Mesoscopic Modelling of Masonry using Weak Discontinuities in the Partition of Unity Framework

B. Vandoren[1 2], K. Heyens[1] and K. De Proft[2]

[1]Vakgroep Bouwkunde, Departement TIW, XIOS Hogeschool Limburg, Belgium
[2]Vakgroep FYS, Faculteit WET, Hasselt University, Belgium

1. Introduction

These days, durability and sustainability are gaining more and more interest. High energy and material costs force us to optimise the energy use in buildings and the use and re-use of building materials. An obvious way to re-use building materials is renovation of buildings. Since in a renovation project the function of the building is not always kept, changes must be made in order to make the new function of the building possible. These changes can be both on the architectural (e.g. changes in division) as on the constructive level (e.g. change of loading conditions). This leads to new mechanical conditions which often include a raise of vertical loads due to higher floor loads, increase of slenderness of the wall due to removal of intermediate floors and the introduction of eccentric loading due to new intermediate floors. In order to preserve the existing load carrying structure, the construction must be recalculated with these new loading conditions.

Most old and historical buildings are constructed using masonry. The recalculation of masonry structures subjected to the new loading conditions shows multiple problems. First of all, the actual material parameters describing the behaviour of masonry are not the same as the initial parameters. Over the years, strength and stiffness of masonry change, altering the overall behaviour of a wall. It is well known that masonry structures can suddenly collapse /1/. Secondly, the current masonry codes are based on new brick materials and mortars. In older buildings, the quality of brick and mortar differs from values nowadays. Finally, the masonry wall may already be damaged or there might be irregularities in the wall (e.g. windows openings).

Consequently, advanced computational modelling techniques are often necessary to compute a realistic value of the ultimate collapse load of a masonry structure. Three major groups of finite element modelling approaches exist: microscopic, mesoscopic and macroscopic /2/. The former approach models each masonry constituent (the units, the mortar and the unit-mortar interface) in high detail, thus leading to many degrees of freedom and high computation times. In the macroscopic approach the joints and bricks are homogenized to one orthotropic material. The main advantage of this method is that not much computational effort is needed to calculate large structures. However, the obtained crack path is less detailed. These drawbacks can be alleviated by the use of mesoscopic models. In this approach, joints and bricks are modelled by separate entities, but in less detail than the microscopic approach. Classically, the joints are incorporated by interface elements, situated on the boundaries of the continuum brick elements (/2/, /3/). When a critical state is reached in a joint, a strong discontinuity (i.e. a jump in the displacement field) is introduced in the interface.

An alternative way to incorporate strong discontinuities is the partition of unity method (/4/, /5/, /6/). Within this method, nodes are locally enhanced to enrich the solution with discontinuous modes. This concept was applied to masonry by De Proft et al. /7/ and will be extended in this paper by the incorporation of weak discontinuities. A weak discontinuity introduces a jump in the strain field, allowing for failure to localise in a zone with finite width (/8/, /9/). The thickness of this failure is in this case linked to the joint thickness. The main advantages of the weak discontinuity approach are the ability to incorporate the real mortar and brick dimensions, and the ability to perform the constitutive modelling in the general stress and strain spaces.

2. Partition of unity concept for weak discontinuities

2.1. Displacement decomposition

The displacement field of a body crossed by a weak discontinuity (Figure 1) is obtained by:

$$\mathbf{u} = \hat{\mathbf{u}} + H_{\Omega^w}\tilde{\mathbf{u}} \tag{1}$$

in which $\hat{\mathbf{u}}$ and $\tilde{\mathbf{u}}$ denote the regular and enhanced displacement field, respectively. H_{Ω^w} is a unit ramp function /10/, defined by:

$$H_{\Omega^w} = \begin{cases} 0 & \text{if } \mathbf{x} \in \Omega^- \\ \dfrac{\xi - \xi^-}{\xi^+ - \xi^-} & \text{if } \mathbf{x} \in \Omega^w \\ 1 & \text{if } \mathbf{x} \in \Omega^+ \end{cases} \tag{2}$$

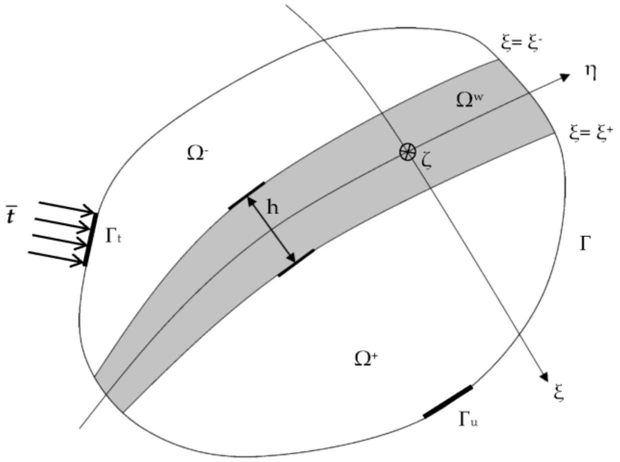

Figure 1: Body crossed by a weak discontinuity

2.2. GFEM discretisation

In this work, the Generalized Finite Element method has been adopted to model the discontinuities /11/. The unit ramp function (Equation 2) is used as an enhanced basis: its value equals unity for a point inside a masonry brick. When a support of a node is crossed by a weak discontinuity (i.e. joint), an enhanced set of degrees of freedom is added to the solution field of that node. Consequently, each brick possesses its own set of enhanced degrees of freedom. Special care has to been taken in the implementation of a meshgenerator, to prevent linear dependency of the enhanced basis functions /11/. In the current model, two linear quadrilateral elements are used to model one brick.

3. Nonlinear modelling

3.1. Material model

The stone behaviour remains linear elastic during the simulations, whereas the joint behaviour is governed by an exponential damage evolution law /12/:

$$\omega = \begin{cases} 0 & \text{if } \kappa < \kappa_0 \\ 1 - \dfrac{\kappa_0}{\kappa} \exp\left[-\dfrac{\kappa - \kappa_0}{\gamma} \right] & \text{if } \kappa \geq \kappa_0 \end{cases} \tag{3}$$

where $\kappa_0 = \dfrac{f_{t0}}{E}$ in which E represents the Young's modulus of the mortar joints. The loading function κ, expressed in terms of strain invariants, is derived from the Drucker-Prager model /13/ :

$$\kappa = \frac{1}{2}\frac{f_{c0} - f_{t0}}{f_{c0}}\frac{I_{1,\varepsilon}}{1-2\nu} + \frac{\sqrt{3}}{2}\frac{f_{c0} + f_{t0}}{f_{c0}}\frac{\sqrt{J_{2,\varepsilon}}}{1+\nu} \tag{4}$$

in which f_{t0} and f_{c0} are the mortar tensile and compressive strengths, respectively. The brittleness of response is governed by γ :

$$\gamma = \frac{G_{fI}}{f_{t0}} - \frac{1}{2}\kappa_0 \tag{5}$$

where G_{fI} denotes the mode I fracture energy.

3.2. Energy release control

An important aspect in modelling masonry and other solids containing many nonlinearities is the use of a robust algorithm which is capable of tracing the whole equilibrium path, particularly the post-peak response of the structure. In this work, an energy release constraint function is used to trace the equilibrium path. The robustness and versatility of this method was shown by Gutiérrez /14/. The constraint function is given by:

$$g = \frac{1}{2}\hat{\mathbf{f}}^T\left(\lambda_0\Delta\mathbf{a} - \Delta\lambda\mathbf{a}_0\right) - \tau \tag{6}$$

in which τ represents the enforced energy dissipation during a loadstep, $\hat{\mathbf{f}}$ is a vector containing the prescribed unit loads, λ is a load scaling factor and $\mathbf{a}$ is a vector containing the displacement field. The subscript $_0$ refers to the converged values of the previous loadstep.

4. Numerical examples

4.1. Three-point bending test

In order to demonstrate the potential of the developed masonry model, a three-point bending test has been carried out. The material parameters (Tables 1-2) and experimental data (Figure 3, dotted curve) are obtained from /15/. The simulation results show a good agreement with those from the experiment.

Table 1: Elastic material parameters for the three-point bending test

	dimensions	*E* [*N/mm²*]	ν
joints	10 *mm*	3360	0,20
bricks	76 · 230 · 110 *mm³*	17500	0,15

Table 2: Inelastic material parameters for the three-point bending test

	f_{t0} [*N/mm²*]	f_{c0} [*N/mm²*]	G_{fI} [*N/mm*]
joints	0,086	7,26	0,002

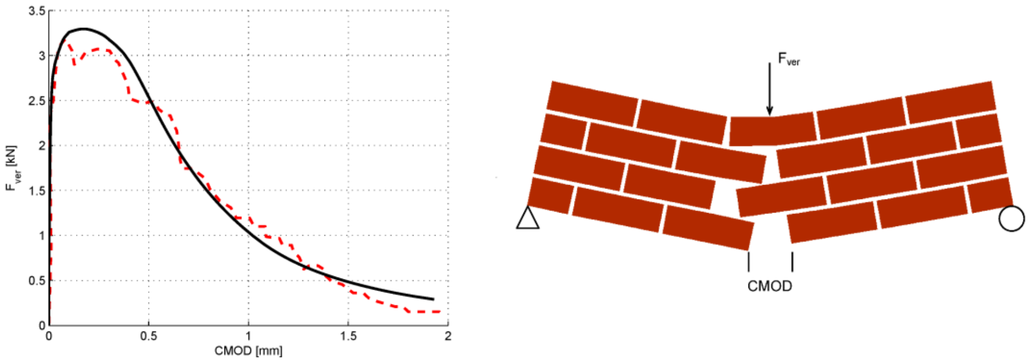

Figure 2: Load-CMOD curve and deformed mesh for the three-point bending test

4.2. Shear wall test

The second example is a shear wall with opening. Tables 3-4 summarise the employed material parameters. A confining stress of 0,30 *N/mm²* is applied on top of the wall. Again a good agreement is observed with the ultimate load capacity found in experimental tests /16/ (Figure 3, dotted curve). The typical stair-step crack pattern is also recovered.

Table 3: Elastic material parameters for the shear wall test

	dimensions	*E* [*N/mm²*]	ν
joints	10 *mm*	782	0,14
bricks	52 · 210 · 100 *mm³*	16700	0,15

Table 4: Inelastic material parameters for the shear wall test

	f_{t0} [N/mm²]	f_{c0} [N/mm²]	G_{fI} [N/mm]
joints	0,25	10,5	0,018

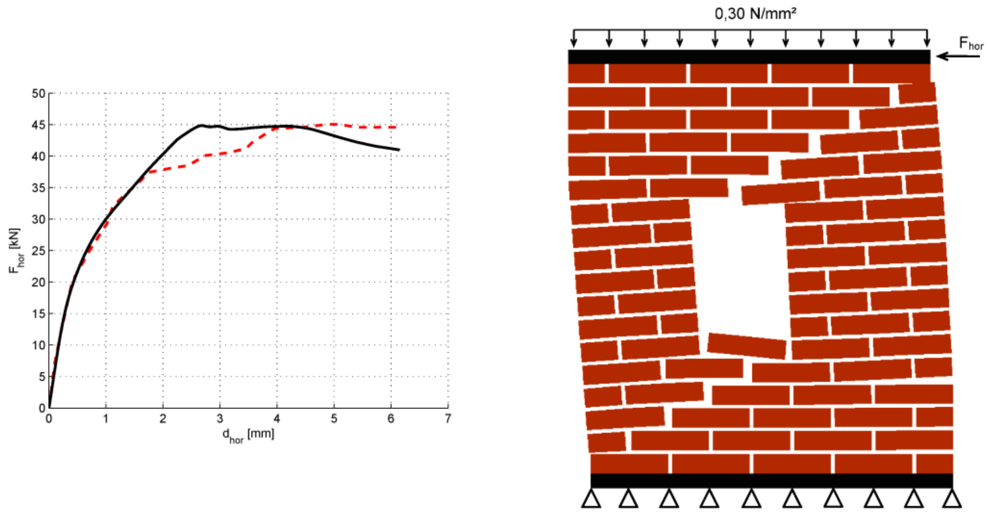

Figure 3: Load-displacement curve and deformed mesh for the shear wall test

5. Conclusion and future works

In this paper, a mesoscopic masonry model is developed in which joints are modelled by weak discontinuities. The discontinuities are incorporated using the Generalized Finite Element Method. A Drucker-Prager damage model is used to describe the failure of the mortar joints, whereas the stone behaviour remains linear elastic. The collapse load and post peak response are traced using an energy release constraint function. A three-point bending test and a shear wall test showed that the presented method leads to realistic load capacities and failure patterns.

The developed methodology will be extended in the future by the implementation of cracking of the bricks. Furthermore, other constitutive laws will be employed and compared, to describe the nonlinear unit and joint behaviour. Finally, the model will be extended to capture the effects of thermal fields.

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