Available online at www.sciencedirect.com

ScienceDirect

Journal of Differential Equations 409 (2024) 881–883

Journal of Differential Equations

www.elsevier.com/locate/jde

Corrigendum

Corrigendum to “A flow box theorem for 2d slow-fast vector fields and diffeomorphisms and the slow log-determinant integral” [J. Differ. Equ. 333 (2022) 361–406]

Freddy Dumortier

Dept. Wiskunde, Universiteit Hasselt, Campus Diepenbeek, Agoralaan-Gebouw D, B-3590 Diepenbeek, Belgium

Received 10 November 2022; accepted 7 August 2024
Available online 26 August 2024

Abstract

In [1] the Theorems 1, 2 and 3, as well as Proposition 1, are incorrect as they are stated. To make them correct it suffices to add the extra condition

$$D(x, 0, \lambda) = 0$$

to the expressions (1.1) and (1.4).

The same holds for Definition 2.

MSC: primary 34C20, 34C45, 34D15, 37E30; secondary 34C07, 34C40, 34C23, 37D10, 34E15

Keywords: Planar vector field; Planar diffeomorphism; Normal form; Invariant manifold; Invariant foliation

DOI of original article: https://doi.org/10.1016/j.jde.2022.06.008.
E-mail address: freddy.dumortier@uhasselt.be.

https://doi.org/10.1016/j.jde.2024.08.028
0022-0396/

1. On the generality of the results

The extra condition on D does not restrict the generality of the results on the normal form of 2-d slow-fast vector fields and diffeomorphisms along normally hyperbolic critical curves with regular slow dynamics. It suffices to add two propositions that can be obtained by straightforward calculation. Proposition 2 corrects the flaw at the end of the proof of Lemma 2 in [1].

Proposition 1. *Let $X_{\varepsilon,\lambda}$ be a family of vector fields, given by the equation*

$$\begin{cases} \dot{x} = y.\bar{C}(x, y, \varepsilon, \lambda) + \varepsilon.\bar{B}(x, \varepsilon, \lambda) \\ \dot{y} = y.\bar{A}(x, y, \varepsilon, \lambda) + \varepsilon.\bar{D}(x, \varepsilon, \lambda), \end{cases} \tag{1.1}$$

with $\bar{A}, \bar{B}, \bar{C}$ and $\bar{D}$ smooth functions, having the property that for all $\lambda \in K \subset \mathbb{R}^p$, with K compact, and for all $x \in [0, x_0]$, with $x_0 > 0$, hold the conditions $\bar{a}(x, \lambda) = \bar{A}(x, 0, 0, \lambda) < 0$, and $(\bar{c}\bar{d} - \bar{a}\bar{b}) > 0$ (resp. < 0), for $\bar{b}(x, \lambda) = \bar{B}(x, 0, \lambda), \bar{d}(x, \lambda) = \bar{D}(x, 0, \lambda)$, and $\bar{c}(x, \lambda) = \bar{C}(x, 0, 0, \lambda)$.
Then there exists, on $[0, x_0] \times [-y_0, y_0]$, for $y_0 > 0$ and ε sufficiently close to 0 and for $\lambda \in K$, a smooth (ε, λ)-family of coordinate changes of the form

$$(x, y) \mapsto (x, y + \varepsilon.g_1(x, \lambda)),$$

for some smooth function g_1, transforming expression (1.1) into:

$$\begin{cases} \dot{x} = y.C(x, y, \varepsilon, \lambda) + \varepsilon.B(x, \varepsilon, \lambda) \\ \dot{y} = y.A(x, y, \varepsilon, \lambda) + \varepsilon.D(x, \varepsilon, \lambda), \end{cases} \tag{1.2}$$

with A, B, C and D smooth functions, having the property that for all $\lambda \in K \subset \mathbb{R}^p$, with K compact, and for all $x \in [0, x_0]$, with $x_0 > 0$, hold the conditions $d(x, \lambda) = D(x, 0, \lambda) = 0$, $a(x, \lambda) = A(x, 0, 0, \lambda) = \bar{a}(x, \lambda) < 0$, and

$$b(x, \lambda) = B(x, 0, \lambda) = ((\bar{a}\bar{b} - \bar{c}\bar{d})/\bar{a})(x, \lambda),$$

hence $b(x, \lambda) > 0$ (resp. < 0).

Proposition 2. *Let $f_{\varepsilon,\lambda} : \mathbb{R}^2 \to \mathbb{R}^2$ be a smooth family of diffeomorphisms, given by the equation*

$$(x, y) \mapsto (x + y.\bar{C}(x, y, \varepsilon, \lambda) + \varepsilon.\bar{B}(x, \varepsilon, \lambda), y.\bar{A}(x, y, \varepsilon, \lambda) + \varepsilon.\bar{D}(x, \varepsilon, \lambda)), \tag{1.3}$$

with $\bar{A}, \bar{B}, \bar{C}$ and $\bar{D}$ smooth functions, having the property that for all $\lambda \in K \subset \mathbb{R}^p$, with K compact, and for all $x \in [0, x_0]$, with $x_0 > 0$, hold the conditions $0 < \bar{a}(x, \lambda) = \bar{A}(x, 0, 0, \lambda) < 1$, and $(\bar{b} - \bar{a}\bar{b} + \bar{c}\bar{d})(x, \lambda) > 0$ (resp. < 0), for $\bar{b}(x, \lambda) = \bar{B}(x, 0, \lambda), \bar{d}(x, \lambda) = \bar{D}(x, 0, \lambda)$, and $\bar{c}(x, \lambda) = \bar{C}(x, 0, 0, \lambda)$.
Then there exists, on $[0, x_0] \times [-y_0, y_0]$, for $y_0 > 0$ and ε sufficiently close to 0 and for $\lambda \in K$, a smooth (ε, λ)-family of coordinate changes of the form

$$(x, y) \mapsto (x, y + \varepsilon.g_1(x, \lambda)),$$

for some smooth function g_1, transforming expression (1.3) into:

$$(x, y) \mapsto (x + y.C(x, y, \varepsilon, \lambda) + \varepsilon.B(x, \varepsilon, \lambda), y.A(x, y, \varepsilon, \lambda) + \varepsilon.D(x, \varepsilon, \lambda)), \tag{1.4}$$

with A, B, C and D smooth functions, having the property that for all $\lambda \in K \subset \mathbb{R}^p$, with K compact, and for all $x \in [0, x_0]$, with $x_0 > 0$, hold the conditions $d(x, \lambda) = D(x, 0, \lambda) = 0$, $0 < a(x, \lambda) = A(x, 0, 0, \lambda) = \bar{a}(x, \lambda) < 1$, and

$$b(x, \lambda) = ((\bar{b} - \bar{a}\bar{b} + \bar{c}\bar{d})/(1 - \bar{a}))(x, \lambda),$$

hence $b(x, \lambda) > 0$ (resp. < 0).

Data availability

No data was used for the research described in the article.

Acknowledgments

Thanks to Samuel Jelbart for informing about the incorrect formulation.

References

[1] F. Dumortier, A flow box theorem for 2d slow-fast vector fields and diffeomorphisms and the slow log-determinant integral, J. Differ. Equ. 333 (2022) 361–406.