speeds of the slow-moving buses would dominate the operation speed of Lane 2, indicating that all of the cars travel at a limited speed, see

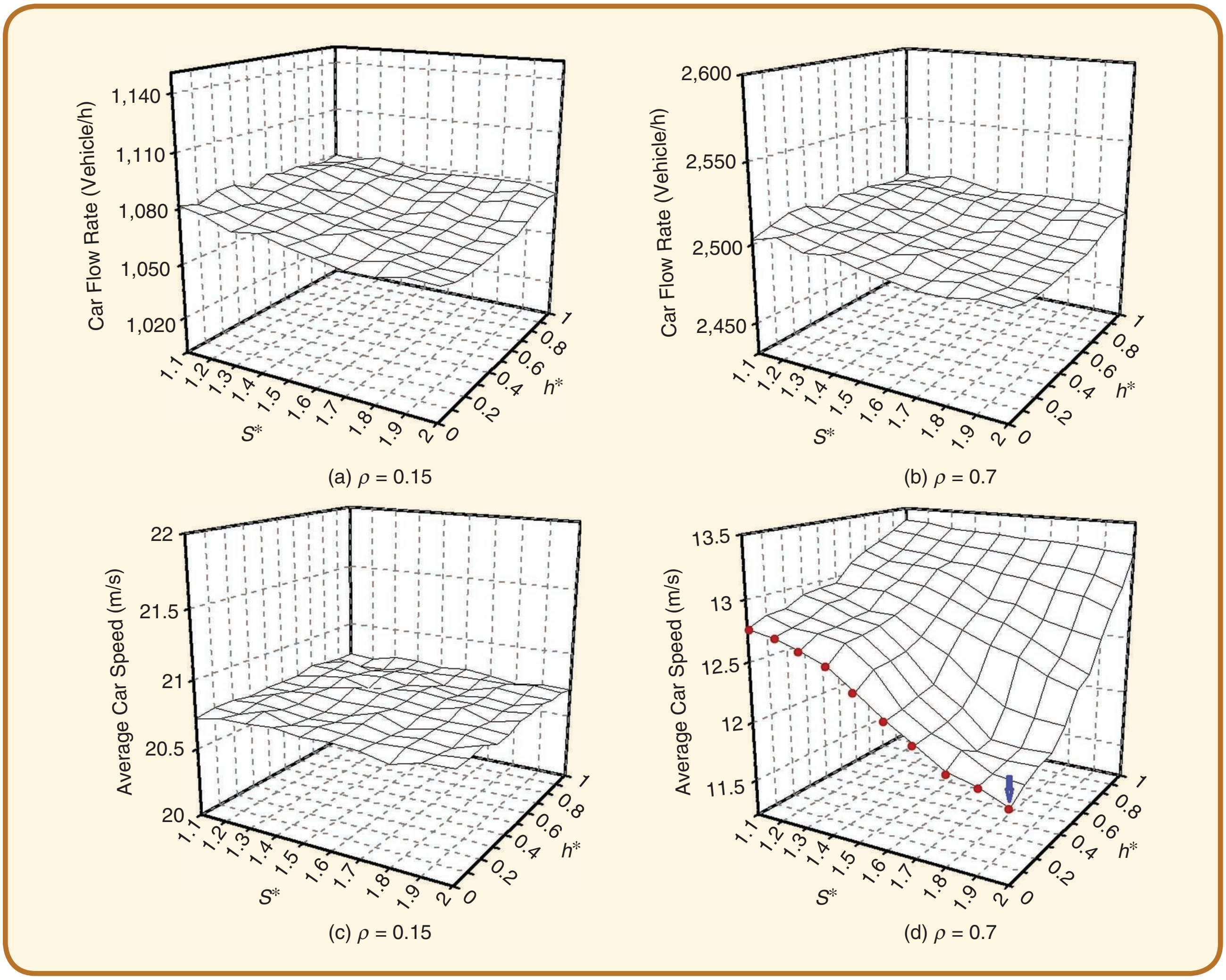

FIG 7 Performance measures for cars when implementing different degrees of H-LBC in the case of $\rho = 0.15$ and $\rho = 0.7$.

the reduced speed in figure 8(b) compared to figure 8(d). While if some controls are implemented, the evenly-distributed bus headways help speed up these delayed buses, and further improve the car operation.

B. The Optimal Values of h^* and s^*

The preferable values of h^* and s^* to optimize system performance regarding minimizing headway standard deviation and passenger waiting time as well as maximizing car flow rates and average car speed, are listed in Table 3. These measures are independent of car entry probabilities. While for other performance indicators that have a relationship with the entry probabilities, Fig. 10 presents their optimal values of the control parameters for different car entry probabilities.

Usually, the selection of the optimal values for the control parameters is determined by an explicit objective, e.g., travel-time minimization, operators' benefit maximization, total cost minimization. In this paper, we set the target is to minimize passenger weighted travel time. For example, the optimal control strength for H-LBC when $\rho = 0.7$ is $h^* = 1.0$ and $s^* = 1.3$.

C. Comparison of Control Strategies

In this subsection, we compare the performance of the following control strategies under different congestion levels:

- HC: $h^* = 1.0$ and $s^* = +\infty$.
- LBC: $h^* = 0$ and $s^* = 1.3$.
- H-LBC: the values of h^* and s^* are shown in Fig. 10(b).

Note that the optimal values of h^* and s^* for HC and LBC are derived from additional simulations. The performance comparison is made on the assumption of each of these strategies being implemented with its optimal degree of control.

Since the car flow rate is independent with degrees of bus controls, it is not discussed herein. Fig. 11 presents the percentage of changes in the other indictors concerning the noncontrol situation under different car entry

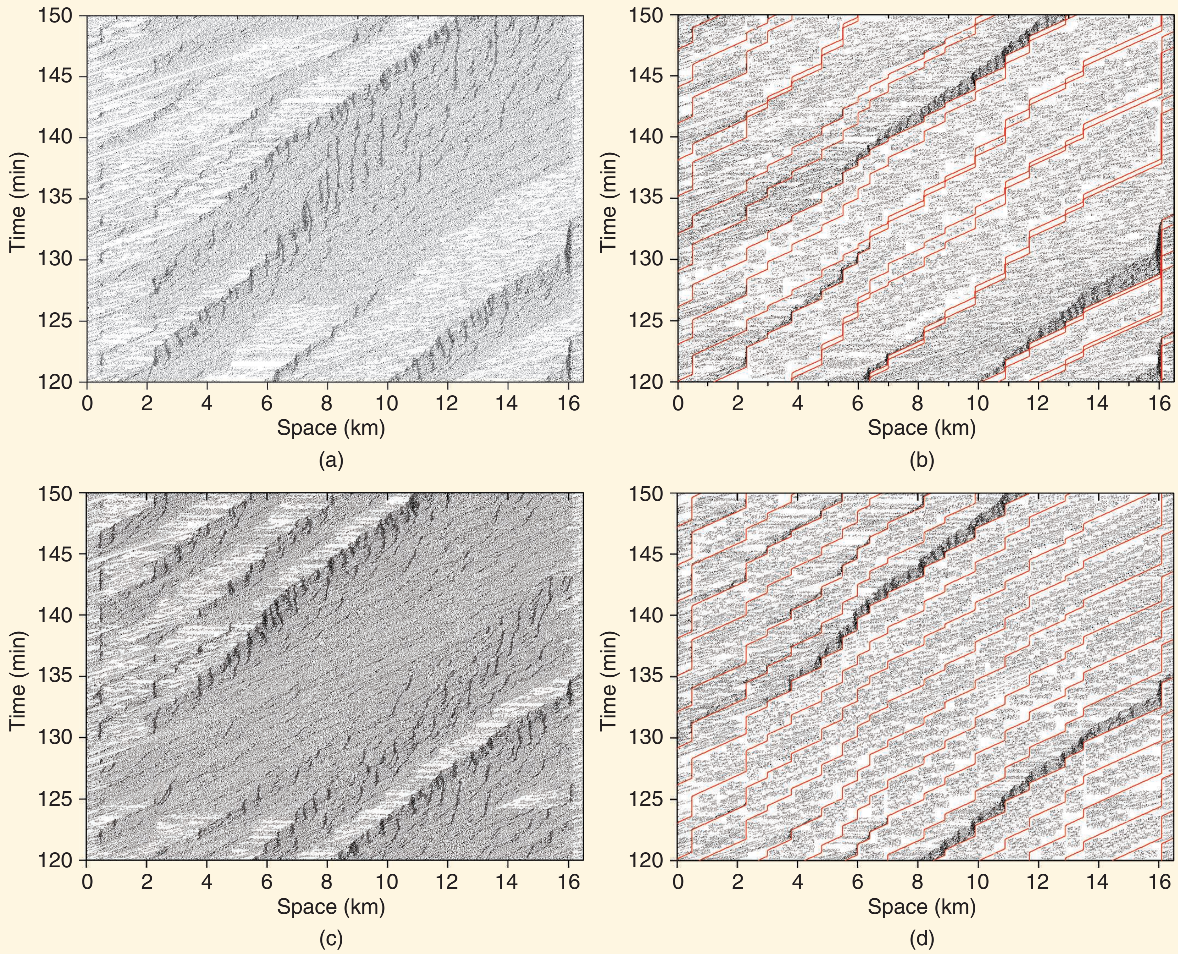

FIG 8 Trajectories of vehicles when $\rho = 0.15$ for (a) Lane 1 under no control; (b) Lane 2 under no control; (c) Lane 1 under H-LBC with $h^* = 1.0$ and $s^* = 1.3$; (d) Lane 2 under H-LBC with $h^* = 1.0$ and $s^* = 1.3$. (The black dots stand for cars, and the red dots represent buses.)

probabilities. From this figure, we make the following key findings.

Firstly, as illustrated in Fig. 11(a), under any traffic congestion levels, LBC has the weakest ability to improve service reliability, followed by HC that makes much more improvement. H-LBC performs slightly better than HC because it does not only hold the early buses but also speeds up the late ones.

Secondly, H-LBC is always the best time-saving strategy either for passengers or for buses. On the one hand, despite the negative influence from boarding limit control actions, which makes passengers wait longer, H-LBC has a good ability in the passenger waiting time reduction and performs just a little weaker than HC, as reflected in Fig. 11(b). On the other hand, H-LBC provides the largest savings in the passenger in-bus time, bus travel time, and passenger weighted travel time, as demonstrated in Fig. 11(c), (d) and (e).

Thirdly, when $\rho \leq 0.15$, the three control strategies show no significant differences in terms of car speed from the noncontrol case, as depicted in Fig. 11(f). This is because few cars can travel freely on the route. While under a more crowded scenario ($\rho > 0.15$), car crowdedness emerges, and the bus control with lower headway deviation will speed up the slow-moving buses and further increase the average car speed. Among these strategies, H-LBC performs best with the largest increase in car speed, and this is followed by HC and LBC.

Fourthly, the reduction of waiting time by H-LBC is more noticeable on a low congestion level ($\rho < 0.35$), see Fig. 11(b). This happens because buses under low congestion conditions suffer larger headway disruptions, which causes passengers to wait for a longer time. On the contrary, more significant improvements of other indicators by H-LBC (e.g., the passenger in-bus time, bus travel time, passenger weighted travel time and average car speed) are found on high congestion

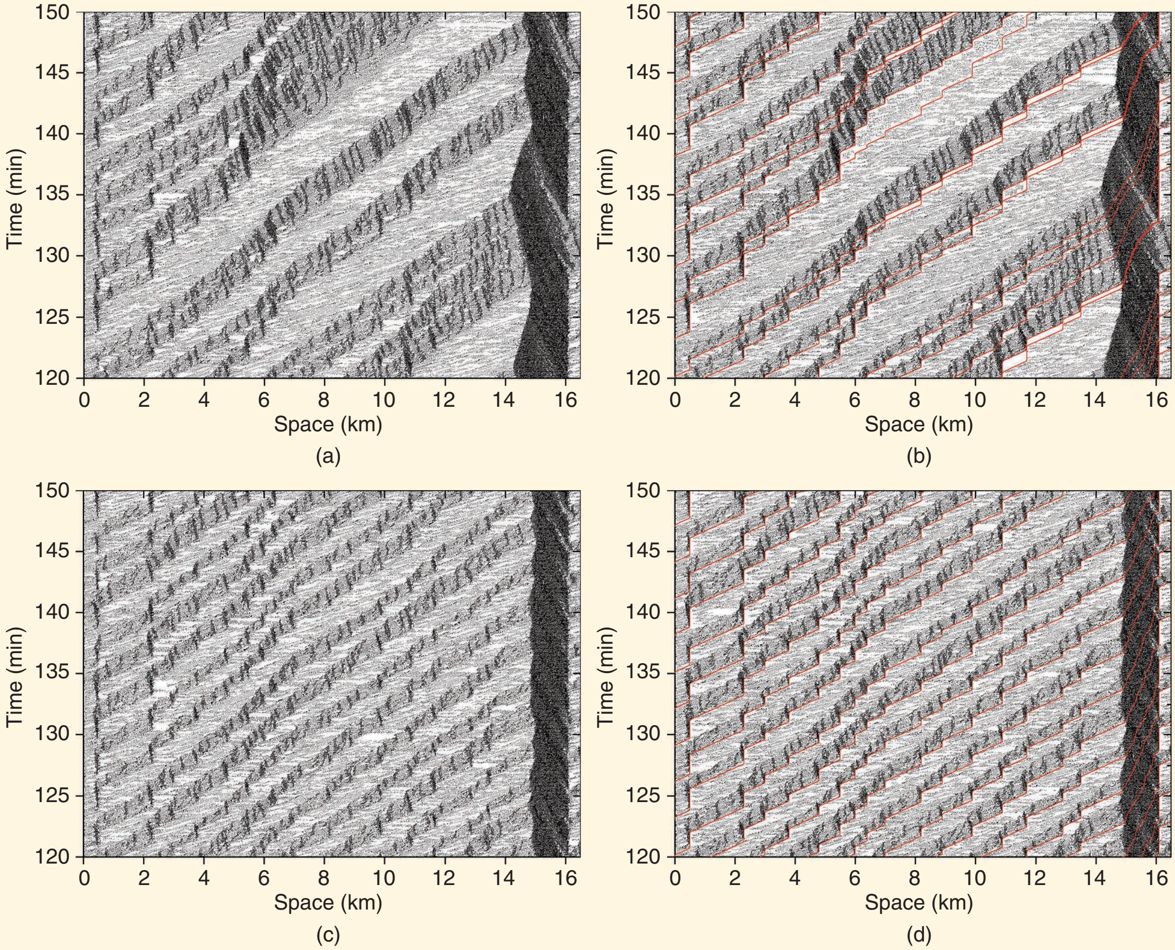

FIG 9 Trajectories of vehicles when $\rho = 0.7$ for (a) Lane 1 under no control; (b) Lane 2 under no control; (c) Lane 1 under H-LBC with $h^* = 1.0$ and $s^* = 1.3$; (d) Lane 2 under H-LBC with $h^* = 1.0$ and $s^* = 1.3$. (Unloading a large amount of passengers at Stop 17 (an important hub station) will compel the bus to occupy the lane 2 for a long time, thus resulting in a serious car jam nearby.)

levels ($\rho \geq 0.35$), see Fig. 11(c)–(f). As aforementioned, increased cars have to follow slow-moving late buses, which leads to more serious jam and further delays the vehicles in turn. This phenomenon goes worse in a more crowded situation. In this case, H-LBC can exert more influences.

VI. Conclusions and Further Work

A. Conclusions

The present research was conducted based on a cellular automaton simulation model built on a two-lane mixed traffic road. In this model, the operation of both cars and buses as well as their interaction are characterized. From the simulation results, the effects of the three threshold-based control strategies, including holding control, limited boarding control, and holding combined with limited boarding control, on the performance of bus route systems have been explored.

Using data collected from a real-world bus line, we implemented H-LBC strategy under different car entry probabilities. The numerical experiments demonstrate that tighter control is good for improving service reliability and speeding up car operation, but may result in both long passenger trip time (including the waiting, in-bus and weighted travel time) and long bus travel time. Moreover, the optimal control settings vary with congestion level. For example, a smaller value of the holding parameter is recommended to minimize the passenger in-bus time under a dedicated lane; while a larger one is preferred under mixed traffic conditions. Furthermore, the comparison among HC, LBC and H-LBC suggests that, H-LBC provides the most stable bus operation, the shortest trip time of buses and passengers, and the highest travel speed of social vehicles. It is also worth noting that, traffic participants, i.e., passengers, buses and cars, can enjoy larger travel time savings or speed increases by H-LBC under more crowded conditions.

B. Further Work

This study can be further enhanced by considering two other realistic cases to make the developed control strategies more applicable in real-world bus operation. Firstly, in the current analysis, bus overtaking is not allowed. However, overtaking is a very common situation in bus operation, especially at stops where the trailing bus finishes its dwelling process before the preceding one [14], [25]. The integration of bus overtaking into the strategies can thus provide valuable

Table 3. The preferable control parameters of H-LBC for some indicators.

Indicator	h^*	s^*
Headway standard deviation	1.0	1.1
Passenger waiting time	1.0	≥ 1.5
Car flow rate	-	-
Average car speed	1.0	1.1

Note: "-" represents the values of control parameters make no obvious difference on flow rate.

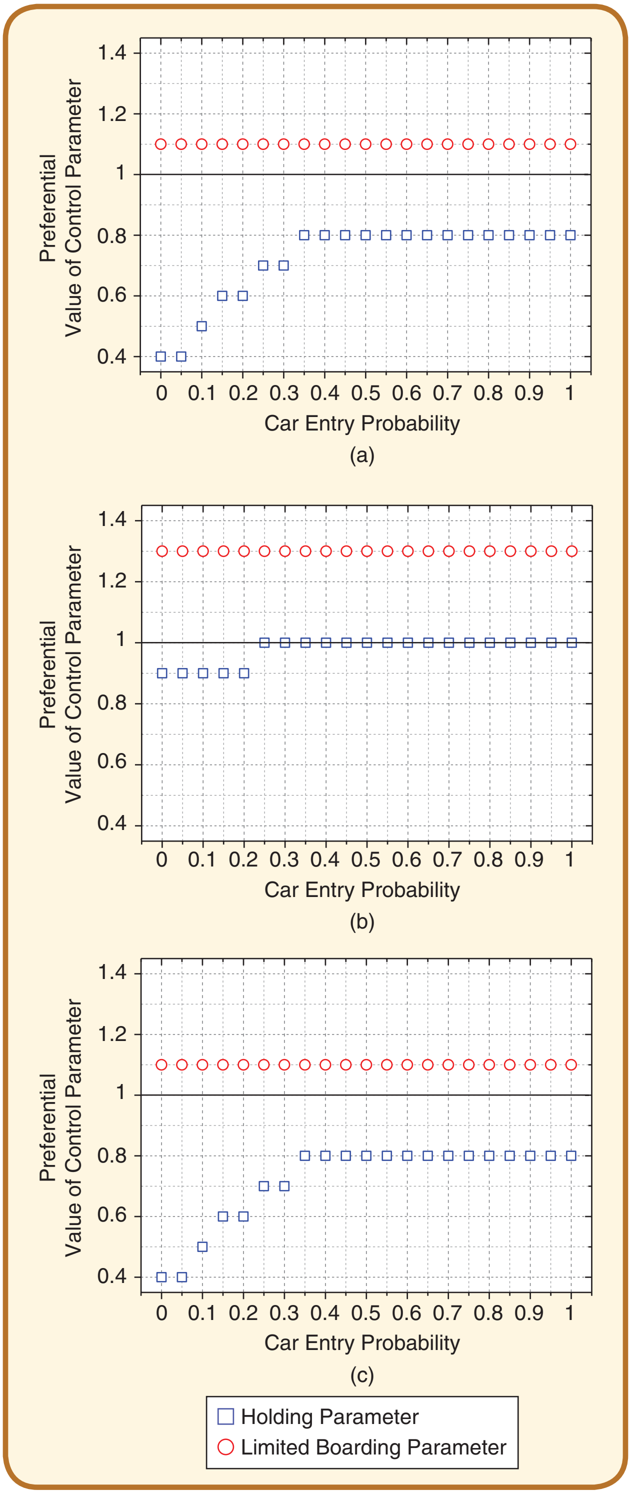

FIG 10 The preferable values of holding and limited boarding control parameters of H-LBC for (a) Passenger in-bus time; (b) passenger weighted travel time; (c) bus travel time under different car entry probabilities.

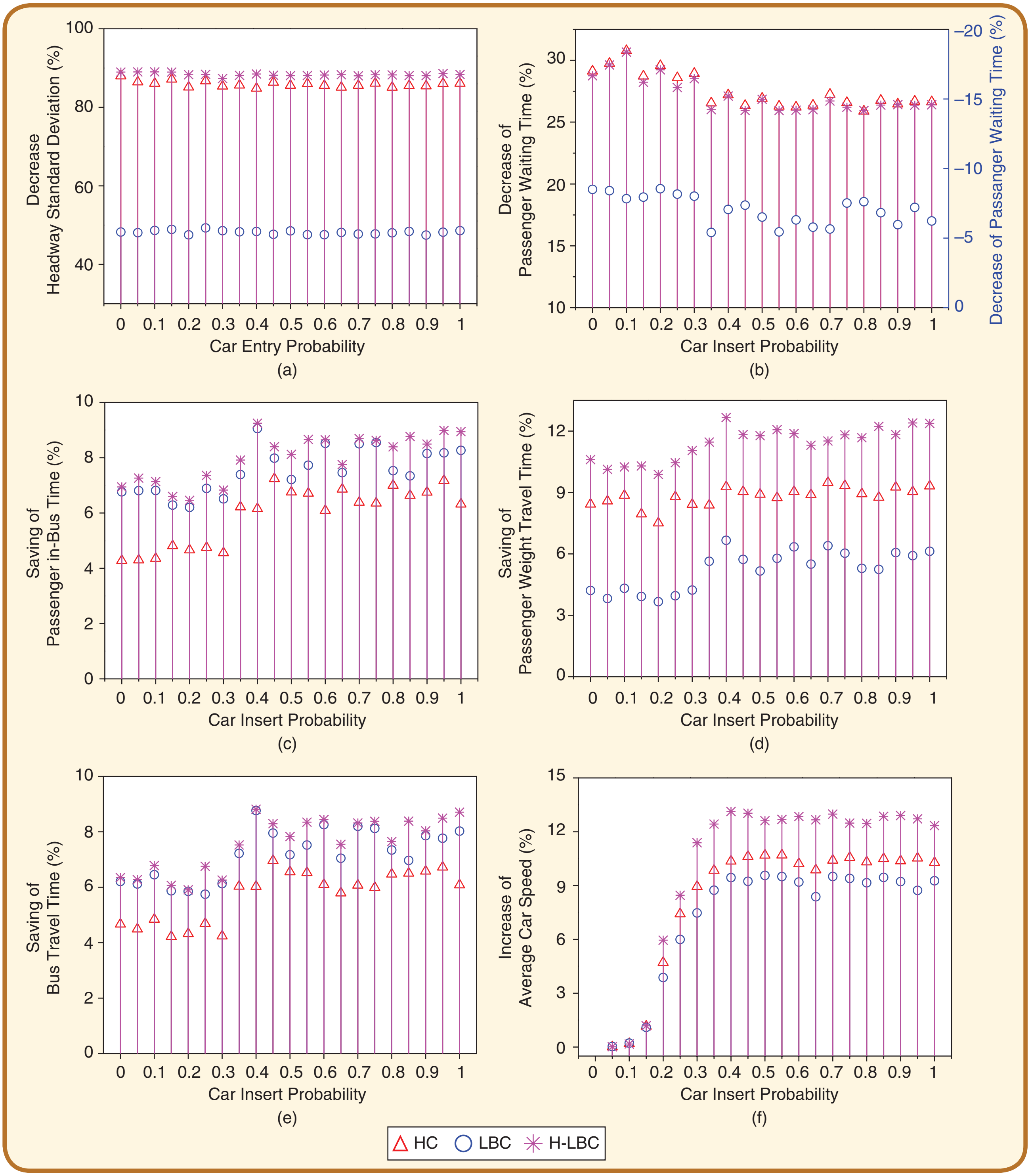

FIG 11 Percentage of (a) headway standard deviation reduction; (b) passenger waiting time saving; (c) passenger in-bus time saving; (d) passenger weighted travel time saving; (e) bus travel time saving; (f) car speed increase, for LBC, HC and H-LBC with respect to the no-control case under different car entry probabilities.

management guidance for the bus lines where overtaking is permitted. Secondly, it is known that bus holding increases the inconvenience of passengers. Selecting appropriate intermediate stops as the holding points under mixed traffic conditions is thus an important topic to investigate in the future.

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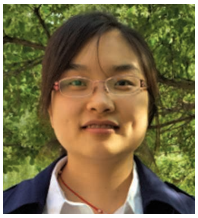

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