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CONTENTS

ECONOMICS SIMULATION

MANAGEMENT SIMULATION

HEALTH CARE MANAGEMENT

ON THE COMPLETION TIME OF A PROJECT WITH RANDOM ACTIVITY DURATIONS BASED ON A MODEL OF STOCHASTIC MARKED GRAPHS

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ABSTRACT

Activities of a project often have random durations. This makes the estimation of the total project more difficult. Simulation, approximation and bouding techniques help the project manager in formulating these estimates. This paper focuses on formulating an upper bound on the project length based on the fact that an activity network can be represented as a stochastic marked graph. The bound, however, includes a part which requires that the distributions of the activity durations are fully specified. As many times, only incomplete information is available on these durations like the interval, the mean, the mode or the standard deviation, this research investigates how the bound should be computed including only incomplete information.

INTRODUCTION AND LITERATURE REVIEW

Scientifically it has become a challenge to develop a model guaranteeing a successful project. But the reality is different: project activities use all available time; activities are finished for 90% during 90% of their time; a carelessly planned project takes three times the time as planned, a carefully planned project only twice (Shtub et al., Chapter 1, 1994). These thoughts might sound humorous, but the fact is that, however, dedicated project management suffers from uncertainties. The effect is even stronger because a project manager cannot have everything under control. Finally, a project which should not have been started cannot be ended successfully by whatever excellent project manager .

Project network analyses concentrate mostly on the total project time (or makespan in a scheduling context). Management is interested in two types of questions: "What is the expected total project time?", and "Which activities are critical in obtaining this total project duration?" Research into finding out which activities 'most probably' lie on the critical path is done by Dodin and Elmaghraby (1985). Projects are confronted in an extended way with the following objectives: be completed by a certain date, for a certain amount of money, within some level of performance (Tuman, 1986). A project can be planned according to a minimum cost criterion, but it can take more time and maybe not reach the required quality or conformance to the specifications. Avots (1984) suggests that time planning is of greater importance in the earlier stages of the project; during the project cost becomes of more importance and, after the project, only the technical quality matters.

Risk in project management exists in all of the three building blocks: time, cost and quality. The project manager has to cope with risk in subsequent steps: first risk identification, then risk management and reduction, and finally risk evaluation (Ho, 1992). All steps make part of a decision-making process. The decision-maker has to judge whether a part of the inherent risk can be avoided, reduced or accepted.

The most widely studied type of risk is *time risk*. Time risk exists due to uncertainties in the duration of some activities. Most studies assume that all activities of the project are known, that the precedence relations are known, but their durations are not fully known. Less attention has been paid to *cost risk*. The reason is that cost risk is not at all specific for project management. The problem is easier than the time risk due to the additivity of the individual components. In the literature hardly any articles exist on the analysis of *quality risk*. However, it has been found that many projects fail because the technical contents of the project have not been controlled sufficiently or not enough at an early stage (Morris, 1988). Time risk is certainly of a more complex nature than cost risk, but still has the advantage that techniques exist to approximate or to simulate the uncertainties. In quality risk measures have to be compared which are not additive because they are expressed in different measuring units.

Duration of activities in a project network is mostly of random nature. Due to the fact that a project is unique, no historical data exist to provide information regarding the probability distribution of an activity duration. In the analyses it is common to estimate a number of moments of the duration of the individual activities. Hardly anyone tries to obtain a full distribution. An *a priori*-distribution is chosen: most authors choose for a Beta-distribution. However, in the context of simulation other distributions have been proposed such as the triangular distribution, because random numbers from these distributions can be drawn in a more efficient way. It is of greater importance whether the choice of the distribution is of any importance. However, MacCrimmon and Ryavec (1964) have shown that the choice of the distribution hardly influences the total project time, except for some very extreme types of distributions.

Bounding techniques are also useful in terms of approximation. Most textbooks today still use the approximation by the authors of the original formulation Malcolm et al. (1959). They estimated the expected project duration as the length of the longest path through the deterministic network obtained by replacing each random arc length by its expected value.

A next review (Adlakha and Kulkarni, 1989), covering the literature from 1966-1987, deals with stochastic PERT networks: they stress the risk aspect with subjects as estimates, errors, bias and Monte-Carlo simulation approximations.

The PERT approach has proposed to collect information from experts in the form of three estimates for the duration: an optimistic, most likely and pessimistic estimate. From these estimates formulae for the expected value and the variance are proposed. The formulae are based on the assumption that a Beta-distribution might be underlying the duration, with the optimistic and pessimistic as the upper and lower bound of the range on which the distribution is defined. The most likely estimate corresponds to the modal value of the distribution (in case it is in fact unimodal).

Some criticism has been formulated on this approach as some combinations of the three estimates with the proposed formulae may lead to bimodal types of the Beta-distribution (Pagnoni, Chapter 4, 1990). The triangular distribution might be a valid alternative with the same interpretation of the three estimates. Any distribution on a finite range is valid as long as the interpretation of the three estimates leads to a unimodal distribution. However the largest type of criticism relates to the determination of the project length distribution in terms of its expected values, its variance or its tail probabilities. To simplify the computation of the expected value of the total project time and its variance, some additional assumptions are made within the PERT-approach. They are: (1) one single path dominates all others. This means that the probability is very low that another path becomes the critical path; and (2) the activity durations are independent random variables.

In the PERT approach the path with the longest (approximate) expected value is chosen as the critical path. It is assumed that the activity durations are independent of each other meaning that the expected total project time and its variance can be obtained by summing the expected values resp. variances of the activities on the critical path. On basis of the central limit theorem it is assumed that the total project length follows a Normal distribution. These assumptions are used to make statements on the completion of the total project within a required deadline. The PERT approach leads to an optimistic value (a lower bound) on the expected value of the project length. Also the normal character of the project length can be questioned. Even if the lengths of the individual paths follow a Normal distribution, the project length is Normally distributed only if the dominance assumption is valid (Elmaghraby, 1977). Therefore a lot of research has been spent on finding approximations which are more realistic or to bounds on the project length's expected value.

In this paper we determine bounds on the expected value of the project length. The lower bound is quite trivial. In the next section it is shown how a project network can be modeled as a stochastic marked graph (SMG). Through an optimisation problem on the SMG, an upper bound on the expected project length is obtained without assuming independence of paths or neglecting some paths. It however does not provide a measure of variability in order to evaluate the risk on the project duration.

A PROJECT NETWORK AS A STOCHASTIC MARKED GRAPH

The activity-on-arc and activity-on-node network representations are two classical representations of a project network.

Petri nets are an established model to represent and analyze concurrent systems. A *Petri net* is a collection of directed arcs connecting places and transitions. These arcs have a default capacity of one unless stated otherwise. Places can contain tokens, and the assignment of tokens to places is called the state or marking of the net. Arcs can only connect places to transitions and vice versa. A transition is said to be enabled if the number of tokens in its input places is at least equal to the arc weight going from the input places to the transition. Once enabled, a transition can fire. When fired, the tokens in the input places are moved to output places, according to the arc weights and place capacities.

A *marked graph* is a Petri net in which each place has at most one input transition and one output transition. Marked graphs constitute a good formalism to model manufacturing systems containing parallel tasks and synchronization or to order activities like in PERT. They are more general than PERT graphs in the sense that places can contain several tokens. Marked graphs have been studied extensively either in a deterministic or in a stochastic context. One of the main problems of timed and stochastic Petri net models for large systems is the explosion of computational complexity algorithms to analyze performance measures (such as the cycle time) of marked graphs. Campos et al. (1992) determine upper and lower bounds on the steady-state

performance of marked graphs to evaluate performance in an efficient way. In this paper we develop a tight upper bound for the cycle time of a stochastic marked graph.

In case the SMG is cyclic, an upper bound on the cycle time can be computed through an optimisation problem in which some function of the stochastic transition times needs to be determined. By connecting the start transition of the network with the finish transition a cycle is created. For the required function an upper bound can be computed depending on available information for the stochastic transition times, for example the range, mean, variance and/or mode (Janssens et al., 2009). In this research, the available information consists of the range and mode, and – in case of a specific type of distribution for the transition times is assumed – also a first and second moment of the distribution.

A project can be modelled as a safe marked graph, which means that each place can hold at maximum one token. In a safe marked graph every transition must have some input place, which means that the graph G is covered with directed circuits. As the number of tokens on a directed circuit is invariant under firing, all transitions on token-free circuits are dead. Therefore, let us assume that the initial marking M_0 places at least one token on each directed circuit in G. In this case, the marked graph is live and, since it is safe, M_0 places exactly one token on each directed circuit in G.
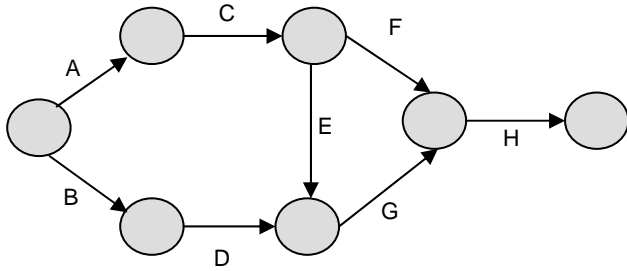

Figure 1: An activity-on-arc network

During the explanation of the theoretical development, the results will be illustrated by means of a project network example, taken from Heizer and Render (2001, Chapter 16, p. 674). An activity-on-arc representation of the project is shown in Figure 1.

Its representation as an SMG is shown in Figure 2. To each activity is associated a transition. One transition (t_Z) is added to produce a cyclic SMG. The concept of immediate predecessors is represented by forks and joins in the SMG. To each transition a stochastic firing time distribution is associated (the firing time of $t_Z = 0$).

Typically in project scheduling, the stochastic analysis is based on three estimates: optimistic, most probable, and pessimistic. The time estimates of the Heizer and Render example are given in Table 1.
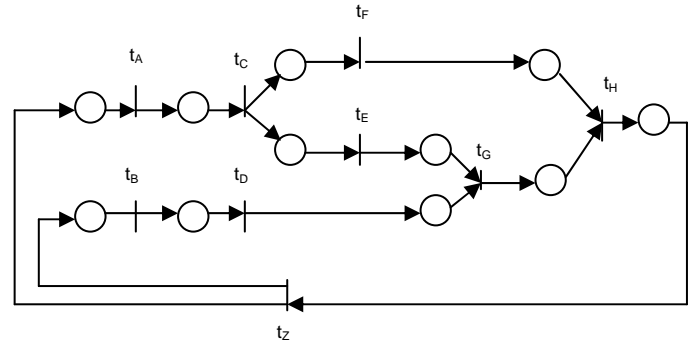

Figure 2: Stochastic marked graph related to Figure 1

Activity	Optimistic	Most probable	Pessimistic
A	1	2	3
B	2	3	4
C	1	2	3
D	2	4	6
E	1	4	7
F	1	2	9
G	3	4	11
H	1	2	3

Table 1: Time estimates

Bounding techniques obtain lower and upper bounds for the expected project length. The lower bound is of less interest but is required to discuss as it appears in the formulation of the upper bound. Prior to determining the lower bound for a stochastic marked graph, the bound is determined for its deterministic counterpart. Following Magott (1984) the minimal cycle time can be found as the solution of a linear program (LP).

When switching from deterministic marked graphs towards SMG, a similar linear optimization program has been formulated by Campos et al. (1992, p. 390). A lower bound for the mean cycle time for live strongly connected marked graphs can be obtained by solving such a linear program. As deterministic timed graphs are a special case of SMG with the mean transition firing time equal to the deterministic firing time, both types of linear programs give the same results in case of deterministic marked graphs.

Campos et al. (1992) prove that for strongly connected marked graphs with arbitrary values of mean and variance for transition firing times, the lower bound for the mean cycle time obtained in their LP cannot be improved. If both mean and variance of the firing time of each transition are known, the lower bound as obtained by the LP cannot be reached (unless all variances are equal to zero).

The asymmetric three-parameter triangular distribution appears in the project scheduling literature already for a long time. The three parameters are in a one-to-one correspondence with the optimistic, most probable and pessimistic estimates. This leads to an intuitive appeal to this distribution, while in reality the shape of the distribution is unknown. The expected value and the variance of the

activity durations, assuming a triangular distribution, are given in Table 2.

Activity	Expected value	Variance
A	2.0	0.167
B	3.0	0.167
C	2.0	0.167
D	4.0	0.667
E	4.0	1.500
F	4.0	3.167
G	6.0	3.167
H	2.0	0.167

Table 2: Expected value and variance using the triangular distribution

The Critical Path Method, making use of the expected values from table 2 as deterministic values, a project length CT = 16 is obtained. The same value appears as a solution of Magott's linear program for the marked graph:

Min CT
subject to
$S(t_A) - S(t_Z) \geq 2$
$S(t_B) - S(t_Z) \geq 3$
$S(t_C) - S(t_A) \geq 2$
$S(t_D) - S(t_B) \geq 4$
$S(t_F) - S(t_C) \geq 4$
$S(t_E) - S(t_C) \geq 4$
$S(t_G) - S(t_E) \geq 6$
$S(t_G) - S(t_D) \geq 6$
$S(t_H) - S(t_F) \geq 2$
$S(t_H) - S(t_G) \geq 2$
$S(t_Z) - S(t_H) + CT \geq 0$

AN UPPER BOUND ON THE PROJECT NETWORK LENGTH

Let $\pi(M_0)$ be the cycle time of the SMG and $\pi^D(M_0)$ be the cycle time of its deterministic equivalent, discussed in the previous section. Sauer and Xie (1993) prove that the following bound holds:

$$\pi(M_0) \leq \pi^D(M_0) + \inf_{z \in E}\left\{\sum_{t \in T} E\left[(X_t - z_t)^+\right]\right\} \quad (1)$$

where the infimum needs to be found in the set E defined as:

$$E = \left\{z \middle| z_t \geq m_t, \forall t \in T \text{ and } \sum_{t \in \gamma} z_t \leq \pi^D(M_0).M_0(\gamma), \forall \gamma \in \Gamma\right\},$$

$$z = [z_1, z_2, ..., z_m]$$

The second term in the right hand side of the inequality is an optimisation problem where a vector z has to be determined within a set of linear constraints E. The objective function however consists of a sum of nonlinear functions in the elements of the vector z. In most cases the partial expectation function $E[(X_t - z_t)^+]$ cannot be expressed analytically in z_t, so the objective function cannot be formulated. In other cases, like the case when the X_t are distributed according to a

triangular distribution, the function can be expressed analytically but is difficult to solve because of the sum of nonlinear terms (Janssens et al., 2009). In cases, in which the distribution is not completely specified, also no objective function can be formulated.

A practical solution to this problem might be found in looking for distributions which have an upper bound on the the partial expectation function $E[(X_t - z_t)^+]$, given a number of constraints like the knowledge on the interval on which the variable is defined, the mode, the expected value of higher moments, or any combination of these characteristics. It is of interests to investigate whether these distributions lead to a feasible and/or easy way to formulate the objective function. Such an upper bound, of course, is only of practical value if the bound is tight.

There exists a similarity between the function of our interest and a function, called the stop-loss premium in insurance mathematics. In insurance mathematics, an insurance company using the option of re-insurance is confronted with a stop-loss premium. A stop-loss premium limits the risk X of an insurance company to a certain amount t. If the claim size is higher than t the re-insurance company takes over the risk $X\text{-}t$. The stop-loss premium is based on the expected value of $X\text{-}t$, which in case of a known claim size distribution may be defined as:

$$\int_0^\infty (x - t)^+ dF(x) \quad (2)$$

where $F(x)$ represents the claim size distribution (Goovaerts, De Vylder, and Haezendonck 1984). A single term of our objective function is the same as this integral.

As the optimistic and pessimistic estimates are finite numbers, the distribution of the duration of an activity t can be represented as defined on an interval $[a_t, b_t]$ with $0 \leq a_t \leq b_t < +\infty$. In case only this knowledge is used, the worst distribution would put all its mass in b_t to obtain an upper bound:

$$E\left[(X_t - z_t)^+\right] = b_t - z_t. \quad (3)$$

This makes the second term in inequality (1) a linear objective, but uses minimal information on the activity duration.

The additional use of the most probable estimate (c_t) disturbs the linear character of the objective function. The upper bound is given by:

$$E\left[(X_t - z_t)^+\right] = \frac{1}{2}\frac{(b_t - z_t)^2}{b - c_t} \text{ if } c_t \leq z_t$$

$$E\left[(X_t - z_t)^+\right] = \frac{1}{2}(b_t + c_t - 2z_t) \text{ if } c_t \geq z_t \quad (4)$$

The function contains a quadratic component in the decision variables z_t. Assuming unimodality, without specific knowledge of the mode c_t, will lead to the case in which the upper bound puts all mass in b_t which becomes also the mode and reduces the bound to equation (3).

In case one has symmetric distributions in mind, the extreme case assigns as much as possible mass to b_t while satisfying

the constraint of symmetry (the same mass to a_t). This assigns 50% mass to b_t and leads to the upper bound:

$$E\left[(X_t - z_t)^+\right] = \frac{1}{2}(b_t - z_t). \qquad (5)$$

This leads again to a linear term and makes the optimization problem a linear programming problem.

The knowledge of the expected value m_t is also of interest. In that case the upper bound is found as:

$$E\left[(X_t - z_t)^+\right] = (b_t - z_t)\frac{m_t - a_t}{b_t - a_t}. \qquad (6)$$

Again this leads to a linear term but, using the three estimates, the problem is that the expected values is not known. However, one might assume that the expected values corresponds to the one when assuming a triangular distribution (leaving open all distributions with variances different from the triangular). If one feels too restricted, the linear program can be run for several feasible values of m_t as the knowledge of the mode limits its range as:

$$\frac{1}{2}(a_t + c_t) \le m_t \le \frac{1}{2}(b_t + c_t). \qquad (7)$$

If the project contains many activities with random durations, the latter option is not efficient.

In case a distribution on a finite interval with known expected value and variance are considered, Janssens et al. (2009) have also shown how to find the upper bound by means of a linear program. The objective function can be formulated as:

$$\inf_{z \in E} \sum_t (b_t - z_t)\frac{\sigma_t^2}{\sigma_t^2 + (b_t - m_t)^2} \qquad (8)$$

where E is defined as in equation (1). It leads to the following inequality for the mean cycle time:

$$\pi(M_0) \le \pi^D(M_0) + \inf_{z \in E} \sum_t (b_t - z_t)\frac{\sigma_t^2}{\sigma_t^2 + (b_t - m_t)^2} \qquad (9)$$

It can be seen that the second term in the r.h.s. is a linear objective function and that the constraints set E also contains only linear constraints. This second term can be written as a constant plus a sum of terms each including one decision variable z_t. The linear program is illustrated by means of the example of Figure 2:

Min $5.962806 - 0.142857\,z_A - 0.142857\,z_B - 0.142857\,z_C - 0.142857\,z_D - 0.142857\,z_E - 0.112426\,z_F - 0.112426\,z_G - 0.142857\,z_H$

subject to

$z_A \ge 2$
$z_B \ge 3$
$z_C \ge 2$
$z_D \ge 4$
$z_E \ge 4$
$z_F \ge 4$
$z_G \ge 6$
$z_H \ge 2$
$z_A \le 3$
$z_B \le 4$
$z_C \le 3$
$z_D \le 6$

$z_E \le 7$
$z_F \le 9$
$z_G \le 11$
$z_H \le 3$
$z_z + z_A + z_C + z_F + z_H \le 16$
$z_z + z_A + z_C + z_E + z_G + z_H \le 16$
$z_z + z_B + z_D + z_G + z_H \le 16$

The model leads to the following solution: $z_A = 2$, $z_B = 3$, $z_C = 2$, $z_D = 5$, $z_E = 4$, $z_F = 9$, $z_G = 6$, $z_H = 2$, $z_Z = 0$. The objective value is equal to 1.70499, which makes the upper bound equal to 17.70499.

CONCLUSIONS

An optimization model has been formulated to compute a tight upper bound for the expected project time in case of activities with random durations. The model is useful both in the case when the distributions of the ectivity durations are fully specified or partially specified. In the former case not all information is used, but the model offers a tractable method of finding a good upper bound. In the latter case the model offers the best solution to obtain this bound. It has been shown that in some cases the model leads to a linear program. In other cases it leads to a mathematical programming model with a non-linear objective function but with a set of linear constraints.

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