

MULTISCALE MODELLING OF MASONRY STRUCTURES USING DOMAIN DECOMPOSITION TECHNIQUES

K. HEYENS^{*‡}, B. VANDOREN[‡] AND L. SCHUEREMANS[†]

^{*‡}Departement IWT, vakgroep Bouwkunde
Xios Hogeschool Limburg
Universitaire campus gebouw H, B-3590 Diepenbeek, Belgium
e-mail: koen.heyens@xios.be

[†]Department of Civil Engineering,
KULeuven
Kasteelpark Arenberg 40-bus 2448, B-3001 Heverlee, Belgium
e-mail: luc.schueremans@bwk.kuleuven.be

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Abstract. This paper describes the application of a domain decomposition technique for multiscale modelling of fracture behaviour in masonry. The use of multiple domains allows for a difference in employed mesh sizes for the macro- and mesoscale. For domains which play a crucial role in the failure process, we apply a mesoscale level meshing, while less critical components can be modelled by a less computationally expensive macroscale mesh. The crack behaviour is modelled by using the GFEM method, while the joint degradation is described using a plasticity based cohesive zone model, with a smooth yield surface. For the purpose of domain decomposition, we propose the use of a FETI method.

1 INTRODUCTION

The design of masonry structures, as it is done today, is still based on codebooks and rules of thumb, which often lead to a lack of control over safety factors and non-optimal structure dimensions. As such, it would be useful to develop reliable numerical tools that predict the behaviour of masonry structures.

The majority of numerical tools currently available for constitutive description of masonry structures are proven to be accurate on small scale structures, but when used on large scale structures, excessive computational effort is required and numerical instabilities occur [14, 7, 5]. Hence, in order to lower the computational cost it is more efficient to focus on regions of the masonry structure where cracks occur. It is also known that the mortar phase is relatively weak, which due to the periodic arrangement of the phases leads

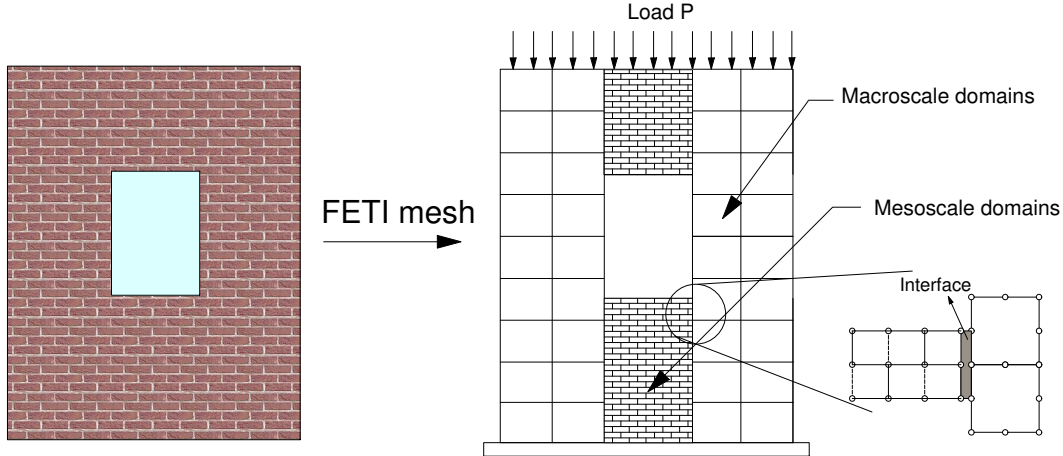


Figure 1: Multiscale approach masonry wall subjected to compression forces

to a stiffness degradation along preferential orientations, i.e. the crack path in masonry (often) follows the joints. A domain decomposition method can be employed to decompose the masonry structure into several domains, and concentrate our computational efforts on the domains which undergo inelastic behaviour.

It is possible to adopt such a technique in two different ways to describe a multiscale model for masonry structures. The first approach consists of initializing the discretized masonry structure as a coarse grid consisting of several domains. Once a domain meets a given criterion, indicating the occurrence of inelastic behaviour, it will be isolated and evaluated on a mesoscale using a finer background mesh. Afterwards, the results from the mesoscale computation will be integrated into the macroscale parent grid using a domain decomposition technique. An other way to use the domain decomposition technique is to define the regions in the discretized structure where possible inelastic behaviour could occur beforehand. These domains will be meshed at mesoscale, while the remainder of the structure will be meshed with a coarse grid on macroscale.

In this contribution the second approach will be presented, as illustrated in Figure 1. As shown in the figure, the mesoscale domains are concentrated under and above the window, since under uniform compression the inelastic behaviour most likely will occur in those regions. The evaluation at macroscale is done with an homogenized stiffness, as described in [14], and the mesoscale crack behaviour is modelled using a discontinuous model based on the Generalized Finite Element Method (GFEM) [18, 10, 5]. Crack growth is given by a plasticity based cohesive zone model, in terms of tractions and displacements. This approach does not constitute a completely new method, but rather an application of domain decomposition techniques on masonry structures, in order to reduce the computational effort and increase the numerical stability within the inelastic regions. The proposed method will serve as a good basis for the future development of

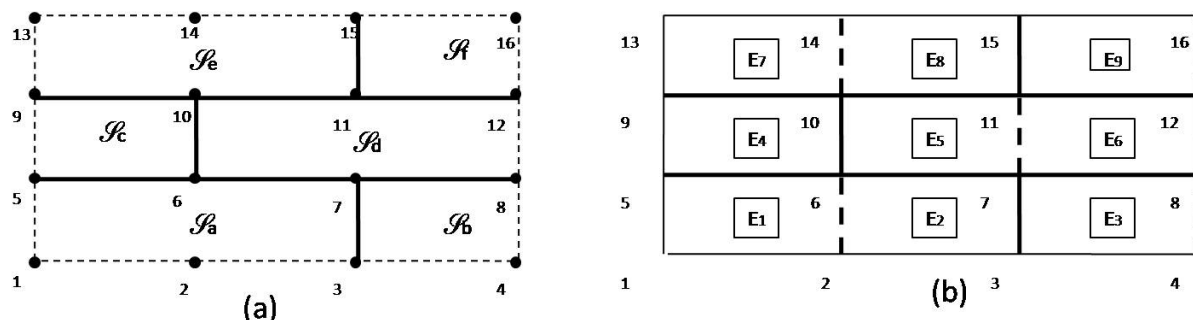


Figure 2: (a) GFEM cells in masonry; (b) background mesh

the automated 'detect-and-refine' approach we have mentioned earlier.

2 THE GENERALIZED FINITE ELEMENT METHOD

The generalized finite element method belongs to the numerical family of discontinuous models. These models are classified as discontinuous, because displacements are represented as discontinuities.

The basic idea of this approach is to enhance the displacement field by discontinuous functions that allow for jumps along the discontinuity surface. A key feature of this method is that the behavior of the crack can be completely captured within the discontinuity, while the surrounding continuum remains elastic. Such a discontinuous function can be added using the partition of unity property of finite element shape functions φ_i [1, 20].

$$\sum_{i=1}^n \varphi_i(\mathbf{x}) = 1 \quad \forall \mathbf{x} \in \Omega \quad (1)$$

When using GFEM, it is necessary to know the possible locations of the crack path before computation. This results in a topology which consists of a number of cells \mathcal{S}_i , defined by possible cracks. Within the partition of unity method, the discontinuity information is processed on the level of a cell [18]. The displacement field for a cell reads

$$\mathbf{u}(\mathbf{x}) = \hat{\mathbf{u}}(\mathbf{x}) + \sum_{i=1}^{N_H} \mathcal{H}_i \tilde{\mathbf{u}}(\mathbf{x}) \quad (2)$$

where: $\hat{\mathbf{u}}(\mathbf{x})$ equals the regular set of displacements and $\tilde{\mathbf{u}}(\mathbf{x})$ equals the enhanced set of displacements.

$$\mathcal{H}_i = \begin{cases} 1 & \text{if } \mathbf{x} \in \mathcal{S}_i \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

It is logical to use the GFEM to describe crack behaviour in masonry walls, as the topology of a masonry wall is mostly known, and it is easy to fit a background mesh where the joints coincide with boundaries of an element boundary (see Figure 2).

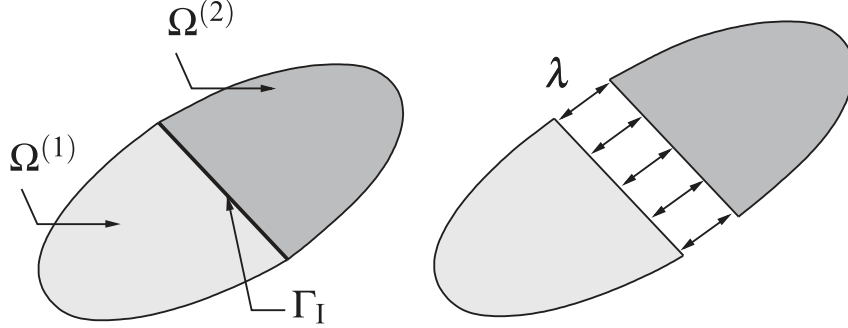


Figure 3: Decomposition in two domains with interface forces λ

3 THE DOMAIN DECOMPOSITION APPROACH

In this section a basic formulation of the Finite Element Tearing and Interconnecting (FETI) method is introduced, a method which belongs to the family of dual domain decomposition methods. Here the given formulation is adopted for a finite element analysis, further details on this method can be found in Farhat et al. [6].

Consider a body divided in two domains ($N_s = 2$) as shown in Figure 3, each domain $\Omega^{(s)}$ has a displacement field $\mathbf{u}^{(s)}$ and after discretization the local equilibrium reads,

$$\mathbf{K}^{(s)}\mathbf{u}(\mathbf{x})^{(s)} = \mathbf{f}^{(s)} \quad (4)$$

where (s) is the number of the respective domain, $\mathbf{K}^{(s)}$ the stiffness matrix and $\mathbf{f}^{(s)}$ the external force vector. For the two domains given in Figure 3, the continuity of the solution field is given by:

$$\mathbf{u}^{(1)} = \mathbf{u}^{(2)} \quad \text{at} \quad \Gamma_I \quad (5)$$

When incorporating the continuity requirement of (5) with (4), the solution reads

$$\begin{bmatrix} \mathbf{K}^{(1)} & \mathbf{0} & \mathbf{B}^{(1)T} \\ \mathbf{0} & \mathbf{K}^{(2)} & \mathbf{B}^{(2)T} \\ \mathbf{B}^{(1)} & \mathbf{B}^{(2)} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{u}^{(1)} \\ \mathbf{u}^{(2)} \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathbf{f}^{(1)} \\ \mathbf{f}^{(2)} \\ \mathbf{0} \end{bmatrix} \quad (6)$$

where the matrices $\mathbf{B}^{(s)}$ contain the values +1 or -1 at those positions that correspond to the interface of the respective domain $\Omega^{(s)}$, and Lagrange multipliers λ are introduced to enforce the compatibility constraints. Since the domains can be part of the macroscale, where the displacement field is continuous or part of the mesoscale, where a discontinuity can be incorporated in the displacement field [1, 5]. This results in two expressions of the displacement vector,

$$\mathbf{u}(\mathbf{x})^{(s)} = \begin{cases} \hat{\mathbf{u}}(\mathbf{x})^{(s)} & \text{if } \mathbf{x} \in \text{macroscale} \\ \hat{\mathbf{u}}(\mathbf{x})^{(s)} + \sum_{i=1}^{N_H} \mathcal{H}_i \tilde{\mathbf{u}}(\mathbf{x})^{(s)} & \text{if } \mathbf{x} \in \text{mesoscale} \end{cases} \quad (7)$$

To solve the interface problem, we solve this for the local displacement field $\mathbf{u}(\mathbf{x})^{(s)}$

$$\mathbf{u}(\mathbf{x})^{(s)} = \mathbf{K}^{(s)+} \left(\mathbf{f}^{(s)} - \mathbf{B}^{(s)T} \boldsymbol{\lambda} \right) - \mathbf{R}^{(s)} \boldsymbol{\alpha}^{(s)} \quad (8)$$

where $\mathbf{K}^{(s)+}$ is the inverse of the stiffness matrix for a domain $\Omega^{(s)}$ with no rigid modes or a generalized inverse if the domain s is floating, in which case $\mathbf{R}^{(s)}$ are the rigid body modes. $\boldsymbol{\alpha}^{(s)}$ are the amplitudes of the rigid body modes [13].

4 A PLASTICITY BASED COHESIVE ZONE MODEL

In a cohesive zone model, the fracture behaviour is regarded as a gradual phenomenon in which separation takes place across a cohesive zone. Such a cohesive zone does not represent any physical material, but rather the cohesive forces which occur when material elements are being pulled apart [2, 20].

The constitutive relationship for the cohesive zone is defined in terms of tractions and separations. First, the basic equations of the plasticity theory in the traction space are presented. Next, the proposed material model will be discussed, based on the plasticity yield surface which was proposed in earlier work [9].

4.1 The cohesive zone model

In this study, the plasticity theory is embedded in a finite element environment using a cohesive zone model. Hence, it is necessary to describe the mathematical relation between the tractions \mathbf{T} and separations $\boldsymbol{\Delta}$, each with a normal and tangent component. Following classical elasto-plasticity, the total deformation rate vector of a discontinuity can be decomposed in two parts, an elastic and a plastic component:

$$\dot{\boldsymbol{\Delta}} = \dot{\boldsymbol{\Delta}}^e + \dot{\boldsymbol{\Delta}}^p \quad (9)$$

where the elastic deformation rate is related to the traction rate by the elastic stiffness matrix \mathbf{D} as

$$\dot{\mathbf{T}} = \mathbf{D} \dot{\boldsymbol{\Delta}}^e \quad (10)$$

A definition of a yield surface is used to bound the elastic region, and can be used as a fracture criterion. As the constitutive model is expressed in terms of tractions and separations, the yields surface needs to be defined in the traction space $\{T_n, T_t\}$, where T_n stands for the normal component of the traction vector, and T_t for the tangential component of the traction vector. A suitable mathematical description for a smooth yield surface F is given by a cubic Bézier function [9, 19], as illustrated in Figure 4.

4.2 Proposed material model

All simulations of plastic deformation are based on the notion of a yield function f , such that $f < 0$ in the elastic regime, $f = 0$ on the yield locus, and $f > 0$ when plastic

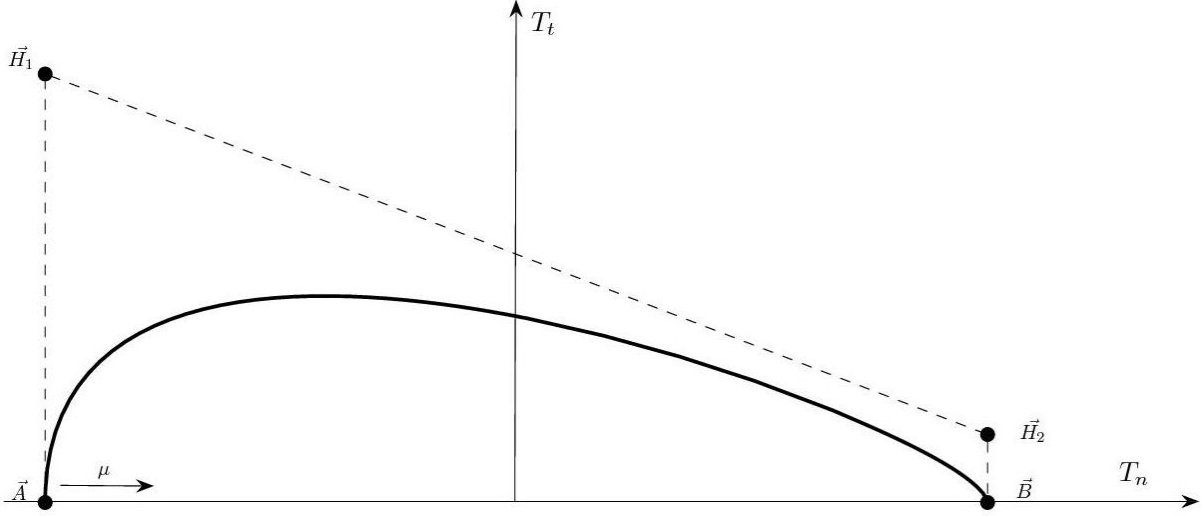


Figure 4: Cubic Bézier curve with two control points $\{A, B\}$ and two hinge points $\{H_1, H_2\}$

deformation occurs. Our employed yield function is described by

$$f(\mathbf{T}, \mathbf{T}^\mu) = \sqrt{T_n^2 + T_t^2} - \sqrt{(T_n^\mu)^2 + (T_t^\mu)^2} \quad (11)$$

In case $f > 0$, the computed traction vector $\mathbf{T}^{(t)}$ is located outside the yield surface, and plastic deformation occurs. For the adopted yield surface, its evolution throughout the computation is governed by the decrease of tensile strength, compression strength and cohesion. The decrease of compression strength is not implemented and remains constant in this work, since the influence of the decrease of tensile strength and cohesion in masonry cracking is substantial in comparison with the decrease of compression strength.

The decrease of tensile strength and cohesion are given by the following expressions:

$$f_t = f_t^p \cdot \exp \left[\frac{-\beta_n \cdot f_t^p}{G_f^I} \cdot \Delta_n^{pl} \right] ; \Delta_n^{pl} = \int_0^t \dot{\Delta}_n^{pl} dt \quad (12)$$

$$c = c_r + \left(\frac{C}{c^p} - c_r \right) \cdot \exp \left[\frac{-\beta_t \cdot \frac{C}{c^p}}{G_f^{II}} \cdot (\Delta_t^{pl}) \right] ; \Delta_t^{pl} = \int_0^t \dot{\Delta}_t^{pl} dt \quad (13)$$

where f_t^p is the initial tensile strength, G_f^I the mode I fracture energy, G_f^{II} the mode II fracture energy, c^p is the value of the cohesion obtained before plastic deformation occurs, c_r is the residual cohesive strength and $\frac{C}{c^p}$ is the initial cohesive strength. The material parameters β_n and β_t control the rate of softening for each internal variable. Δ_t^{pl} and Δ_n^{pl} are respectively the tangent- and normal plastic deformations of the displacement vector Δ .

5 IMPLEMENTATION NON-LINEAR ANALYSIS

The algorithms described in the previous sections are implemented in a Matlab[®] environment. An algorithm outline of the implemented code considering the non-linear analysis with a domain decomposition technique is presented in Table 1.

6 CONCLUSIONS

In this paper, we have presented a multiscale approach to modelling masonry fractures using a domain decomposition technique. The fracture behaviour is based on a plasticity cohesive zone model with a smooth yield surface, and discontinuities are incorporated using the Generalized Finite Element Method. Implementation of the algorithms was done in a Matlab[®] environment and the domains are chosen in such way that there is a minimal computational effort.

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Algorithm outline of the non-linear conversion analysis

For each loading step Δt do:

1. Initialize solution field $\Delta \mathbf{u}_0^{(s)}$ and interface connectivity $\mathbf{B}^{(s)}$
2. Compute strain $\Delta \boldsymbol{\epsilon}_i^n \leftarrow \Delta \mathbf{u}_i^{(s)}$ and the displacement jumps $\boldsymbol{\Delta}_i \leftarrow \Delta \mathbf{u}_i^{(s)}$ in the cohesive zones for each integration point n in $\Omega^{(s)}$.
3. Compute stress $\Delta \boldsymbol{\sigma}_i^n \leftarrow \Delta \boldsymbol{\epsilon}_i^n$ and the tractions $\mathbf{T}_i \leftarrow \boldsymbol{\Delta}_i$ in the cohesive zones for each integration point n in $\Omega^{(s)}$.
 - check if $f < 0$ else plastic deformation occurs
4. Compute internal force vector $\Delta \mathbf{f}_{int,i}^{(s)}$
5. Update internal force vector $\mathbf{f}_{int,i+1}^{(s)} = \mathbf{f}_{int,i}^{(s)} + \Delta \mathbf{f}_{int,i}^{(s)}$
6. Apply prescribed displacements or forces.
7. FETI solver
 - compute rigid body modes $\mathbf{R}^{(s)}$.
 - compute Lagrange multipliers $\boldsymbol{\lambda}_i$ and the amplitudes $\boldsymbol{\alpha}_i^{(s)}$.
 - update external force vector with interdomain forces $\mathbf{f}_{ext,t+\Delta t}^{(s)} = \mathbf{f}_{ext,t+\Delta t}^{(s)} - \mathbf{B}^{(s)T} \boldsymbol{\lambda}_i$
 - Compute displacements increment $\delta \mathbf{u}_{i+1}^{(s)} = \mathbf{K}_i^{(s)+} \left(\mathbf{f}_{ext,t+\Delta t}^{(s)} - \mathbf{f}_{int,i+1}^{(s)} \right) - \mathbf{R}^{(s)} \boldsymbol{\alpha}_i^{(s)}$
8. Update displacement vector $\mathbf{u}_{i+1}^{(s)} = \Delta \mathbf{u}_i^{(s)} + \delta \mathbf{u}_{i+1}^{(s)}$
9. Assemble domain quantities $\mathbf{u}_{i+1}^{(s)}$ in global quantities \mathbf{u}_{i+1}
10. check for convergence $\| \delta \mathbf{u}_{i+1} \| \leq \epsilon \cdot \| \Delta \mathbf{u}_1 \|$ Else go to 2 if criterion is met go to the next loading step

Table 1: Algorithm outline of the non-linear conversion analysis

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