Selection of Control Variates for Variance Reduction in a Multiresponse Simulation with a small Number of Replications

An Caris and Gerrit K. Janssens
Operations Management and Logistics
Hasselt University
Agoralaan - Building D
B3590 Diepenbeek
e-mail: {an.caris,gerrit.janssens}@uhasselt.be

Abstract

Using simulation, excessive run lengths or replications may be necessary to yield estimators with acceptable precision. A variety of variance reduction techniques (VRTs) have been developed to improve the efficiency of simulations. Variance reduction techniques attempt to reduce the variance of an output random variable and thus obtain greater precision for the same amount of simulation time, or to achieve a desired precision with less simulation time. This research focuses on one specific VRT, called the method of Control Variates (CV), which takes advantage of correlation between certain random variables and response variables to obtain a variance reduction. Control variates are auxiliary random variables with known properties and a strong stochastic correlation with the performance measure(s) of interest.

The selection of control variates for simulation experiments with multiple response variables is discussed. Multiple responses might include performance measures of different resources involved or might be different measures of a single resource (like averages, variances or quantiles). A criterion is developed to determine the number of control variates and how to select the most appropriate ones. The aim is to reduce the variance of several response variables as much as possible. Response variables are weighted according to their coefficient of variation. An initial ranking of control variates is attained by summing their weighted minimum variance ratios. When deciding how many control variates to use, a loss factor has to be taken into account. The loss factor measures the loss in variance reduction due to the estimation of the regression coefficients, inherent to the CV-method.

In simulation experiments with a small number of replications, due to computational costs, outliers can seriously distort the estimation of the optimal weights for the control variates. A robust regression technique (least median of squares) is applied to eliminate the influence of outliers and achieve more significant estimates of the regression coefficients. The 'least median of squares'-method proves to be highly effective for detecting outliers. A re-weighted ordinary least squares regression is performed, in which outliers receive a zero weight.

The selection method described is tested on several simulation models, such as a job shop system with priorities, a production-inventory system with an unreliable production facility and a supply chain with transport alternatives. Special attention is paid to exploit control variates when extreme quantiles (in the tails of the distribution of system performance, e.g. customer service levels in inventory management systems) are of interest.

In practice the comprehensible selection method proves to be effective and computationally efficient. Most variance reduction is achieved after introducing a single control variate. The use of a second control variate sometimes only leads to a restricted gain in variance reduction. This result confirms the advise of limiting the number of control variates, especially with a small number of replications.

Keywords: variance reduction, multiresponse simulation, control variates, robust regression