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MULTI-SCALE MODELLING OF MASONRY STRUCTURES

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1. Introduction

Masonry is a widespread and common form of construction used throughout history. Thus it is important to know the structural behaviour of those buildings, especially when speaking in terms of maintenance of cultural heritage preservation or structural restoration of an historic building. The design of masonry structures, as it is done today, is still based on codebooks and rules of thumb, which often lead to a lack of control over safety factors and non-optimal structure dimensions. As such, it would be useful to develop reliable numerical tools that predict the behaviour of masonry structures.

The majority of numerical tools currently available for constitutive description of masonry structures have proven to be accurate on small scale structures, but when used on large scale structures, excessive computational effort is required and numerical instabilities occur /4,6,9/. Hence, in order to lower the computational cost it is more efficient to focus on regions of the masonry structure where cracks occur. It is also known that the mortar phase is relatively weak, which due to the periodic arrangement of the phases leads to a stiffness degradation along preferential orientations, i.e. the crack path in masonry (often) follows the joints. A domain decomposition method can be employed to decompose the masonry structure into several domains, and concentrate the computational efforts on the domains which undergo inelastic behaviour.

It is possible to adopt such a technique in two different ways to describe a multi-scale model for masonry structures. The first approach consists of initializing the discretized masonry structure as a coarse grid consisting of several domains. Once a domain meets a given criterion, indicating the occurrence of inelastic behaviour, it will be isolated and evaluated on a meso-scale using a finer background mesh. Afterwards, the results from the meso-scale computation will be integrated into the macro-scale parent grid using a domain decomposition technique. Another way to use the domain decomposition technique is to define the regions in the discretized structure where possible inelastic behaviour could occur on beforehand. These domains will be meshed at meso-scale, while the remainder of the structure will be meshed with a coarse grid on macro-scale.

In this contribution the second approach will be presented, as illustrated in Figure 1. As shown in Figure 1, the meso-scale domains are concentrated under and above the window, since under uniform compression the inelastic behaviour most likely will occur in those regions. The evaluation at macro-scale is done with an homogenized stiffness, as described in /9/, and the meso-scale crack behaviour is modelled using a

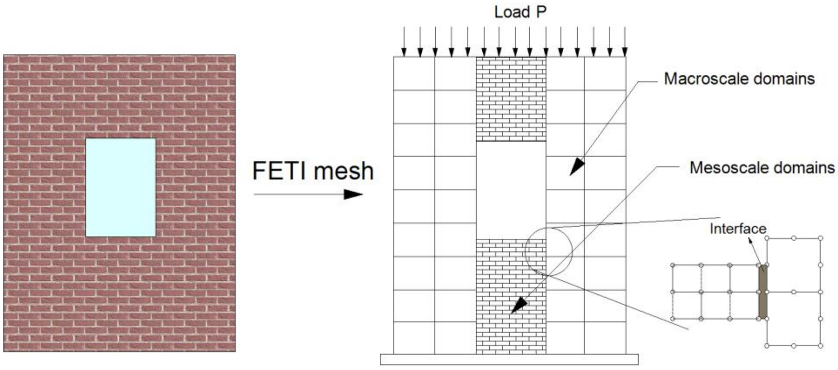


Figure 1: Multi-scale approach masonry wall subjected to compression forces and the FETI mesh (Finite Element Tearing and Interconnecting).

discontinuous model based on the Generalized Finite Element Method (GFEM) /4,7,11/. Crack growth is given by a plasticity based cohesive zone model, in terms of tractions and displacements. This approach does not constitute a completely new method, but rather an application of domain decomposition techniques on masonry structures, in order to reduce the computational effort and increase the numerical stability within the inelastic regions. The proposed method will serve as a good basis for the future development of the automated 'detect-and-refine' approach we have mentioned earlier.

2. Meso-scale approach

The meso-scale framework consists of a background mesh fitted on a masonry topology, using the simplified micro-model as proposed by Lourenço in /9/ Cracking of the mortar joints is modelled by using GFEM in combination with a plasticity based cohesive zone law.

2.1 The generalized finite element method

The generalized finite element method belongs to the numerical family of discontinuous models. These models are classified as discontinuous, because displacements are represented as discontinuities.
The basic idea of this approach is to enhance the displacement field by discontinuous functions that allow for jumps along the discontinuity surface. A key feature of this

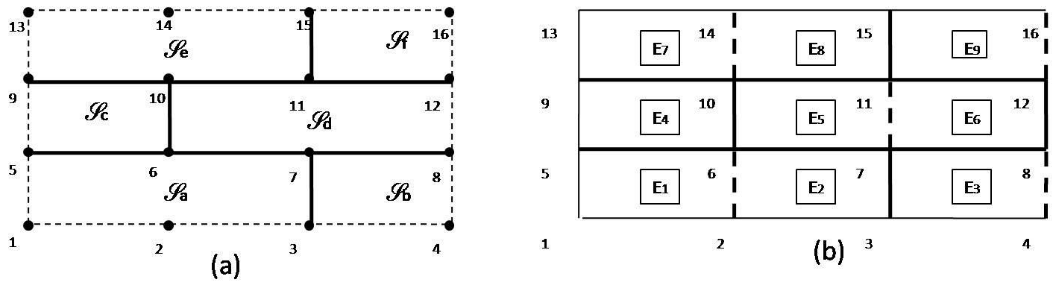


Figure 2: (a) GFEM cells in masonry; (b) background mesh

method is that the behaviour of the crack can be completely captured within the discontinuity, while the surrounding continuum remains elastic. Such a discontinuous function can be added using the partition of unity property of finite element shape functions φ_i /2,11/, that yields the following equation

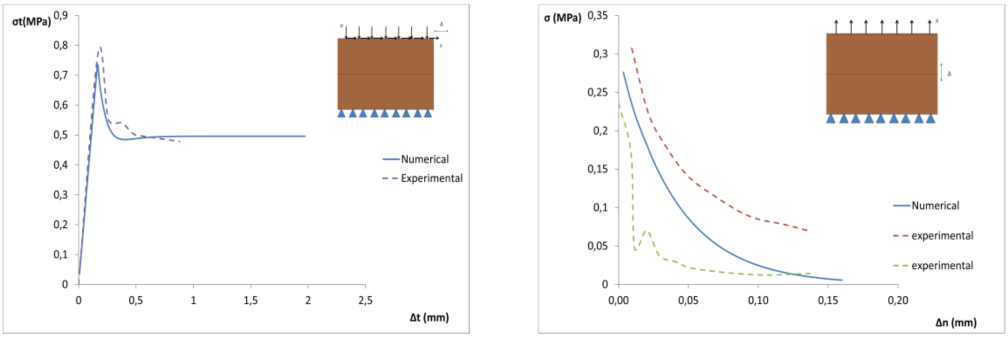
$$\sum_{i=1}^{n} \varphi_i(x) = 1 \qquad \forall x \in \Omega \tag{2.1}$$

When using GFEM, it is necessary to predefine the possible locations of the crack path before computation. This results in a topology which consists of a number of cells $\mathcal{S}_i$, defined by possible cracks. Within the partition of unity method, the discontinuity information is processed on the level of a cell /11/ The displacement field for a cell reads.

$$\mathbf{u}(x) = \hat{\mathbf{u}}(x) + \sum_{i=1}^{N_H} \mathcal{H}_i \tilde{\mathbf{u}}(x) \tag{2.2}$$

where: $\hat{\mathbf{u}}(x)$ equals the regular set of displacements en $\tilde{\mathbf{u}}(x)$ equals the enhanced set of displacements.

$$\mathcal{H}_i = \begin{cases} 1 & \text{if} \quad x \in \mathcal{S}_i \\ 0 & \text{otherwise} \end{cases} \tag{2.3}$$



(a) (b)

Figure 3: Benchmark tests: (a) confined shear test; (b) tensile test

Table 1: Material parameters of the cement mortar and brick continuum (/1,10/)

	tensile strength [MPa]	cohesion [MPa]	$G_{f\,I}$ [N/mm]	$G_{f\,II}$ [N/mm]	Young's mod[MPa]
Continuum	-	-	-	-	14525
Interface (mortar)	0,3	0,62	0,012	0,564	11320

2.2 A plasticity based cohesive zone model

In a cohesive zone model, the fracture behaviour is regarded as a gradual phenomenon in which separation takes place across a cohesive zone. As such, a cohesive zone does not represent any physical material, but rather the cohesive forces which occur when material elements are being pulled apart /3,12/.

The constitutive relationship for the cohesive zone is defined in terms of tractions and separations. The plasticity evolution law is based on a smooth yield surface which was proposed in earlier work /7,8/.

The proposed yield surface is constructed using the material parameters of the mortar; tensile strength, compressive strength and cohesion. In this work the material parameters of a cement mortar are used /1,10/ (Table 1). The softening laws for the strength parameters are based on an exponential function. Figure 3 shows the

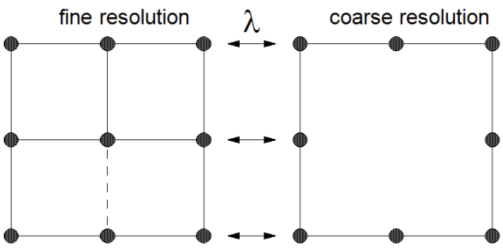


Figure 4: Decomposition in two domains with interface forces λ; fine resolution = GFEM background mesh and coarse resolution = standard FE mesh

validation of the meso-scale model for a simple tensile and shear test, applying a cement mortar. The experimental data is retrieved from Van der Pluijm /10/ for the tension test and Chaimoon /1/ for the shear test.

It should be noted that upon using another mortar composition, the material parameters derived from simple small scale tests can easily be substituted into the plasticity model.

3. Multi-scale approach: FETI

3.1 Basic theory

Domain decomposition techniques are used to partition the computation of large systems, where the interfaces between the domains are iterative solved. In this contribution, a basic formulation of the Finite Element Tearing and Interconnecting (FETI) method has been implemented, which belongs to the family of dual domain decomposition methods. The given formulation is adopted for finite element analysis. For further details on this method, the authors refer to the work of Farhat et al. /5/.

The FETI approach uses Lagrange multipliers to fulfil the compatibility between different domains $\Omega^{(s)}$. Hence the global system $\mathbf{Ku} = \mathbf{f}$ is split into a set of subsystems (N_s) which are connected using the Lagrange multipliers, so the local equilibrium of a domain reads:

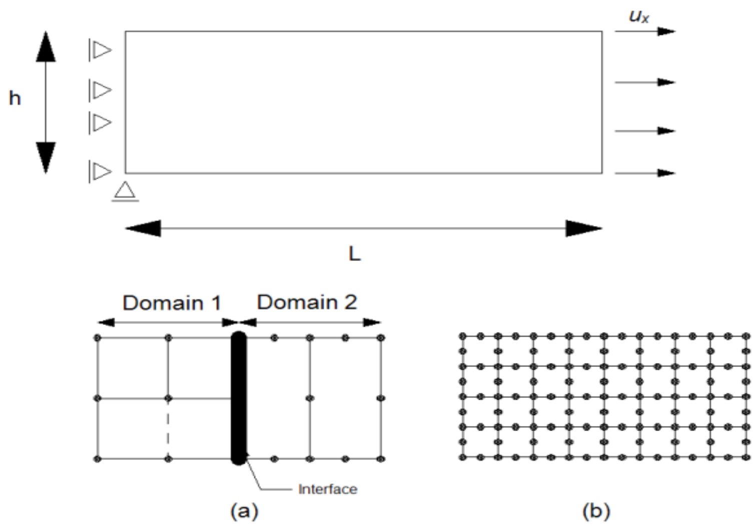


Figure 5: Cantilever beam with h=20mm and L=60mm; (a) FETI mesh and (b) FE mesh.

$$\mathbf{K}^{(s)}\mathbf{u}(x)^{(s)} = \mathbf{f}^{(s)} \tag{3.1}$$

The continuity between the displacements fields is given by Boolean matrices **B** containing the values +1 or -1 at those positions that correspond to the interface of the respective domain $\Omega^{(s)}$, so that:

$$\sum_{s=1}^{N_s} \mathbf{B}^{(s)}\mathbf{u}^{(s)} = 0 \tag{3.2}$$

When adopting this method it is possible to combine both fine and coarse meshes by using the Lagrange multipliers to enforce the compatibility constraints. Thus it is possible to use the GFEM background mesh as the fine mesh for a meso-scale computation and quadrilateral elements as the coarse mesh for a macro-scale computation, Figure 4. Hence both computation are independently executed and bridged by using the FETI method.

3.2 Numerical example of the FETI approach

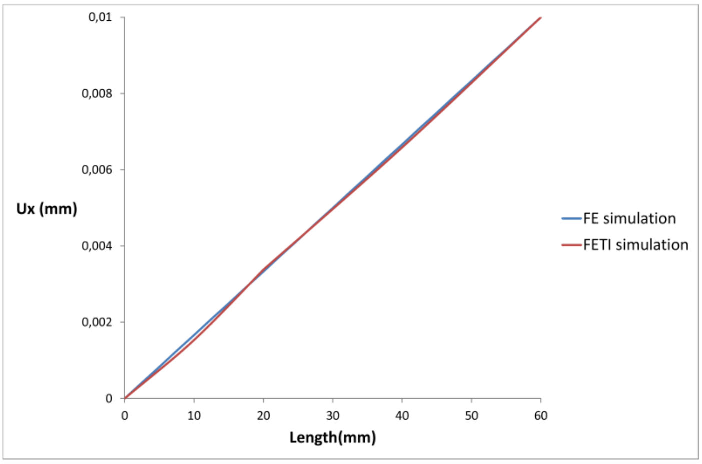


Figure 6: Displacement field at height 10 mm

A numerical example of a cantilever beam, Figure 5, illustrates the bridging of a GFEM background mesh with a standard coarse FE mesh (Figure 5(a)). This will be compared with a basic FE mesh generated with 32 elements (Figure 5 (b)).
When comparing the results, it is important to have a continuous displacement field between both domains in the FETI mesh. Figure 6 presents the displacement field at height 10 mm along the length of the beam of both the FE mesh and the FETI mesh. It indeed can be observed that the displacement field of both meshes coincide and that there is no discontinuity in the displacement field between both domains.

4. Conclusions and future works

In this work, we have introduced a novel multi-scale model for masonry structures. We have shown that GFEM can be employed as a suitable tool for describing crack behaviour of the masonry mortar at the meso-scale, while simultaneously zones in which no cracking occurs can be modelled by a computationally inexpensive coarse finite element mesh. We have shown that both scales can be bridged using the FETI method, guaranteeing a continuous displacement field. This allows for the combination of both methods into a multi-scale technique, in which regions where possible inelastic behaviour can occur are predefined with a meso-scale mesh, while other regions are meshed with a coarse grid.

Further optimization of this method will include a 'detect-and-refine' method, in which the entire structure is meshed initially using a coarse grid, but local meso-scale refinements are created as inelastic behaviour occurs.

Acknowledgments

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