Mathematical relations of the h-index with other impact measures in a Lotkaian framework

Peer-reviewed author version

EGGHE, Leo (2011) Mathematical relations of the h-index with other impact measures in a Lotkaian framework. In: MATHEMATICAL AND COMPUTER MODELLING, 53(5-6). p. 610-616.

DOI: 10.1016/j.mcm.2010.09.012
Handle: http://hdl.handle.net/1942/11519

Mathematical relations of the h-index with other impact measures in a Lotkaian framework

L. Egghe

Universiteit Hasselt (UHasselt), Campus Diepenbeek, Agoralaan, B-3590 Diepenbeek, Belgium[1]
and
Universiteit Antwerpen (UA), Stadscampus, Venusstraat 35, B-2000 Antwerpen, Belgium
leo.egghe@uhasselt.be

ABSTRACT

In a Lotkaian framework there exist formulae for impact measures such as the h-index, g-index, R-index and Randić's H-index.

Given two situations in which the h-indices are equal, we establish the functional relation of the R-, g- and H-index of Randić with the total number of papers. In all cases we have a decreasing relationship which can also be explained from a concentration point of view. Variants of relations between these measures are also proved.

We also prove a Lotkaian formula for the π-index of Vinkler. We indicate that the square root of this index is better in line with the other indices and we also show that the previously established results on the h-, g-, R- and H-index are not valid for the π or $\sqrt{\pi}$ -index.

[1] Permanent address

Key words and phrases: Hirsch-index, h-index, Lotka, impact measure, g-index, R-index, Randić, π -index

I. Introduction

The h-index (or Hirsch-index, Hirsch (2005)) is a remarkable impact measure of an author (or a journal, topic, institute,… - see the review Egghe (2009a)) establishing, e.g., the citation impact of the author's articles (in the sense of received citations).

It is generally recognized that the main disadvantage of the h-index is its insensitivity for the number of citations to papers in the h-core (i.e. the h most highly cited papers where h is the highest rank such that all papers on rank $1,...,h$ receive h or more citations (definition of the h-index), where papers are arranged in decreasing order of the number of received citations). As long as the papers in the h-core received h or more citations, we have an h-index equal to h, independent of the actual number of citations that these papers have.

This has lead several authors to the definition of variants of the h-index that take more into account the actual number of citations to highly cited papers.

1. The g-index (Egghe (2006)): Again (here and further) order the papers of an author in decreasing order of the number of received citations. Then g is the highest rank such that the first g papers together received at least g^2 citations (i.e. the first g papers received, on the average, at least g citations).
2. The R-index (Jin, Liang, Rousseau and Egghe (2007)): The R-index is the square root of the sum of all citations to papers in the h-core. So this definition – although simple – uses the h-index itself.
3. The H-index of Randić (Randić (2009)) (to distinguish this index from the h-index of Hirsch we will denote this index with capital H, as in Randić (2009)): H is the sum of a finite sequence of h-indices. The first term is the h-index itself; the second term is the h-index of the same table but where ranks are doubled; the third term is the h-index of the same original table but where ranks are multiplied by 4 and so on, until the procedure ends in zero (for more details, see Randić (2009)).
4. The π-index (Vinkler (2009)): the π-index is the sum of the number of citations to the top square root of the number of papers.

Other approaches to measure impact is to look at the top 1% or 0.1% of the total number of papers (as e.g. used in the Web of Science (WoS)).

In all these definitions it is clear that the number of citations to the highest cited papers are better used than in the definition of the h-index of Hirsch. So these impact measures are expected to discriminate better between scientists with equal h-index. This and other aspects will be studied in this paper.

This paper will study these indices in the Lotkaian framework and in general IPPs: in general we have an information production process (e.g., as in the above example, an author) in which we have sources (e.g. the papers of this author) and items (e.g. the citations to these papers).

Lotkaian informetrics supposes that the source-item relation (number of sources with j items but where the variables are continuous densities) is given by a decreasing power law (Lotka's law)

$$f(j) = \frac{C}{j^\alpha} \tag{1}$$

$C > 0$, $j \geq 1$, $\alpha > 1$.

In this framework it was shown in Egghe and Rousseau (2006) that the "Lotkaian" formula for the h-index is

$$h = T^{\frac{1}{\alpha}} \tag{2}$$

where T is the total number of sources and (2) is valid for $\alpha > 1$.
Similar results on the relation between the h-index and the number of publications and citedness of publications can be found in Iglesias and Pecharromán (2007) and Glänzel (2006)

If $\alpha > 2$ it was shown in Egghe (2006) that the Lotkaian formula for the g-index is

$$g = \left(\frac{\alpha-1}{\alpha-2}\right)^{\frac{\alpha-1}{\alpha}} T^{\frac{1}{\alpha}} \tag{3}$$

$$g = \left(\frac{\alpha-1}{\alpha-2}\right)^{\frac{\alpha-1}{\alpha}} h \tag{4}$$

((4) follows from (3), using (2)).

Also for $\alpha > 2$ it was shown in Jin, Liang, Rousseau and Egghe (2007) that the Lotkaian formula for the R-index is

$$R = \sqrt{\frac{\alpha-1}{\alpha-2}T^{\frac{1}{\alpha}}} \tag{5}$$

$$R = \sqrt{\frac{\alpha-1}{\alpha-2}h} \tag{6}$$

For Randić’s H-index, for $\alpha > 2$, it was shown in Egghe (2009b) that

$$H = \frac{1}{1-2^{-\frac{\alpha-1}{\alpha}}} T^{\frac{1}{\alpha}} \tag{7}$$

$$H = \frac{h}{1-2^{-\frac{\alpha-1}{\alpha}}} \tag{8}$$

For the π-index, no Lotkaian formula is proved in the literature; therefore we provide a formula in this paper: we will show that, if $\alpha > 2$

$$\pi = \frac{\alpha-1}{\alpha-2} T^{\frac{\alpha}{2(\alpha-1)}} \tag{9}$$

$$\pi = \frac{\alpha - 1}{\alpha - 2} h^{\frac{\alpha^2}{2(\alpha-1)}} \tag{10}$$

, using (2), but in the sequel we will explain why it is more logical to use $\sqrt{\pi}$.

In the next section we will prove that, if we have two IPPs with equal h-index $h_1 = h_2$, we have that R decreases with T, the total number of sources. An intuitive explanation is also provided. We also show that, if $R_1 = R_2$, then h increases with T. Also here an intuitive explanation is given.

We also prove the same results for the g-index and the H-index.

In the third section we prove the above mentioned formulas (9) and (10) for π and we also show that the above results for R, g and H are not valid for π or $\sqrt{\pi}$. This shows that π and $\sqrt{\pi}$ are impact measures of a different nature.

The paper closes with conclusions and suggestions for further research.

II. The h-index in relation with the R-, g- and H-index

II.1 Relations between the h-index and the R-index

The most important problem in this context is: given two situations (further on indicated by indices $i = 1, 2$) such that their h-indices are equal: $h_1 = h_2$, how do the R-values R_1 and R_2 relate to each other ? We have the following theorem

Theorem II.1: For two Lotkaian IPPs with Lotkaian exponent $\alpha_1, \alpha_2 > 2$ respectively such that their h-indices are equal: $h_1 = h_2$, we have the following equivalencies (T_1, T_2 denote the total number of sources in the two IPPs):

(i) $T_1 > T_2 \Leftrightarrow R_1 < R_2$

(ii) $T_1 < T_2 \Leftrightarrow R_1 > R_2$

(iii) $T_1 = T_2 \Leftrightarrow R_1 = R_2$

Proof: $T_1 = T_2$ and $h_1 = h_2$ imply that $\alpha_1 = \alpha_2$, by (2). Hence, by (6) we have $R_1 = R_2$. Conversely, if $R_1 = R_2$ and $h_1 = h_2$, (6) implies that $\alpha_1 = \alpha_2$ and hence, by (2): $T_1 = T_2$. This proves (iii).

Suppose now that $T_1 > T_2$. Then, since $h_1 = h_2$ and by (2) we have $\alpha_1 > \alpha_2$. Since the function $f(x) = \frac{x-1}{x-2}$ decreases in x (easy proof) we have by (6) that $R_1 < R_2$ (since $h_1 = h_2$). Conversely, let $R_1 < R_2$ and $h_1 = h_2$. Then, since $f(x)$ decreases, we have that $\alpha_1 > \alpha_2$. This together with $h_1 = h_2$ leads to $T_1 > T_2$. This proves (i). The proof of (ii) is obtained by interchanging the indices 1 and 2. □

A shorter proof of the inequality assertions in (i) and (ii) goes as follows. An increase of T, for h constant implies an increase of α (by (2)) hence a decrease of R, by (6), since h is constant. Conversely, a decrease of R, for h constant, implies an increase of α, by (6). Since h is constant this implies an increase of T. Both proofs are, essentially the same.

Intuitive explanation: We can give an intuitive explanation in the connection of inequality (concentration) theory – cf. Egghe (2005), Chapter IV. For constant h and increasing T, α must increase (again by (2)). In the rank-order distribution this means that the data are distributed more equally (see Egghe (2005), Corollary IV.3.2.1.5, p. 204-205: the Lorenz curve is strictly decreasing). Since h is constant there are, hence, less items in sources in the h-core and hence R is smaller.

We can also study what a constant R-index means for the h-index. We have the following result.

Theorem II.2: For the same Lotkaian situations as in Theorem II.1, supposing $R_1 = R_2$, we have

(i) $T_1 > T_2 \Leftrightarrow h_1 > h_2$

(ii) $T_1 < T_2 \Leftrightarrow h_1 < h_2$

(iii) $T_1 = T_2 \Leftrightarrow h_1 = h_2$

Proof: Suppose $T_1 = T_2$. If $\alpha_1 > \alpha_2$ then we have, by (2) and the fact that $f(x) = \frac{x-1}{x-2}$ decreases, that $R_1 < R_2$ (by (6)). This is false since $R_1 = R_2$. Similarly $\alpha_1 < \alpha_2$ cannot be true. Hence $\alpha_1 = \alpha_2$ and hence, since $T_1 = T_2$, $h_1 = h_2$. Conversely, if $R_1 = R_2$ and $h_1 = h_2$ we have, by (6)

$$\frac{\alpha_1 - 1}{\alpha_1 - 2} = \frac{\alpha_2 - 1}{\alpha_2 - 2}$$

from which we have $\alpha_1 = \alpha_2$. By (2) we conclude $T_1 = T_2$. This proves (iii).

Let now $T_1 > T_2$. If $\alpha_1 \le \alpha_2$ then, since $f(x)$ decreases and by (2) we have $\sqrt{\frac{\alpha_1 - 1}{\alpha_1 - 2}} \ge \sqrt{\frac{\alpha_2 - 1}{\alpha_2 - 2}}$ and $T_1^{\frac{1}{\alpha_1}} > T_2^{\frac{1}{\alpha_2}}$ and hence $R_1 > R_2$, contrary to $R_1 = R_2$. Hence $\alpha_1 > \alpha_2$. But $h_1 = \frac{R_1}{\sqrt{\frac{\alpha_1 - 1}{\alpha_1 - 2}}}$ and $h_2 = \frac{R_2}{\sqrt{\frac{\alpha_2 - 1}{\alpha_2 - 2}}}$. Since $R_1 = R_2$ and since $f(x)$ decreases, it follows that $h_1 > h_2$. Conversely, $h_1 > h_2$ implies, by (6) and since $R_1 = R_2$ that $\sqrt{\frac{\alpha_1 - 1}{\alpha_1 - 2}} < \sqrt{\frac{\alpha_2 - 1}{\alpha_2 - 2}}$. Since $f(x)$ decreases we have $\alpha_1 > \alpha_2$. This, together with $h_1 > h_2$ implies $T_1 > T_2$, by (2). □

Intuitive explanation:

Since $R_1 = R_2$ we have by (5) that α increases. As in the previous intuitive explanation, we have that in the rank-order distribution the data are distributed more equally and hence less items are in first ranks. By definition of R_1, R_2 and since $R_1 = R_2$ we must have $h_1 > h_2$.

II.2 Relations between the h-index and the g-index

The formula for the g-index ((3), (4)) has the same structure as the one of the R-index, namely

$$g = \varphi(\alpha) T^{\frac{1}{\alpha}} \tag{11}$$

($\varphi(\alpha) = \left(\frac{\alpha-1}{\alpha-2}\right)^{\frac{\alpha-1}{\alpha}}$ for the g-index and $\varphi(\alpha) = \sqrt{\frac{\alpha-1}{\alpha-2}}$ for the R-index). If we look carefully to the proofs of Theorem II.1 and II.2 we see that we can copy these proofs to the g-index, provided that

$$\varphi(\alpha) = \left(\frac{\alpha-1}{\alpha-2}\right)^{\frac{\alpha-1}{\alpha}} \tag{12}$$

is decreasing. The following Lemma gives the proof.

Lemma II.3: The function

$$f(x) = \left(\frac{x-1}{x-2}\right)^{\frac{x-1}{x}} \tag{13}$$

strictly decreases in $x > 2$.

Proof:

$$f'(x) = \frac{x-1}{x}\left(\frac{x-1}{x-2}\right)^{-\frac{1}{x}}\left(\frac{-1}{(x-2)^2}\right) + \left(\frac{x-1}{x-2}\right)^{\frac{x-1}{x}}\left(\ln\left(\frac{x-1}{x-2}\right)\right)\frac{1}{x^2}$$

$$= \left(\frac{x-1}{x-2}\right)^{-\frac{1}{x}} \frac{1}{x}\left[-\frac{x-1}{(x-2)^2} + \frac{x-1}{x-2}\left(\ln\left(\frac{x-1}{x-2}\right)\right)\frac{1}{x}\right]$$

< 0 if and only if

$$\frac{x-1}{x-2}\ln\left(\frac{x-1}{x-2}\right) < \frac{x(x-1)}{(x-2)^2} \tag{14}$$

But

$$\frac{x(x-1)}{(x-2)^2} > \left(\frac{x-1}{x-2}\right)^2 > \frac{x-1}{x-2}\ln\left(\frac{x-1}{x-2}\right)$$

is valid (hence (14)) and hence the proof of the Lemma is finished, if

$$\frac{x-1}{x-2} > \ln\left(\frac{x-1}{x-2}\right)$$

is true. But this is always true as can be seen geometrically ($\ln y < y$ for all $y > 0$). □

So we have proved the following Theorems on the g-index.

Theorem II.4: Under the same assumptions as in Theorem II.1 we have, if $h_1 = h_2$

(i) $T_1 > T_2 \Leftrightarrow g_1 < g_2$

(ii) $T_1 < T_2 \Leftrightarrow g_1 > g_2$

(iii) $T_1 = T_2 \Leftrightarrow g_1 = g_2$

Theorem II.5: Under the same assumptions as in Theorem II.2 we have, if $g_1 = g_2$

(i) $T_1 > T_2 \Leftrightarrow h_1 > h_2$

(ii) $T_1 < T_2 \Leftrightarrow h_1 < h_2$

(iii) $T_1 = T_2 \Leftrightarrow h_1 = h_2$

II.3 Relations between the h-index and the H-index of Randić

Also Randić's H-index is of the form

$$H = \varphi(\alpha) T^{\frac{1}{\alpha}} \tag{15}$$

but now with

$$\varphi(\alpha) = \frac{1}{1 - 2^{-\frac{\alpha-1}{\alpha}}} \tag{16}$$

which is clearly seen to be strictly decreasing. Hence we have the following results.

Theorem II.6: Under the same assumptions as in Theorem II.1 we have, if $h_1 = h_2$

(i) $T_1 > T_2 \Leftrightarrow H_1 < H_2$

(ii) $T_1 < T_2 \Leftrightarrow H_1 > H_2$

(iii) $T_1 = T_2 \Leftrightarrow H_1 = H_2$

Theorem II.7: Under the same assumptions as in Theorem II.2 we have, if $H_1 = H_2$

(i) $T_1 > T_2 \Leftrightarrow h_1 > h_2$

(ii) $T_1 < T_2 \Leftrightarrow h_1 < h_2$

(iii) $T_1 = T_2 \Leftrightarrow h_1 = h_2$

In the next section we will study the π- and $\sqrt{\pi}$-index and we will see that these indices are structurally different from the indices R, g and H.

III. Study of the π-index of Vinkler and of the $\sqrt{\pi}$-index

For the other indices we had proved the Lotkaian formulae in previous work. This is not so for the π-index; so we will give (and prove) the formula here.

Theorem III.1 : The π-index equals, for $\alpha > 2$

$$\pi = \frac{\alpha - 1}{\alpha - 2} T^{\frac{\alpha}{2(\alpha-1)}} \tag{17}$$

$$\pi = \frac{\alpha - 1}{\alpha - 2} h^{\frac{\alpha^2}{2(\alpha-1)}} \tag{18}$$

Proof : It is well-known (Egghe (2005), Exercise II.2.2.6 or see Egghe and Rousseau (2006), Appendix for an explicite proof) that (1) is equivalent with

$$g(r) = \frac{B}{r^{\beta}} \tag{19}$$

B, $\beta > 0$, $r \in [0,T]$, where $g(r)$ is the rank-frequency function linked with the size-frequency function (1). The relation between the parameters of $f(j)$ and $g(r)$ is

$$B = \left(\frac{C}{\alpha - 1} \right)^{\frac{1}{\alpha-1}} \tag{20}$$

$$\beta = \frac{1}{\alpha - 1} \tag{21}$$

By definition of the π -index, we have that π equals the total number of items in the first $\sqrt{T}$ sources. Hence we have

$$\pi = \int_0^{\sqrt{T}} g(r)\,dr$$

$$\pi = \frac{B}{1-\beta} T^{\frac{1-\beta}{2}} \tag{22}$$

if $\beta < 1$ (equivalently : $\alpha > 2$). Using (20) and (21), (22) reduces to (17) as is readily seen and (18) follows, by (2). □

Second proof : A second proof can be given as follows : by definition of the π -index we have

$$\pi = \int_{j_0}^{\infty} jf(j)\,dj \tag{23}$$

for

$$j_0 = g\left(\sqrt{T}\right) = \frac{B}{T^{\frac{\beta}{2}}} \tag{24}$$

Since $\alpha > 2$, (23) equals

$$\pi = \frac{C}{\alpha - 2} \frac{B^{2-\alpha}}{T^{\frac{\beta}{2}(2-\alpha)}} \tag{25}$$

Using (20) and (21), formula (25) reduces to (17), using also that

$$T = \int_1^{\infty} f(j)\,dj = \frac{C}{\alpha - 1} \tag{26}$$

□

A good property for an impact measure that improves the h-index as discussed in the Introduction is that it equals h in the situation that we have h papers each with h citations (and the other papers have no citations) (i.e. h-index is h). This is true for R and g and for Randić's H-index we have (as is readily seen):

$$H = h + \frac{h}{2} + \frac{h}{2^2} + ...$$

$$H = 2h \quad (27)$$

(so one had better defined $\frac{H}{2}$ instead of H in Randić (2009)).

For the π-index we obviously have, in the above simple case $\pi = h^2$ (since $\sqrt{T} \geq h$: indeed, by (2) $h = T^{\frac{1}{\alpha}} < \sqrt{T}$ since $\alpha > 2$ here)[1]. Therefore I suggest to use, henceforth, the square root of the π-index, defined as the $\sqrt{\pi}$-index (that was also the reason for the square root in the definition of R). Its Lotkaian formula is, by (17) and (18)

$$\sqrt{\pi} = \sqrt{\frac{\alpha - 1}{\alpha - 2}} T^{\frac{\alpha}{4(\alpha-1)}} \quad (28)$$

$$\sqrt{\pi} = \sqrt{\frac{\alpha - 1}{\alpha - 2}} h^{\frac{\alpha^2}{4(\alpha-1)}} \quad (29)$$

Note that in our model $\alpha > 2$ and hence, by (2), $\sqrt{T} > h$. By definition of $\sqrt{\pi}$ we then have $\sqrt{\pi} > R$. This is also seen from (5) and (28) :

$$\frac{\sqrt{\pi}}{R} = \frac{T^{\frac{\alpha}{4(\alpha-1)}}}{T^{\frac{1}{\alpha}}}$$

[1] As requested by one of the referees, we underline that $\sqrt{T} > h$ is only valid for $\alpha > 2$. In Schreiber (2010) there are examples of the opposite inequality $\sqrt{T} < h$.

Hence

$$\sqrt{\pi} = RT^{\frac{(\alpha-2)^2}{4\alpha(\alpha-1)}} > R$$

The results on the relations of R, g and H versus h are not true for π or $\sqrt{\pi}$ (henceforth we will use $\sqrt{\pi}$ for the reasons described above but all statements are also true for π).

Suppose that h is constant. In contrast with R, g or H we now have that $\sqrt{\pi}$ is not decreasing in α. Indeed, from (29) it follows that, given that h is constant,

$$\lim_{\substack{\alpha \to 2 \\ >}} \pi = +\infty \tag{30}$$

and

$$\lim_{\alpha \to +\infty} \pi = +\infty \tag{31}$$

(since $h > 1$).

and hence the theorems on the relation between R, g, H and h cannot be proved. Stronger, these results are false for π or $\sqrt{\pi}$. Indeed let h be constant and let T increase. By (2) α must increase. But, according to (30), (31), $\sqrt{\pi}$ can increase or decrease, dependent on the α-values. The same argument can be given to disprove the other results, obtained for R, g and H.

IV. Conclusions and suggestions for further research

The Lotkaian framework offers some possibilities to study the behavior of some variants of the h-index h (such as R, g, H or $\sqrt{\pi}$) in function of h. We could prove that, for h constant the indices R, g and H (of Randić) are decreasing with respect to T (the total number of sources). Also, given that R, g or H are constant, the h-index is increasing with respect to T.

We also studied the π -index of Vinkler and remarked that it is more natural (in comparison with the h-index) to work with $\sqrt{\pi}$. In this article we prove a Lotkaian formula for π and $\sqrt{\pi}$. Then we show that the above results on R, g and H are not valid for π or $\sqrt{\pi}$, showing the different nature of this impact measure.

All these results are limited to the case $\alpha > 2$ for reasons of the convergence of some integrals. Nevertheless they give a good insight into the T-dependencies of these measures. Formulas for Lotkaian informetrics in case $\alpha \leq 2$ are available (see Egghe (2005)), involving finite item densities. It is an open problem to study these more intricate cases, hereby covering the case $\alpha \leq 2$ but also refining the case $\alpha > 2$.

Of course, it is known that deviations from the Lotkaian model exist in the paper-citation relationship (see van Raan (2001a,b), Radicchi, Fortunato and Castellano (2008), Brantle and Fallah (2007), Lehman, Jackson and Lautrup (2008) and Egghe (2009c)). How to generalise our results to these cases is also an open problem. But it is our conviction that the Lotkaian framework serves as a first approximation of reality.

References

T.F. Brantle and M.H. Fallah (2007). Complex innovation networks, patent citations and power laws. In: Proceedings of the Portland International Conference on Management of Engineering and Technology (PICMET 2007), 540-549. Portland International Center for Management of Engineering and Technology, Portland, OR, USA.

L. Egghe (2005). Power Laws in the Information Production Process: Lotkaian Informetrics. Elsevier, Oxford, UK.

L. Egghe (2006). Theory and practise of the g-index. Scientometrics 69(1), 131-152.

L. Egghe (2009a). The Hirsch-index are related impact measures. Annual Review of Information Science and Technology, to appear.

L. Egghe (2009b). Mathematical results on the H-index and H-sequence of Randić. Preprint.

L. Egghe (2009c). A rationale for the Hirsch-index rank-order distribution and a comparison with the impact factor rank-order distribution. Journal of the American Society for Information Science and Technology 60(10), 2142-2144.

L. Egghe and R. Rousseau (2006). An informetric model for the Hirsch-index. Scientometrics 69(1), 121-129.

W. Glänzel (2006). On the h-index – A mathematical approach to a new measure of publication activity and citation impact. Scientometrics 67(2), 315-321.

J.E. Hirsch (2005). An index to quantify an individual's scientific research output. Proceedings of the national Academy of Sciences of the USA 102, 16569-16572.

J.E. Iglesias and C. Pecharromán (2007). Scaling the h-index for different scientific ISI fields. Scientometrics 73(3), 303-320.

B. Jin, L. Liang, R. Rousseau and L. Egghe (2007). The R- and AR-indices : Complementing the h-index. Chinese Science Bulletin 52(6), 855-863.

S. Lehmann, A.D. Jackson and B.E. Lautrup (2008). A quantitative analysis of indicators of scientific performance. Scientometrics 76(2), 369-390.

F. Radicchi, S. Fortunato and C. Castellano (2008). Universality of citation distributions: Towards an objective measure of scientific impact. Proceedings of the National Academy of Sciences of the USA 105(45), 17268-17272.

M. Randić (2009). Citations versus limitations of citations: beyond Hirsch index. Scientometrics 80(3), 809-818.

M. Schreiber (2010). Twenty Hirsch index variants and other indicators giving more or less preference to highly cited papers. Annalen Der Physik 19(8), 536-554.

A.F.J. van Raan (2001a). Competition amongst scientists for publication status: Toward a model of scientific publication and citation distributions. Scientometrics 51(1), 347-357.

A.F.J. van Raan (2001b). Two-Step competition processes leads to quasi power-law income distributions. Application to scientific publication and citation distributions. Physica A, 298, 530-536.

P. Vinkler (2009). The π-index: a new indicator for assessing scientific impact. Journal of Information Science 35(5), 602-612.