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The power of power laws and an interpretation of Lotkaian informetric systems as self-similar fractals

by

L. Egghe

Limburgs Universitair Centrum (LUC), Universitaire Campus, B-3590 Diepenbeek, Belgium[1]
and
Universiteit Antwerpen (UA), Campus Drie Eiken, Universiteitsplein 1, B-2610 Wilrijk, Belgium

leo.egghe@luc.ac.be

ABSTRACT

Although already defined in 1926 by A. Lotka, power laws are becoming more and more important since they have been found valid in social networks such as the Internet.

In this paper we prove some unique properties of power laws. We show that they characterise functions with the scale-free property (also called self-similarity property) as well as functions with the product property. They also have some desirable properties above exponential ones.

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[1] Permanent address: Limburgs Universitair Centrum, Universitaire Campus, B-3590 Diepenbeek, Belgium.
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Further, in an old paper [S. Naranan. Nature, vol. 227, N° 5258, p. 631-632, August 8, 1970], Naranan proves the validity of Lotka's law based on the exponential growth of articles in journals and of number of journals. His argument is reproduced here and also a discrete-time argument is given, yielding the same law of Lotka.

We then show that this argument makes it possible to interpret such an information production process as a self-similar fractal and show the relation between Lotka's exponent and the (self-similar) fractal dimension of the system. We hereby show that Lotkaian informetric systems are self-similar fractals, a fact that was only revealed by Mandelbrot but for random texts which are a very special case of an informetric system.

I. Introduction

The historical law of Lotka (see Lotka (1926)) states that if f(p) denotes the number of authors with p=1,2,3,… publications (in a certain field), then

$$f(p) = \frac{C}{p^{\alpha}} \tag{1}$$

where C, $\alpha > 0$ are constants: hence (1) is a decreasing power law. In recent years Lotka's power law has regained a lot of attention since one has shown that this power law is valid in social networks e.g. the Internet as well, now considering websites versus their number of in- or out-links (see e.g. Adamic, Lukose, Puniyani and Huberman (2001) or Huberman (2001)).

As in other "-metrics" theories (such as econometrics), instead of (1), one can use functions of a continuous variable p (ranging in an interval) as well (see Egghe (1989, 1990), Egghe and Rousseau (1990)).

It is a debate for many years now how to "prove" Lotka's law, i.e. to present a mathematical explanation of the regularity (1). Of course, no arguments are possible without assumptions (or "axioms"). One trivial way to proceed is to treat (1) as an axiom and stating hereby that the theory based on (1) will be called "Lotkaian informetrics", hence hereby allowing for "non-Lotkaian-informetrics", in case f(p) is given by another regularity than a power law as in

(1). This approach could be compared with the way Euclidean geometry is built: there the Axiom of Euclid “For every straight line and every point not on this line there exists exactly one straight line, through this point, that is parallel with the first straight line” is stated without proof (all arguments based on this axiom then belong to Euclidean geometry) but one allows for other non-Euclidean axioms, giving rise to non-Euclidean geometries. All these geometries can even have practical applications, dependent on the type of space one is working in.

There is – formally – nothing wrong with such an approach: once Lotkaian informetrics (i.e. informetrics based on (1)) has been established one has a clear view on results that follow from (1) and other ones that cannot be proved using (1). Furthermore Lotkaian informetrics can also reveal regularities that are universally true, also in non-Lotkaian informetrics, as can be checked by performing experiments in such systems.

Having said this, it is not forbidden to check for important properties of functions of the type (1) since such properties could give insight in the importance of a power law for informetrics. This is done in the next section where we characterise power laws as scale-free (i.e. self-similar) functions (i.e. functions satisfying f(Dx)=Ef(x), for all x, where D and E are constants). Scale-free functions hence have the same behavior at any scale of the variable x. We also characterise power functions as functions having the product property (i.e. functions satisfying f(xy)=Ff(x)f(y), for all x,y, where F is a constant.

In section III we indicate some more properties of power functions in “forgetting” processes and we show that e.g. an exponentially decreasing function does not have such a property.

In section IV, we start by reproducing an old argument of Naranan (see Naranan (1970)) showing that Lotka’s law can be derived from the exponential growth of the number of articles in a journal and the exponential growth of the number of journals. Also a discrete-time argument is given, yielding again the same law of Lotka. Its exponent, ∀, is further proved to be equal to $\alpha = 1 + \frac{\ln a_1}{\ln a_2}$, where a_1 is the exponential growth rate of the journals (more generally: sources) and where a_2 is the exponential growth rate of the articles (more generally: items) in these journals (sources). In this section we will also show that the dynamics of Naranan’s argument defines a self-similar fractal (see Feder (1988)) for the underlying informetric system and its fractal dimension is calculated. We show that the fractal dimension, D_f, equals $\alpha - 1 = \frac{\ln a_2}{\ln a_2}$. This considerably extends a result of Mandelbrot which is only valid for random texts (Mandelbrot (1977)). The link of Lotkaian informetrics with self-similar fractals is important: fractal dimensions measure the degree of complexity of systems.

II. Functional properties of power functions.

The following property of a power law of type (1) (we do not even need the fact that f decreases here) is trivial: multiply p by Dp, where D>0 is any positive constant. We then have

$$f(Dp) = \frac{C}{(Dp)^{\alpha}}$$

$$f(Dp) = \frac{\frac{C}{D^{\alpha}}}{p^{\alpha}}$$

$$f(Dp) = E\frac{C}{p^{\alpha}} = Ef(p)$$

hence, up to a constant, the same power law as in (1). Hence power laws satisfy the following property.

Definition II.1: A function f is called scale-free if, for every positive constant D, there is a positive constant E such that

$$f(Dx) = Ef(x) \tag{2}$$

for all x in the domain of f (i.e. the set on which f is defined). Note that the constant E depends on D but not on x.

This is a very important property of f: it garantees that, if there is a change of scale (e.g. change of currency, change of scientific discipline, change of books, change of species, or any transformation $x \to Dx$, i.e. where ratios are constant and the value zero is the same – this is called a ratio scale - see Roberts (1979)), we still measure by the same function f. Hence derived properties such as concentration, complexity (see sections III, IV) or, more simply, the informetrics theory as a whole, remain the same. Hence this also covers evolution in time where the same informetric system is studied but over different time intervals (e.g. a bibliography over 10 or over 30 years).

We have the following important result which can be found in Roberts (1979) (section 4.2).

Theorem II.2: The following assertions are equivalent:

(i) f is a function on $\mathbb{R}^+$ to $\mathbb{R}^+$, is continuous and scale free

(ii) f is a power function, i.e. there exist constants $a \in \mathbb{R}^+$, $b \in \mathbb{R}^+$ such that

$$f(x) = ax^b \tag{3}$$

for all x in the domain of f.

So, for decreasing functions f we have that (i) in Theorem II.2 is equivalent with a decreasing power function as in (1) for $p \in \mathbb{R}^+$, hence with Lotka's law. Next we note the equivalency of the scale free property with the so-called product property.

Definition II.3: A function $f : \mathbb{R}^+ \to \mathbb{R}^+$ is said to have the product property if there exists a positive constant F such that

$$f(xy) = Ff(x)f(y) \tag{4}$$

for all x,y in the domain of f.

Proposition II.4: The following assertions are equivalent for a function $f : \mathbb{R}^+ \to \mathbb{R}^+$:

(i) f is scale free

(ii) f has the product property.

Proof:

(ii)$\Rightarrow$(i) Since f(xy)=Ff(x)f(y) by (4) we can take D=y yielding f(Dx)=Ff(D)f(x)=Ef(x) with E=Ff(D) which is independent of x.

(i)$\Rightarrow$(ii) The scale-free property of f implies the existence of a function E(y), only dependent on y, such that, for all x

$$f(xy) = E(y)f(x) \tag{5}$$

(namely take D = y and E = E(y) in the definition II.1). For x = 1, (5) yields

$$f(y) = E(y)f(1) \tag{6}$$

Now (5) and (6) together yield

$$f(xy) = Ff(x)f(y)$$

where $F = \frac{1}{f(1)}$, an absolute constant. ~

Note that, in the above proof, we did not need the continuity of f. Of course, in order to have a power function, we need the continuity of f (Theorem II.2).

The product property is, essentially, the same as the property repeatedly discussed by Bookstein in Bookstein (1977, 1990, 2001) but there the constant E is taken as 1 due to the requirement that f(1)=1. There is, however, no reason to require this since in case f is a distribution $f(1) \neq 1$ and also if f denotes the actual number of sources, then also $f(1) \neq 1$.

Furthermore Proposition II.4, together with the previous results show that (4) is the more natural property, characterising power functions of the type (1).

A synonym for the scale-free property is the self-similarity property: (2) expresses that, when using another magnitude (say when going from x to Dx) we, essentially, measure properties with the same function f. This will be linked (in section I.3) with self-similar fractals.

We refer the reader to Katz (1999), Bilke and Peterson (2001), Jeong, Tombor, Albert, Ottval and Barabási (2000) and Barabási and Albert (1999) for further discussions on scale-free systems (incl. scale-free aspects of networks, which is very important with respect to size changes).

III. Further comparison of power functions with exponential functions

In this section we will continue the comparison of power functions with exponential functions but in the framework of size-frequency functions, i.e. where f(p) describes the (number or density of) sources with production p. To fix the ideas we will work with

$$f_1(x) = \frac{C}{x^\alpha} \tag{7}$$

$C, \alpha > 0$, $x \geq 1$ (as in (1)) and with

$$f_2(x) = ca^x \tag{8}$$

$c > 0, \quad 0 < a < 1, x \geq 1,$ a decreasing exponential function.

Due to the given characterisation of power functions, it is immediately clear that e.g. exponential functions are not scale-free: the characteristic parameter: a in $f(x) = ca^x$ is changed when applying another scale:

$$f(Dx) = ca^{Dx} = cb^{x} \tag{9}$$

with $b = a^{D} \neq a$ for $D \neq 1$. In this case, measurements are different, when changing scales, a bad property.

Another elementary observation, made in Anderson and Tweney (1997) is also interesting. We have, for the functions f_1, f_2 as in (7), (8):

$$f_1^{'}(x) = -\alpha C x^{-(1+\alpha)} = K\frac{f_1(x)}{x} \tag{10}$$

$$f_2^{'}(x) = (c \ln a) a^{x} = Q f_2(x) \tag{11}$$

where K, Q<0 are constants. In terms of memories f, the function f' measures the "forgetting" process and the function $\frac{f'}{f}$ measures the rate of this change. It is generally accepted that this rate declines in x (e.g. x=t=time in this application) as is the case for f_1:

$$\frac{f_1^{'}(x)}{f_1(x)} = \frac{K}{x} \tag{12}$$

but this is not the case for f_2:

$$\frac{f_2^{'}(x)}{f_2(x)} = Q\,. \tag{13}$$

As required by one of the referees this needs some more explanation, which we find in Anderson and Tweney (1997). Jost's law (1897), as cited in McGeoch (1942) p. 140, expresses that the decay rate for a memory depends on the age of the memory: given two memories of equal strength, the younger memory decays more rapidly than the older one does. According to (12) and (13), the power function is compatible with Jost's law while the exponential one is not (see also the same remarks in Simon (1966)).

So far for some advanteous properties of power functions above exponential functions as size-frequency functions. It is remarkable, however, that exponential growth implies power-type size-frequency functions as will be explained in the next section.

IV. Proof of Lotka's law based on exponential growth

The paper Naranan (1970) has until now – as far as the author can see this – not really been understood for its high informetric value. Naranan supposes exponential growth of sources (journals) as well as of items in sources (articles in journals). He furthermore assumes that the growth rate of items in a source is the same for each source. Although we cannot explain this now, these 3 assumptions are the basis for the description of an informetric system as a self-similar fractal (see later in this section). Even Naranan himself seems to be unaware of this aspect of his theory. In subsection IV.1, we will give Naranan's argument leading to a power law for the size-frequency function f. We will deduce a formula for Lotka's exponent ∀ in function of both growth rates. This is also done in Naranan (1970) but the fractal argument, based on this formula is missing there; we will give it in subsection IV.3, then showing the full importance of Naranan's work. In subsection IV.2 we will present a discrete-time variant of the argument of Naranan in subsection IV.1, leading to the same law of Lotka.

Of course, one remark, as given in Egghe and Rousseau (1990) can be made: "explaining" Lotka's power law by "assuming" an exponential growth is replacing the problem of explaining Lotka's power law to explaining exponential growth. This is the negative way of looking at such arguments. One can argue in a positive way that such an argument makes a link between an informetric theory based on exponential growth and Lotkaian informetrics, which is important and not fully understood in Egghe and Rousseau (1990), let alone its link with fractals. This is the reason why we give Naranan's argument here, hereby also indicating that the cumbersome criticism of Hubert (1976) is not in order.

IV.1 Naranan's argument

Naranan's paper deals with a "classical" informetric system being a bibliography consisting of journals (as sources) and articles (as items) in these journals. We will follow the general source-item terminology, since the argument is universal. He considers an informetric process that grows in time. Naranan's assumptions are 3-fold:

(i) The number of sources grows exponentially in time t. Let us denote this as

$$N(t) = c_1 a_1^t \tag{14}$$

(ii) The number of items p(t) in each source grows exponentially in time t.

(iii) The growth rate and the number of items in a source at t=0 in (ii) is the same for every source.

Because of (ii) and (iii), we have

$$p(t) = c_2 a_2^t \tag{15}$$

where p(t) is the number of items in each source at time t (c_2 and a_2 are the same for all sources).

We need to derive, from (14), the age distribution of the sources. Via the transformation (15) we will then obtain the production distribution of these sources, i.e. the size-frequency distribution. At any time t_0 and $0 < t \le t_0$.

$$\frac{N(t) - N(0)}{N(t_0)} \tag{16}$$

is the fraction of the sources (at t_0) that started (were "born") in the time interval]0,t]. Hence

$$\frac{N'(t)}{N(t_0)} \tag{17}$$

is the density (at t_0) of the sources that started at t itself: indeed

$$\int_0^t \frac{N'(\tau)}{N(t_0)} d\tau = \frac{N(t) - N(0)}{N(t_0)} \tag{18}$$

being equal to (16). So, taking $t = t_0 - \tau$,

$$\frac{N'(t_0 - \tau)}{N(t_0)} \tag{19}$$

is the density of sources that are (at t_0) ϑ time units old. (19) equals

$$\frac{c_1 \left(\ln a_1\right) a_1^{t_0 - \tau}}{c_1 a_1^{t_0}}$$

$$= \left(\ln a_1\right) a_1^{-\tau} \tag{20}$$

Since this is independent of t_0, (20) represents the overall age density. To go from the variable ϑ (age of source) to the variable p (number of items in a source) we use the cumulative distribution F of sources that are less than or equal to ϑ time units old:

$$\int_0^\tau \left(\ln a_1\right) a_1^{-\tau} d\tau \tag{21}$$

We now use (15) twice:

$$p(\tau) = c_2 a_2^\tau$$

implies

$$\frac{dp}{d\tau} = c_2 \left(\ln a_2\right) a_2^\tau$$

and

$$\ln\left(\frac{p}{c_2}\right) = \tau \ln a_2$$

which yields

$$d\tau = \frac{dp}{c_2\left(\ln a_2\right)a_2^{\tau}} = \frac{dp}{p\ln a_2} \tag{22}$$

and

$$\tau = \frac{\ln\left(\frac{p}{c_2}\right)}{\ln a_2}. \tag{23}$$

(22) and (23) in (21) yields

$$F(p) = \int_0^p \frac{\ln a_1}{\ln a_2}\frac{1}{p'}a_1^{-\frac{\ln\left(\frac{p'}{c_2}\right)}{\ln a_2}}dp'$$

$$F(p) = \int_0^p \frac{\ln a_1}{\ln a_2}\left(\frac{p'}{c_2}\right)^{-\frac{\ln a_1}{\ln a_2}}\frac{dp'}{p'}$$

$$F(p) = \int_0^p \frac{\ln a_1}{\ln a_2} c_2^{\frac{\ln a_1}{\ln a_2}} p'^{-\left(1+\frac{\ln a_1}{\ln a_2}\right)} dp' \tag{24}$$

being the cumulative distribution function of sources with less than or equal to p items. Hence, the size-frequency distribution f(p) is nothing else than FN(p). Hence

$$f(p) = \frac{\ln a_1}{\ln a_2} c_2^{\frac{\ln a_1}{\ln a_2}} p^{-\left(1+\frac{\ln a_1}{\ln a_2}\right)} \tag{25}$$

, hence Lotka's law with exponent

$$\alpha = 1 + \frac{\ln a_1}{\ln a_2} \tag{26}$$

So we have shown that Lotka's law can be derived from exponential growth and, furthermore, the exponent α is related to this (double) exponential growth via formula (26), which will play a crucial role in subsection IV.3: (26) will be basis for the fractal description of Lotkaian informetric systems as self-similar fractals. This fact was not mentioned in Naranan and we think it is revealed here for the first time.

IV.2 Discrete-time argument

We now present the same argument but treating time as a discrete variable (t=0,1,2,3,...) and we will show that the same result is valid, hence also showing that the differential calculus of Naranan (giving rise to the dispute in Hubert (1976)) is not needed: an elementary argument of counting provides the same result, showing again that the critcism of Hubert (1976) is not correct. So we suppose we have (14) and (15) for discrete t. Let $t \in \mathbb{N}$ be fixed but arbitrary. At time t there is a fraction of $\frac{N(t-i)}{N(t)}$ sources that exist i or more time units (e.g. a year), hence with p or more items, where by (iii) $p = c_2 a_2^i$, hence $i = \frac{\ln\left(\frac{p}{c_2}\right)}{\ln a_2}$. Hence the fraction $\frac{N(t-i)}{N(t)}$ equals (by(i))

$$\frac{N(t-i)}{N(t)} = \frac{c_1 a_1^{\,t - \frac{\ln\left(\frac{p}{c_2}\right)}{\ln a_2}}}{c_1 a_1^t}$$

$$= \frac{1}{a_1^{\frac{\ln\left(\frac{p}{c_2}\right)}{\ln a_2}}}$$

$$= \left(\frac{c_2}{p}\right)^{\frac{\ln a_1}{\ln a_2}}$$

One can write that

$$G(p) = \left(\frac{c_2}{p}\right)^{\frac{\ln a_1}{\ln a_2}} \tag{27}$$

which is the cumulative fraction of sources with p or more items (i.e. G=1-F with F as in the previous (continuous) argument). Approximating the size-frequency function f(p) by -GN(p) hence yields a power law with the same exponent α as in (26). Indeed

$$G(p) = \int_p^\infty f(p')dp'$$

and so

$$f(p) = -G'(p)$$

$$f(p) = \frac{\ln a_1}{c_2 \ln a_2}\left(\frac{c_2}{p}\right)^{1+\frac{\ln a_1}{\ln a_2}}$$

, hence Lotka's law $\frac{C}{p^\alpha}$ identical with (25) with α as in (26). This shows the validity of Naranan's argument. We rephrase this important result as a theorem (for discrete and continuous time).

<u>Theorem IV.1</u>: In any informetric system: let the sources grow exponentially in time with rate $a_1 > 1$ and let the items in the sources grow exponentially in time with fixed rate $a_2 > 1$, then the size-frequency function of this system is of power type (hence Lotkaian) $f(p) = \frac{C}{p^\alpha}$ where

$$\alpha - 1 = \frac{\ln a_1}{\ln a_2}, \tag{28}$$

the quotient of the logarithms of the growth rate of the sources and the one of the items in the sources. Reversely, any Lotkaian system with size-frequency function $f(p) = \frac{C}{p^{\alpha}}$ can be considered as a system described by Naranan by defining a_1 and a_2 by (28): note the freedom that is there: given any a_1, a_2 satisfying (29), any a_1^b, a_2^b $(b \neq 0)$ satisfies (28) as well.

Apart from its explanatory value for the law of Lotka, the full depth of this result will be explored further on in subsection IV.3.

We have the following trivial but interesting corollary.

Corollary IV.2: If the exponential growth rate of the sources equals the exponential growth rate of the items in the sources, then we have Lotkaian informetrics with Lotka exponent ∀=2.

IV.3 Complexity in informetrics

Complexity in any system is expressed by using fractals and their fractal dimension (see e.g. Feder (1988)). The “easiest” (to handle) fractals are so-called self-similar fractals in which each “level” is constructed in an identical way. Let us give the example of the triadic Koch curve, depicted (only the first levels) in Fig. IV.1: at each level a line piece (as in n=0) is cut into 3 equal pieces and is replaced by a triangular construction of 4 of these line pieces (as in n=1) and this continues at infinity.

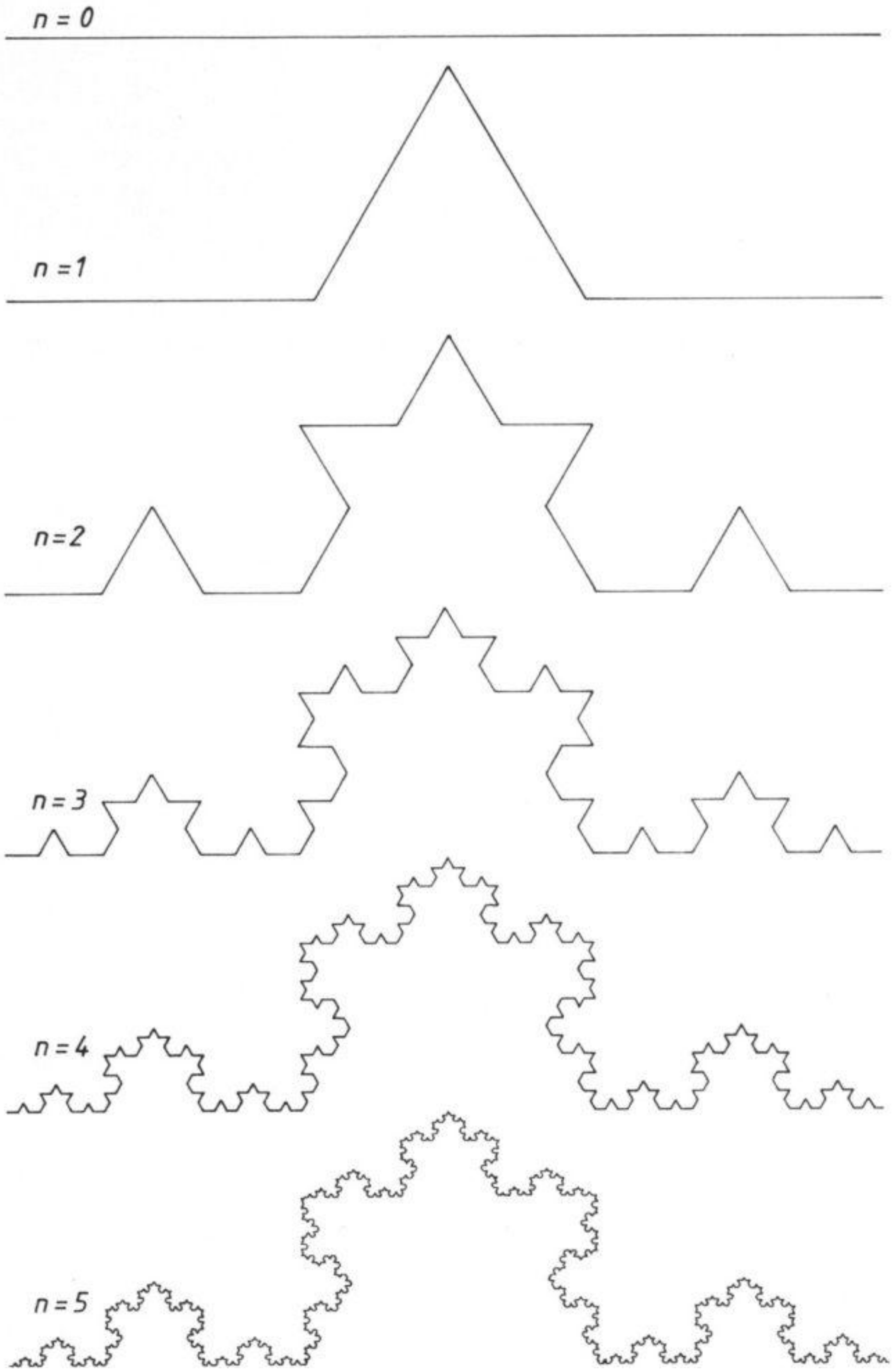

Fig IV.1. Construction of the triadic Koch curve

This construction of the triadic Koch curve can be rephrased as follows (time=level=0,1,2,3,...).

(i) The number of line pieces grows exponentially in time t, here proportional with 4^t (generalisation: a_1^t)

(ii),(iii) 1/length of each line piece is the same for every line piece and grows exponential in time t, here proportional with 3^t (generalisation: a_2^t)

The reader can compare this formalism by the formalism (i), (ii), (iii) of Naranan in subsection IV.1, leading to Lotka's law. This shows that Lotkaian informetric systems can be considered as self-similar fractals, in the following way. In each self-similar fractal, the constituting parts grow (at each level) with a certain rate (say a_1) while their sizes decline with a rate such that $\frac{1}{\text{size}}$ (hence increasing) grows with a rate (say a_2) that is different from the first growth rate. Note also that the decline of the sizes is the same for all constituting parts. Interpreting this for the Koch curve: the constituting parts are line segments and their growth rate at each level is $a_1 = 4$ while for their sizes we have that $\frac{1}{\text{size}}$ grows with a rate $a_2 = 3$ (i.e. sizes decline with a rate $\frac{1}{3}$: at each level the size of the line segment is $\frac{1}{3}$ of the size of the line segments at the previous level and hence it is a constant for each line segment). For Lotkaian informetrics, as proved by the above results we have, in Naranan's interpretation, that the number of sources grows with a growth rate a_1 and that the number of items in each source grows, with equal rate for each source, with a rate a_2. This shows the link of Lotkaian informetrics with the theory of self-similar fractals. In fact, as shown in Section II, Lotkaian informetrics is characterized (amongst all informetric systems) as <u>the</u> type of informetric system where the size-frequency function is a power law.

The triadic Koch curve is an example of a proper fractal. This means that if our scale, say, doubles (e.g. an airplane, from where we watch a coastline, halves its height) the length multiplies with more than 2. The fractal dimension is a way to measure this "more than" and is the alternative for measuring lengths which, strictly speaking, is not possible if we do not indicate from which height we are watching. A classical result in self-similar fractal theory is that its fractal dimension is given by (for the triadic Koch curve)

$$D = \frac{\ln 4}{\ln 3} = 1.26186 > 1$$

and more genrally (using a_1, a_2 above)

$$D = \frac{\ln a_1}{\ln a_2} \tag{29}$$

(cf. Feder (1988)). Note that D can be >1, =1 or <1. Combining the above considerations we have the following theorem.

Theorem IV.3: Suppose we have an informetric system of Lotkaian type, i.e. where the size-frequency function f satisfies

$$f(p) = \frac{C}{p^{\alpha}}$$

where C, $\alpha > 0$, $p \geq 1$. Such a system can be considered as a self-similar fractal with fractal dimension $D_f = \alpha - 1$.

Proof: This follows from Theorem IV.1 and the fact that (29) is the formula for the fractal dimension of a self-similar fractal. ~

This result was earlier seen by Mandelbrot but only in the context of random texts, see Mandelbrot (1977) but the arguments are clarified in Egghe and Rousseau (1990).

In an intuitive way, the theorem above is a consequence of the self-similarity of power functions, described in section II. An example is given by the distribution of website sizes: if one is looking at the distribution of site sizes for one arbitrary range, say sites that have between 1,000 and 2,000 pages, it would look the same as that for a different size range, say from 10 to 100 pages. In other words, zooming in or out in the scale at which one studies the web, one keeps obtaining the same result, just as in the case of the Koch curve (Fig. (IV.1)) (Huberman (2001)).

Examples of fractal dimensions: D=1 for a line, D=2 for a surface, D=1.52 for the coastline of Norway (Feder (1988)). In informetrics: D=1 (hence $\forall=2$) if $a_1=a_2$ (by Corollary IV.2), hence if the growth rate of the sources is the same as the one of the items. Also the higher $\forall$, the higher D and hence the higher the complexity of the informetric system, apparently going together with increasing concentration, as explained in section III. Such high values are common in the description of the connectivity (i.e. the number f(p) of points with p links) of so-called social networks (such as the internet, intranets, citation networks, collaboration networks) as can be seen in Bilke and Peterson (2001), Jeong, Tombor, Albert, Ottval and Barabási (2000), Barabási, Jeong, Néda, Ravasz, Schubert and Vicsek (2002), Adamic, Lukose, Puniyani and Huberman (2001) and Barabási and Albert (1999).

V. Conclusions

In this paper we showed the importance of power laws and its appplicability in informetrics (incl. social networks such as the Internet). The importance lies in the fact that power laws (and only power laws) satisfy the so-called scale-free property which makes systems self-similar at any measuring scale.

This leaded us to self-similar fractals: an old argument of Naranan (1970) for continuous time is extended here so that it is also valid in discrete time. In both cases it is shown that exponential growth both of articles in journals as well as of journals leads to a power law (Lotka function). Further we show that this argument defines a self-similar fractal (Feder (1988)) for the underlying informetric system and we prove that $\alpha - 1$ is the fractal dimension of such a system (where α is the exponent in Lotka's power law).

Hence this paper links Lotkaian informetrics with the mathematical theory of fractals. This is important in itself but gives us also the possibility to measure the complexity of such an informetric system.

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