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Peer-reviewed author version

RAMAEKERS, Katrien & JANSSENS, Gerrit K. (2006) Towards a best strategy in inventory decision making and demand forecasting for intermittent demand. In: 4th International Industrial Simulation Conference 2006. p. 409-414..

Handle: http://hdl.handle.net/1942/1375

Towards a best strategy in inventory decision making and demand forecasting for intermittent demand

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Keywords: intermittent demand, forecasting, inventory management, simulation, optimisation

Abstract

Demand forecasting is one of the most crucial aspects of inventory management. For intermittent demand, i.e. demand peaks follow several periods of zero or low demands, forecasting is difficult. A simulation model is used to study a single-product inventory system facing demand of the intermittent type. In this paper, a research approach is described to find a best strategy in combining inventory decision making and demand forecasting for intermittent demand.

1 Introduction

Inventory systems have to cope with uncertainty in demand. The inventory control literature mostly makes use of the Normal or Gamma distributions for describing the demand in the lead-time. The Poisson distribution has been found to provide a reasonable fit when demand is very low (only a few pieces per year). Less attention has been paid to irregular demand. This type of demand is characterised by a high level of variability, but may be also of the intermittent type, i.e. demand peaks follow several periods of zero or low demands.

Demand forecasting is one of the most crucial aspects of inventory management [9]. However, for intermittent demand, forecasting is difficult, and errors in prediction may be costly in terms of obsolescent stock or unmet demand [8]. The standard forecasting method for intermittent demand items is considered to be Croston's method [2]. This method builds estimates taking into account both demand size and the interval between demand occurrences. Despite the theoretical superiority of such an estimation procedure, empirical evidence suggests modest gains in performance when compared with simpler forecasting techniques [7]. Furthermore, the choice of the forecasting method can have an impact on the inventory management policy that is best used.

Preliminary research demonstrates the presence of an interaction between demand forecasting and inventory decision making for intermittent demand using a simulation model to study a single-product inventory system facing demand of the intermittent type. Therefore, in this paper, a research approach is described to optimise the simulation model in order to obtain the best strategy in combining inventory decision making and demand forecasting. Furthermore, some initial results are discussed.

2 Experimental framework

This section describes the inventory systems and forecasting methods that are used in this research. The study focuses on a single-product inventory system facing demand of the intermittent type. To generate intermittent demand, demand occurrence and demand size are separately generated. The demand occurrence is generated according to a first-order Markov process with transition matrix

$$\mathbf{P} = \begin{pmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{pmatrix},$$

where p_{00} is the probability of no order in the next period when there has been no order in this period and p_{10} is the probability of no order in the next period when there has been an order in the current period. Individual order sizes are generated using a Gamma distribution with shape parameter γ and scale parameter β.

The simulation run length is set to 52 periods and 10 replications are made for each simulation run.

The total costs and the performance (number of stock-out periods and number of stock-out units) of the inventory system are determined.

2.1 Inventory systems

There are two general types of inventory systems: continuous review models and periodic review models. In continuous review models, the stock level is always known whereas in periodic review models, the stock level is determined only every R time units.

In this research, two periodic review models are used. The first one is the (R, s, S) system. This means that every R units of time, the inventory level is checked. If it is at or below the reorder point s, a sufficient quantity is ordered to raise it to S. The second system (R, s, Q) is similar to the (R, s, S) system but uses a fixed order quantity Q instead of an order-up-to-level S.

A deterministic lead-time L is assumed. Three possible review periods are considered: review period equal to lead-time, review period equal to twice the lead-time and review period equal to half the lead-time.

The following costs are considered: unit holding cost per period C_h, ordering cost C_o and unit shortage cost per period C_s. The simulation starts with an initial inventory level I_0.

2.2 Forecasting methods

The standard forecasting method for intermittent demand items is considered to be Croston's method. However, in practice, single exponential smoothing and simple moving averages are often used to deal with intermittent demand.

Exponential smoothing is probably the most used of all forecasting techniques. The single exponential smoothing (SES) method is easy to apply, the forecast is calculated as:

$$F_t = \alpha X_{t-1} + (1-\alpha) F_{t-1} \quad (1)$$

where α is the smoothing constant that determines the weight given to the most recent past observations and therefore controls the rate of smoothing or averaging. It is commonly constrained to be in the range of zero to one.

The assumption of the *moving average* forecasting method is that a future value will equal an average of past values. The number of past values used to calculate the forecast can vary. The four-period moving average forecast is calculated as:

$$F_t = W_4 X_{t-4} + W_3 X_{t-3} + W_2 X_{t-2} + W_1 X_{t-1}. \quad (2)$$

Croston's method [2] was developed to provide a more accurate forecast of the mean demand per period. Croston's method applies exponential smoothing separately to the intervals between nonzero demands and their sizes. Let I_t be the smoothed estimate of the mean interval between nonzero demands, and let S_t be the smoothed estimate of the mean size of a nonzero demand. Let q be the time interval since the last nonzero demand. Croston's method works as follows: if $X_t = 0$ then

$$S_t = S_{t-1}; I_t = I_{t-1}; q = q + 1 \quad (3)$$

else

$$S_t = \alpha X_t + (1-\alpha) S_{t-1}; I_t = \alpha q + (1-\alpha) I_{t-1}; q = 1. \quad (4)$$

where α is the smoothing parameter. Combining the estimates of size and interval provides the forecast:

$$F_t = S_t / I_t. \quad (5)$$

2.3 Experimental design

The experimental design includes three qualitative factors: the forecasting method, the inventory management policy and the review period. In addition, depending on the choice of the qualitative factors, a set of quantitative factors are part of the experimental design. If the (R, s, Q) inventory management policy is used, the safety stock SS and order quantity Q are the parameters to optimise. If the (R, s, S) inventory management policy is used, the safety stock SS and order-up-to-level S are the optimising parameters. For single exponential smoothing and Croston's method, the smoothing parameter α is optimised and for moving averages, the weights of the past values are optimised.

This research aims to decide on the optimal combination of forecasting method, inventory management policy and review period. Furthermore, the optimal settings for the safety stock, the fixed order quantity or order-up-to-level and the parameter(s) of the forecasting method are determined.

3 Approach

Because of the dependence of the quantitative factors on the choice of the qualitative factors, we use the research approach described in this section.

For every combination of forecasting method, inventory management policy and review period, the optimal values of the quantitative factors are determined. This is done using two different optimisation methods: Taguchi method and tabu search. These two methods are shortly described below. Once the optimal values are found, the best combination of forecasting method, inventory management policy and review period is chosen.

3.1 Design of experiments: Taguchi's method

Design of Experiment (DOE) Techniques [5] provide a way to set up the complete experimental design before the experimentation process begins. The experimental points are chosen in order to cover the search space as completely as possible. Design of experiment methods can in general only be applied to discrete variables, so the first step before applying a DOE-method consists of choosing a limited number of discrete values in the domain of each continuous variable.

Several schemes for setting up experimental designs are known from literature. The Taguchi design is an interesting technique. The first step is to rank the n relevant values of each decision variable and give them a level number from 1 to n. The next stage is to set up the experiments. This is usually done using specially constructed orthogonal arrays containing a number of rows. Each row defines one experiment to be carried out with the corresponding levels for the variables.

Three discrete values are chosen in the domain of each of the three quantitative, continuous factors. These values are shown in Table 1, where SS, S and Q are calculated using the following formulas:

$$SS = z\sigma_{R+L} \tag{6}$$

where σ_{R+L} is the standard deviation of demand over the review period and the lead-time. The value z depends on the desired service level.

The fixed order quantity Q is determined using the formula of the Economic Order Quantity EOQ:

$$Q = \sqrt{\frac{2\overline{X}C_o}{C_h}}. \tag{7}$$

The order-up-to-level S is the sum of the safety stock and the average demand over the vulnerable period:

$$S = SS + \overline{X}(R + L). \tag{8}$$

3.2 Tabu search

Tabu search uses a local or neighbourhood search procedure to iteratively move from one solution to the next in the neighbourhood of the first, until some stopping criterion has been satisfied. To explore regions in the search space that would be left unexplored by the local search procedure and escape local optimality, tabu search modifies the neighbourhood structure of each solution as the search progresses. The solutions admitted to the

Variable	1	2	3
α	0.2	0.5	0.8
Weight 1	0.1	0.25	0.4
Weight 2	0.2	0.25	0.3
Weight 3	0.3	0.25	0.2
Weight 4	0.4	0.25	0.1
Safety Stock	SS-2	SS	SS+2
Order-up-to-level	S-5	S	S+5
Fixed Order Quantity	Q-5	Q	Q+5

Table 1: Discrete values of the variables in the Taguchi method

new neighbourhood are determined through the use of special memory structures. Tabu search uses both long-term and short-term memory, and each type of memory has its own special strategies [3, 4].

Tabu search is a heuristic optimisation technique developed specifically for combinatorial problems. Very few works deal with the application to the global minimization of functions depending on continuous variables. The method we propose in this paper is based on [1, 6]. The purpose in these papers is to keep as close as possible to original tabu search. Two issues must be examined: the generation of current solution neighbours and the elaboration of the tabu list.

To define a neighbourhood of the current solution, a set of hyperrectangles is used for the partition of the current solution neighbourhood. The k neighbours of the current solution are obtained by selecting one point at random inside each hyperrectangular zone.

Once a new current solution is determined, the immediate neighbourhood of the previous solution is added to the tabu list.

As a starting point, the safety stock SS, fixed order quantity Q and order-up-to-level S are calculated using the formulas above. A neighbourhood consists of 5 neighbours and the tabu list contains 5 tabu areas. 10 simulation runs are made for each experimental choice. The tabu search is stopped after 200 iterations.

4 Experimental environment

The experimental environment contains the costs of the inventory system and the parameters for generating intermittent demand. The research approach described above, is executed using a single combination of the costs of the inventory system and demand. However, these factors can have an effect on the results that are obtained. An experimental design is set up for these factors and the optimisation phase is repeated for each experimental point.

Demand occurrence is generated using a first-order Markov process with transition matrices:

$$\mathbf{P_1} = \begin{pmatrix} 0.7875 & 0.2125 \\ 0.85 & 0.15 \end{pmatrix}$$

or

$$\mathbf{P_2} = \begin{pmatrix} 0.5667 & 0.4333 \\ 0.65 & 0.35 \end{pmatrix}.$$

They correspond with a probability of 20% to have demand in a certain period for the first matrix and a probability of 40% to have demand in a period for the second matrix. The *size of demand* is generated using a Gamma distribution with 4 different combinations of the scale parameter γ and the shape parameter β. These values are summarized in Table 2.

Combination	γ	β
1	6	1
2	12	1
3	3	2
4	24	0.5

Table 2: Parameters of the Gamma distribution

The levels of the costs of the inventory system are given in Table 3. The initial inventory level I_0 equals 5.

This experimental design makes it possible to determine the impact of uncontrollable factors as the cost structure and the demand on the optimal strategy in inventory decision making and demand forecasting for intermittent demand.

Level	C_o	C_h	C_s
1	100	2	5
2	200	4	10

Table 3: Levels for the costs of the inventory system

5 Results

The basic configuration of the factors of the experimental environment is set as follows: demand occurrence is generated using a first-order Markov process with transition matrix

$$\mathbf{P_1} = \begin{pmatrix} 0.7875 & 0.2125 \\ 0.85 & 0.15 \end{pmatrix}.$$

For the demand size, a gamma distribution with scale parameter 6 and shape parameter 1 is used. The ordering cost equals €100 per order, the unit shortage cost €5 per period and the unit holding cost €2 per period.

When using Taguchi's method, the optimal solution for this experimental environment is an order-up-to-level inventory management system with a review period equal to twice the lead-time. Croston's method with $\alpha = 0.5$ is best used as forecasting method. The safety stock is equal to 6 units and the order-up-to-level is also equal to 6 units for the optimal solution. This means an order is placed every time the inventory level drops below the safety stock and enough is ordered to raise it again to the size of the safety stock.

When the demand size is doubled, the review period is best set equal to the lead-time. The safety stock and order-up-to-level have higher values but they are still equal to each other. When the demand frequency is doubled, the smoothing constant $\alpha = 0.8$ gives the best results. The safety stock is 4 and the order-up-to-level equals 9.

Changes in the costs of the system also cause differences in the conclusions. When the unit holding cost is doubled, α should be set to 0.8. The order-up-to-level is in this situation best lower than the safety stock. When the unit shortage cost or the ordering cost is doubled, the only difference in the conclusions is the order-up-to-level being smaller than the safety stock.

When Tabu search is used as optimisation method for the quantitative factors, the optimal strategy for the basic configuration of the factors of the experimental environment is an order-up-to)level inventory management policy with a review period equal to the lead-time. Exponential smoothing is the forecasting method that leads to the lowest costs. The smoothing parameter α does not have significant impact on the results. The order-up-to-level S is equal to 1 which implies that the safety stock SS is negative and the reorder point is 0.

When the demand frequency is doubled, the best inventory management policy depends on the cost structure. However, instead of an order-up-to-level S of 1 unit, the order-up-to-level S or fixed order quantity Q is a value between 15 and 30. It can also be noted that when the demand frequency is doubled, Croston's method becomes less useful as forecasting method. Changing the parameters of the demand size does not have a significant impact on the results.

Changes in the cost structure of the inventory system have a significant impact on the results. If the ordering cost is doubled, it is better to have an order-up-to-level or fixed order quantity that is between 15 and 30, except when the unit holding cost is high and the unit shortage cost is low. When the unit holding cost is doubled, an order-up-to-level of 1 is the best choice, unless the ordering cost and unit shortage cost are also high and the demand frequency is high. Doubling the unit shortage cost leads to an order-up-to-level or fixed order quantity between 15 and 30, except when the unit holding cost is high and the demand frequency is low.

6 Conclusions

Overall, it can be concluded that both optimisation methods lead to roughly similar results but when tabu search is applied, continuous values are used which leads to more accurate results.

The factors of the experimental environment have an impact on the best strategy for combining inventory decision-making and demand forecasting and there is also interaction between those factors.

For intermittent demand, the best inventory management policy is an order-up-to-level policy with an order-up-to-level S equal to 1. The reorder point equals 0 and the safety stock is negative. The best forecasting method depends on the cost structure. The parameters of the forecasting method do not influence the results significantly.

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Biography

Katrien Ramaekers graduated as Master of Business Economics, option Technology at the Limburg University Centre in 2002. In October 2002, she started as a Ph.D.-student at Hasselt University. Her main research interest is on the integration of simulation and optimisation, especially as a support for complex logistics decision-making and for decision support with limited information in supply and demand. She is a member of the Data Analysis and Modelling research group and of the Transportation Research Institute of Hasselt University.

Gerrit K. Janssens received degrees of M.Sc. in Engineering with Economy from the University of Antwerp (RUCA), Belgium, M.Sc. in Computer Science from the University of Ghent (RUG), Belgium, and Ph.D. from the Free University of Brussels (VUB), Belgium. After some years of work at General Motors Continental, Antwerp, he joined the University of Antwerp until the year 2000. Currently he is Professor of Operations Management and Logistics at Hasselt University within the Faculty of Business Administration. He also holds the CPIM certificate of the American Production and Inventory Control Society (APICS). During the last fifteen years he has repeatedly been visiting faculty member of universities in Thailand, Vietnam, Cambodia and Zimbabwe. His main research interests include the development and application of operations research models in production and distribution logistics.