# Faculteit Bedrijfseconomische Wetenschappen

master handelsingenieur

***Masterthesis***

***Real options versus buyback contracts: A comparison in terms of risk***

**Thibaud Boden**

Scriptie ingediend tot het behalen van de graad van master handelsingenieur, afstudeerrichting operationeel management en logistiek

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Master thesis: 21370 Real options versus buyback contracts: A comparison in terms of risk
**Promoter:** Prof. Dr. Inneke Van Nieuwenhuyse

Real options versus buyback contracts: A comparison in terms of risk

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In this paper, a simulation study attempts to compare two kinds of supply chain coordinating contracts in the context of a newsvendor problem. The contracts are compared in terms of a risk measure well known in the financial literature as the Conditional Value at Risk (CVaR). The paper provides a detailed account of how to compute CVaR as well as the profits of all members of the supply chain by means of a simulation study iterating two thousand observations of demand. The simulation study's results suggest that there is no difference between the contracts in terms of risk, but they do suggest that different members of the supply chain are subjected to different levels of risk exposure. One party in our model, the retailer, appears to only benefit from signing a contract in terms of both reduced risk and increased profits. The retailer's supplier however, the manufacturer, will have to accept an increase of risk while previously not being exposed to any in exchange for an increase of his expected profits.

*Keywords:* newsvendor problem, VaR, CVaR, risk, real options, buyback, simulation

## 1. Introduction and Problem statement

Many companies choose to specialise in a particular part of the supply chain, such as the retailer, the manufacturer, the supplier, etc. This allows different players to specialise in certain processes that ultimately lead to the supply of products to the final consumer. However, each player faces demand uncertainty, which can have a major impact on their economic performance. If the player in question faces this hardship in the context of a sales season, he will need to try to make arrangements before the season in anticipation of the coming demand uncertainty. Suppose for instance a retailer who suffers from customer demand uncertainty during a sales season. He does not know the exact quantities his customers expect yet has to order these quantities from his manufacturer before the season begins. He can't order too few and order them during the season, because the lead time of the manufacturer is too great, or for some other reason. He wouldn't want to order too many either, as he would find himself with stocks of a product that has lost a lot of its value after the season has ended. This is a classic problem known as the newsvendor problem (Surti, Celani, & Gajpal, 2020).

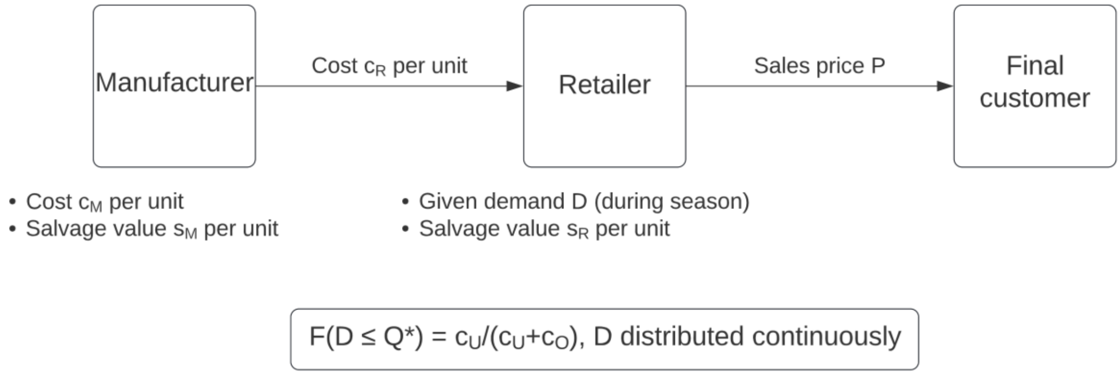


**Figure 1: Basic Newsvendor Problem**

The newsvendor problem can be easily described in the following way (Cachon, 2003) (see Figure 1 for a summary). Take a supply chain consisting of three players: a retailer, a manufacturer and the final customer market. During a certain period (the sales season), there is stochastic market demand $D$ for the seasonal good. This good has a particular value for the customer which allows the retailer to charge a certain sales price $P$ for it during the season. When the season is over, however, the value will drop and the retailer can only liquidate the goods at a lower price per unit, called the salvage value $s_R$. The main challenge for the retailer is that he must pay for and receive the required number of goods before the season starts, knowing additional goods cannot be ordered during the season. Consequently, a rational retailer will aim to optimise his ordered quantity. To accomplish this, he maximises his revenues during the season by selling as many goods as possible, yet minimises the number of goods he can't sell, which he will have to liquidate afterwards. He does so by identifying an overage cost $c_O$ and an underage cost $c_U$. The overage cost is the retailer's cost of carrying too many goods relative to what the customer is willing to buy. It is in contrast with the underage cost, which is associated with carrying too few goods and thereby losing revenue. The retailer determines his optimal order quantity $Q^*$ by minimising these expected overage and underage costs. The optimal order quantity is determined such that

$$F(D \leq Q^*) = \frac{c_U}{c_U + c_O}$$

This quantity will have to be ordered from the manufacturer before the season because he has a certain lead time that the retailer must take into account. This means that the retailer must place orders before the real demand is known. Notice that these costs are mismatched, as their impacts on the retailer's optimal order quantity are in opposite directions. The underage cost tends to increase $Q^*$ while the overage cost tends to decrease $Q^*$. Additionally, the manufacturer also imposes a certain price for the goods he supplies to the retailer, called the wholesale price (or the retailer's cost) $c_R$. He produces these goods at the manufacturer's cost per unit $c_M$. If the manufacturer has built up too much stock and thus cannot sell all the goods to the retailer, he too can liquidate them at a lower than wholesale price, called the manufacturer's salvage value $s_M$.

It has been shown that, when the retailer in such a chain maximises his own profit independently, the total expected profit of the supply chain $\pi_{SC}$ will be less than the total expected profit of the coordinated supply chain $\pi_{CO}$what it could have been had the supply chain been coordinated (Chiu, Choi, & Tang, 2011). With coordination, the retailer and manufacturer work together as if they are one entity. When this is the case, the retailer chooses an order quantity and a price that is optimal for the supply chain, meaning that the expected total profit of the whole supply chain is maximised (Cachon, 2003).

The literature provides many ways the supply chain in a newsvendor setting can be attempted to be coordinated. The vast majority of them employ some type of contract signed by both parties (the manufacturer and the retailer). In order for the supply chain to be coordinated by the contract, there are at least two necessary conditions to be satisfied. One of them is that the contract terms are determined such that $\pi_{SC}$ is maximised, meaning it is equal to the total expected profit of the coordinated supply chain. The other necessary condition is that both parties are as well as off or better in the case they sign the contract. That is, their individual profits ($\pi_M$ for the manufacturer, $\pi_R$, for the retailer) must be equal to or higher than their profits without the contract. There are many ways the supply chain in a newsvendor setting can be attempted to be coordinated. This paper focuses on two types of contracts often encountered in the literature: real option contracts and buyback contracts (Adhikari, Bisi, & Avittathur, 2020). These will be discussed in further detail in the literature review of this paper. Moreover, this paper will focus on expanding the literature on these contracts by evaluating the differences in risk between the contracts and how the risk can be measured.

# 2. Literature review

This section lays out the literature review that has been conducted in this paper.

Section 2.1 reviews the literature concerning real option contracts applied to attempt to coordinate the newsvendor problem that was introduced in section 2.

Section 2.2 reviews the literature concerning buyback contracts and their coordination of the newsvendor problem.

Section 2.3 explores the concepts of risk-aversion and risk measurement in the context of real option and buyback contract.

## 2.1. Literature review: real option contracts

A well-known type of contract is a real options contract (Luo, Li, Johnny Wan, Qu, & Ji, 2015). Instead of ordering $Q$ goods before the season starts, the retailer will merely order $Q$ options. The options can be bought at a certain option price $c_R$ from the manufacturer. At the beginning of the season, when demand will become known, the retailer will have the possibility to exercise any number equal to or less than the $Q$ options he had previously bought at a certain exercise price per unit $E$. When the retailer exercises these options, the manufacturer has the obligation to deliver him the same number of goods. This implies that a real option contract allows the retailer to commit the manufacturer to reserve some amount of goods for him. The manufacturer will have to liquidate by himself the stock of goods that he has built up but that the retailer does not order. The retailer effectively shifts his entire overstocking risk to the manufacturer, whom in return is guaranteed a pre-payment in the form of the retailer's $c_R$ (Burnetas & Ritchken, 2005; Cheng, Ettl, Lin, Schwarz, & Yao, 2003; Luo et al., 2015). It follows that the retailer does not incur any overage costs anymore, they are instead completely covered by the manufacturer.

Most of the literature on real options relevant to supply chain management focuses on proposing mathematical formulations for coordinating the supply chain. The vast majority look at a two-echelon supply chain in a single period (or season). One exception is Adhikari et al (Adhikari et al., 2020). Adhikari et al, 2020 include a five-echelon supply chain into their model, with a retailer, a manufacturer, followed by three suppliers in a sequence (one supplying to the other). By doing so, they show that the real option model can be expanded for more complex supply chains. However, like all the other real options papers to our knowledge, they fail to incorporate competing firms at any stage of the supply chain.

The real options papers usually introduce their methodology by formulating a mathematical benchmark which can later be used to compare their real options model and assess whether it can coordinate the supply chain. The chosen benchmark is usually the situation in which the supply chain functions solely through the setting of the wholesale price (also called a "price-only" contract)(Adhikari et al., 2020; Chen, Hao, & Li, 2014; Luo et al., 2015; Wang & Liu, 2007; Zhao, Wang, Cheng, Yang, & Huang, 2010). After having formulated the profit functions of all parties, the real options studies choose the mechanism by which parameters of the newsvendor problem are reached. In a real context situation, a negotiation takes place where one party may or may not have more bargaining power over the other. In the literature, negotiations are simulated through a Stackelberg game in the case of two-echelon supply chains (Cheng et al., 2003; Luo et al., 2015; Wang & Liu, 2007). Yet, other approaches are possible, as demonstrated by Zhao et al., 2010 by using Nash's bargaining power and Eliashberg's model (Zhao et al., 2010). In the case of the five-echelon supply chain proposed by Adhikari et al., 2020, the negotiation takes place by sequentially letting the most downstream party offer a contract to the party directly upstream. The latter will, in turn, offer a contract to the party directly upstream to himself, and so on (Adhikari et al., 2020). A commonality in the negotiation process for all these papers is that they choose one party to be the dominant one, meaning one party has the most negotiation power to set the terms of the contract. In many papers, the most upstream party is dominant (typically a manufacturer)

(Luo et al., 2015), but others seem to prefer a downstream-led (typically a retailer) supply chain (Adhikari et al., 2020; Wang & Liu, 2007; Zhao et al., 2010). This decision is always motivated by the fact that, in the specific industry of interest of the study in question, most supply chains are dominated by either a powerful supplier, manufacturer, or retailer.

Some papers take an interest in the information-asymmetry between supply chain members and how it challenges the arrival to a coordinating real options contract. Indeed, Wang & Liu, 2007 who study a retailer-led supply chain explain that retailers in some supply chains have an "information edge", i.e. a key piece of information that the downstream parties lack or do not know with certainty (Wang & Liu, 2007). The researchers explain that, in their scenario, the retailer decides on pricing, exploiting their more complete knowledge on demand, while the upstream manufacturer only has a veto power (Wang & Liu, 2007). Other studies provide similar insights (Cheng et al., 2003; Fu, Lee, & Teo, 2010; Luo et al., 2015; Zhao et al., 2010). Wang & Liu, 2007 also notably mention that once coordination has been reached, increasing the option price any further will decrease the manufacturer's extra profits and increase the retailer's (Wang & Liu, 2007). In the extreme case that the option price is zero, the manufacturer takes all the extra profits. This is confirmed by Luo et al., 2015 (Luo et al., 2015).

### 2.2. Literature review: Buyback contracts in supply chains

As an alternative to real option contracts, there are also buyback contracts (Basu, Liu, & Stallaert, 2019). Analogous to real options contracts, buyback contracts aim to coordinate the supply chain by providing an incentive for the parties to behave in a way aligned with each other's interests. The key element in these contracts' terms is the manufacturer's offer to the retailer of buying any unsold goods back after the season. In fact, the manufacturer incentivises the retailer to increase his ordered quantity by increasing his salvage value $s_R$ by the buyback price per unit $b$. The aim of the contract is to set $b$ and the whole sale price $c_R$ to maximise the expected profit of the entire supply chain (Basu et al., 2019). The party with the most negotiation power will be able to set $c_R$ such that it receives the largest share of the coordinated supply chain's profit (Basu et al., 2019). As with real options, most of the literature concerning buyback contracts focuses on newsvendor problems characterised by a two-echelon supply chain (one manufacturer and one retailer), and a single-period stochastic demand (Basu et al., 2019; Chen et al., 2014; Venkataraman & Asfaw, 2020; Wu, 2013; Zhou, Chen, Chen, & Wang, 2014). This part of the literature focuses on deriving mathematical models that coordinate the supply chain. The most notable findings are risk-related, which are reviewed in section 2.3 of this paper.

### 2.3. Literature review: Risk measures in supply chain coordination

The literature on real options and buyback contracts sometimes also addresses the notion of risk preference. For instance, some studies on real options include the risk preferences of the retailer, which is usually risk-averse (Adhikari et al., 2020; Chen et al., 2014). The retailer is often described as biased to order a smaller number of options than a risk-neutral retailer would. The manufacturer can correct this in the case of an option contract by accepting only lower option prices and higher exercise prices than in the optimal risk-neutral case (Adhikari et al., 2020). The risk preference is almost exclusively represented by von Neumann and Morgenstern's utility functions (Chen et al., 2014; Zhao et al., 2010). Utility functions will not be discussed in this paper, but the interested reader is referred to Chen et al., 2014 and Zhao et al., 2010 for more details.

The research involving buyback contracts often also deals with risk preference. The manufacturer is usually a risk-neutral party who offers the contract to the retailer, the risk-averse party (Basu et al., 2019; Venkataraman & Asfaw, 2020; Wu, 2013; Zhou et al., 2014). One difficulty for the manufacturer in this position is that the retailer often does not share how risk-averse he is (Basu et al., 2019). The retailer could lie and pretend to be more risk-averse than he really is, signalling to the manufacturer that the retailer's optimal order quantity is lower than the risk-neutral situation, causing the manufacturer to offer a higher buyback price and losing some of the extra revenue gained from the contract (Basu et al., 2019). In fact, when this happens, there is no guarantee that the contract will coordinate the supply chain (Basu et al., 2019). The literature tackles this by having the manufacturer offer a menu of different contracts with a parameter that offsets any buyback price that is higher than it should be for any given retailer's risk-aversion (Basu et al., 2019; Venkataraman & Asfaw, 2020). Venkataraman & Asfaw, 2020 assume a fixed value of $k$ such that a more risk-averse retailer will only require a buyback price of $k * b$, while other retailers would require the full buyback price $b$. The retailer will reveal its preference by selecting one contract or the other; the manufacturer will adjust $c_R$ accordingly to coordinate the supply chain (Venkataraman & Asfaw, 2020). Basu et al., 2019 incorporate a buyback premium (a kind of put price) with different values to choose from in their menu of contracts. Paying a higher buyback premium before the season gives the retailer the right to receive a higher buyback price for every good he sends back after the season. Any of these contracts coordinate the supply chain (Basu et al., 2019; Venkataraman & Asfaw, 2020). The contracts in question offer different pairs of $b$ and $c_R$, but the mathematical formulations are complex and difficult to compute (Basu et al., 2019).

However, risk preference merely acknowledges how risk-averse a decision maker is. It does not quantify the risk the decision maker incurs by pursuing a given project. In fact, one dimension the literature is particularly lacking is the measurement of risk. That is, the risk associated with any given type of contract (be it real options or buyback contracts). One way of measuring risk is mean-variance, as first developed by Markowitz in the financial literature (Markowitz, 1959). Mean-variance is a valuation tool that helps select optimal parameters for a decision maker, especially used in the context of risk-averse decision makers (whose utility functions are concave) (Ma, Liu, Li, & Yan, 2012). Mean-variance selects the project with the lowest variance and the largest profit. One shortfall of this mean-variance approach is that it only considers the profits in the right tail of the profit distribution curve (the highest profits) without any regard for the left tail (the lowest profits) (Ma et al., 2012). A rational decision maker would likely not care about variance causing his profits to be higher than the mean. He would only want to consider and minimise the variance causing his profits to fall or to turn into losses. One way of focusing on a decision maker's worst outcomes is using the Value-at-Risk (VaR) measure (see Figure 2), who's origin also lies in the financial literature (Soleimani, Seyyed-Esfahani, & Kannan, 2014).

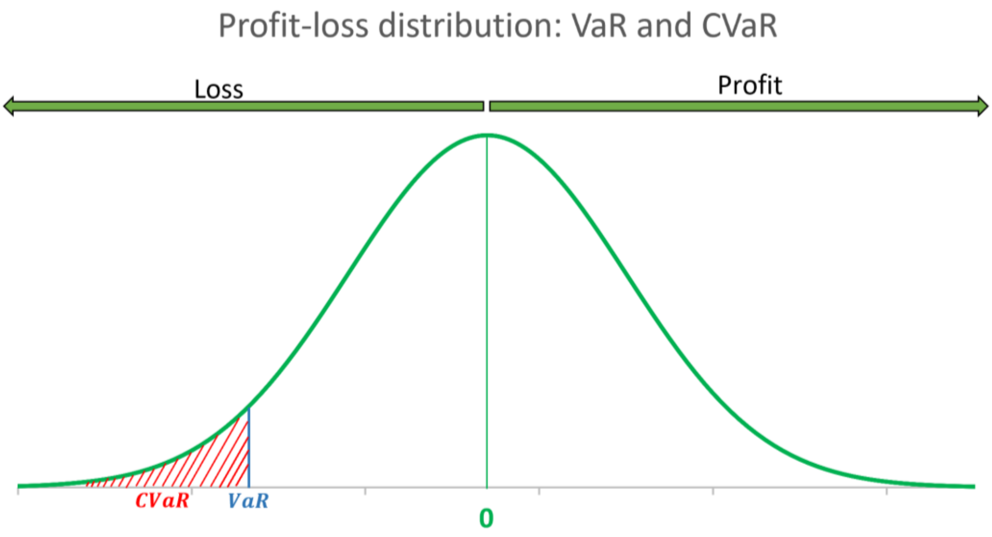


**Figure 2: Profit-loss distribution displaying risk in terms of VaR and CVaR**

VaR outputs a number that specifies the maximum loss a decision maker could suffer from a given decision under a given confidence level (Soleimani et al., 2014). In their definition, VaR can be formulated as follows:

$$VaR_{\alpha}(Z) = \inf(\eta \in R : F_Z(\eta) \geq \alpha),$$

where a $(1-\alpha)\%$ confidence level applies and $F$ is the cumulative distribution function of a random variable $Z$. $Z$ is the profit (when $Z$ is positive) or loss (when $Z$ is negative) occurring over a certain time horizon, due to market uncertainties, when for instance choosing a particular contract as a retailer. If the retailer for instance computed a 30 day $VaR_{5\%}(Z) = 3\%$, that would mean that the retailer can be 95% confident he will not lose more than 3% over the period. Equivalently, he has a 5% chance to lose 3% or more within the next 30 days. One important criticism of VaR is that it ignores the size of the losses worse than the given $\alpha$ level (Hong, Hu, & Liu, 2014). In our example, the retailer does not get any information from VaR on the losses beyond the 5% quantile under the distribution curve of $F$. In other words, the worst 5% of his losses are not considered. To solve this, CVaR (Conditional Value at Risk) was introduced (Rockafellar & Uryasev, 2002). Soleimani et al., 2014 formulate CVaR in the following manner:

$$CVaR_{\alpha}(Z) = E\big(Z \big| Z \geq VaR_{\alpha}(Z)\big)$$

CVaR (also known as expected shortfall) captures tail risk[1] better than VaR (Hong et al., 2014). This is because VaR only considers a lower bound of the distribution while CVaR returns the average of tail losses beyond VaR (Hong et al., 2014). Looking back at our example where a retailer is a risk decision maker, our retailer could for instance compute a 30 day $CVaR_{5\%}(Z) = 4.5\%$. In that case, the retailer can expect that in the worst 5% of his losses in the next 30 days his average loss will be 4.5%.

Surprisingly, in the literature, the use of VaR and CVaR to measure the risks of real options or buyback contracts is hardly used. There is, to the best of our knowledge, one recent study that has approached real options contracts to coordinate the supply chain and measured the risk by means of CVaR (Fan, Feng, & Shou, 2020). Fan et al., 2020 develop a mathematical formulation to theoretically prove that a two-echelon supply chain can be coordinated by a real options contract, while minimising CVaR (Fan et al., 2020). Notably, they find that the supply chain cannot be coordinated if the buyer of the options is more risk-averse than the supplier (Fan et al., 2020). The researchers explain that any increase in the option price when the buyer is risk-averse results in extra risk for the buyer that outweighs the resulting reduced risk for the supplier (Fan et al., 2020). The total risk of the supply chain is therefore increased. On the flip side, a risk-averse supplier will analogously result in a reduction in the total risk of the supply chain (Fan et al., 2020).

Less recently, Zhou et al. (2014) used CVaR in a context similar to Fan et al. (2020) but with buyback contracts. An important distinction however is that they did not measure the risk of a particular project. They measured the risk-aversion of the decision makers themselves. The researchers found that the manufacturer's risk-aversion is negatively related to his optimal buyback price (Zhou et al., 2014). The buyback price serves as a risk sharing instrument for the manufacturer so it should come as no surprise that he would be less willing to increase it if he is more averse to losses (Zhou et al., 2014). As for the retailer, they found his risk-aversion to be positively correlated with the manufacturer's optimal buyback price (Zhou et al., 2014). So, the more a retailer is averse to losses, the more it is in the manufacturer's best interest to increase the buyback price so as to incentivise the retailer to order larger quantities of goods (Zhou et al., 2014).

In conclusion, the literature on the differences in risks between real options and buyback contracts is scarce. This master's thesis will compare both contracts by use of a simulation that will attempt to compare both contracts in terms of their associated risk, measured by CVaR.

## 3. Methodology

This section establishes a simulation method for comparing the expected profits of the different parties in a two-echelon supply chain, and the CVaR of these profits, for the different contract types.

[1] Tail risk is the chance of a loss occurring due to a rare event, as predicted by a probability distribution.

In section 3.1, we present a brief explanation of the supply chain examined (the model), accompanied by the notation of all the parameters relevant to the simulation, followed by an overview of the formulae leading up to the calculation of the supply chain's expected profits.

Section 3.2 explains how the formulae are applied in the context of a simulation with various scenarios. In

3.3, the calculation of CVaR is discussed. The simulation is conducted using Microsoft Excel.

## 3.1. Model and notation

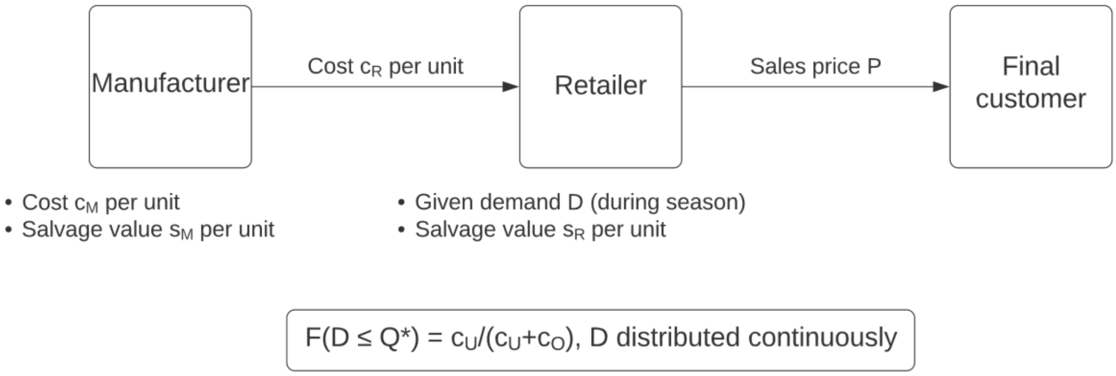


**Figure 1: Basic Newsvendor Problem**

The supply chain comprises a single retailer who buys a good from a single manufacturer in the context of a newsvendor setting, as detailed in section 1 and summarised in Figure 1. The following parameters are of interest:

$p$: the price of the good (which the retailer receives when selling during the season)

$c_R$: the retailer's cost of the good (charged by the manufacturer)

$c_M$: the manufacturer's cost of the good

$s_R$: the salvage value of the good that the retailer can obtain after the season is over

$s_M$: the salvage value of the good that the manufacturer can obtain after the season is over

$Q^*$: the optimal number of goods that the retailer procures from the manufacturer and will attempt to sell during the season, if demand allows it

$E(D)$: the expected demand during the season, known only when the season starts

$\sigma_D$: the standard deviation of the demand during the season

$D$: the demand actually observed during the season

Initially, the parties do not enter any form of coordinating contracts. If the retailer orders more goods than the demand, he will incur an overage cost $c_O$ of:

$$c_O = c_R - s_R$$

Similarly, if he orders less goods than demanded, the retailer will sell less goods than he could have at $p$ and at a cost of $c_R$, incurring an underage cost $c_U$ of:

$$c_U = p - c_R$$

From $c_O$ and $c_U$, the retailer can derive his optimal customer service level $CSL$ as follows:

$$CSL = \frac{c_U}{c_U + c_O}$$

The $CSL$ is the probability that the demand during the season $D$ is less or equal to his optimal ordering quantity $Q^*$. The retailer therefore chooses his $Q^*$ as follows:

$$Q^* \text{such that } F(D \leq Q^*): \frac{c_U}{c_U + c_O} = CSL$$

Assuming the demand is normally distributed, with a mean $E(D)$ and a standard deviation $\sigma_D$, $Q^*$ can be computed by the taking the sum of $E(D)$ and a safety stock. The safety stock is determined by a Z-score $Z_{CSL}$ dependent on the $CSL$, revealing how many standard deviations ($\sigma_D$) from the mean ($E(D)$) the retailer should account for[2]. $Z_{CSL}$ can be calculated using a Z-table or through the use of a mathematical formula. For the sake of precision, this paper opts for the latter, using the INV.NORM function available in Microsoft Excel, with a mean of 0 and a standard deviation of 1. In summary, $Q^*$ is defined as:

$$Q^* = E(D) + Z_{CSL} * \sigma_D$$

Knowing his $Q^*$, the retailer is able to find his expected understock $E(Understock)$ using the standard normal loss function, which is the expected number of lost sales as a fraction of the standard deviation:

$$E(Understock) = \sigma_D * L\left(\frac{Q - E(D)}{\sigma_D}\right)$$

$L\left(\frac{Q-E(D)}{\sigma_D}\right)$ can be calculated using a standard normal loss function table, or with a mathematical formula. In the latter case, Excel's NORM.S.DIST function can be used with different values of its "cumulative" argument. Setting the cumulative argument to FALSE and TRUE returns the probability mass function and the cumulative distribution function, respectively. The complete expression to be entered is:

$$= \text{NORMS.S.DIST}\left(\frac{\mathrm{Q} - \mathrm{E(D)}}{\sigma_\mathrm{D}}, \text{FALSE}\right) - \frac{\mathrm{Q} - \mathrm{E(D)}}{\sigma_\mathrm{D}} * \left(1 - \text{NORMS.S.DIST}\left(\frac{\mathrm{Q} - \mathrm{E(D)}}{\sigma_\mathrm{D}}, \text{TRUE}\right)\right)$$

Given the expected understock, the expected overstock $E(Overstock)$ can be obtained:

$$E(Overstock) = Q - E(D) + E(Understock)$$

The retailer now has all the necessary information to determine his expected profit $\pi_R$:

[2] Note that the safety stock $Z_{CSL} * \sigma_D$ is negative when $CSL < 50\%$, because the Z-score then becomes negative.

$$\pi_R = (p - c_R) * \left(Q^* - E(Overstock)\right) - (c_R - s_R) * E(Overstock)$$

During the season, the retailer's margin is $p - c_R$, which he gains for each of his ordered quantities $Q^*$ less the quantities he did not sell ($E(Overstock)$). After the season, the remaining goods ($E(Overstock)$) are liquidated at the reduced price $s_R$.

The manufacturer's expected profit $\pi_M$ is simply the difference between what it charges the retailer and what its production cost, multiplied by how much the retailer orders:

$$\pi_M = Q^* * (c_R - c_M)$$

The sum of the parties' expected profits is the supply chain's total (expected) profit $\pi_{SC}$:

$$\pi_{SC} = \pi_R + \pi_M$$

Under these conditions, it has been shown that the supply chain is not coordinated (Chiu et al., 2011). This can be verified in the model by adapting it, so the two parties are treated as belonging to one entity. This is referred to as vertical integration ($VI$). In practice, $c_O$ and $c_U$ then depend on the manufacturer's cost and salvage parameters instead of the retailer's:

$$c_{O_{VI}} = c_M - s_M$$

$$c_{U_{VI}} = p - c_M$$

The resulting total profit of the supply chain in the case of vertical integration $\pi_{SC_{VI}}$ can be used as a benchmark to verify a contract's ability to coordinate the supply chain. One such contract is the real options contract, addressed in subsection 3.1.1. The other is the buyback contract, addressed in subsection 3.1.2.

### 3.1.1. Real options model

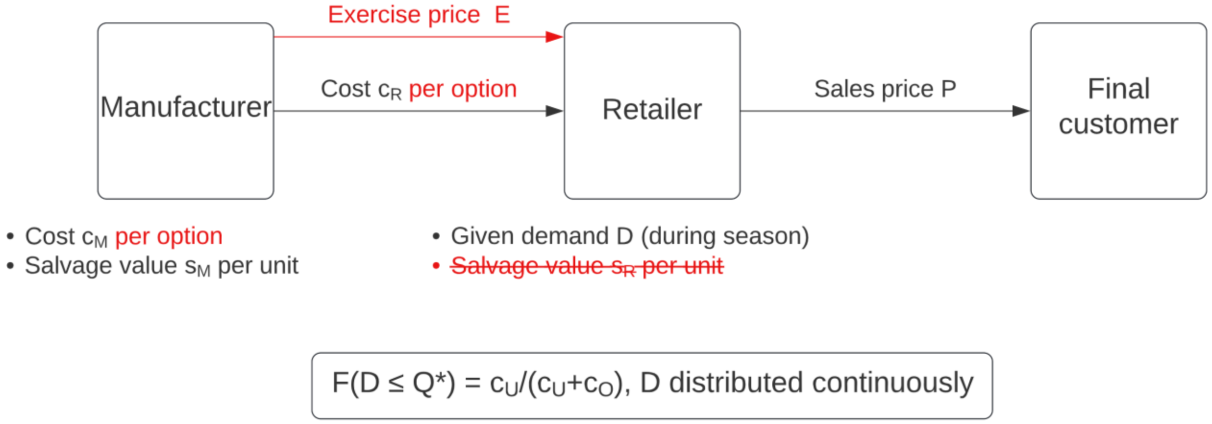


**Figure 3: Newsvendor Problem with a real options contract**

This section details this paper's real options model. Figure 3 visualises the model analogously to Figure 1, marking the changes in red. Upon entering the real options contract ($RO$), the retailer's cost $c_R$ no longer refers to the amount the manufacturer charges him for each good, but to the amount he's charged for each option he buys. Since the retailer will only purchase the amount of goods needed to satisfy the season's demand (or less, if he bought fewer options), the retailer will not have any goods to salvage. As a result, the overage cost becomes:

$$C_{O_{RO}} = c_R$$

The introduction of a real options contract also adds an exercise price $E$ to be paid by the retailer to the manufacturer each time an option is used, which weighs on the underage cost:

$$C_{U_{RO}} = p - c_R - E$$

$E$ can be optimally calculated by deriving a formula from the new customer level:

$$CSL_{RO} = \frac{C_{U_{RO}}}{C_{U_{RO}} + C_{O_{RO}}}$$

$$= \frac{p - c_R - E}{p - c_R - E + c_R}$$

$$\leftrightarrow E = c_R * \frac{p - s_M}{s_M - c_M} + p \tag{1}$$

Note that Equation (1) shows there must be a fixed relation between $c_R$ and $E$.

The profits of the parties are impacted by how many options the retailer exercises, which is referred to as the expected calls $E(calls)$. These depend on how many sales are expected, which is the difference between the ordered quantity and the expected overstock:

$$E(calls) = Q^* - E(Overstock)$$

The retailer's margin is dictated by the difference between $p$ and $E$, for each call. His margin is decreased by his cost of ordering options. It follows that the retailer's profit is equal to:

$$\pi_{R_{RO}} = -Q^* * c_R + E(calls) * (p - E) \tag{2}$$

The manufacturer gains $E$ from each call but must bear the cost of manufacturing, partially compensated by the $c_R$ he charges to the retailer. Additionally, the manufacturer liquidates any goods that were not sold at the end of the season, at his salvage value. The manufacturer's profit then is:

$$\pi_{M_{RO}} = Q * (c_R - c_M) + E(calls) * E + s_M(Q - E(calls)) \tag{3}$$

It has been shown that there is always at least one combination of $E$ and $c_R$ that will allow the total supply chain profit to be maximised [Add source]. To determine these combinations, both parties must be better off with the contract than without it, or indifferent. The retailer's profit from Equation (2) therefore has to be equal to or larger than his profit in the uncoordinated situation, which can be expressed as:

$$\pi_{R_{RO}} = -Q^* * c_R + E(calls) * (p - E) \geq \pi_R$$

Equation (1) can be inserted into Equation (2) by substituting $E$ to obtain:

$$-Q^* * c_R + E(calls) * \left(p - c_R * \frac{p - s_M}{s_M - c_M} + p\right) \geq \pi_R$$

This expression can be simplified to:

$$c_R \geq \frac{\pi_R}{-Q^* + E(calls) * \left(-\frac{p - s_M}{s_M - c_M}\right)} \quad (4)$$

Equation (4) is a lower limit on the $c_R$ that would allow supply chain coordination, as a result of the retailer's necessary condition of being better off than without the contract. Note that none of the parameters of (4) are unknown, meaning that (4) returns a single value.

Analogously, the manufacturer's willingness to participate in the contract must be taken into account, such that Equation (3) be larger than or equal to $\pi_M$:

$$\pi_{M_{RO}} = Q * (c_R - c_M) + E(calls) * E + s_M\big(Q - E(calls)\big) \geq \pi_M$$

Substituting $E$ with (1) into (3) yields:

$$Q * (c_R - c_M) + E(calls) * \left(c_R * \frac{p - s_M}{s_M - c_M} + p\right) + s_M\big(Q - E(calls)\big) \geq \pi_M$$

This expression can be simplified to:

$$c_R \leq \frac{\pi_M + Q^* c_M - E(calls) * p - s_M * (Q - E(calls))}{Q^* + E(calls) * \frac{p - s_M}{s_M - c_M}} \quad (5)$$

Equation (5) represents an upper limit on the value of $c_R$ that would allow supply chain coordination, as a result of the manufacturer's necessary condition of being indifferent or better off than without the contract. As with (4), (5) returns a single value.

Using the bounds on $c_R$ in (4) and (5), all the values[3] of $c_R$ that coordinate the supply chain can be computed. By substituting the results in (1), all possible combinations of $c_R$ and $E$ are obtained.

### 3.1.2. Buyback model

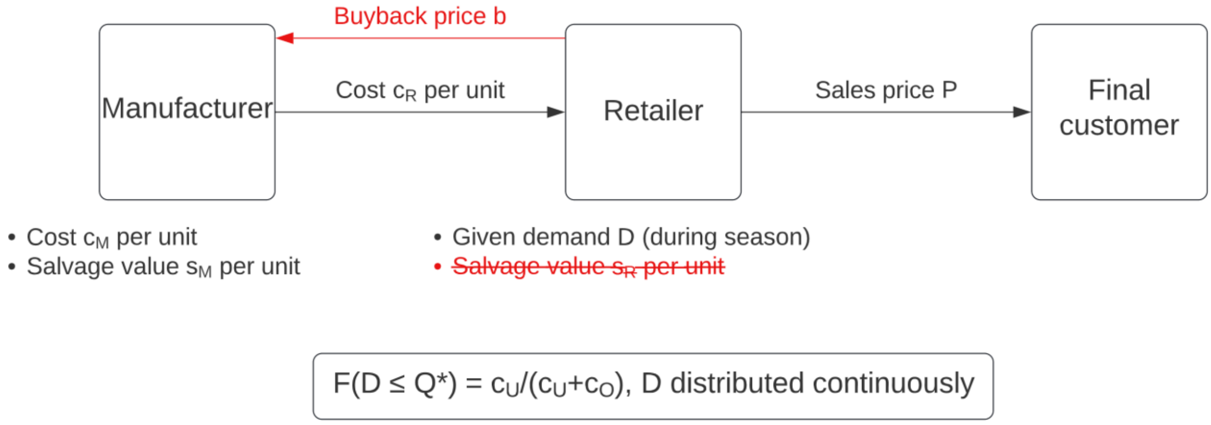


**Figure 4: Newsvendor Problem with a buyback contract**

This section details this paper's buyback model. Figure 4 visualises the model analogously to Figure 1, marking the changes in red. Upon entering the buyback contract ($BB$), the retailer no longer salvages his leftover goods himself, but rather sells them to the manufacturer at the end of the season at a buyback price $b$, which changes the overage cost to:

[3] These values can be continuous but for the sake of simplicity this study only considers integer values

$$C_{O_{BB}} = c_R - b$$

$b$ can be optimally calculated by deriving a formula from the new customer level:

$$CSL_{BB} = \frac{C_U}{C_U + C_{O_{BB}}}$$

$$= \frac{p - c_R}{p - c_R + c_R - b}$$

$$\leftrightarrow b = -\left(\frac{p - c_R}{CSL} - p\right) \tag{6}$$

The formula for the retailer's profit only changes by switching $s_R$ for $b$:

$$\pi_{R_{BB}} = (p - c_R) * \left(Q^* - E(Overstock)\right) - E(Overstock) * (c_R - b) \tag{7}$$

The formula for the manufacturer's profit in the case of buyback contracts must also include the cost he bears for buying back leftover goods:

$$\pi_{M_{BB}} = Q^* * (c_R - c_M) - E(Overstock) * (b - s_M) \tag{8}$$

It has been shown that there is always at least one combination of $b$ and $c_R$ that will allow the total supply chain profit to be maximised (Cachon, 2003). To determine these combinations, both parties of the supply chain must be better off with the contract than without it. The retailer's profit from Equation (7) therefore has to be equal to or larger than the one in the uncoordinated situation, which can be expressed as:

$$\pi_{R_{BB}} = (p - c_R) * \left(Q^* - E(Overstock)\right) - E(Overstock) * (c_R - b) \geq \pi_R$$

Substituting $b$ from Equation (6) into (7) yields:

$$(p - c_R) * \left(Q^* - E(Overstock)\right) - E(Overstock) * \left(c_R + \left(\frac{p - c_R}{CSL} - p\right)\right) \geq \pi_R$$

This expression can be simplified to:

$$c_R \leq \frac{\pi_R * CSL - p * Q^* * CSL + p * E(Overstock)}{-Q^* * CSL + E(Overstock)} \tag{9}$$

Equation (9) represents an upper bound on $c_R$ that ensures the retailer is indifferent or better off when entering a buyback contract. Note that (9) returns a single value for $c_R$.

Likewise, the manufacturer must be indifferent or better off when entering the contract. Analogously, using Equation (8):

$$\pi_{M_{BB}} = Q^* * (c_R - c_M) - E(Overstock) * (b - s_M) \geq \pi_M$$

Substituting $b$ from (6) into (8) yields:

$$Q^* * (c_R - c_M) - E(Overstock) * \left(-\left(\frac{p - c_R}{CSL} - p\right) - s_M\right) \geq \pi_M$$

This expression can be simplified to:

$$c_R \geq \frac{CSL * (\pi_M + c_M * Q^*) + E(Overstock) * p * (CSL - 1) - s_M * CSL * E(Overstock)}{Q^* * CSL - E(Overstock)} \tag{10}$$

Equation (10) represents a lower bound on $c_R$ that ensures the manufacturer is indifferent or better off when entering a buyback contract. Note that (10) returns a single value for $c_R$.

After computing the values from (9) and (10), an array of $c_R$ values will be obtained that can be paired with corresponding $b$ values by substituting them in (6). The result yields all the combinations of $b$ and $c_R$ that coordinate the supply chain using buyback contracts.

### 3.2. Applying the model to the context of a simulation

The formulas established in section 3.1 aim to find pairs of values of $(c_R, E)$ (in the case of real option contracts) and $(c_R, b)$ (in the case of buyback contracts) that coordinate the supply chain and observe the expected profit for both parties of the supply chain. In this section, the reader is presented how these formulas are applied to a simulation in which observations of demand are simulated.

This paper's simulation generates 2000 random numbers as a basis to compute an equal number of observations of demand that can be used to compare various scenarios. The random numbers are continuous values between 0 and 1 (excluding both 0 and 1), generated by Microsoft Excel's RAND function. This same sequence of numbers subsequently serves as the input to Excel's NORM.INV function as a probability argument. NORM.INV takes a probability, a mean, and a standard deviation and returns the inverse of the normal cumulative distribution for those arguments. In practice, a given random number below 0.5 (corresponding to a probability of less than 50%) in tandem with a certain mean and standard deviation result in a number below the mean. Naturally, the higher the standard deviation specified, the farther from the mean the result will be. At 0.5, the result is exactly the mean. Above 0.5, the result is greater than the mean. In this paper, for each of the 2000 random numbers, a mean of exactly 800 goods is specified, along with a standard deviation of 150. The calculations yield 2000 observations of a normally distributed demand, as visualised in Figure 5.

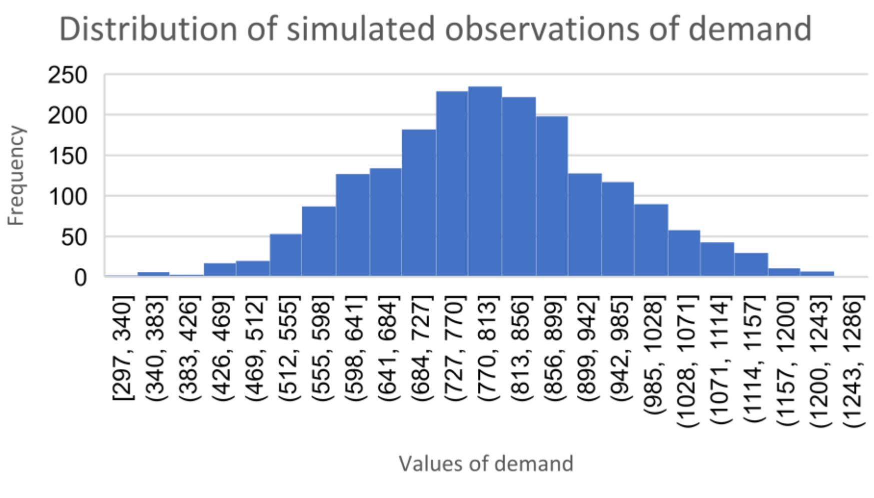


**Figure 5: Distribution of simulated observations of demand**

Each observation is an actual level of demand that could be observed during the season by the retailer and the manufacturer. For each observation, the resulting profits of both parties can be calculated under three kinds of scenarios. The scenarios in question are the uncoordinated scenario (no contract is signed), the real options scenarios and the buyback scenarios. The real options and buyback scenarios considered coordinate the supply chain through their use of contracts, as will be discussed later in this section.

The uncoordinated scenario serves as a base case and is composed of the following parameters:

$$p = 200$$

$$c_R = 135$$

$$c_M = 50$$

$$s_R = 10$$

$$s_M = 10$$

Using those parameters and the formulas from section 3.1, the retailer computes ex-ante (before the season and thus before the demand is known) his optimal order quantity $Q^*$ (and other relevant variables) and makes his arrangements with the manufacturer who will plan accordingly. Once the season starts, the demand becomes known and both parties are subjected to the observations of demand of this simulation. For each observation, the level of demand results in a particular profit for both parties. Hence, any particular level of demand will be associated with a corresponding retailer's profit and a corresponding manufacturer's profit. Those profits rely on a single value of $Q^*$, $c_R$, $c_M$, etc which are computed only once per scenario and used for all 2000 observations. Some variable's computations however need to be repeated for each observation. These are the variables that result in expected values in section 3.1. because there they are evaluated ex-ante, knowing only some mean demand and a standard deviation of demand. In the simulation, the demand is known for each observation. Therefore, these variables' formulas must be adapted. The expected understock $E(Understock)$ and expected overstock $E(Overstock)$ for instance (renamed $Understock_k$ and $Overstock_k$, respectively) are calculated by evaluating the difference between $Q^*$ and the demand level of a particular observation:

$$Understock_k = d_k - Q^*,\ if\ d_k > Q^*, else\ 0 \text{ with } k = 0, \dots, 2000$$

$$Overstock_k = Q^* - d_k,\ if\ d_k < Q^*, else\ 0 \text{ with } k = 0, \dots, 2000$$

With $d_k$ being the $k^{th}$ observation of demand, $Understock_k$ being the Understock associated with the $k^{th}$ observation of demand, $Overstock_k$ being the Overstock associated with the $k^{th}$ observation of demand.

Similarly, in the real option scenarios, the expected calls variable $E(Calls)$ is not used in the same way in the simulation as in section 3.1. Once the retailer knows his demand, he will not order the $E(Calls)$ he calculated ex-ante, but rather the actual demand that he observes during the season. Hence, $E(Calls)$ is replaced by $d_k$.

Having all parameters set up for the simulation, 2000 observations of profits can be calculated for the retailer and the manufacturer, repeated for each considered scenario. As mentioned, the base scenario is the uncoordinated scenario under which no contract is signed. The real options scenarios are those scenarios for which pairs of $(c_R, E)$ coordinate the supply chain if both parties agree to a real option contract. Similarly, the buyback scenarios are those scenarios for which pairs of $(c_R, b)$ coordinate the supply chain if both parties agree to a buyback contract. The real options and buyback scenarios will be referred to as feasible scenarios. The feasible scenarios use the same parameters as the uncoordinated scenario (and the same observations of demand) except for the parameter $c_R$. Each feasible scenario is associated with a different $c_R$. The pairs of $(c_R, E)$ for the real option scenarios are obtained by listing all integer[4] values of $c_R$ between the bounds for $c_R$ yielded from Equations (4) and (5). The pairs of $(c_R, b)$ for the buyback scenarios are obtained analogously using the bounds for $c_R$ yielded from Equations (9) and (10).

### 3.3. Formulation of CVaR

Once all retailer and manufacturer profits for each scenario are computed for all 2000 observations, the CVaR can be calculated for each profit at a given confidence level. Three confidence levels are considered: 95%, 99% and 99.9%.

The profits are sorted from lowest to highest in a spreadsheet. Every observed profit, once sorted, is associated with what we refer to as a "sequence number". The lowest profits have a sequence number of 1, the second to lowest are associated with a value of 2, and the highest profits have a value of 2000. The sequence numbers are used to find the "critical sequence number", which will serve as an input in the calculation of CVaR. The critical sequence number is the sequence number that is associated with the profit of interest in the calculations for a given confidence level. This profit of interest is the VaR. It is calculated with the following formula:

$$VAR = (1 - \alpha) * n$$

with $(1 - \alpha)$ being the confidence level and $n$ the number of observations.

In the case of a 95% confidence level, the critical sequence number is (1-95%) multiplied by 2000 (rounded to the nearest integer). This equals 100. Therefore, the 100th worst profit will be $VaR_{5\%}$. Analogously, $VaR_{1\%}$ is the 20th worst profit. For $VaR_{0.1\%}$, the 2nd worst profit is of interest.

Taking the sum of the nth worst profits (n corresponding to the critical sequence number at the given confidence level) and dividing the sum by the critical sequence number, the average of the nth worst profits is obtained. That corresponds to CVaR. For example, $CVaR_{5\%}$ is determined by summing the 100th worst profits and dividing the result by 100.

[4] In this paper's simulation, only integer values are considered in the search for pairs of $(c_R, E)$ and $(c_R, b)$ but positive real values could be used as well.

# 4. Results

This section presents the results of the simulation that was conducted using the methodology from section 3. All monetary values are assumed to be in euros.

The results of the uncoordinated scenario are explained in section 4.1. It is followed by section 4.2 and 4.3 which respectively present the results for the real options scenarios and the buyback scenarios. Finally, section 4.4 relates all results to risk in terms of CVaR at a confidence level of 95%. The interested reader can find charts and tables considering confidence levels of 99% and 99.9% in the appendix.

## 4.1. Results: Uncoordinated scenario

Let us begin by considering the base case scenario: the uncoordinated scenario. After plugging our inputs into the formulas from section 3.1, Table 1 is produced.

**Table: Uncoordinated scenario – Ex-ante calculations**

| Inputs | $p$ | $c_R$ | $c_M$ | $s_R$ | $s_M$ |
|---|---|---|---|---|---|
| | 200 | 135 | 50 | 10 | 10 |
| Base variables | $c_O$ | $c_U$ | $CSL$ | $Z_{CSL}$ | $Loss\ function$ |
| | 125 | 65 | 34.21% | -40.67% | 63.48% |
| Base variables | $Q^*$ | $E(Understock)$ | $E(Overstock)$ | | |
| | 739 | 95 | 34 | | |
| Profits | $\pi_R$ | $\pi_M$ | $\pi_{SC}$ | | |
| | 41,532 | 62,815 | 104,347 | | |

**Table 1: Uncoordinated scenario – Ex-ante calculations**

Table 1 presents the inputs first introduced in section 3.1 applied to the formulas of that section to compute the profits that the retailer and the manufacturer would expect before the season. Hence, the calculation is referred to as ex-ante. The variables that are computed from the inputs (and the mean demand of 800 with a standard deviation of 150) which are in turn used to compute the profits are dubbed "base variables". These base variables will serve in the next step, in which the supply chain parties are subjected to 2000 iterations of actual revealed demand. Notice that ex-ante the overage cost $c_O$ is almost twice as high as the underage cost $c_U$, incentivising the retailer to pick an optimal order quantity $Q^*$ well below the demand mean of 800. Rather unsurprisingly, more understock is expected than overstock.

Using the base variables of Table 1 and the 2000 simulated levels of demand, 2000 possible levels of retailer and manufacturer profits are obtained. These are displayed in Figure 6.

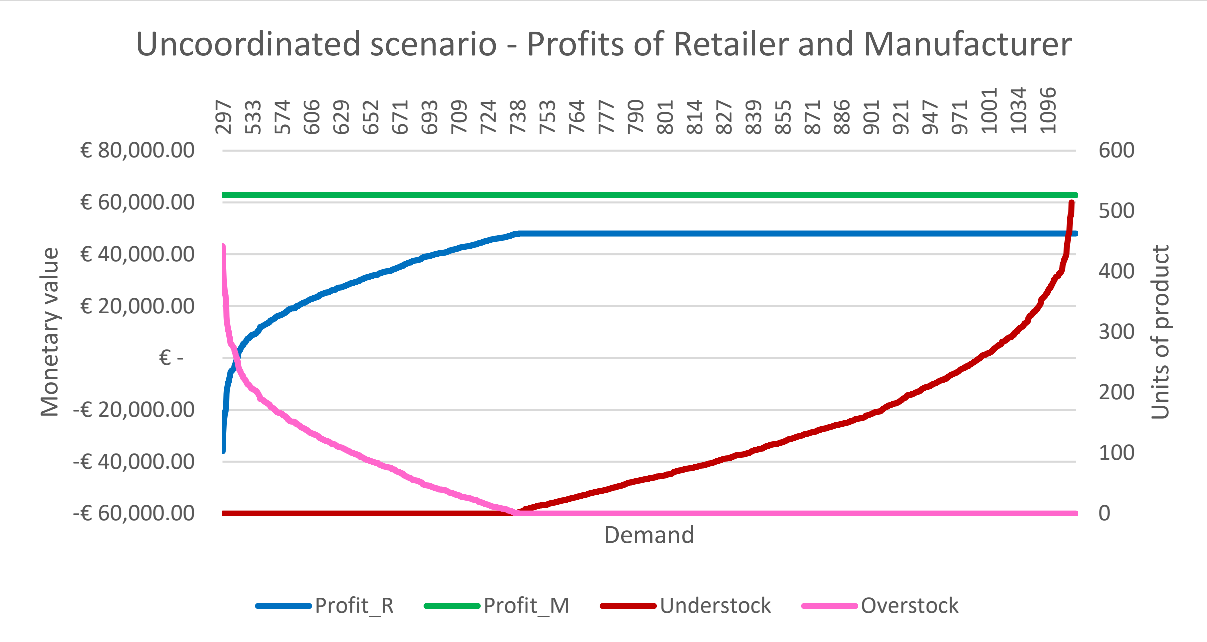


**Figure 6: Uncoordinated scenario – Profits of the retailer and the manufacturer**

The blue line shows the retailer's profit which is contrasted by the green line showing the manufacturer's profit in function of observed levels of demand. The demand is on the horizontal axis and spans a value of 297 to 1096, with a mean of approximately 800. The left-hand vertical axis applies to the supply chain parties' profits in monetary units (here assumed to be euros) while the right-hand vertical axis applies to the understock and overstock lines, coloured in red and pink, respectively. Figure 6 illustrates that high levels of overstock plunge the retailer's profits. This misfortune on the retailer's end occurs when observed demand happens to be drawn from the lower tail of the demand distribution from Figure 5. When the demand is closer to the optimal quantity $Q^* = 739$ that the retailer calculated ex-ante (see Table 1), the overstock shrinks closer to 0. At a demand level of exactly $Q^*$, the retailer's profit is at its peak at about € 48,000. Above that level, the retailer misses out on revenue because his stock is insufficient to match demand. His average profit level is at about € 41,300, unsurprisingly similar to the corresponding value in Table 1. Remarkably, the manufacturer is not affected by any swings in demand, as his profit remains constant at a level of about € 62,800 (also similar to the corresponding value in Table 1). The reason behind this phenomenon is simply that the manufacturer's revenue and costs are only dependent on the quantity the retailer orders before the season, with a fixed cost of production $c_M$ and charged price $c_R$. This stability does not imply, though, that this scenario is optimal for the manufacturer. There might be a scenario in which both the retailer's and the manufacturer's average profit are greater than in this uncoordinated scenario.

### 4.2. Results: Real options scenarios

Let us now explore scenarios in which both parties would sign a real options contract. First and foremost, we must identify a pair of $(c_R, E)$ that allows for a feasible scenario (one in which both parties are indifferent or better off). Using the inputs from Table 1 and the formulas from section 3.1.1, all relevant variables are calculated, in particular the exercise price $E$ and the expected calls $E(Calls)$. From that set of variables, Microsoft Excel's Solver is utilised to change $c_R$ to maximise the profits of both parties of the supply chains (with the relevant constraints). The result is summarised in Table 2.

**Table: Real options scenarios – Ex-ante calculation of one feasible scenario**

| Inputs | $p$ | $\boldsymbol{c_R}$ | $c_M$ | $s_R$ | $s_M$ |
|---|---|---|---|---|---|
| | 200 | **16** | 50 | 10 | 10 |
| Base variables | $c_O$ | $c_U$ | $CSL$ | $Z_{CSL}$ | $Loss\ function$ |
| | 16 | 60 | 78.95% | 80.46% | 11.88% |
| Base variables | $Q^*$ | $E(Understock)$ | $E(Overstock)$ | $\underline{E}$ | |
| | 921 | 18 | 139 | **124** | |
| Profits | $\pi_{R\,RO}$ | $\pi_{M\,RO}$ | $\pi_{SC\,RO}$ | | |
| | 44,709 | 67,064 | 111,774 | | |
| Δ | + 3,176 | + 4,249 | + 7,426 | | |

**Table 2: Real options scenarios – Ex-ante calculation of one feasible scenario**

Table 2 shows that starting from the base scenario of Table 1 and signing a real option contract is beneficial to the whole supply chain. If both parties can negotiate a reduction of $c_R$ (to the benefit of the retailer) to € 16 in exchange of the introduction of an exercise price $E$ of € 124, the retailer gains € 3,176 and the manufacturer gains € 4,249. Notice that $Q^*$ is actually higher than the mean demand, contrary to the situation of Table 1. Likewise, $c_O$ is diminished to the point it is lower than $c_U$. It can also be noted that the customer service level $CSL$ is doubled from the level of Table 1, so the supply chain can service the final customer twice as well.

Using Equations (4) and (5) from section 3.1.1, other feasible combinations of $(c_R, E)$ can be found. In this paper, only integer values of these variables are considered, but positive real values are equally valid. The feasible combinations are summarised in Table 3.

**Table: Real options scenarios – Feasible pairs of $(\boldsymbol{c_R}, \boldsymbol{E})$**

| $\boldsymbol{c_R}$ | $E$ | $\pi_{R\,\mathrm{RO}}$ | $\pi_{M\,\mathrm{RO}}$ | $\pi_{SC\,RO}$ | $R\%$ | $M\%$ |
|---|---|---|---|---|---|---|
| 15 | 129 | 41,720 | 70,054 | 111,774 | 37.33% | 62.67% |
| 16 | 124 | 44,710 | 67,064 | 111,774 | 40.00% | 60.00% |
| 17 | 119 | 47,700 | 64,075 | 111,774 | 42.67% | 57.33% |

**Table 3: Real options scenarios – Feasible pairs of $(\boldsymbol{c_R}, \boldsymbol{E})$**

Table 3 shows that, in the case of a real options contract, a higher $c_R$ (and by extension lower $E$ since both variables are linked through Equation (1)) is associated with a higher retailer's profit and a lower manufacturer's profit. The share of the total profit enjoyed by the retailer is shown in the column $R\%$, the manufacturer's share is shown in the column $M\%$. In this study, given the input parameters that were chosen as well as the mean demand and its standard deviation, real options contracts are only feasible for $c_R$ values between 15 and 17, yielding three feasible real options scenarios[5]. All three of these scenarios coordinate the supply chain because both the retailer and manufacturer are better off in all cases than without a contract. The negotiation power of both parties will determine which contract (with which distribution of the total profits) will be signed, but all contracts yield the same total profit for the supply chain. Knowing the feasible pairs of $(c_R, E)$, they can be used to compute the resulting profits of the supply chain parties for all 2000 iterations of demand. We note that the average profits correspond approximately with the profits found ex-ante in Table 3. These results will be of further use in section 4.4 when discussing the results related to CVaR.

### 4.3. Results: Buyback scenarios

Let us now explore scenarios in which both parties would sign a buyback contract. Analogously to section 4.2, we need to find a pair of $(c_R, b)$ that allows for a feasible scenario (one in which both parties are indifferent or better off). To do so, all variables are recalculated in the context of buyback contracts, also taking into account the buyback price $b$. From that set of variables, Microsoft Excel's Solver is once again utilised to change $c_R$ to maximise the profits of both parties of the supply chains (with the relevant constraints). The result is summarised in Table 4.

**Table: Buyback scenarios – Ex-ante calculation of one feasible scenario**

| Inputs | $p$ | $\boldsymbol{c_R}$ | $c_M$ | $s_R$ | $s_M$ |
|---|---|---|---|---|---|
| | 200 | **144** | 50 | 10 | 10 |
| Base variables | $c_O$ | $c_U$ | $CSL$ | $Z_{CSL}$ | $Loss\ function$ |
| | 15 | 56 | 78.95% | 80.46% | 11.88% |
| Base variables | $Q^*$ | $E(Understock)$ | $E(Overstock)$ | $\boldsymbol{b}$ | |
| | 921 | 18 | 139 | **129** | |
| Profits | $\pi_{R\,BB}$ | $\pi_{M\,BB}$ | $\pi_{SC\,BB}$ | | |
| | 41,720 | 70,054 | 111,774 | | |
| Δ | + 187 | + 7,239 | + 7,426 | | |

**Table 4: Buyback scenarios – Ex-ante calculation of one feasible scenario**

Table 4 shows that starting from the base scenario of Table 1 and signing a buyback contract is beneficial to the whole supply chain. A $c_R$ of € 144 is paired with a buyback price $b$ of € 129, resulting in the retailer being indifferent (the gain of € 187 is smaller than 0.5% of the retailer's total profit) and the manufacturer gaining € 7,239. Notice that $Q^*$, $CSL$ and the supply chain's profit remain unchanged in comparison with the real options scenarios of Table 2 and Table 3. Therefore, buyback contracts seem to be just as fit to coordinate the supply chain as real options contracts.

[5] If we consider real values instead of integers, the actual interval is [14.82, 17.52] and there is an infinite number of solutions.

Using Equations (9) and (10) from section 3.1.2, other feasible combinations of $(c_R, b)$ can be found. We note again that this paper only considers integer values of these variables, yet positive real values are equally valid. The feasible combinations are summarised in Table 5.

**Table: Buyback scenarios – Feasible pairs of $(\boldsymbol{c_R}, \boldsymbol{b})$**

| $c_R$ | $b$ | $\pi_{R_{BB}}$ | $\pi_{M_{BB}}$ | $\pi_{SC_{BB}}$ | $R\%$ | $M\%$ |
|---|---|---|---|---|---|---|
| 135 | 118 | 48,482 | 63,292 | 111,774 | 43.37% | 56.63% |
| 136 | 119 | 47,700 | 64,075 | 111,774 | 42.67% | 57.33% |
| 137 | 120 | 46,917 | 64,857 | 111,774 | 41.98% | 58.02% |
| 138 | 121 | 46,135 | 65,639 | 111,774 | 41.28% | 58.72% |
| 139 | 123 | 45,492 | 66,282 | 111,774 | 40.70% | 59.30% |
| 140 | 124 | 44,710 | 67,064 | 111,774 | 40.00% | 60.00% |
| 141 | 125 | 43,927 | 67,847 | 111,774 | 39.30% | 60.70% |
| 142 | 127 | 43,284 | 68,490 | 111,774 | 38.72% | 61.28% |
| 143 | 128 | 42,502 | 69,272 | 111,774 | 38.02% | 61.98% |
| 144 | 129 | 41,720 | 70,054 | 111,774 | 37.33% | 62.67% |

**Table 5: Buyback scenarios – Feasible pairs of $(\boldsymbol{c_R}, \boldsymbol{b})$**

Table 5 is analogous to Table 3 but shows more feasible combinations that coordinate the supply chain (as there happen to be more for our example[6]). This is the case because both the retailer and the manufacturer are better off for than in the uncoordinated situation for each pair of $(c_R, b)$. These pairs can be used to compute the resulting profits of the supply chain parties for all 2000 iterations of demand. As in section 4.2 when computing the real options scenarios, the average profits of the buyback scenarios correspond approximately with the profits found ex-ante in Table 4. These results will be of further use in section 4.4 when discussing the results for CVaR.

## 4.4. Results: VaR and CVaR

This section presents how the results of sections 4.1, 4.2, and 4.3 are combined to analyse the risks associated with each scenario across the three scenario types: uncoordinated scenario, real options scenarios and buyback scenarios.

The first step to be able to calculate VaR and in turn CVaR, is to sort all obtained profits for all scenarios. Table 6 offers an illustration.

[6] Note that if we looked at real values then there would always be an infinite number of feasible combinations between the upper and lower bounds of $c_R$ from Equations (9) and (10).

| | $\pi_R$ |
|---|---|
| **Sequence Number** | **Uncoordinated** |
| 1 | -35,961 |
| 2 | -30,892 |
| 3 | -26,119 |
| 4 | -23,888 |
| 5 | -23,305 |
| 6 | -21,503 |
| 7 | -20,622 |
| 8 | -20,538 |
| 9 | -17,518 |
| 10 | -13,854 |
| … | … |
| 2000 | 48,035 |

**Table 6: Uncoordinated scenario – Retailer profits sorted from smallest to largest**

Table 6 displays a portion of the retailer's obtained profits for each iteration of encountered demand level, sorted from smallest to largest. After the sorting, the sequence is noted in an adjacent column. In this paper, we are ultimately interested in obtaining CVaR at the 95%, 99%, and 99.9% confidence levels, for all profits. To obtain them, as explained in section 3.3, the critical sequence numbers must be found accordingly. As mentioned in that section, the critical sequence numbers corresponding with those confidence levels are 100, 20, and 2 respectively. These do not change across scenarios because the same 2000 demand levels are used for all of them. To find the retailer's VaR(99.9) in the uncoordinated scenario, it suffices to look up its 2$^{nd}$ (as that is the critical sequence number) worst profit (or loss) level, which is -€ 30,892. Furthermore, one can quite simply find CVaR(99.9) by averaging the 2 worst profits, yielding -€ 33,426. This same principle is repeated for all profits computed in all scenarios, at the three confidence levels. Figure 7 illustrates CVaR and VaR graphically in one real options scenario for the retailer's profits.

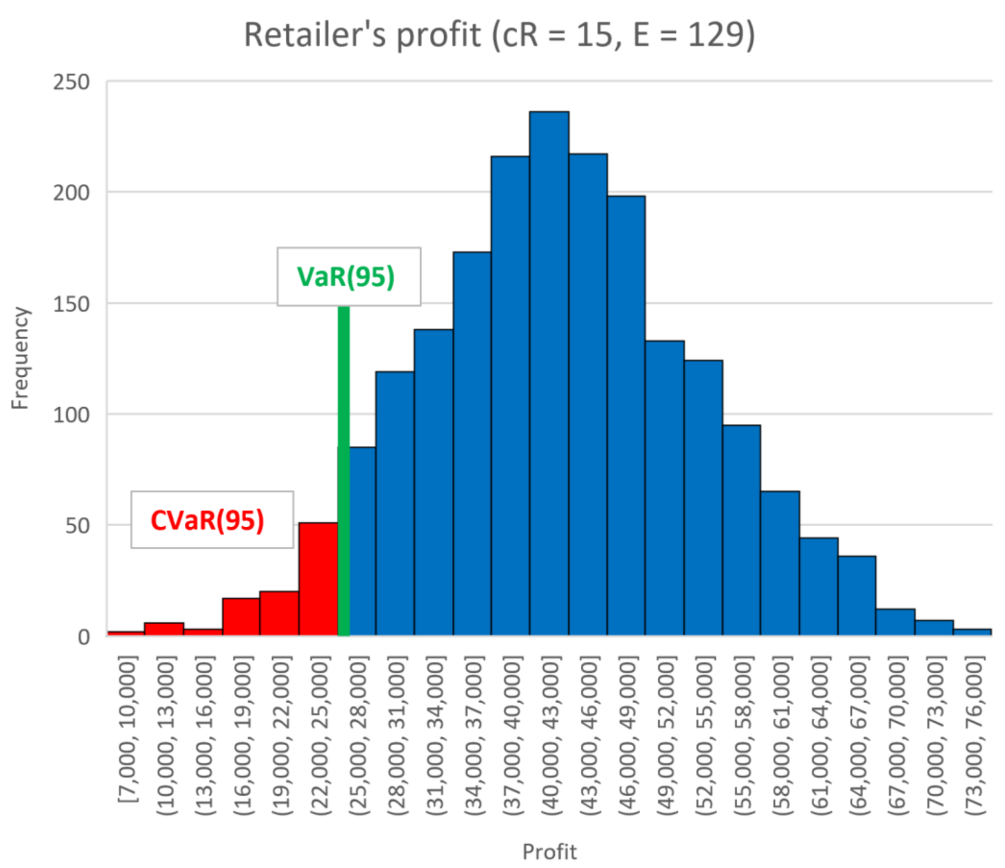


**Figure 7: Distribution of the retailer's profits in a real options scenario with** $(\boldsymbol{c_R = 15, E = 129})$

Figure 7 illustrates risk measured in terms of CVaR analogously to Figure 2 from section 2.3. It shows the distribution of profits that the retailer earns through 2000 instances of observed demand (in a real options contract with $c_R$ at € 15 and $E$ at € 129). VaR(95) is located in the left-tail of that distribution, at a value of approximately € 25,500. Therefore, the retailer's 5% worst profits are at and below € 25,500. CVaR(95) contrary to VaR(95) is not located at just one point in the distribution. Instead, it refers to the average of the left-tail of the distribution. In other words, CVaR(95) represents the average of the values below VaR(95). In this case, CVaR(95) equals approximately € 21,300. This means that the retailer's 5% worst profits average at € 21,300. That is the risk that the retailer incurs by signing the real options contract. Of course, that risk must be compared with the CVaR(95) in the uncoordinated scenario. This comparison is made in Figure 8.

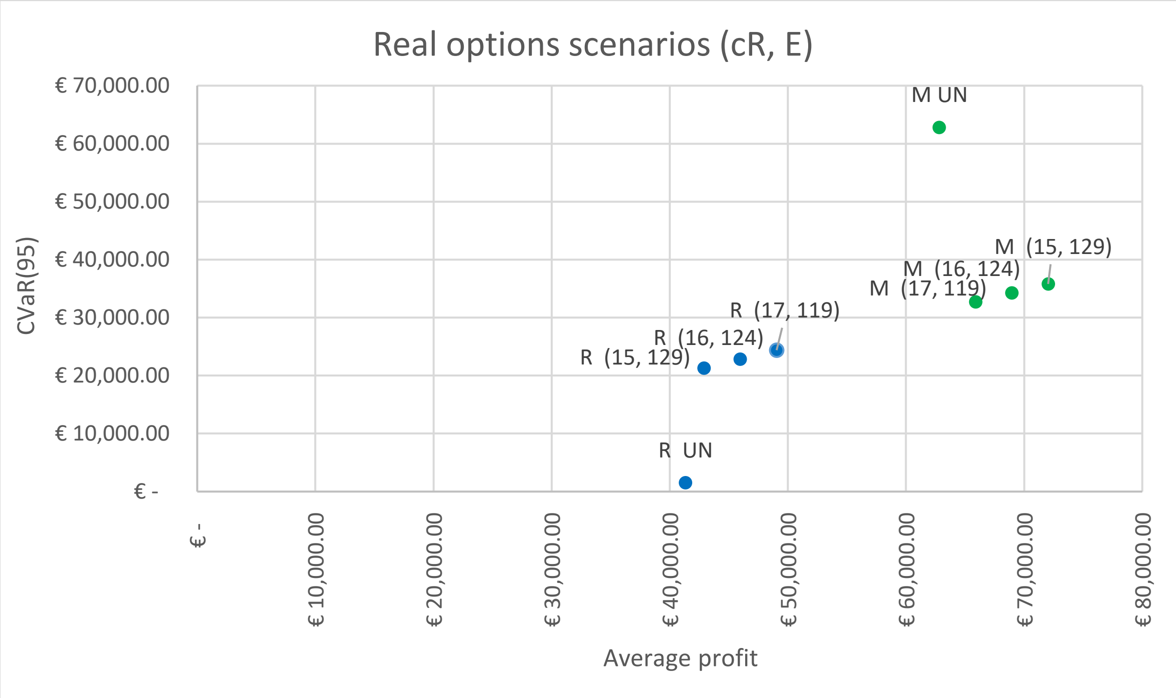


**Figure 8: Risk of three real options scenarios in terms of CVaR(95), paired with the average profit**

Figure 8 depicts the three feasible scenarios in which the retailer and the manufacturer sign a real options contract, compared with the uncoordinated scenario. R denotes the retailer, M denotes the manufacturer, and UN stands for the uncoordinated scenario. R UN for instance displays the retailer's CVaR(95) and the average profit he made over the 2000 simulated seasons in the uncoordinated scenario, while R (15, 129) displays the same but in a real options scenario with an associated combination $(c_R, E)$.

The uncoordinated points R UN and M UN provide a valuable insight. The retailer can considerably increase his CVaR(95) (from about € 1,500 to at least € 21,200) by signing any contract. So, his risk is reduced. What's more, his average profit increases as well. The retailer therefore has a large incentive to sign the contract. The same cannot be said so easily for the manufacturer. Before signing the contract, his CVaR(95) is equal to his average profit, meaning he is exposed to no risk at all. He may of course wish to sign one of the three possible contracts, seeing as they all increase his average profit. But doing so exposes him to a rather great deal of risk that would not arise otherwise. If he can choose the contract, he would take the one maximising his average profit M (15, 129) and would have to accept that his 5% worst profits would average about € 35,800; about 57% of the € 62,800 that would have otherwise been guaranteed. If the manufacturer is risk-averse, he may not wish to sign the contract despite being able to secure a higher expected profit.

Another insight of interest is that the contracts symmetrically benefit one party over the other. The best contract for the retailer is uses the combination (17, 119) because it yields him the highest average profit and highest CVaR(95), but this same combination produces the worst outcome for the manufacturer in terms of both profit and risk (excluding the uncoordinated scenario). This could be a subject of controversy among the parties during the negotiations of the contracts.

Figure 9 depicts the ten feasible scenarios in which the supply chain parties sign a buyback contract. In the same manner as Figure 8, it denotes the retailer with R and the manufacturer with M, and the uncoordinated case with UN.

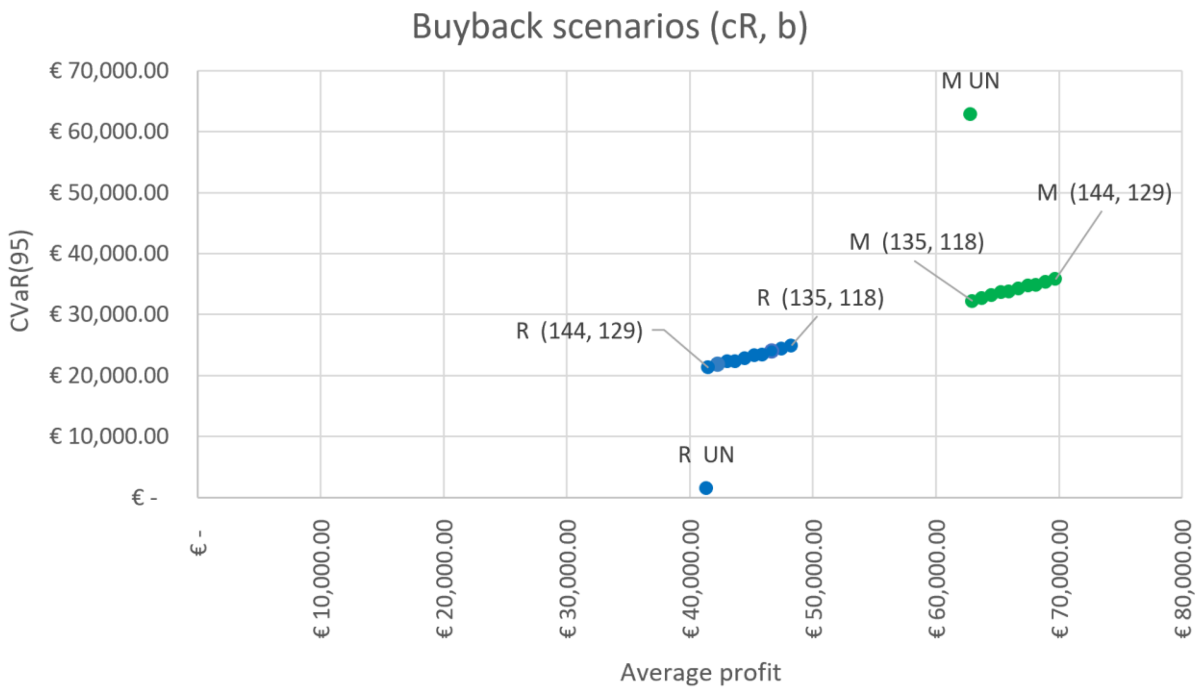


**Figure 9: Risk of ten buyback scenarios in terms of CVaR(95), paired with the average profit**

Figure 9 shows virtually no difference with Figure 8. There are indeed more observations, but that is only because only integer values of $c_R$ were pursued, and there just happen to be more integers between the interval of $c_R$ [135, 144] in our buyback contracts than in the interval of $c_R$ [15, 17] in our real options contracts. If one were to take into account the decimal values between $c_R = 15$ and $c_R = 17$ in the real options contracts, Figure 8 and Figure 9 would be identical except for switching pairs of $(c_R, E)$ with pairs of $(c_R, b)$. Therefore, all results for our real options scenarios also apply to our buyback scenarios. It appears there is no difference in risk between these types of contracts in terms of CVaR.

For the sake of further inquiry, this paper also looked at what would happen if a lower standard deviation of demand was chosen. Originally, a standard deviation of demand of 150 was used across all scenarios. Let us now consider a standard deviation half of that, at 75. The direct consequence for the model is that there is a lot less risk involved in all scenarios. If there is less variation, therefore less risks, then there must be less risks of overstocking in an uncoordinated scenario. Given that contracts that employ real options or buyback schemes that reduce risks of overstocking, one might expect a fall in standard deviation of demand to cause the signing of these contracts to become less appealing. Figure 10 displays the same graph as Figure 8 except the standard deviation of demand (sd) is halved.

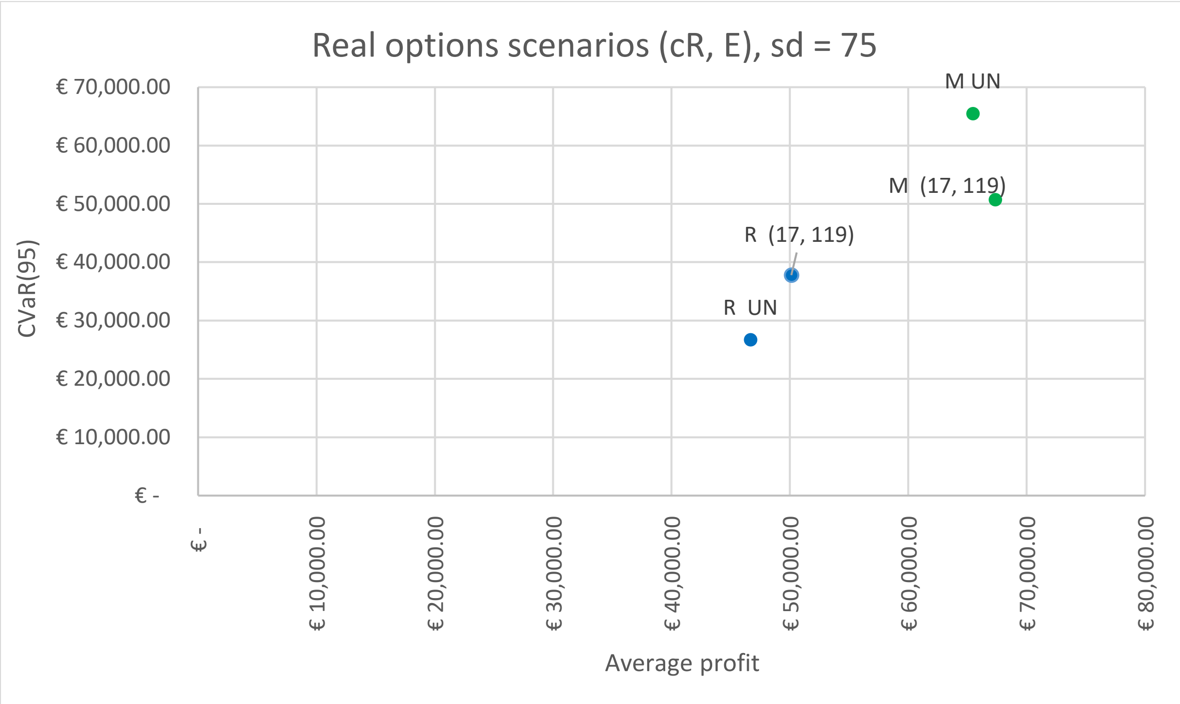


**Figure 10: Risk of one real options scenario in terms of CVaR(95), paired with the average profit (sd = 75)**

Figure 10 shows just one feasible combination of a real options scenario analogously to what was displayed in Figure 8, but for a standard deviation of demand of 75 instead of 150. Because there is less variation, the interval for $(c_R, E)$ is smaller. In this case, it is constricted to the interval [16.14,17.41]. Since this paper only considers integers, only one feasible scenario is computed for the real options contracts. However, both Figure 8 and Figure 10 display the combination $(c_R = 17, E = 119)$. Noticeably, the levels of CVaR(95) are higher in all situations, including the uncoordinated scenario. There is less risk in this setting. Stated differently, the worst average profits are higher at a lower standard deviation of demand (as one might have expected). Average profits in turn have stayed almost exactly equal. Because of the reduced risk, the points R UN and R(17, 119) are closer together at sd=75 and their difference in CVaR(95) is indeed smaller than in the situation of sd=150. At sd=75 the difference equals € 11,071, while at sd=150 it equals € 22,817. In other words, at sd=75 the retailer can only secure an increase in the average of his 5% worst average profits of about half the increase he would have had at sd=150 . Therefore, real options indeed appear to be less appealing for the retailer in the case of less variation in demand yet would still prefer to sign the contract than to remain in the uncoordinated scenario as he still mitigates risk through a higher CVaR(95), all the while increasing his average profit. Perhaps counter-intuitively however, the situation is reversed for the manufacturer. He is actually more inclined to sign the real options contract. The reason behind this lies in the fact that the manufacturer, at sd=150 and sd=75, starts with a CVaR(95) that his equal to his average profit, but in any contract, he has to accept a lower CVaR(95) (so more risk). At sd=150 and $(c_R = 17, E = 119)$, he has to accept his CVaR(95) to fall by € 30,137 while in the same scenario at sd=75 he only has to accept a fall of € 14,771. A very risk-averse manufacture might not have signed a contract when demand variation was relatively high at sd=150 as that would have hypothetically been associated with too much risk in return for the increase in average profit. Conversely, at sd=75, this same manufacturer might consider the increase in average profit to be worth the risk.

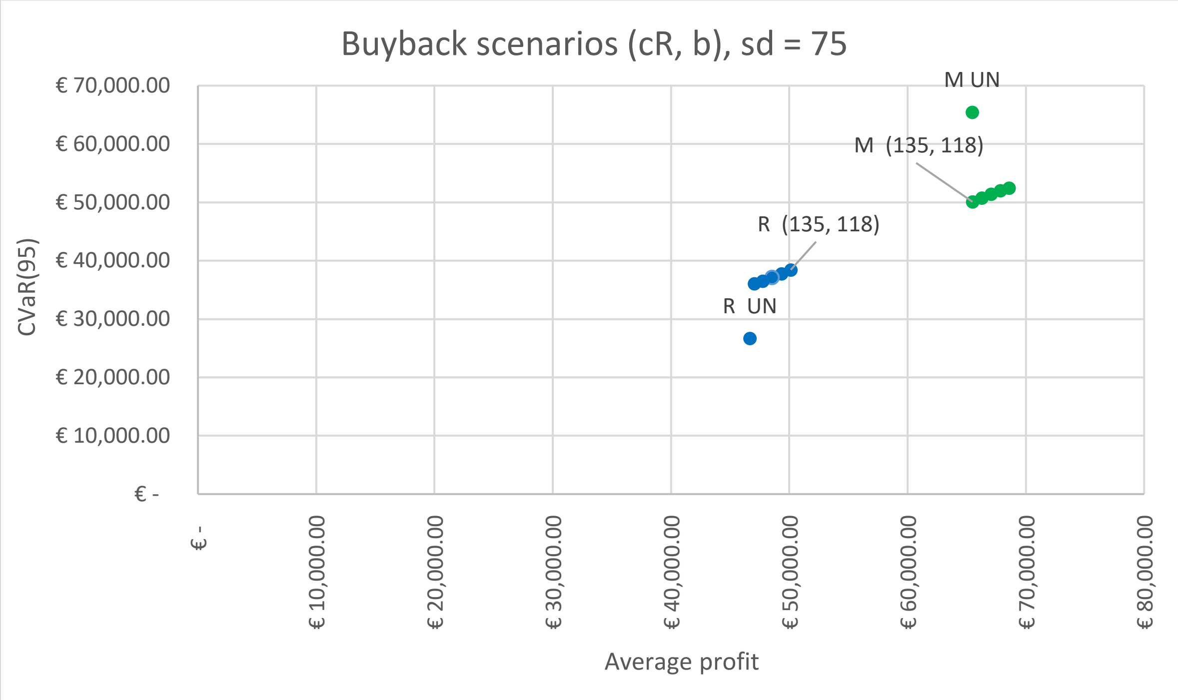


**Figure 11: Risk of five buyback scenarios in terms of CVaR(95), paired with the average profit (sd=75)**

Figure 11 displays the same effort of lowering the standard deviation of demand to sd=75 as Figure 10 did, but with buyback scenarios. In parallel with the real options scenarios, the overall risk is lower in these buyback scenarios since CVaR(95) has increased in each one of them, maintaining almost exact levels of average profits. Comparing the difference in CVaR(95) at sd=75 and sd=150 between R UN and R(135, 118), and between M UN and M(135, 118), the results are similar to the ones derived from Figure 10. The retailer loses some incentive to sign a buyback contract at sd=75 because there is less risk to mitigate via the contract while the average profits remain about the same. The manufacturer however sees his mitigation of risk increase at sd=150. At the point ($c_R = 135, b = 118$) there is almost no increase in profit to be gained from the contract, but other contracts promise larger average profits.

This section has only addressed a confidence level of 95%, but the interested reader is referred to the appendix to find charts that address confidence levels of 99% and 99.9%. The insights are analogous to the ones discussed at the 95% confidence level, but with generally lover values of CVaR (except for the manufacturer’s uncoordinated case which is always equal to its average profit). Additionally, the appendix contains Tables 7, 8 and 9, which provide the precise figures obtained in the simulation study for all scenarios. The figures in question are VaR(95), VaR(99), VaR(99.9), CVaR(95), CVaR(99), CVaR(99.9) and the average profits computed for the retailer and the manufacturer. Table 7 displays these figures in the uncoordinated case, Table 8 for all real options scenarios, and Table 9 for the buyback scenarios. Table 9 omits some of the scenarios to save space.

# 5. Conclusions and insights

The literature dealing with contracts that coordinate a newsvendor problem's supply chain often concerns itself with analysing how to adapt the problem to more realistic or more complex settings. The newsvendor problem is expanded to include greater than two-echelon supply chains with more complex relations between the members of the supply chain, or to account for information asymmetry, limited production capabilities, or even risk-aversion. Often in these settings, the objective is to formulate a model of an uncoordinated supply chain model and to find a path to achieving coordination. Coordination is considered to be achieved if all members are indifferent or better off, such that the expected total profit of the supply chain is greater than in the uncoordinated situation. It is reasonable to use this measure, but its limitation is that expected values are by their nature rarely realised. Before the sales season starts, if one supply chain player expects a certain amount of profit, the actually observed value after the season is highly likely to be below or above the expected value. But by how much? And is there a difference in risk depending on the contract chosen? Those are the questions which this paper has attempted to tackle. It has done so by borrowing a risk measuring instrument from the financial literature, known as CVaR (Conditional Variance at Risk). This approach appears to be a rather new and rare one in the subject of supply chain coordination through contracts. By means of a simulation study, the results of this paper suggest that while there does not any appear to be any difference relating to risk (measured in CVaR) between the real options contracts and the buyback contracts, there is a difference in risk exposure between the supply chain parties. In our model, the retailer can only benefit from a contract (be it by virtue of real options or a buyback mechanism) since it increases his CVaR (or the average of his worst profits) while simultaneously increasing his average earned profit. His manufacturer on the flip side starts off with a riskless situation and must accept some amount of risk (a reduction in his CVaR) in exchange for an increase of the average profit. If the manufacturer were to be characterised by a particular level of risk-aversion, he may object to this exposure to risk and choose not to sign any contract. Furthermore, by allowing the simulation's standard deviation of demand to vary, this paper's results suggest that a situation in which there is less variability of demand, a risk-averse manufacturer may be more inclined to sign a coordinating contract. This stems from the fact that, in our model, a lower standard deviation of demand does not alter the average profit significantly, yet it does increase CVaR across the board. The retailer is impacted differently by this change in standard deviation of demand. His CVaR increases as well, but in comparison with the situation subjected to more demand fluctuation his gain of CVaR achieved via a coordinating contract is diminished. However, compared with an uncoordinated situation, the retailer still makes gains on his risk exposure as well as his bottom line, even in the case of a relatively lower demand fluctuation. Overall, this paper has argued for a more nuanced approach to coordinating the supply chain through contracts. It has also highlighted CVaR as risk-measuring instrument. Managers could apply the methods of this paper's simulation study to carry out their own simulation and measure their risk, as long as they have solid assumptions about their model, particularly the mean of their seasonal products' demand (if it is approximately normally distributed) and its standard deviation (as long as the supply chain they are a part of can be characterised as a newsvendor problem).

The simulation carried out in this paper can serve as a starting point for further research. Since there was no difference to be found in terms of risk between the real options and buyback contracts, it would prove interesting to challenge this finding. Perhaps an example with certain input parameters could yield a contradicting result. Should this paper's finding indeed be generalisable, then an analytical study could be able to assert this through mathematical proofs.

**Acknowledgements**

I would like to express my gratitude towards promoter Prof. Dr. Inneke Van Nieuwenhuyse for her expertise, guidance and valuable comments that have helped to shape this paper and improve it greatly. I would also like to thank my friends and colleagues for providing feedback about the clarity of this paper's presentation. Finally, I would like to thank my parents for their unconditional support of my studies over the years and, by extension, of this very paper.

# 6. Appendix

| | Uncoordinated | |
|---|---|---|
| | R | M |
| | UN | UN |
| VAR(95) | 12,919 | 62,815 |
| VAR(99) | -5,508 | 62,815 |
| VAR(99.9) | -30,892 | 62,815 |
| CVaR(95) | 1,551 | 62,815 |
| CVaR(99) | -16,016 | 62,815 |
| CVaR(99.9) | -33,426 | 62,815 |
| Average profit | 41,331 | 62,815 |

**Table 7: Obtained simulation results for the uncoordinated case for the retailer (R) and the manufacturer (M)**

| | Real options scenarios $(c_R, E)$ | | | | | |
|---|---|---|---|---|---|---|
| | R | M | R | M | R | M |
| | (15, 129) | (15, 129) | (16, 124) | (16, 124) | (17, 119) | (17, 119) |
| VAR(95) | 25,517 | 42,897 | 27,365 | 41,048 | 29,214 | 39,199 |
| VAR(99) | 18,631 | 31,356 | 19,995 | 29,992 | 21,359 | 28,628 |
| VAR(99.9) | 9,145 | 15,458 | 9,841 | 14,762 | 10,537 | 14,066 |
| CVaR(95) | 21,269 | 35,777 | 22,818 | 34,228 | 24,368 | 32,678 |
| CVaR(99) | 14,704 | 24,775 | 15,792 | 23,687 | 16,879 | 22,600 |
| CVaR(99.9) | 8,198 | 13,870 | 8,827 | 13,241 | 9,457 | 12,612 |
| Average profit | 42,884 | 72,006 | 45,956 | 68,934 | 49,028 | 65,862 |

**Table 8: Obtained simulation results for the real options scenarios for the retailer (R) and the manufacturer (M)**

| | Buyback scenarios $(c_R, b)$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | R | M | R | M | R | M | R | M |
| | (135,118) | (135, 118) | (137, 120) | (137, 120) | (139, 123) | (139, 123) | (144, 129) | (144, 129) |
| VAR(95) | 29,769 | 38,645 | 28,660 | 39,754 | 27,920 | 40,494 | 25,517 | 42,897 |
| VAR(99) | 21,816 | 28,171 | 20,902 | 29,085 | 20,452 | 29,535 | 18,631 | 31,356 |
| VAR(99.9) | 10,861 | 13,742 | 10,214 | 14,390 | 10,165 | 14,438 | 9,145 | 15,458 |
| CVaR(95) | 24,863 | 32,184 | 23,874 | 33,172 | 23,313 | 33,733 | 21,269 | 35,777 |
| CVaR(99) | 17,281 | 22,198 | 16,477 | 23,002 | 16,194 | 23,286 | 14,704 | 24,775 |
| CVaR(99.9) | 9,767 | 12,302 | 9,146 | 12,922 | 9,138 | 12,931 | 8,198 | 13,870 |
| Average profit | 48,237 | 62,969 | 46,678 | 64,528 | 45,262 | 65,944 | 41,508 | 69,699 |

**Table 9: Obtained simulation results for the buyback scenarios for the retailer (R) and the manufacturer (M) (some data points are omitted to save space)**

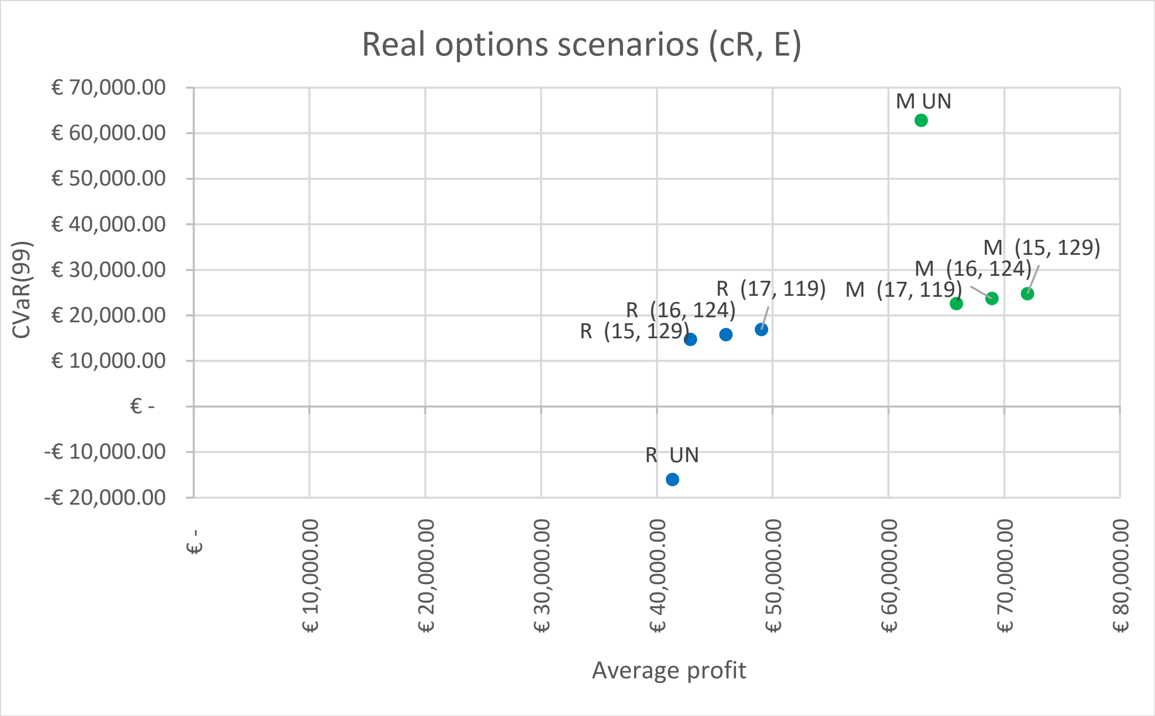


**Figure 12: Risk of three real options scenario in terms of CVaR(99), paired with the average profit**

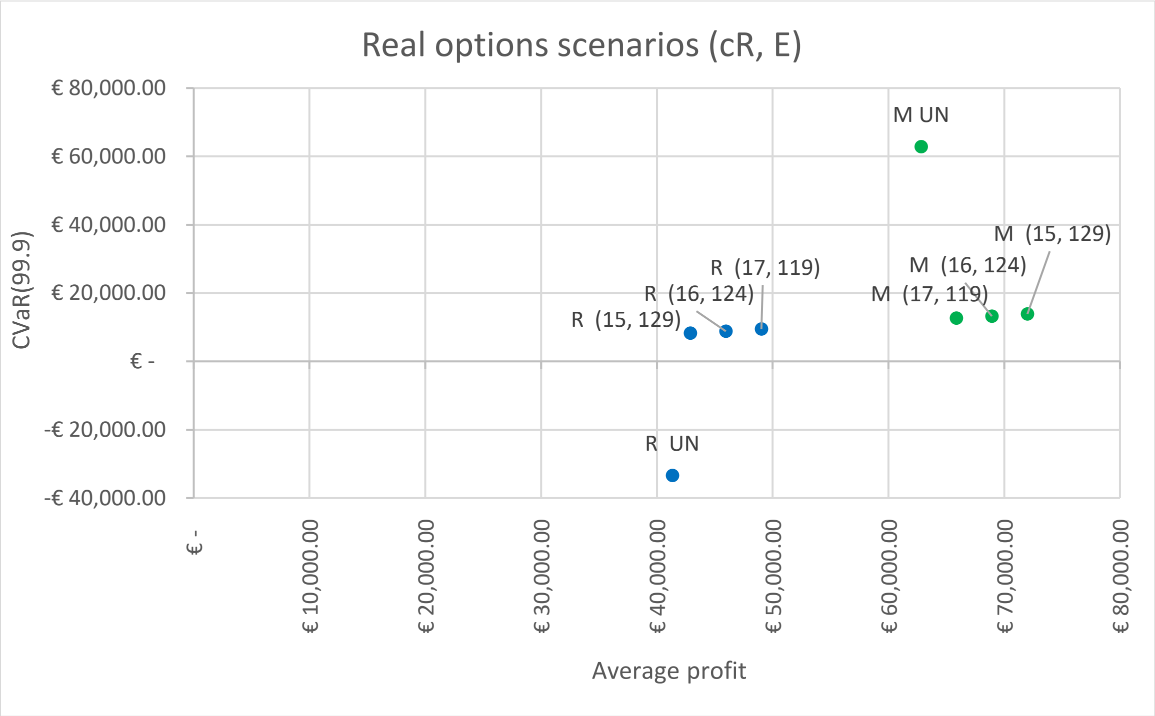


**Figure 13: Risk of three real options scenario in terms of CVaR(99.9), paired with the average profit**

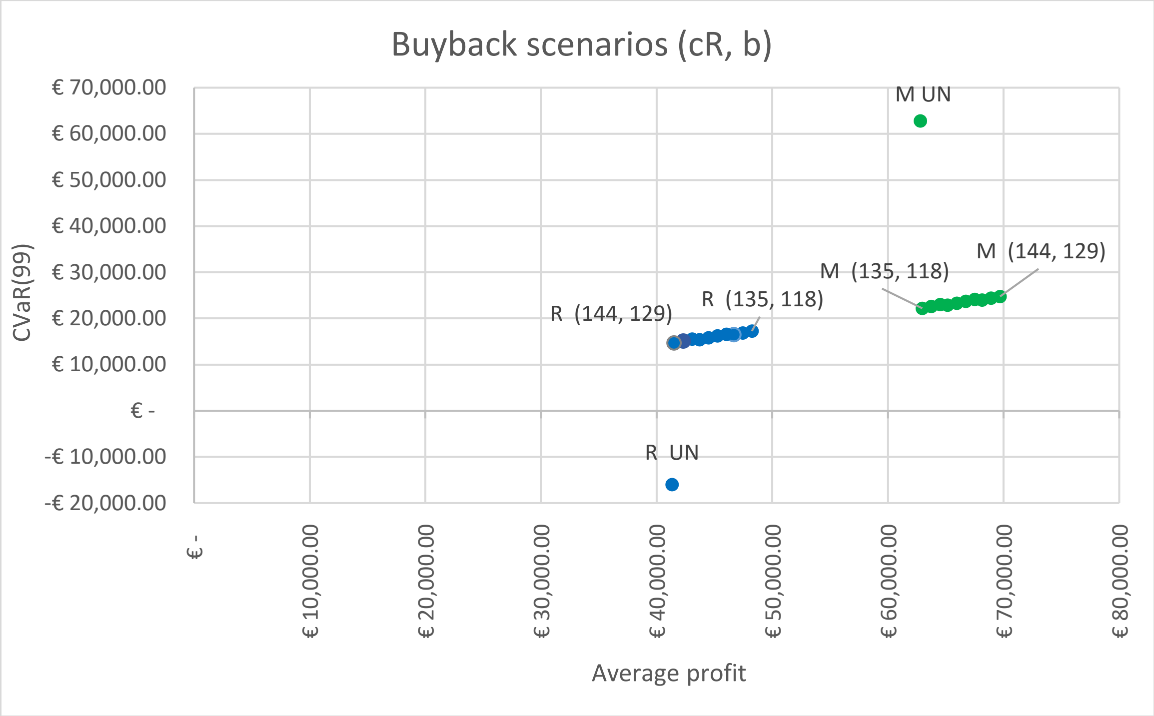


**Figure 14: Risk of ten buyback scenarios in terms of CVaR(99), paired with the average profit**

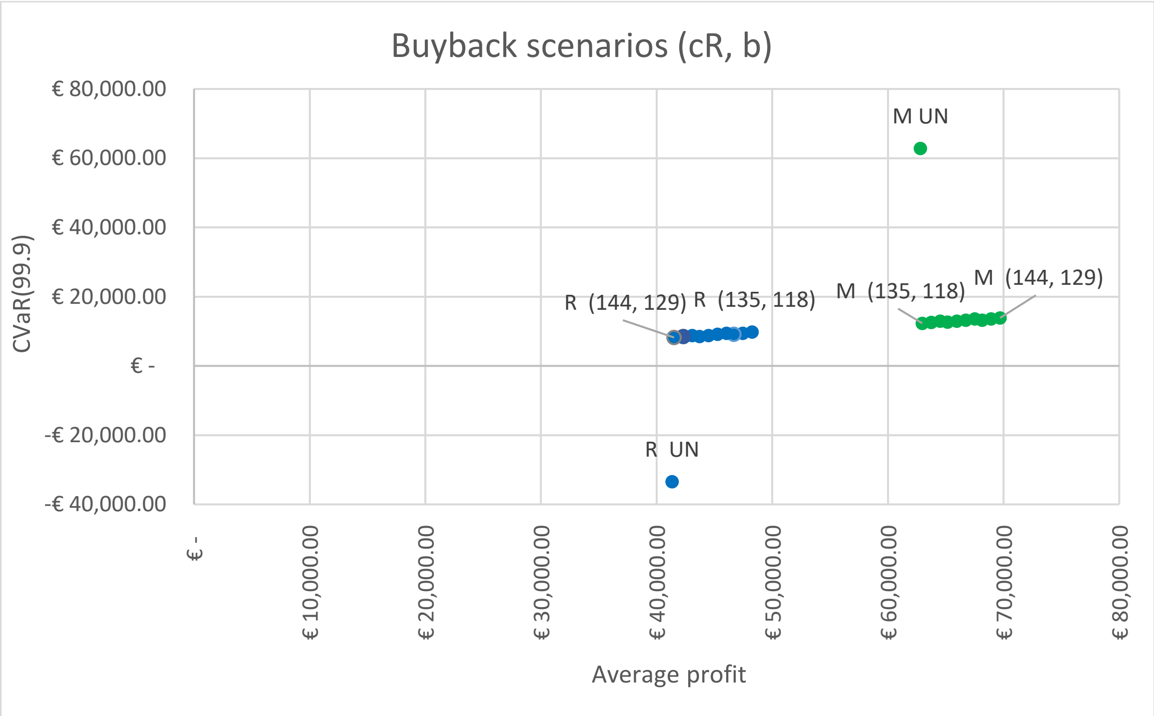


**Figure 15: Risk of ten buyback scenarios in terms of CVaR(99.9), paired with the average profit**

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