A Bayesian Hierarchical Approach to Model the Rank of Hazardous Intersections for Bicyclists Using the Gibbs Sampler

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Samenvatting

In deze paper worden Bayesiaanse hiërarchische modellen gebruikt om in Leuven, een kleine universiteitsstad in België, gevaarlijke kruispunten voor fietsers te identificeren en te rangschikken. De doelstelling van deze paper is de studie van het proces dat gevolgd wordt om lijsten op te stellen van gevaarlijke kruispunten, gebaseerd op de beschikbare ongevallendata. Een hiërarchisch model met random effecten laat toe verschillende bronnen van variatie te specificeren, namelijk de variatie tussen de kruispunten en de variatie binnen elk kruispunt afzonderlijk. De Gibbs sampler wordt gebruikt om de verdeling van de fietsongevallenproporties te onderzoeken. Een belangrijk voordeel van de Gibbs sampler is de mogelijkheid om steekproeven te nemen uit complexe functies van de fietsongevallenproporties, zoals de rangorde van een kruispunt. In de tekst wordt aangetoond dat de rangorde zelf als een dichtheid kan worden beschouwd. Omdat de fietsongevallenproportie een stochastisch karakter heeft, kan de rangorde, gebaseerd op de gemiddelde a posteriori proportie, niet deterministisch zijn. Het ordenen van gevaarlijke locaties is een interessante manier om inzicht te verwerven in het concept van gevaarlijke punten, maar er bestaat niet zoiets als "de" juiste rangorde. Deze tekst onderzoekt de vraag of een rangorde alleen genoeg motivatie kan geven voor de selectie van een gevaarlijk punt.

Summary

In this paper, Bayesian hierarchical modeling techniques are used to identify and rank hazardous intersections for bicycles in Leuven, a small university town in Belgium. The objective of this paper is to infer the process of listing the most dangerous intersections, based on the available accident data. The hierarchical random effects model allows the specification of different sources of variation, namely the variation between intersections and the variation within each intersection. The Gibbs sampler is used to explore the distribution of the bicycle accident proportions. An important advantage of the Gibbs sampler is the possibility to sample complex functions of the bicycle accident proportions, like the rank of an intersection. It is shown that the ranking itself could be seen as a density. Since the bicycle accident proportions have a stochastic character, the ranking of intersections based on the mean posterior proportion cannot be deterministic. Ranking hazardous sites is an interesting means to get insight in dangerous locations, but there is no such thing as "the" correct ranking. This paper investigates the question whether a ranking alone can give enough evidence for the selection of dangerous sites.

Inhoudsopgave

1. INTRODUCTION

In the year 2000, almost 10% of all seriously and fatally injured road accident victims in Belgium were bicyclists. Moreover, 34% of all seriously and fatally injured victims are to be found in urban areas (*1*). One may conclude that the accident risk for vulnerable road users is too high. It is important to improve their situation, and to rank order the sites that are particularly dangerous for this subgroup.

In this paper, Bayesian binomial hierarchical modeling techniques are used to identify and rank hazardous intersections for bicycles in Leuven, a small university town in Belgium. The objective of this paper is to obtain a ranking of the most dangerous intersections, based on the available accident data. The Gibbs sampler is used to explore the distribution of the bicycle accident proportions. An important advantage of the Gibbs sampler is the possibility to infer not only an estimated proportion or count, but also complex functions of the bicycle accident proportions, like the rank of an intersection (*2*). Since the bicycle accident proportions have a stochastic character, the ranking of intersections based on the posterior mean proportion cannot be deterministic. When decision makers base their investment choices on a ranking of dangerous sites, they should recognize that the ranking is probabilistic in nature. The strength of a particular ranking scheme may be questioned when taking into account the accompanying uncertainty.

The choice for a small university city to investigate bicycle accidents at intersections can be easily explained. First, since most of the students at university stay in Leuven on weekdays, it is to be expected that they move around by bike. Because of the higher concentration of bicycles, it would be interesting to know the most dangerous sites. Second, the local government is working on a mobility plan. Authorities want to stimulate the use of bicycles in the inner city. In order to provide a save infrastructure, a selection of dangerous road sections is made, and investments are done according to a priority list (*3*). Because intersections have always been dangerous locations, it makes sense to look for an intersection prioritization.

At first sight, the selection of hazardous intersections seems to be a straightforward task. Hauer and Persaud (*4*) describe a two-stage procedure to identify hazardous locations, which can be applied to intersections as well. In a first step, accident history of different intersections is reviewed to select a number of apparently dangerous sites. In the second step, the selected intersections are studied in more detail in order to optimally assign investments. However, the intersections with the highest observed bicycle accident rate are not always the most dangerous ones. It is known that the number of accidents at a specific location is fluctuating from year to year. By chance, one year an intersection may have an extremely high count of accidents, without being really hazardous. If investments are to be spread over different locations, decision makers are more interested in sites where accidents are caused by permanent problems. Using empirical accident rates as estimates, without taking into account this uncertainty, one cannot guarantee that the most dangerous sites will be dealt with first.

Recently, Bayesian techniques have been used to solve these problems in accident ranking. Although the problem of hazardous intersection identification has been widely discussed in literature (*5*), the interest in Bayesian methods to improve the process only originated in the eighties. Ever since, many applications used in some way an Empirical Bayes approach. Hauer (*6*) presented the Empirical Bayes approach as a better estimate of the expected number of accidents, because of the enhanced accuracy of the estimates. Hauer and Persaud (*4*) examined the performance of some identification procedures. Empirical Bayes methods were used to estimate proportions of correctly and falsely identified deviant road sections. Belanger (*7*) applied Empirical Bayes methods to estimate the safety of four-legged unsignalized intersections. The results were used to identify black spot locations. Hauer (*5*) reviewed the development of procedures to

identify hazardous locations in general. Vogelesang (8) gives a very comprehensive overview of Empirical Bayes methods in road safety research.

The use of hierarchical Bayesian models in traffic safety is less widespread. Schlüter et al. (9) deal with the problem of selecting a subset of accident sites based on a probability assertion that the worst sites are selected first. They propose three criteria for site selection: (a) the posterior probability that the underlying accident rate for a site is larger than the underlying accident rates of the other sites by a given amount, (b) the probability that the number of accidents at a site during the next period will exceed some specified threshold and (c) the expected number of future accidents. To estimate accident frequencies, a hierarchical Bayesian Poisson model has been used. Christiansen et al. (*10*) developed a hierarchical Bayesian Poisson regression model to estimate and rank accident sites, using a modified posterior accident rate estimate as a selection criterion. Davis and Yang (*11*) combined hierarchical Bayes methods with an induced exposure model to identify intersections where the crash risk for a subgroup is relatively high. Point and interval estimates of the relative crash risk for older drivers were obtained using the Gibbs sampler.

The paper is organized as follows. First, some concepts of hierarchical Bayes, Markov Chain Monte Carlo simulation and Gibbs sampling are briefly described. It is also shown how the rank distribution for each intersection can be sampled. This section is followed by a description of the data set. Next, the results of an empirical study on 168 intersections are presented and discussed. The paper is completed with some conclusions and directions for future research.

2. BAYESIAN HIERARCHICAL MODELS AND MCMC (GIBBS) SAMPLING

The problem of bicycle accident proportions can be expressed in a binomial hierarchical model. A group of 168 intersections in the inner city of Leuven is examined and the number of accidents in the period 1991 to 1998 is counted. In some of the accidents at an intersection, bicyclists might have been involved. The objective is to find, for each intersection, an estimate of the proportion of bicycle accidents.

2.1 Hierarchical Models

Suppose that the true proportion of bicycle accidents at intersections is θ. In this case, it is implicitly assumed that accidents at all intersections happened in similar circumstances. This is not a very realistic setting, and it makes sense to assume that accidents at different intersections occur in slightly different conditions. Indeed, it is natural to suppose that the intersections and their accidents are not completely similar. Accidents may occur at different times, with different drivers. Intersections are to be found at different locations, with a specific infrastructure. It is to be expected that the proportions for the intersections will differ, but it might be acceptable to assume that they are drawn from the same distribution. Instead of using θ as the proportion of bicycle accidents, a proportion θ_i is used for intersection i. All θ_i values are sampled from one population, called a hyper population.

Based on Gelman et al. (*12*), this setting can be schematically represented as in Figure 1. Here, the value r_i ($i = 1, 2, …, p$) is the observed number of bicycle accidents at intersection i. The estimated proportion of bicycle accidents at intersection i is equal to r_i / n_i. This is an estimate for the "true" (but unobserved) proportion θ_i at intersection i.
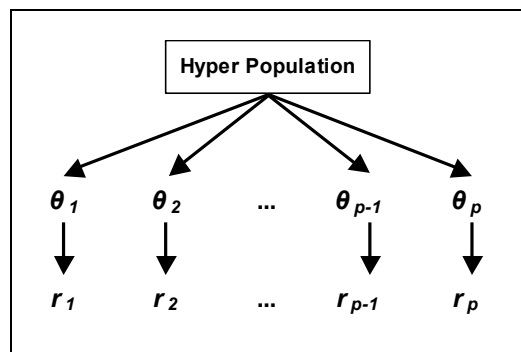

FIGURE 1 A Diagram of the Hierarchical Bayesian Model

Once the hierarchical structure is determined, a prior distribution should be chosen for the hyper population and its parameters. When there is no information available for the hyper parameters, a non-informative prior is preferred. One frequently used approach is to obtain point estimates for the parameters of the hyper prior from the data. This is the Empirical Bayes approach. Since the hyper parameters are replaced by their point estimates, this model cannot fully take into account their uncertainty. In a full Bayesian approach, a prior distribution is specified for the hyper parameters, to express the uncertainty about their true value.

If no further prior information is available, a non-informative prior should be assigned to the hyper parameters (*12*). Although the use of informative priors might be useful, there is always some reason to use non-informative priors (*13*). First, in many cases it is simply not possible to obtain an informative prior distribution, not only because of

organizational constraints such as time limits or financial restrictions, but also because of the lack of expert knowledge. Second, even if it is possible to elicit prior expert knowledge, there is the danger of ending up with a poor definition of the prior information. Once the knowledge is available, it should still be presented in a suitable functional form, which is not always straightforward. Moreover, elicitation of prior knowledge typically ends up with only some features of the prior information. It should be possible then to obtain the rest of the prior definition in a convenient way. Third, in statistical analysis one is often attracted by the possibility of presenting objective results. The use of informative prior information may be considered as a reduction in objectivity. The use of non-informative priors enables the researcher to maintain a high level of objectivity in the analysis. At the same time, he can benefit from the advantages of a Bayesian approach.

2.2 MCMC Techniques and Gibbs Sampling

The main interest in this study is in the posterior probability for bicycle accidents at each intersection. The joint posterior distribution of these probabilities is given by (*14*):

$$p(\underline{\theta} \mid \underline{r}) = \frac{p(\underline{r} \mid \underline{\theta})p(\underline{\theta})}{\int p(\underline{r} \mid \underline{\theta})p(\underline{\theta})d\underline{\theta}} \, .$$

Here $\underline{\theta}$ is the vector of unobserved bicycle accident proportions, and $\underline{r}$ is the vector of the observed number of bicycle accidents (data). Since a ranking of intersections would be a useful outcome of this analysis, inference is required on single proportions, say θ_k for intersection k. This is achieved by integrating the joint posterior distribution over all other parameters θ_i ($i = 1, 2, …, k\text{-}1, k\text{+}1, …, p$).

$$p(\theta_k \mid \underline{r}) = \int p(\underline{\theta} \mid \underline{r})d\theta_1 d\theta_2 … d\theta_{k-1}d\theta_{k+1}…d\theta_p$$

One way to solve these marginal posterior probabilities is to work out the integrals analytically, for which the calculations may become very cumbersome. Another approach is to sample from the posterior distribution. This is the basic principle of Monte Carlo simulation. Numerical calculations are carried out using simulation. Instead of analytically calculating exact or approximate estimates, the Markov Chain Monte Carlo (MCMC) technique generates a stream of simulated values for the quantities of interest (*2*). Marginal posterior inference then involves the classical data summaries like the mean and the standard deviation.

In general, samples should be drawn from the joint posterior distribution. Whereas independent sampling from the posterior may be difficult, it is possible to sample from a Markov Chain with the joint posterior as its stationary distribution. A sequence of random variables $\theta^{(0)}$, $\theta^{(1)}$, $\theta^{(2)},...$ is a Markov Chain if $\theta^{(i+1)} \sim p(\theta \mid \theta^{(i)})$. This implies that, conditional on the value of $\theta^{(i)}$, $\theta^{(i+1)}$ is independent of $\theta^{(i-1)}$, ..., $\theta^{(0)}$. Monte Carlo simulation may be applied for Bayesian posterior inference, using simulated values of all the unknown posterior bicycle accident proportions, generated from a Markov Chain with the joint posterior as its stationary distribution. To design a Markov Chain with this property, Gibbs sampling may be used.

If the vector of parameters $\underline{\theta}$ consists of p sub-components, then starting values $\theta_1^{(0)}, \theta_2^{(0)},...,\theta_p^{(0)}$ should be defined. The following sampling scheme is then repeated

thousands of times, to eventually obtain a sample from $p(\underline{\theta} \mid \underline{r})$, the stationary distribution of the Markov Chain:

Sample $\theta_1^{(1)}$ from $p\!\left(\theta_1 \mid \theta_2^{(0)}, \theta_3^{(0)}, \ldots, \theta_p^{(0)}, \underline{r}\right)$;

Sample $\theta_2^{(1)}$ from $p\!\left(\theta_2 \mid \theta_1^{(1)}, \theta_3^{(0)}, \ldots, \theta_p^{(0)}, \underline{r}\right)$;

...

Sample $\theta_p^{(1)}$ from $p\!\left(\theta_p \mid \theta_1^{(1)}, \theta_2^{(1)}, \ldots, \theta_{p-1}^{(1)}, \underline{r}\right)$.

After a large number of iterations, the distribution of the bicycle accident proportion at each intersection will have been sampled. Posterior summary statistics may be calculated to infer the newly obtained distributions.

One particular application of the Gibbs sampler is the ability to make inferences on arbitrary functions of unknown model parameters (*2*). An example of this feature is the computation of the rank probability of bicycle accidents for each intersection. The result is a sample from the posterior distribution of the ranks. This sample may then be summarized by the mean or median rank for each intersection. Moreover, a 95% credibility interval may be added to express the uncertainty associated with the rank position of each intersection.

At every other iteration, the Gibbs sampler provides an estimate of the rank for a given intersection. For an intersection *i*, all other intersections are scanned to find those with an equal or lower estimated bicycle accident probability. The number of these intersections corresponds to the rank of intersection *i*, because they all have a lower posterior bicycle accident probability.

3. DATA

This study is based on the data set of traffic accidents obtained from the National Institute of Statistics (NIS) for the region of Flanders (Belgium) over the years 1991 to 1998. Out of this data set, all accidents on intersections in the inner city of Leuven were selected. For each accident, a flag indicated whether bicyclists were involved. Since Leuven is a small university city, it is expected that accidents with bicyclists frequently occur. In total, 642 accidents at 168 intersections were identified, with accident counts ranging from 1 to 30. In 369 accidents, at least one bicyclist was involved. This overall percentage of more than 50% bicycle accidents indicates the importance of the study of vulnerable road users.

Some remarks about the data set should be added. First, data is available only for intersections where accidents happened. All results should therefore be interpreted conditional on the occurrence of accidents.

Second, the data set used in this study contains the accidents with slightly, seriously and fatally injured road users. Since the data set reports on intersections in the inner city of Leuven, it is reasonable to assume that the reported numbers are an underestimation of the real accident counts. Moreover, no distinction is made according to accident gravity.

Third, it is assumed that the state (infrastructure, traffic direction…) of the intersections did not change during the period of analysis. Therefore, abstraction is made of the order of the accidents over the years. Only counts and proportions are considered at the intersection level.

Fourth, the model does not consider spatial correlations among intersections. One could argue that neighboring sites might have an influence on the safety of each other. Distances and geographical neighborhood should be measured in order to take correlations into account. This complex extension, however, is not worked out in the paper.

Fifth, no explanatory variables are used. Although the framework of Bayesian hierarchical models offers all necessary tools to include covariates, and the use of explanatory variables may indeed be useful in the context of traffic safety for bicyclists, there are some major problems with the use of covariates in the current setting. Since the data are obtained from intersections in the inner city of a university town, the characteristics are almost equal for all intersections. As a result, the study objects are quite homogeneous, and the explanatory power of the covariates will probably be limited. Also, because all results are conditional on the occurrence of accidents, explanatory variables in this model would not be generally significant.

4. RESULTS

4.1 Binomial Hierarchical Model

A full Bayesian approach to the problem is followed in this paper. A Binomial hierarchical model is presented, using non-informative priors for the hyper parameters (*12*). The number of bicycle accidents, out of the total number of accidents, is modeled as a binomial distributed random variable. The proportion of bicycle accidents is used as a vague measure of "bicycle risk". The choice for this measure is partly dictated by the available data. Since data are on hand only for intersections with accidents, zero-accident intersections are not taken into account.

Strictly speaking, the bicycle accident proportion used in the analysis is a measure of bicycle risk, conditional on the occurrence of accidents, and it is only a vague indicator of the degree of deficiency of the intersection. If an accident happens, the proportion can be interpreted as the probability of a bicycle accident. Although it is known that this proportion is not the best measure of bicycle risk, it is well suited to illustrate the possibilities of the Bayesian framework in the context of ranking hazardous intersections.

Formally, the problem may be described as follows. The observed data are, for each intersection i, the total number of accidents, n_i, and the number of accidents with bicyclists involved, r_i. For each intersection, this number is modeled as a binary response variable with a true bicycle accident proportion θ_i.

$$r_i \sim Binomial(\theta_i, n_i) \qquad i = 1,...,168$$

In the model, the parameters of interest are θ_i, the probability of the outcome "accident with a bicycle involved" at intersection i. In the binomial setting, there are two mutually exclusive and exhaustive outcomes, namely "accident with a bicycle involved" and "accident without bicycle involved". Each intersection has its own bicycle accident proportion. It is assumed, however, that the parameters of interest, θ_i, are independently sampled from a beta hyper-distribution, with hyper parameters a and β.

$$\theta_i \sim Beta(\alpha, \beta)$$

Because of the lack of extra information about the distribution of the bicycle accident proportions, a non-informative hyper prior density is used. Gelman et al. (*12*) propose the use of a uniform prior on the transformations $a/(a+\beta)$ and $(a+\beta)^{-1/2}$, which are the mean and the approximate standard deviation of the beta density for the bicycle accident proportions.

Instead of plugging in values for a and β, estimated from the data, a and β are defined in terms of prior distributions. That is, the beta distribution in a and β is not known and has to be estimated. The hierarchical Bayes scheme can be presented as in Figure 2.
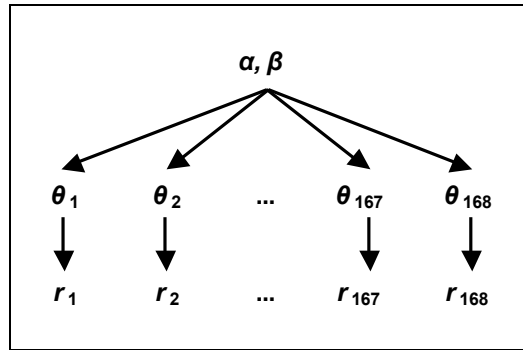
FIGURE 2 A Diagram for the Bicycle Accident Study

Sampling the posterior distribution is done using the Gibbs sampler in WINBUGS[©]. Uniform priors were assigned to the transformed variables in terms of a and β. The results of the sampling are shown below. After 5000 burn-in iterations, results were monitored and reported for another 5000 iterations for five chains of the Gibbs sampler (25000 iterations after burn-in). Burn-in iterations are necessary to obtain converging chains. A Markov Chain should be approaching convergence to yield acceptable estimates. For the model built in this paper, convergence for the parameters was achieved with the given burn-in period and number of iterations, as confirmed by the Gelman and Rubin's convergence diagnostic (*15*). The given number of iterations also provided adequate precision, as was indicated by the Monte Carlo standard error of the mean (MC error). All MC errors are much smaller than 1% of the corresponding estimated value, confirming an acceptable level of precision. Using the posterior mean estimated values for a and β, one could get an idea of the shape of the Beta distribution of the bicycle accident proportions. For a and β, the posterior distributions shown in Figure 3 were obtained.
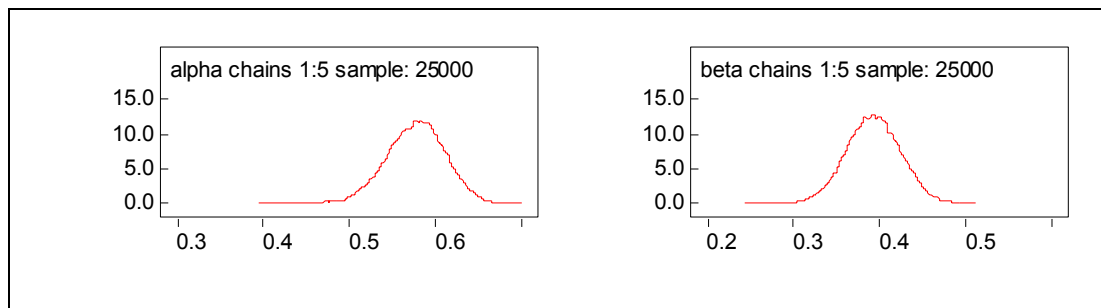

FIGURE 3 Posterior Densities for the Hyper Parameters a and β

Detailed results are presented for three intersections (sequence numbers 25, 26 and 27), each with a different posterior density for the bicycle accident proportions. The first intersection has an observed zero bicycle accident proportion (0 bicycle accidents out of 2 accidents). The second has an observed proportion equal to 1 (3 accidents, all of them with bicycles). The third intersection has 9 bicycle accidents observed out of 15. The sampled posterior densities for the bicycle accident proportions at these three intersections are given in the left column of Figure 4. For the other intersections, similar graphs can be made.
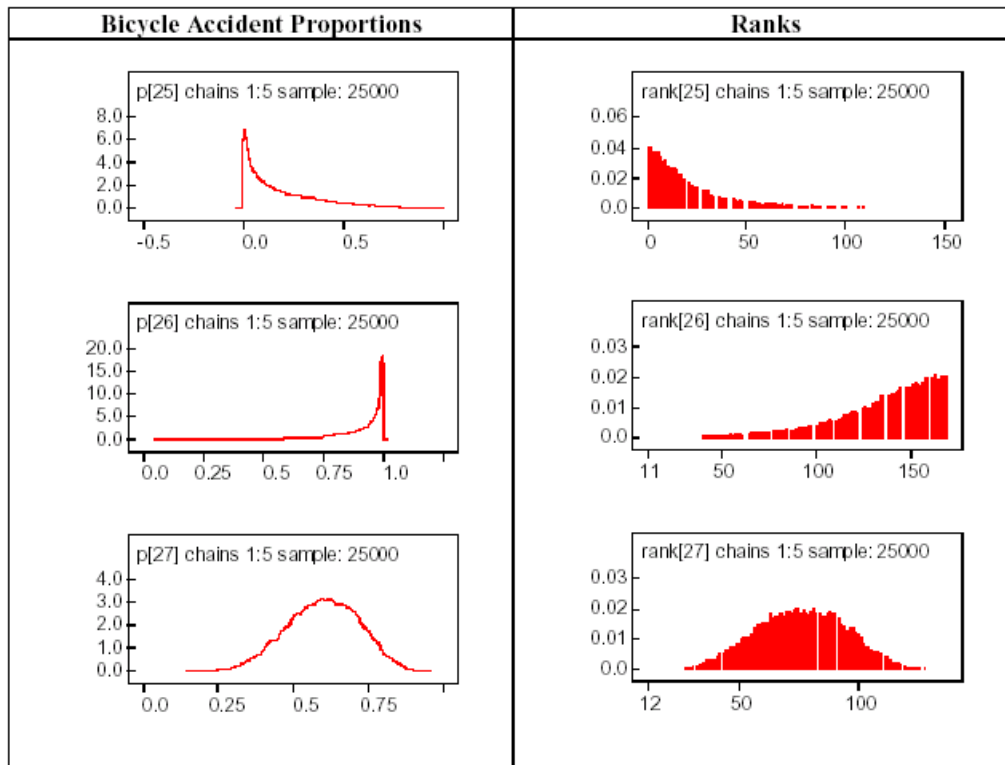
FIGURE 4 Posterior Densities for Bicycle Accident Proportions and Ranks at Three Intersections

Observe that, even with a very low empirical bicycle accident rate, the probability for the proportions being much higher is different from zero. Also note that the posterior distribution of the proportions is highly influenced by the data, because of the use of non-informative prior distributions. Changing the prior distribution is a way to perform sensitivity analyses, or to include prior expert knowledge into the model.

Table 1 contains the posterior summary information for the three selected intersections. The table shows the posterior mean of the bicycle accident proportion (column 2), together with a 95% credibility interval (column 3). A 95% credibility interval for the bicycle accident proportion indicates the part of the posterior distribution that contains 95% of all sampled values. Due to the uncertainty brought into the model, the credibility intervals are quite large. All MC errors, shown in column 4, are small enough to assume adequate precision.

TABLE 1 Posterior Information for Three Intersections

1	2	3	4	5	6	7
Site	Posterior Mean	Credibility Interval (95%)	MC Error	Posterior Mean Rank	Credibility Interval (95%)	MC Error
25	0.1936	[0.0006; 0.7060]	0.001313	24.59	[1; 93]	0.1601
26	0.9019	[0.5145; 1.0000]	0.000856	134.0	[63; 167]	0.1783
27	0.5985	[0.3556; 0.8183]	0.000750	75.92	[40; 112]	0.1186

The credibility interval should be kept in mind when ranking hazardous intersections according to the posterior mean. Figure 5 demonstrates this ranking for all intersections. Intersections on the horizontal axis are ordered by increasing posterior mean. The dashed lines indicate the upper (UCL) and lower (LCL) credibility limit. The posterior mean is a summary statistic for an underlying distribution, and ranking intersections

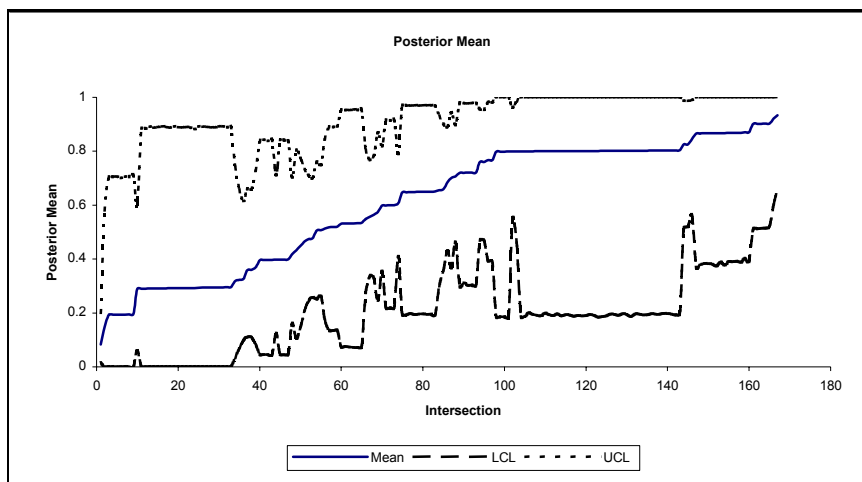according to this value is only one out of many possible options. Other examples are presented in Schlüter et al. (*9*).

FIGURE 5 Posterior Mean Bicycle Accident Rate and 95% Credibility Intervals for All Intersections

As mentioned in Schlüter et al. (*9*), the ranking of intersections based on the posterior mean will differ from the ranking based on observed rates. The reason for these differences may be found in the different sample sizes for each intersection and the well-known "regression to the mean" effect. In the selection of accident sites, some intersections have a lower number of accidents than others. It is to be expected that more variability is present at intersections with smaller sample sizes, resulting in a stronger shrinkage to the overall posterior mean proportion. As a consequence, intersections with the same observed accident rates will not get the same ranking, because of the specific variation at each location.

The random effects hierarchical model (each intersection having its own random effect) allows taking into account both the variation within each intersection (each having its own sampled density of bicycle accident proportions) and the variation of all intersections (all being sampled from the same hyper prior distribution). The first variation indicates the uncertainty degree with which the danger for a bicyclist can be estimated at each intersection (how accurate is the estimate at each intersection?). The second indicates how much the intersections differ. Figure 6 shows both sources of variation.
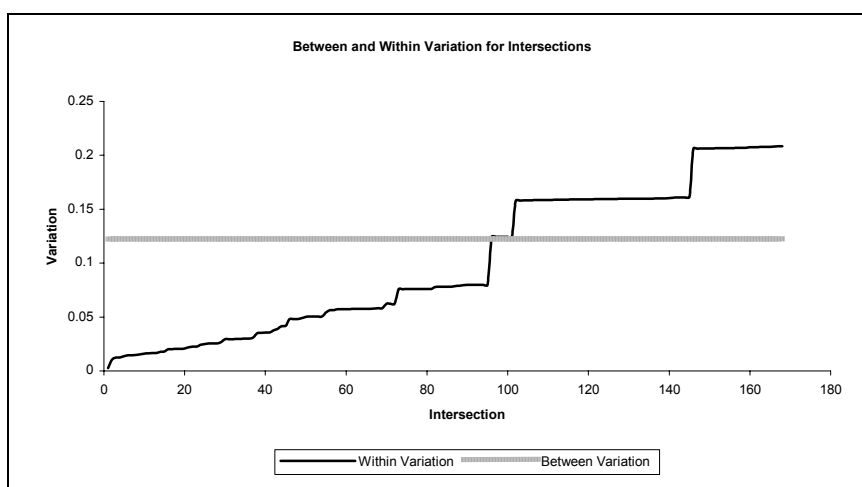

FIGURE 6 Between and Within Variation for All Intersections

The within variation at one intersection is typically different from the within variation at another. They are at very distinguished levels, indicating that it is useful to model each intersection with its own distribution for the bicycle accident proportion. The between variation is calculated based on the sampled posterior mean values for α and β. A considerable number of within variations is below the between variation, and some of them are at a reasonably low level. Therefore, the random effect for most intersections could be estimated with an acceptable precision. If this were not the case, it would be difficult to assess whether intersections are indeed different, and thus whether a random effects model is useful. The assumption of different intersections seems to be valid, and it might be interesting to investigate the differences among the intersections.

4.2 Ranking of intersections

One advantage of the Gibbs sampler is the ability to make inferences on the rank probability of bicycle accidents for each intersection. Table 1 contains the posterior mean of the ranking for the three intersections discussed earlier (column 5). The MC errors are shown in column 7, and the 95% credibility intervals in column 6 express the uncertainty associated with the rank positions. For the selected intersections, the order is the same as obtained by the ranking of the posterior mean proportion. Observe, however, the high uncertainty associated with the rank distribution. Figure 7 shows the posterior mean rank and the 95% credibility interval for the estimated bicycle accident proportion at each intersection.
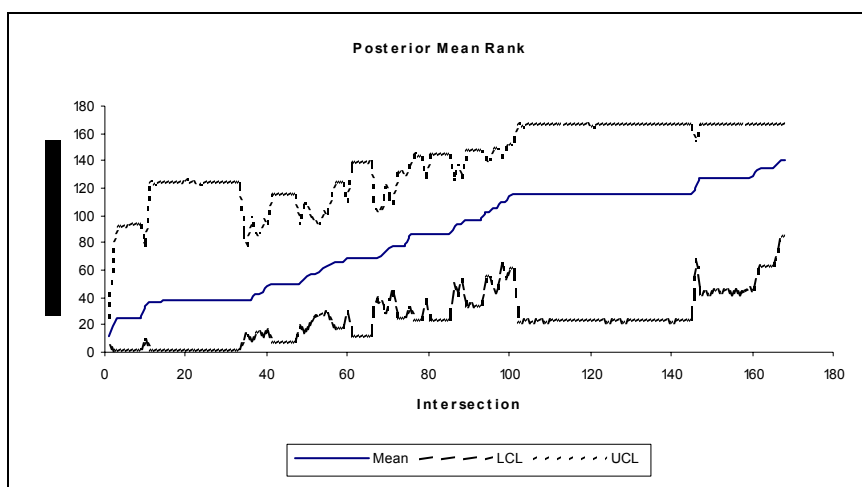

FIGURE 7 Posterior Mean Rank and 95% Credibility Intervals for All Intersections

The interval estimates illustrate the high uncertainty associated with rankings. Having 168 intersections in the data set, the median rank should be at 84.5. Almost all intersections have the median rank included in their credibility interval. This finding stresses the importance of a thorough analysis before the selection of hazardous sites. The uncertainty can also be found when looking at the distribution of the sampled rank values for some intersections. The graphs in the right column of Figure 4 show that for each intersection, many ranks are possible with a certain probability. These graphs look like those in the left column. The posterior densities for the ranks are constructed as a function of the posterior densities for the bicycle accident proportions. Intersection 25 will probably not be very dangerous for bicyclists, although the exact ranking is not known. The mean rank of 24.59 (see column 5 in Table 1) is a way of summarizing the posterior density. Taking into account the credibility interval, one can say that the intersection is ranked between position 1 and 93 in 95% of all sampled cases. A similar

way of reasoning is valid for the other intersections. The credibility intervals in column 6 of Table 1 almost cover 100 ranks for each intersection. The third intersection has the smallest credibility interval. It is also the intersection with the largest observed number of accidents. Clearly, if uncertainty is added to the model for bicycle accident proportions, this is reflected in the rank.

5. CONCLUSIONS AND FUTURE RESEARCH

In this paper, a Bayesian binomial hierarchical model is estimated using the Gibbs sampler. The objective was to infer the process of identifying and ranking dangerous intersections, based on the available accident counts. The use of the bicycle accident proportion as a measure of risk leaves room for improvement, but is well suited to illustrate the usefulness of Bayesian models in the context of ranking hazardous intersections.

The random effects hierarchical model allows the specification of different sources of variation. It was shown that the intersections in the study might be treated as if they all have their own distribution of bicycle accident proportions, with the parameters being sampled from the same hyper prior distribution. As a consequence, differences between intersections are captured by the (between) variation, while each intersection has its own (within) variation. The comparison of the within and between variation allows the assessment of the random effects model. If the effects at the intersections are different, and the estimates of the effects are accurate enough, then it makes sense to use intersection-specific risk parameters.

The Gibbs sampler implemented in WINBUGS© was used to get insight in the distribution of the bicycle accident proportions, which are the main parameters of interest. In the approach followed in the paper, a joint distribution over the parameters and the data is assumed. Conditioning on the data yields a posterior distribution over the parameters of interest. To obtain a density of a bicycle accident proportion at a given intersection, a marginal distribution for the intersection should be derived. This marginalization of the posterior is done using the Gibbs sampler, which is a Markov Chain - Monte Carlo simulation technique.

The Gibbs sampler also offers the possibility to sample complex functions of the bicycle accident proportions, like the rank of an intersection. Although the posterior mean is a well-known criterion to rank order hazardous intersections, it was shown in the paper that the ranking itself could be seen as a density. Since the bicycle accident proportions have a stochastic character, the ranking of intersections based on the posterior proportion cannot be deterministic. This conclusion may put the existing ranking schemes in another light. As long as the ranking is uncertain, it is not possible to have one ranking being definitely better than the others. The possibility to take into account all natural sources of uncertainty is one of the main advantages of the Bayesian framework over the more classical approaches.

It should be clear that ranking hazardous sites is an interesting tool to get insight in dangerous locations, but it is by no means an exact enumeration, in ascending order of danger. When investment resources are limited, decision makers are interested in the most hazardous intersections. Making a rank order of dangerous sites will undoubtedly be helpful to this end. However, since the ranking is in itself a stochastic term, the obtained rank order may be, to a certain extend, a lucky coincidence. Therefore, other criteria like investment capacity or specific mobility and safety objectives should influence the final ranking.

Because of the need for useful intersection ranking, together with the limited informative value of ranking schemes, further research should be done in this area. If more and better data are available, other measures of bicycle risk could be developed and explanatory variables could help in identifying and selecting an acceptable ranking. The incorporation of spatial correlation effects among intersections is another interesting extension of this research. Also the use of other summary statistics for the posterior density should be explored. These extensions should help the decision maker in finding a well-considered choice of intersections to be treated. This choice should be statistically sound and practically feasible, and should result in a considerable increase of the overall safety level.

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