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The influence of publication delays on the observed aging distribution of  
scientific literature

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Abstract

Observed aging curves are influenced by publication delays. In this article we show how the 'undisturbed' aging function and the publication delay combine to give the observed aging function. This combination is performed by a mathematical operation known as convolution. Examples are given such as the convolution of two Poisson distributions, two exponential distributions and of two lognormal ones. A paradox is observed between theory and real data.

## 1. Introduction

In (Luwel & Moed, 1997) the authors study the influence of publication delays on the aging of scientific literature. In their article aging is understood as a decrease in use as shown by journal references. Hence the term 'aging curve' is here synonymous with 'age distribution of references'. Further, Luwel and Moed (1997) define the publication delay as the time between the submission of a manuscript and the actual publication. It is clear that publication delays have an influence on the aging distribution of an article, or on a larger scale, on the impact factor of a journal. This, in turn can influence the output of evaluation procedures. Moreover, if publication delays differ between fields, this is another factor, next to e.g. citation behavior, that makes comparisons between fields very difficult. In their article Luwel and Moed pose the following research problems:

- (i) Obtain insight in the distribution of publication delays, both among and between scientific subfields ;
- (ii) Estimate the publication delay of a journal from the age distribution of its references ;
- (iii) What would the age distribution of references be if there were no publication delays, or at least if they would decrease radically?

In this article we will study the relation between publication delays and the observed aging distribution. Our investigations will mostly be theoretical, but based on observed data (Luwel and Moed, 1997) and with practical applications in mind.

## 2. General considerations about the observed age distribution of references

An observed citation  $n$  time units after publication is the result of a potential citation  $m$  time units after publication ( $m < n$ ) and a publication delay of  $n - m$  time units. Here all  $m$ 's ( $> 0$ ) have to be considered. This means that the observed aging distribution is the convolution of the undisturbed aging distribution (if there were no publication delays) and the distribution of publication delays. We will assume that the delay distribution itself does not depend on time. Recall (Oppenheim et al., 1983) that the convolution operation for sequences is defined as:

$$(a * b)_n = \sum_{i=-\infty}^{+\infty} a_i b_{n-i} = \sum_{i=-\infty}^{+\infty} a_{n-i} b_i = (b * a)_n \quad (1)$$

Often sequences start at the point zero (or one): then the convolution becomes:

$$(a * b)(n) = \sum_{k=0}^n a(k)b(n-k) \quad (2)$$

Similarly, the convolution of the functions  $f_1(x)$  and  $f_2(x)$  is defined as:

$$(f_1 * f_2)(x) = \int_{-\infty}^{\infty} f_1(u) f_2(x-u) du = \int_{-\infty}^{\infty} f_1(x-u) f_2(u) du = (f_2 * f_1)(x) \quad (3)$$

and if  $f$  and  $g$  are zero (or undefined) on the negative real axis, this expression becomes:

$$(f_1 * f_2)(x) = \int_0^x f_1(u) f_2(x-u) du \quad (4)$$

From a stochastic point of view, we can say that the output of this process, namely the number of citations  $n$  time units after publication of the cited article, is a stochastic variable. This stochastic variable is the sum of two other stochastic variables: the number of citations  $n$  time units after publication of the cited article, measured at the time of acceptance, and the publication delay. In this system the publication delay plays the role of the system function (or impulse response, cf. (Rousseau, 1998)). Assuming that these two stochastic variables are independent, we know that the probability density function of the output is the convolution of the probability density function of the input and the probability density function of the system function. For more details about the convolution operation and its potential in informetrics we refer the reader to (Rousseau, 1998).

From this analysis we can already say that it is in general impossible to answer research problem two of Luwel and Moed. Indeed, as the observed aging curve is the convolution of the undisturbed aging curve and the delay distribution, it follows that the only way to obtain the delay distribution –

without collecting delay data – is by a deconvolution of the aging curves. However, the undisturbed aging curve is not observable, hence this deconvolution can not be performed. On the other hand, knowing the delay distribution and the observed aging distribution leads, via deconvolution (Rousseau, 1998), to the undisturbed aging distribution.

As the undisturbed aging distribution and the delay distribution are assumed to be independent, the mean of the convolution, i.e. the observed aging distribution is the sum of the means of each of its components (Rousseau, 1998). So, the larger the average publication delay, the larger the shift of the average of the aging curve. Similarly, the variance of the convolution is the sum of the variance of each of its components. So, by publication delays citations are more spread in time than without delays. Moreover, the more uneven the delays are (larger variance), the larger the influence on the citation curve. In particular, as the influence of the variance can only be felt by a flattening of the citation curve in the direction of later times, fields with more irregular delay times suffer a larger influence on the short-time impact factor (typically the two-year impact factor).

In this article we will consider a number of special cases. Convolutions of Poisson distributions, exponential ones, and of lognormal distributions will be studied. We will also make some observations concerning the aging rate.

### 3. A discrete approach: the Poisson distribution

#### 3.1 A Poisson distribution describes undisturbed aging and delay

Assume that both the input (aging distribution),  $f(k)$ , and the delay distribution (i.e. the system function),  $h(l)$ , are Poisson distributed. We recall that a Poisson distribution for citations was put forward by Pauline Brown (1980):

$$P(X = k) = f(k) = \frac{\alpha^k}{k!} e^{-\alpha} \quad (5)$$

and

$$P(Y = l) = h(l) = \frac{\beta^l}{l!} e^{-\beta} \quad (6)$$

Here,  $\alpha$  and  $\beta$  ( $> 0$ ) are the parameters of the Poisson distributions;  $k$  and  $l$  take integer values  $0, 1, \dots$ . It is well-known (Feller, 1970, p. 268) that the convolution of two Poisson distributions with parameters  $\alpha$  and  $\beta$  is again a Poisson distribution, this times with parameter  $\alpha + \beta$ . In particular, we know that

$$E(Z=X+Y) = E(X) + E(Y)$$

and

$$\text{Var}(Z=X+Y) = \text{Var}(X) + \text{Var}(Y)$$

The mode of a Poisson distribution occurs when  $k$  (resp.  $l$ ) is the largest integer smaller than the Poisson parameter  $\alpha$  ( $\beta$ ) (if  $\alpha$  ( $\beta$ ) is an integer then  $\alpha$  and  $\alpha-1$  ( $\beta$  and  $\beta-1$ ) are tie modes). As  $E(X) = \alpha$  and  $E(Y) = \beta$ , this implies that the mode of the convolution is at least equal to the mode of each of the

contributing distributions, and is usually larger. Moreover, the values at the mode are decreasing in  $\alpha$  ( $\beta$ ), so that the top of the convolution is smaller than the highest values of each of the Poisson distributions that are convolved.

### 3.2 The aging rate of the convolution of two Poisson distributions

The aging rate of the observed citation distribution  $g = f * h$  is given as:

$$\frac{g(k+1)}{g(k)} = \frac{(\alpha + \beta)^{k+1} e^{-(\alpha+\beta)} k!}{(k+1)! (\alpha + \beta)^k e^{-(\alpha+\beta)}} = \frac{\alpha + \beta}{k+1} \quad (7)$$

So we see that at any one moment the aging of the observed distribution (the convolution) is larger than that of the publication delay distribution (which is equal to  $\beta/k+1$ ) and that of the 'undisturbed' aging distribution (equaling  $\alpha/k+1$ ).

## 4. A first continuous approach: the exponential distribution

### 4.1 Undisturbed aging and delay are described by an exponential distribution

As a first continuous case we will consider exponential distributions. Although, for small values, they do not describe real aging distributions, it is well known that they are generally good approximations for large values (Brookes, 1970; Egghe & Rousseau, 1990). Assume first that the two exponential distributions have different parameters,  $\lambda$  and  $\kappa$  :

$$f(t) = \lambda e^{-\lambda t} \text{ and } h(t) = \kappa e^{-\kappa t} \quad (8).$$

Their convolution is:



$$g(t) = (f * h)(t) = \int_0^t \lambda \kappa e^{-\lambda(t-s)} e^{-\kappa s} ds$$

or:

$$g(t) = \frac{\lambda \kappa}{\kappa - \lambda} (e^{-\lambda t} - e^{-\kappa t}) \quad (9)$$

With the same parameter  $\lambda$ , this leads to a gamma or Erlang distribution of order two with parameter  $\lambda$ :

$$g(t) = (f * h)(t) = \int_0^t \lambda^2 e^{-\lambda s} ds = \lambda^2 e^{-\lambda t} t \quad (10)$$

Both forms are unimodal: first a concave increase, followed by a concave decrease and finally a convex decrease (Fig. 1). This seems remarkable: a convolution of two decreasing curves yields indeed a unimodal curve. Avramescu (1979) proposed the resulting curve (9) as the citation curve, see also (Egghe & Rousseau, 1990).

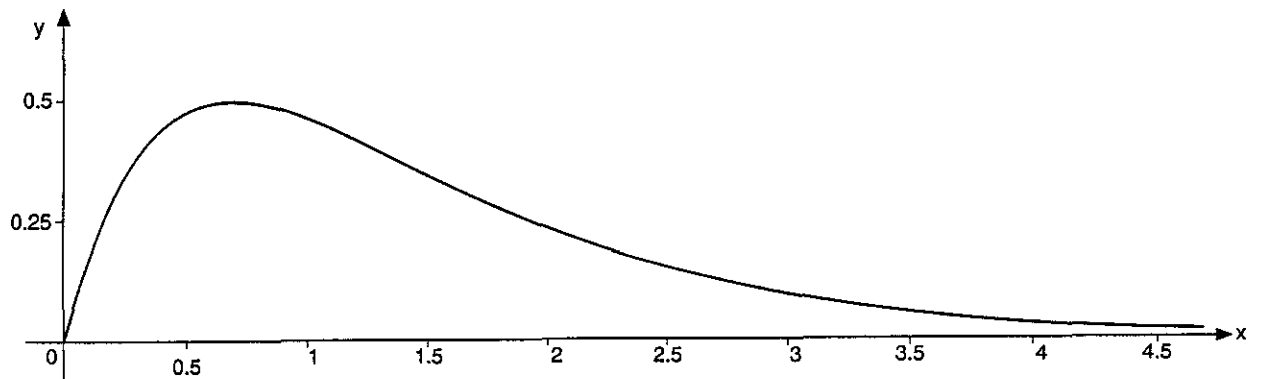


Fig. 1 The Avramescu function, with  $\kappa = 2$  and  $\lambda = 1$

## 4.2 Aging of the Avramescu distribution

We assume that  $\kappa > \lambda$  : then

$$\frac{g(t+1)}{g(t)} = \frac{e^{-\lambda(t+1)} - e^{-\kappa(t+1)}}{e^{-\lambda t} - e^{-\kappa t}}$$

We can rewrite this as:

$$\frac{g(t+1)}{g(t)} = \frac{e^{-\lambda(t+1)} (1 - e^{-(\kappa-\lambda)(t+1)})}{e^{-\lambda t} (1 - e^{-(\kappa-\lambda)t})} \quad (11)$$

so that, in the limit, the aging rate becomes equal to  $e^{-\lambda}$ . Hence we conclude that always the distribution which is initially dominant has the least influence for large values. Note thus that, although this is not likely, it is possible in this model that the publication delay becomes the major explaining factor for long-term aging.

In the continuous case aging can also be characterized by the continuous aging rate (Egghe, 1994):

$$e^{\frac{g'(t)}{g(t)}} \quad (12)$$

Note that this approach is preferable for continuous distributions. The conclusion, however, is the same. Indeed, for  $\kappa > \lambda$ , we have:

$$\frac{g'(t)}{g(t)} = \frac{-\lambda e^{-\lambda t} + \kappa e^{-\kappa t}}{e^{-\lambda t} - e^{-\kappa t}} = \frac{-\lambda + \kappa e^{-(\kappa-\lambda)t}}{1 - e^{-(\kappa-\lambda)t}} \quad (13)$$

Consequently,

$$\lim_{t \rightarrow \infty} e^{\frac{g'(t)}{g(t)}} = e^{-\lambda} \quad (14)$$

## 5. General considerations on the initial part of a convolution curve

We could go on in this way and study distributions such as Pareto's, the lognormal, Weibull's, combined with the exponential distribution or with themselves. Yet, we will first investigate the general shape of a convolution of continuous distributions. In particular, we will be interested to know whether such a convolution starts in a convex or in a concave way. For short, such distributions will be called convex and concave distributions, irrespective of their behavior further on. Note that we will always assume that the distribution functions are zero for negative values, hence the initial value (value at zero) of the convolution is always zero.

### 5.1 Case I: two decreasing distributions functions $f(t)$ and $h(t)$

More precisely, the assumptions are:  $f$  is differentiable on some interval  $[0, t_0[$  and  $h$  is twice differentiable on this interval. Moreover,  $f > 0$ ,  $f' < 0$ ,  $h > 0$ ,  $h' < 0$  and  $h'' > 0$  ( $h$  is a convex function), where these derivatives exist.

Then:

$$(f * h)(t) = \int_0^t f(s)h(t-s)ds \quad (15)$$

and (Apostol, 1974, p.220):

$$(f * h)'(t) = \int_0^t f(s)h'(t-s)ds + f(t)h(0) = (f * h')(t) + f(t)h(0) \quad (16)$$

As, for  $0 \leq s \leq t$ ,  $f(0) \geq f(s) \geq f(t)$ , and as  $h'$  is negative, we have for  $t \geq 0$ :

$$0 > f(t) \int_0^t h'(t-s)ds \geq \int_0^t f(s)h'(t-s)ds \geq f(0) \int_0^t h'(t-s)ds \quad (17)$$

Integrating the outer integrals in (17) leads to:

$$0 > f(t)(h(t) - h(0)) \geq \int_0^t f(s)h'(t-s)ds \geq f(0)(h(t) - h(0))$$

Hence,

$$(f * h)'(t) = \int_0^t f(s)h'(t-s)ds + f(t)h(0) \geq f(0)h(t) - f(0)h(0) + f(t)h(0)$$

Denoting  $f(0)h(t) - f(0)h(0) + f(t)h(0)$  as  $F(t)$ , we see that  $F(0) = f(0)h(0) > 0$ .

Consequently, there is an interval  $[0, t_1]$  such that  $(f * h)'(t) > 0$ . In other words the convolution of  $f$  and  $h$  starts as an increasing function.  $F(t) = 0$  in the point  $t_M$  satisfying  $f(0)h(t_M) + f(t_M)h(0) = f(0)h(0)$ . Consequently, the mode of  $(f * h)$  is larger than this point  $t_M$ . This can be checked e.g. for the convolution of two

exponential distributions, one with parameter 1 and the other with parameter 2. Its mode is situated at  $\ln(2) = 0.693$ . The zero of the corresponding function  $F(t)$  is at  $t = 0.31$ .

Now, for every  $t \in [0, t_0[$  (Apostol, 1974, p.220):

$$(f * h)''(t) = \int_0^t f(s)h''(t-s) ds + f'(t)h(0) + f(t)h'(0) \quad (18)$$

Then:

$$(f * h)''(t) \leq f(0)(h'(t) - h'(0)) + f'(t)h(0) + f(t)h'(0)$$

Consequently,  $(f * h)''(t) < 0$ , if  $G(t) = f(0)h'(t) - f(0)h'(0) + f'(t)h(0) + f(t)h'(0) < 0$ . As  $G(0) = f'(0)h(0) + f(0)h'(0) < 0$ , we conclude that by continuity  $G(t) < 0$  on an interval of the form  $[0, t_2]$ . This proves that the convolution of two decreasing functions (where at least one of the two is convex) is always concave (see Fig.2).

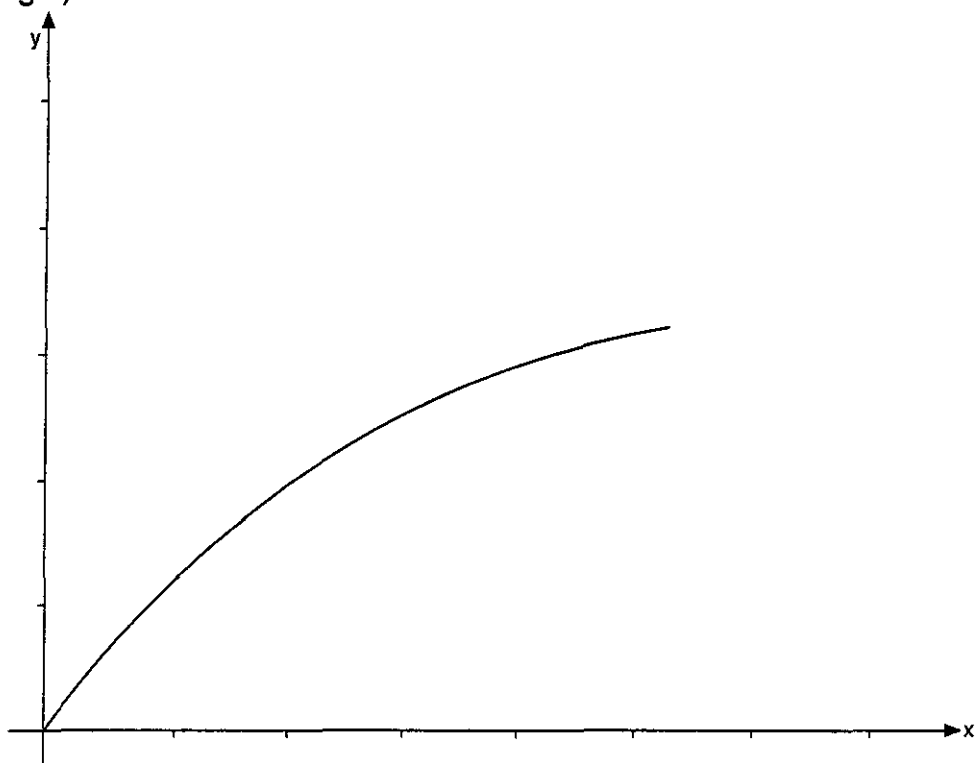


Fig.2 Initial part of the convolution of two decreasing distributions

4.2 Case II At least one of the distribution functions  $f$  and  $h$  is zero at the origin and hence increases.

We assume that  $f$  is a probability distribution that is not constantly zero in a neighborhood of zero. The other function,  $h$ , is assumed to be zero at the origin ( $h(0) = 0$ ), and increases on an interval  $[0, T_0]$  ( $h'(t) > 0$ ). Under these assumptions it is easy to see that  $(f * h)$  must be increasing on the interval  $[0, T_0]$ . Indeed,

$$(f * h)'(t) = (h * f)'(t) = \int_0^t f(t-s)h'(s) ds \quad (19)$$

and expression (19) is clearly positive on  $[0, T_0]$  as both  $f$  and  $h'$  are positive on this interval. Further, we will consider the cases

- (i)  $h'' \geq 0$  on some interval  $[0, T_1]$ ;
- (ii)  $h'' < 0$  on some interval  $[0, T_1]$  and  $f(0) = 0$  (hence  $f' > 0$  on some interval near 0)

Note that these are the only two cases possible (given the general assumptions that  $h(0) = 0$  and  $h' > 0$  on  $[0, T_0]$ ). Now

$$(f * h)''(t) = \int_0^t f(s)h''(t-s) ds + f(t)h'(0) \quad (20)$$

The second term on the right in (20) is clearly positive and if  $h''$  is non-negative on some interval  $[0, T_1]$ ,  $(f * h)''$  will also be non-negative on this interval. Hence the convolution is convex, whatever the shape of  $f(t)$ .

If now  $f(0) = 0$  and  $h'' < 0$ , then we note that

$$(f * h)''(t) = (f' * h')(t) + f'(t)h(0) + h'(t)f(0) = (f' * h')(t). \quad (21)$$

As  $f' > 0$  (if  $f$  is zero in the origin, then it must be increasing in some interval beginning at 0) and also  $h' > 0$  in some neighborhood of zero, this shows that also under these circumstances  $f * h$  is convex (see Fig.3 ).

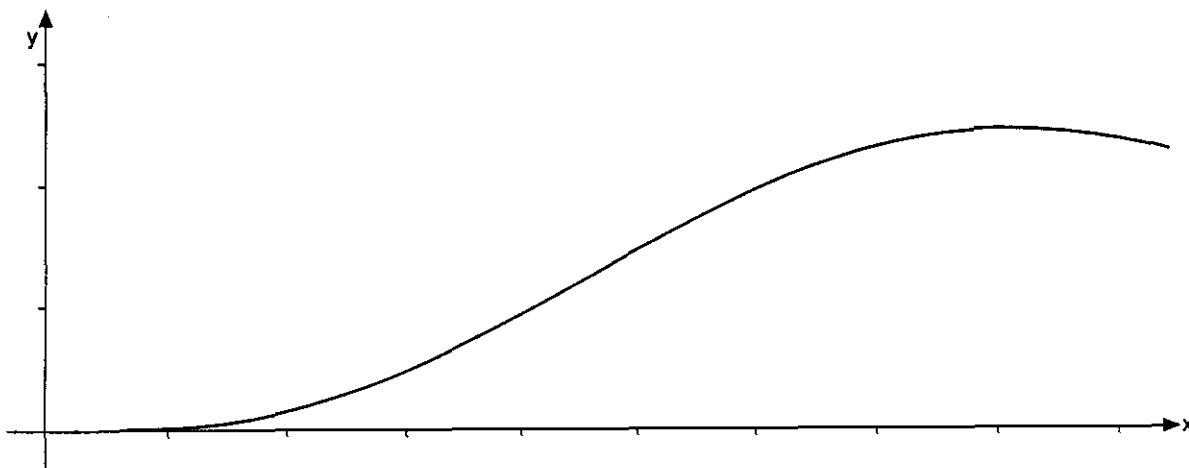


Fig.3 Initial part of the convolution of two distributions, where at least one of the two is increasing

This covers all cases, so that we conclude that if not both distributions are decreasing, then the convolution is convex. This makes it much more likely that an observed aging distribution is convex than otherwise. This reasoning practically eliminates Avramescu's function. As such it constitutes a corroboration of the Egghe-Rao observation (1992), based on the behavior of the aging function.

## 6. The lognormal distribution

### 6.1 Undisturbed aging and delay are described by a lognormal distribution

As another example we will study the case that both distributions are lognormally distributed. In (Egghe & Rao, 1992) and (Matriccioni, 1991) the lognormal distribution was proposed as the (synchronous) citation distribution or aging curve. This means that this case is more realistic than that of convolving two exponential distributions.

We will assume that both  $f(t)$  and  $h(t)$  are lognormally distributed. From the preceding considerations we know already that their convolution is convex, but here we will obtain more detailed information. Note, however, that from (Crow & Shimizu, 1988) it is clear that there is no hope of analytically evaluating the convolution of two lognormal distributions.

We assume that

$$f(t) = \frac{1}{\sqrt{2\pi} \sigma t} e^{-\frac{(\ln(t) - \mu)^2}{2\sigma^2}} \quad (22)$$

for  $t > 0$  and  $f(t) = 0$  otherwise. The function  $h(t)$  is also assumed to be lognormally distributed, but with parameters  $\mu'$  and  $\sigma'$ . Their convolution  $g = f * h$  is:



$$g(t) = \frac{1}{2\pi\sigma\sigma'} \int_0^t \frac{1}{s(t-s)} e^{-\left(\frac{\ln(s)-\mu)^2}{2\sigma^2} + \frac{(\ln(t-s)-\mu')^2}{2\sigma'^2}\right)} ds \quad (23)$$

The integral in (23) cannot be solved using elementary functions. Therefore, we will study (23) from a qualitative point of view. As  $g(t) = (f * h)(t) = (h * f)(t)$ , we see that the derivative of  $g(t)$ ,  $g'(t) = (f' * h)(t)$ , as  $f(0) = h(0) = 0$ . However,

$$f'(t) = -\frac{1}{\sqrt{2\pi}\sigma t^2} e^{-\frac{(\ln(t)-\mu)^2}{2\sigma^2}} \left(1 + \frac{\ln(t)-\mu}{\sigma^2}\right) \quad (24)$$

From this we derive that  $f'(t) > 0$ , whenever  $t < e^{\mu - \sigma^2} = \text{Mo}(f)$ , the mode of  $f(t)$ . The derivative is negative for values larger than  $\text{Mo}(f)$ . Since  $h(t)$  is everywhere positive, for  $t > 0$ , we see that, for  $0 < t < \text{Mo}(f)$ ,  $g'(t)$ , which is then an integral of a strictly positive function, is also strictly positive. As  $g'(t)$  is continuous, this means that there exists a number  $a > 0$ , such that  $g'(t)$  is strictly positive for  $t \leq e^{\mu - \sigma^2} + a$ . This result shows that the convolution of two lognormal distribution increases beyond the mode of one of the two lognormal distributions. By the commutativity of the convolution operator (Rousseau, 1998) this implies that  $g(t)$  increases beyond the mode of both.

Furthermore, from

$$g'(t) = \int_0^t f'(t-s)h(s) ds \quad (25)$$

it follows that:

$$g''(t) = \int_0^t f''(t-s)h(s)ds + f'(0)h(t) \quad (26)$$

Since  $f''(t) > 0$  for  $t$  in a certain interval  $[0, t_f]$ ,  $f'(0) > 0$  and  $h(t) \geq 0$ , we see that

$g''(t) > 0$  for  $t$  in  $[0, t_f]$ . As  $g''(t)$  is continuous, this implies that  $g''(t)$  is strictly

positive on an interval  $[0, t'']$ , with  $t'' > t_f$ . Again, we can reverse the roles of  $f$

and  $h$  and conclude that  $g(t)$  is convex on an interval of the form  $[0, t_g]$ , with  $t_g$

$> \max(t_f, t_h)$ . Since  $g$  is a distribution function it must tend to zero at infinity.

Moreover, since  $f'$  only changes sign once we observe from (25) that this must

also be the case for  $g$ . Hence the convolution of two lognormal distributions is

unimodal. Bringing everything together shows that  $g(t)$  has the following

shape:

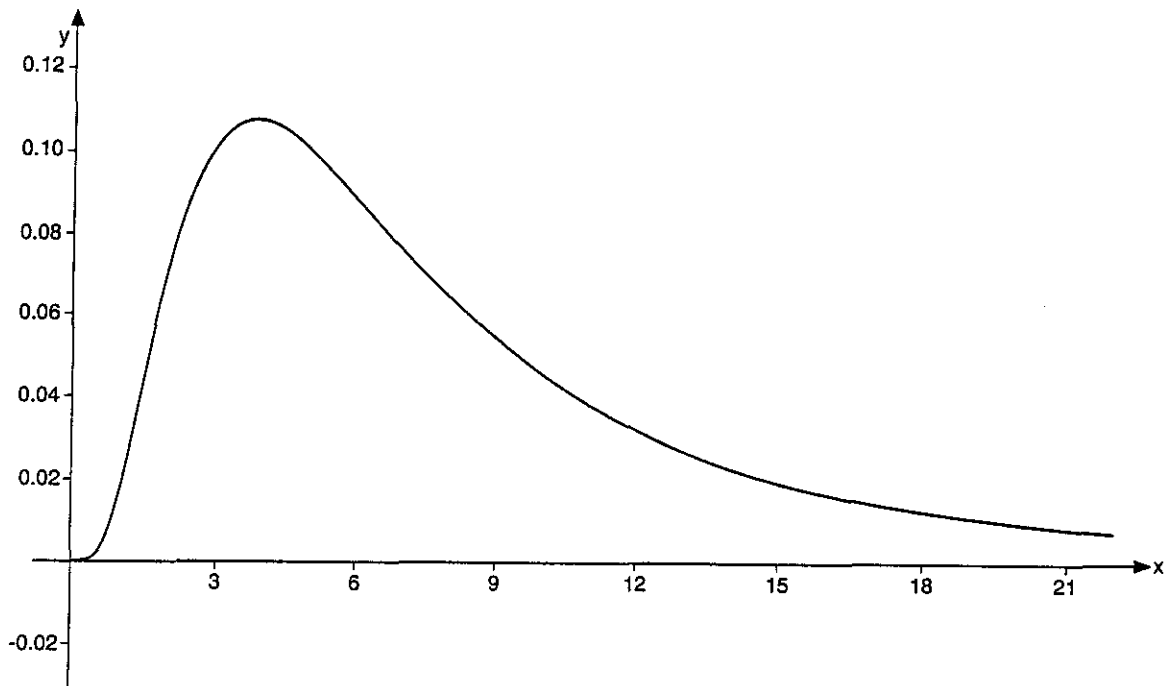


Fig.4 The convolution of two lognormal distributions with  $\mu = \mu' = \sigma = \sigma' = 1$

Its mode is strictly larger than  $\max(\text{Mo}(f), \text{Mo}(h))$ . Fig.4 clearly shows that under these assumptions the observed aging curve is a retarded 'lognormal-like' curve. More details about the lognormal distribution can be found in e.g. (Aitchison and Brown, 1969).

## 7. Some observations about other combinations

### 7.1 The convolution of two Pareto distributions

The Pareto distribution, which is the continuous version of Lotka's (which is the reason why we mention it) has the following probability distribution:

$$f(x) = \frac{c}{x^{c+1}}$$

for  $x \geq 1$  ( $c > 0$ ) and zero elsewhere. Taking the convolution of two Pareto distributions yields a curve which is defined on the interval  $[2, +\infty[$ , and has a similar shape as Avramescu's distribution. Hence, it is not a likely candidate for an aging function.

### 7.2 Other combinations

We finally note that the convolutions of an exponential and a Weibull distribution, or the convolution of two Weibull distributions have a shape that is very similar to that of two lognormal distributions (see Figs.5,6). We recall (Rousseau, 1993) that the Weibull distribution is also a good candidate to describe citation distributions.

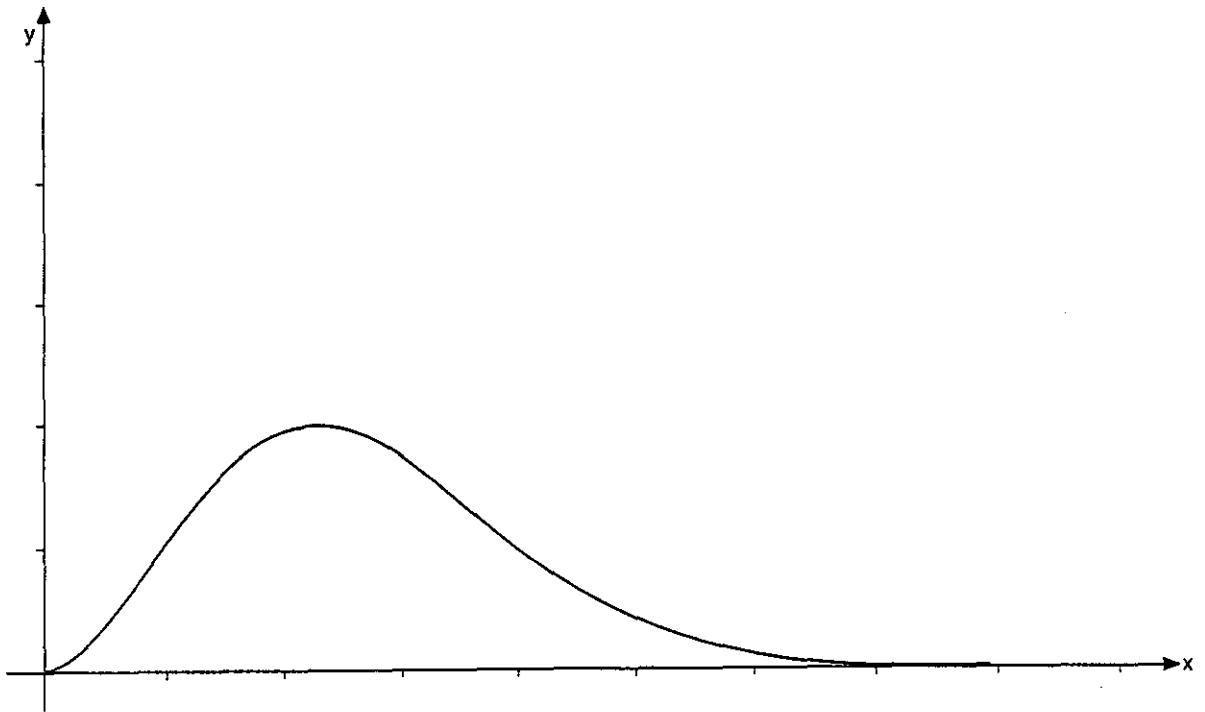


Fig.5 General shape of the convolution of a Weibull (with parameters 2 and 5) and an exponential distribution (with parameter 1)

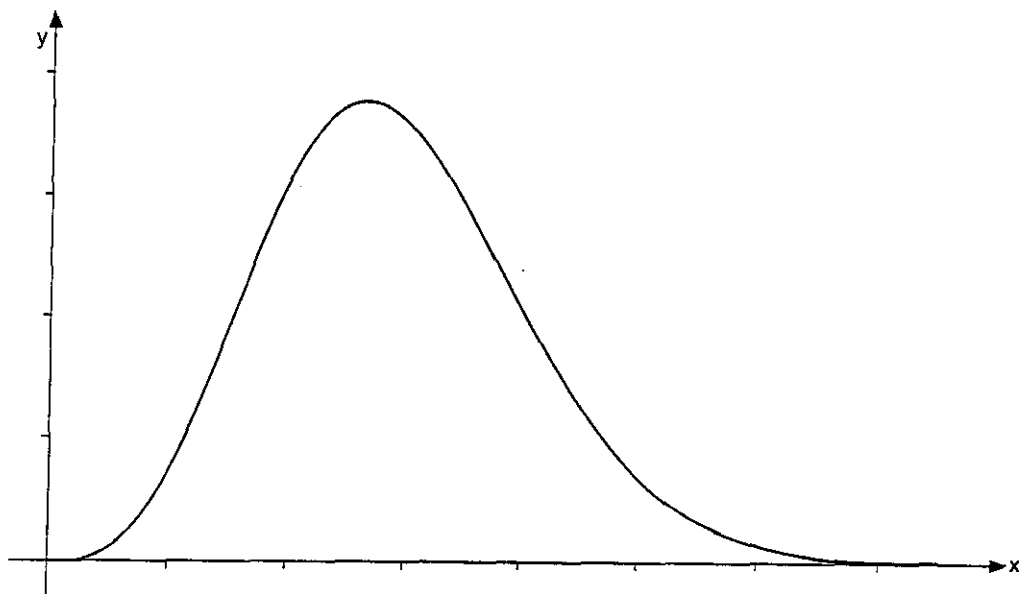


Fig.6 General shape of the convolution of two identical Weibull distributions (with parameters 2 and 5)

## 8. A paradox

### 8.1 Considering the research-citation cycle leads to a paradox

The research-citation cycle (Wouters, 1997) has many stages: each of which can be described by a distribution function; see Fig.7 for our own adaptation of it in the publication-citation context. Even if all of these distributions are decreasing, the resulting one is convex at the origin. So, unless these stages are described by non-classical distributions, the resulting observed aging distribution must be convex.

Real yearly aging data as presented by Nakamoto (1988) - synchronous as well as diachronous - and Brown (1980) are concave. We checked this in the JCR 1994 and 1995 for the journals J BIOL CHEM, P NATL ACAD SCI, NATURE, SCIENCE and J AM CHEM SOC. These journals too show the same concave pattern. Only occasionally we find a convex citation distribution, e.g. for Transactions of the American Mathematical Society (1995 cited journal data). So why are the observations not in accordance with the theory?

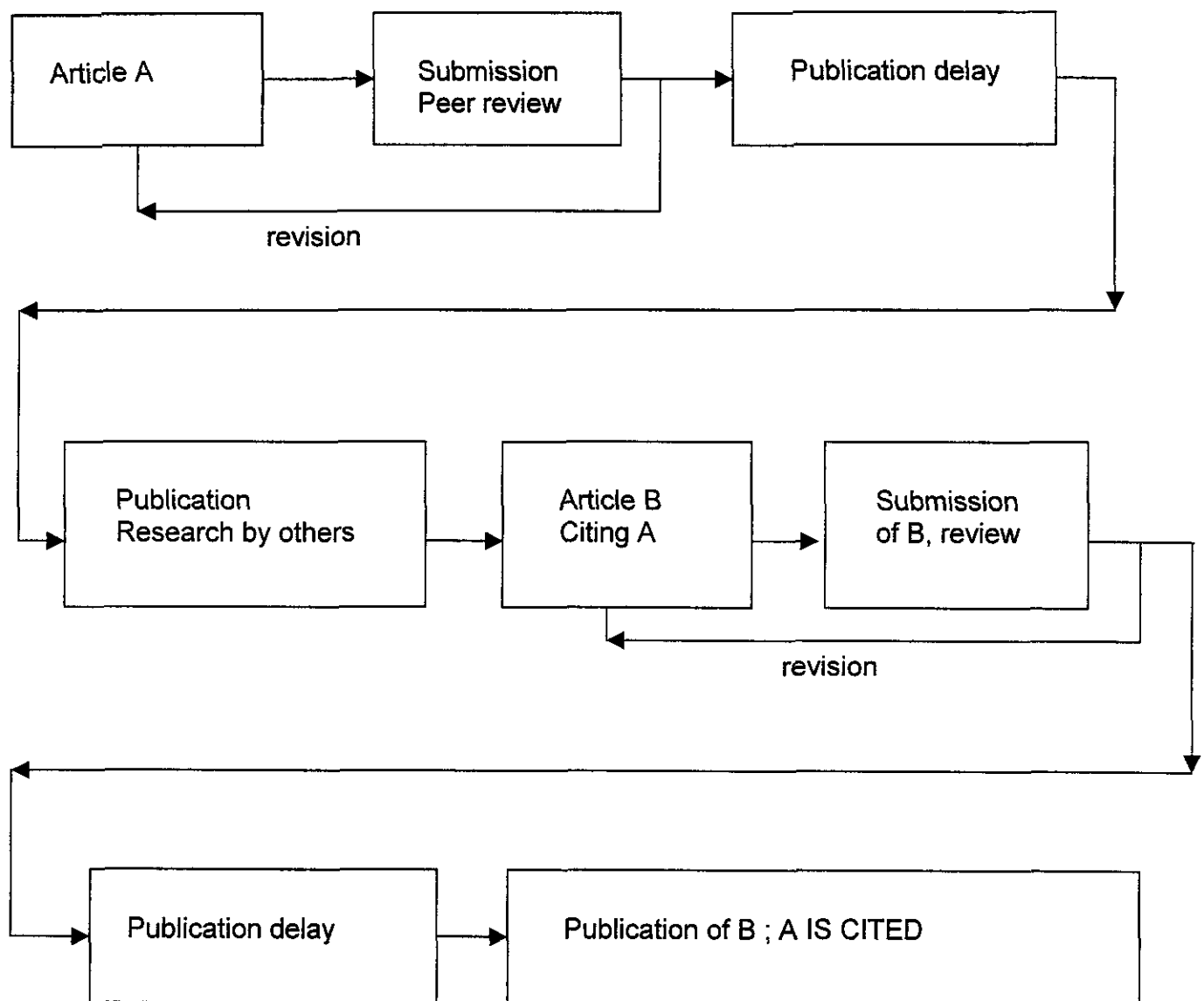


Fig. 7 The research-citation cycle

## 8.2 A possible solution

We assume that this 'paradox' is largely the result of discretizing otherwise continuous phenomena. See Fig.8 However, we leave this as an open problem. Note also that the convex hull of a Poisson distribution can be convex as well as concave, depending on the parameter, so that the Poisson distribution seems to be good contender to describe yearly citation data.

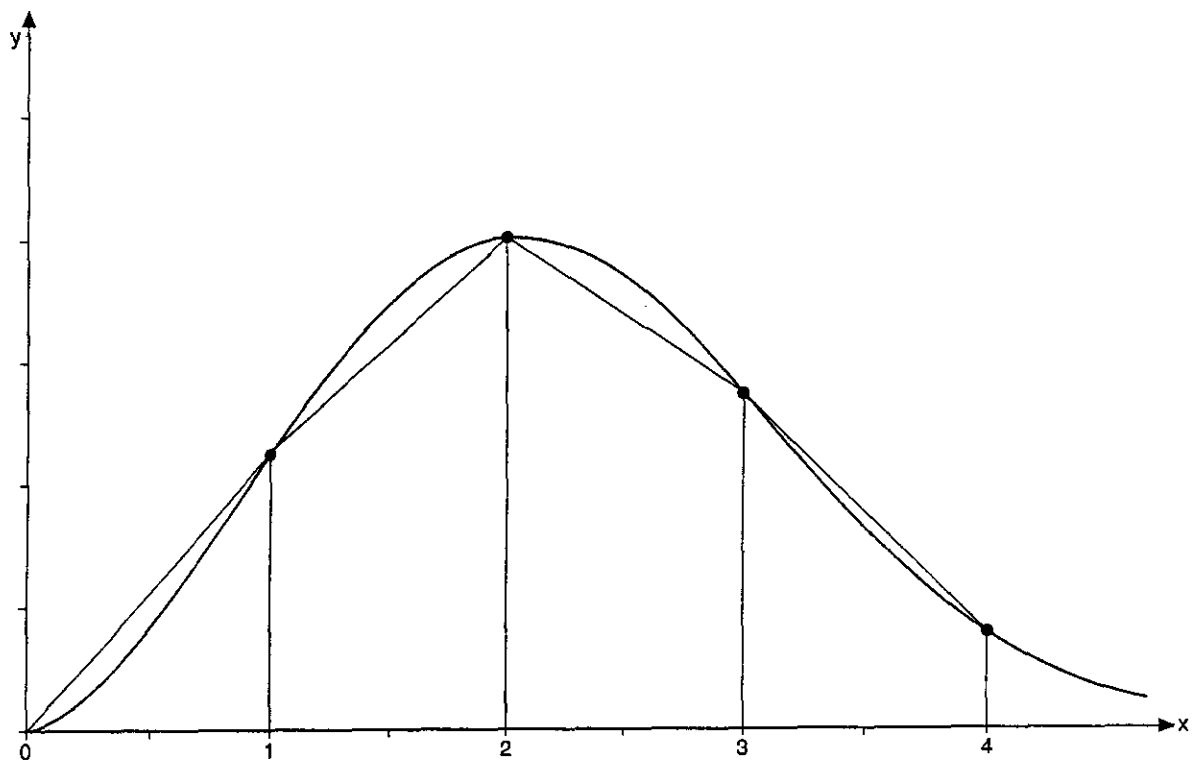


Fig.8 Discretizing a convex distribution may yield  
a function with a concave hull.

## 9. Conclusions and suggestions for further research

We have explained how the publication delay function interacts with the aging distribution. A convolution of the publication delay and the 'undisturbed' aging function yields the observed aging function. A number of likely candidates such as the Poisson and the lognormal distribution were studied in this respect. Generally we found that publication delays shift the mode of the aging curve to later times. Depending on the exact distribution parameters publication delays can have an influence on long-term aging. This effect, however, takes not always place,

As Luwel and Moed do not give details on the delay curve, more research and collection of detailed data is needed, both for publication delays and observed aging data. In particular, we need citation data on a finer scale (finer than yearly), perhaps even monthly. Moreover, on such a fine scale self-citations will have to be separated from other citations.

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