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The analysis of (negatively) correlated non-Gaussian dyadic outcomes: non-multilevel-based alternatives?

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Abstract

When two people interact in a relationship, the outcome of each person can be affected by both his or her own inputs and his or her partner's inputs. For Gaussian dyadic outcomes, linear mixed models taking into account the correlation within dyads, are frequently used to estimate actor's and partner's effects based on the actor-partner interdependence model. In this paper, we explore the potential of generalized linear mixed models (GLMMs) for the analysis of non-Gaussian dyadic outcomes. Several approximation techniques that are available in standard software packages for these GLMMs are investigated. Despite the different modeling options related to these different techniques, none of these have an overall satisfactory performance in estimating actor and partner effects and the within-dyad correlation, especially when the latter is negative and/or the number of dyads is small. In contrast, a generalized estimating equations approach for the analysis of non-Gaussian dyadic data turns out to perform well in all situations considered.

KEY WORDS: binary data, count data, dyadic data, generalized estimating equations, generalized linear mixed models, multilevel analysis

1 Introduction

Dyadic research has become immensely popular in the social and behavioral sciences. When two people interact in a relationship, the outcome of each person can be affected by both his or her own inputs and his or her partner's inputs. The Actor-Partner Interdependence Model (APIM) offers an appealing approach to model such dyadic behavior (Kenny, Kashy, & Cook, 2006). Indeed, it allows to simulataneously study the influence of a person's own predictor variable on his or her own outcome variable, which is called the *actor effect*, and on the outcome variable of the partner, which is called the *partner effect*, while allowing for non-independence in the two persons' responses. Typically, two types of dyads are considered. Dyads are called distinguishable when the two persons from all the dyads can be ordered in the same way (for example, for hetero couples, persons within a dyad can be ordered by gender). Indistinguishable dyads occur when no ordering of persons exists within a dyad (like twins, for example). The left panel of figure 1 shows a graphical presentation of the APIM with two distinguishable dyad members and an X and Y variable for each. The variables X_1 and X_2 represent the predictor variables of persons 1 and 2 of a dyad, respectively, whereas Y_1 and Y_2 represent the outcome variables for the two members. The model contains two actor effects a_1 and a_2 (represented by the horizontal arrows), and two partner effects p_{12} and p_{21} (represented by the diagonal arrows). The curved arrow on the left reflects the correlation between the predictor variables, while the one on the right represents the correlation between the error terms. An alternative but underutilized model (Ledermann & Kenny,

2012) to explore dyadic influences is the Common Fate Model (CFM). When a construct representing a common fate variable exists at the level of the dyad rather than at the individual level, the CFM is more appropriate (right panel of Figure 1). In contrast, self-referential or partner-referential measures that are expected to represent individual behaviors or attitudes are more suitable for the APIM. We will focus here on the APIM, which has clearly dominated the dyadic literature with more than 150 publications over the last 3 years (Kenny & Ledermann, 2012).

Multilevel modeling, also referred to as hierarchical linear modeling, has been shown to be a useful technique for the estimation of actor and partner effects in dyadic data (Kenny et al., 2006). In these multi-level models two different levels are distinguished: the lower level, or level 1, refers to the case of persons nested within a dyad. The lower-level unit is person, whereas the upper level, or level 2, is the dyad. Linear mixed models (LMM) are frequently used and well understood for the analysis of such dyadic data but their use is limited to (Gaussian) outcomes measured at the interval level. The analysis of non-Gaussian dyadic data on the other hand has received little attention in the literature so far. The generalized linear mixed model (GLMM), which is an extension of both the generalized linear model (GLM) (Nelder & Wedderburn, 1972) and the LMM, is potentially suitable for the analysis of clustered observations from the exponential dispersion family distribution (Agresti, 2000). McMahon, Pouget, and Tortu (2005) and Spain, Jackson and Edmonds (2012) provide guidance on fitting the GLMM for dyadic data with binary outcomes but pay little attention to its properties. Two important issues need to be accounted for when modeling dyadic data. While researchers in psychological or social sciences are often faced with clustered data (in educational measurement applications for example, when several test items are administered to students; in longitudinal studies when psychological measurements are repeatedly assessed over time, etc.), the cluster size of two when analyzing dyads is a first important feature that needs consideration. Second, the possible negative correlation between observations within a dyad also warrants further exploration. Indeed, while in an item-response or longitudinal setting, measurements are typically positively correlated, negative correlations may occur within dyads. The strictness of parental supervision is one example of such negative correlation within dyads, where the more extreme in strictness one parent becomes, the more extreme in permissiveness the other parent is likely to become (Cook, 2001).

In this paper, we investigate in detail the performance of multilevel modeling of non-Gaussian outcomes in a dyadic setting. We first discuss the traditional use of the GLMM and its interpretation, point to its limitations, and explore the potential of some other rather non-standard estimation techniques for these GLMMs. Next, we introduce the generalized estimating equations (GEE) approach (Liang and Zeger, 1986) as a viable alternative. While multilevel models have become immensely popular for the analyses of correlated data, GEE is relatively unused in the educational and behavioral sciences (Bauer & Sterba, 2011). GEE can be considered as an extension of the generalized linear model (GLM) that accommodates correlated outcome data too; but whereas multilevel models explicitly specify the joint distribution of the outcomes, GEE only models the univariate marginal expectations as a function of explanatory variables and empirically accounts for the presence of correlation in the data. Simulations for binary and count dyadic data are performed to compare under a wide range of within-dyad correlations and for typical APIM sample sizes the performance of the GEE-approach with different estimation techniques for the multilevel approach. Focus in these simulation studies lies on both the estimation of the actor and partner effects and on the estimation of the within-dyad correlation. We end with an application of the different approaches to data from the Interdisciplinary Project for the Optimization of Separation Trajectories conducted in Flanders (IPOS) and present two illustrations. A first example illustrates the analysis of negatively correlated binary data and investigates the effect of actor's and partner's levels of feeling guilty, during the break-up, on showing so-called forcing behavior or not during the post break-up negotations in 29 ex-couples. The second example presents the analysis of positively correlated count data and explores in 33 ex-couples the effect of the actor's and partner's level of anxious attachment in their relationship with their ex-partner prior to the break-up on the number of unwanted pursuit behavior (UPB) perpetrations after separation.

2 Multilevel models

2.1 Linear mixed models

Let $\mathbf{Y}_i$ denote a 2-dimensional vector of measurements available for dyad $i = 1, \ldots, N$ with components Y_{i1} and Y_{i2} (for example the measurement for a male and female partner in a heterosexual couple [1]). Using linear mixed models for the actor-partner interdependence model, two different formulations are typically considered (Kenny et al., 2006). The first one takes a hierarchical view (i.e. a multilevel approach) and specifies the so-called random-intercepts model, with the random intercept capturing the correlation within a dyad:

$$Y_{ij} = \mathbf{x}_{ij}^t \boldsymbol{\beta} + b_i + \epsilon_{ij} \tag{1}$$

with $b_i \sim N(0, \tau)$ and $\epsilon_{ij} \sim N(0, \sigma^2)$. In APIM (1) $\mathbf{x}_{ij}$ is a vector of known covariates, typically including the actor's predictor variable x_{act}, the partner's predictor variable x_{par}, a distinguishing variable x_{dis} (like gender) in case of distinguishable dyads , and their interactions; $\boldsymbol{\beta}$ a vector of coefficients, called fixed effects, and b_i the random intercept. In a standard multilevel model, the assumption is made that the variance of the random effect is positive (i.e. $\tau \geq 0$).

The second formulation of the APIM takes a marginal view, which does not

[1] Throughout the manuscript we will assume distinguishable dyads but all models and estimation techniques that are presented can easily accommodate for indistinguishable dyads as well.

incorporate random effects, i.e.

$$Y_{ij} = \mathbf{x}_{ij}^t \boldsymbol{\beta} + \epsilon_{ij} \qquad\qquad j = 1, 2 \tag{2}$$

but simply models the variance-covariance in the data [2]. Model (2) assumes in its most general form that the residuals ϵ_{ij} are bivariate normally zero-mean distributed with an unstructured variance-covariance

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}.$$

Here, ρ reflects the within-dyad correlation [3] and can take any value in the interval $[-1, +1]$.

It is important to note here that *marginal models* like (2) describe so-called population averaged effects which refer to an averaging over dyads particular levels of predictors while dyad-specific models like (1) are *conditional models* that describe effects at the dyad level. However, since the marginal and conditional expectation of Y_{ij} are the same here (table 1), i.e. $E(Y_{ij}) = E(Y_{ij} \mid b_i)$, the parameters $\boldsymbol{\beta}$ in (1) and (2) share their interpretation. In other words, if on average *within dyads*, a 1-unit increase in the actor predictor x_{act} for example causes a shift of size β_1 for the actor's outcome Y (i.e. 'the conditional effect'), then this coefficient β_1 can also be interpreted as the effect on the population level, and the estimated *overall sample* means (i.e. 'the marginal effect') will also change with the same coefficient β_1 for such 1-unit increase. The marginal

[2]This method is sometimes referred to as the R-side covariance method.

[3]More precisely it measures the residual intra-cluster correlation (ICC), i.e. the correlation between the measurement of the first person of the dyad and the measurement of the second person of the dyad that is left after accounting for the predictor effects in model (2).

variance-covariance matrix V under model (1) has a compound symmetry structure with correlation equal to $\tau/(\sigma^2 + \tau)$, and so in contrast to the marginal model formulation (2), the hierarchical formulation with the restriction $\tau \geq 0$ does not allow for negative correlation within dyads. The latter can be a serious restriction in dyadic settings where negative within-dyad correlations are not uncommon and therefor formulation (2) is typically preferred above formulation (1). If one takes a marginal view on model (1) though, negative values for τ are perfectly possible (Molenberghs & Verbeke, 2011) [4]. We will further refer to the latter approach as the 'unconstrained approach' as opposed to the more standard 'constrained approach'.

2.2 Generalized linear mixed models: a conditional approach

While the APIM was considered for the Gaussian outcomes in the previous section, we now focus on modeling dyadic binary and count data. Similar to model (1) for Gaussian outcomes, we consider the logistic-normal random intercept model (Snijder & Bosker, 1999) for binary dyadic data with a logit link[5] and assume no overdispersion:

$$\text{logit}[E(Y_{ij} \mid b_i)] = \mathbf{x}_{ij}^t \boldsymbol{\beta} + b_i \quad \text{with } b_i \sim N(0, \tau), \quad \tau > 0 \tag{3}$$

[4] In such approach the conceptual interpretation of the random effect is abandoned and the hierarchical model approach merely used as a vehicle for estimation. The only restriction is that $\tau \geq -\sigma^2/2$ for V to be positive definite.

[5] Other link functions like the probit or log-log could be considered as well, but we will restrict attention to the logit-link here.

Unlike the linear mixed model (1) - the marginal interpretation of $\boldsymbol{\beta}$ is different from the conditional interpretation here (table 1), unless the random intercept variance τ equals zero. The outcomes Y_{i1} and Y_{i2} from dyad i are conditionally independent (i.e. given b_i) but are marginally nonnegatively correlated (table 1). The marginal correlation is not straightforward to calculate for Bernoulli outcomes, given the dependence of the variance on the mean. Pryseley, Tchonlafi, Verbeke and Molenberghs (2011) derive an easy-to-calculate first order approximation of the intra-cluster correlatioin (ICC) in the absence of any predictor effects,

$$\rho \approx \frac{\tau}{\tau + \exp(\beta_0)(1 + \exp(-\beta_0))^2}, \tag{4}$$

where β_0 is the intercept of model (3) (assuming centered predictors). Observe that $\rho = 0$ when $\tau = 0$ and $\rho \to 1$ as $\tau \to +\infty$.

Next, we consider the Poisson-normal random intercept model for count data with a log link and assume no overdispersion

$$\log\left[E(Y_{ij} \mid b_i)\right] = \mathbf{x}_{ij}^t \boldsymbol{\beta} + b_i \quad \text{with } b_i \sim N(0, \tau), \quad \tau > 0 \tag{5}$$

The marginal effects of the explanatory variables are the same as the dyad-specific effects in model (5); Y_{i1} and Y_{i2} are marginally non-negatively correlated when $\tau > 0$ (table 1). Here too, the variance depends on the mean, but the ICC can be approximated (Pryseley et al., 2011) by

$$\rho = \frac{\exp\left(\beta_0 + \frac{1}{2}\tau\right)(\exp\tau - 1)}{1 + \exp\left(\beta_0 + \frac{1}{2}\tau\right)(\exp\tau - 1)} \tag{6}$$

where β_0 is the intercept of model (5) (assuming again no effect of the centered predictors). Again, $\rho = 0$ when $\tau = 0$ and $\rho \to 1$ as $\tau \to +\infty$.

In summary, we have that - similar to the LMM with random intercept - the generalized linear mixed model (GLMM) with random intercept, leads to non-negative marginal correlations too when $\tau > 0$ is restricted to be positive. The similarity between marginal and conditional interpretation of the fixed effect parameters in the GLMM on the other hand depends on the link-function [6]. So far, we have only discussed the counterpart of model (1) for non-Gaussian outcomes. In the next paragraph we will see how we can get to a marginal formulation similar to (2) in the GLMM-framework.

2.3 Generalized linear mixed models: a marginal approach

Fitting GLMMs like models (3) and (5) proceeds by integrating over the random effects. Broadly speaking 3 different strategies have historically been considered to overcome the integration over the (normally) distributed random effects: (i) approximation of the integral using Gaussian quadrature, (ii) approximation of the integrand using Laplace's method, and (iii) a quasi-likelihood approach based on a linearized approximation. We refer the interested reader to Tuerlinckx, Rijmen, Verbeke and De Boeck (2006) for an in-depth review and

[6] It can be shown in general that when the conditional mean is additive in a random effect on the log scale, the marginal mean equals the conditional mean plus a constant, such that slope parameters have the same interpretation in both formulations. No further distributional assumptions are needed in this case. When a logit or probit link is used with a normal random effect, the marginal mean parameters become attenuated by a factor which depends on parameters of the distribution of the covariates. For example for the binary case the marginal effect can be approximated by $\beta/\sqrt{c^2\tau + 1}$ with β the conditional effect from the logistic-normal model and $c = 16\sqrt{3}/(15\pi)$, Molenberghs & Verbeke, 2005)

discussion of these different approximation methods. These 3 approximation techniques are available in standard software package like SAS for example. We skip the technical details of (i) and (ii) here, but elaborate a bit further on (iii) as it will allow us the specification of a marginalized GLMM. To explain the linearized approximation method, we can re-write models (3) and (5) for example as

$$Y_{ij} = h(\mathbf{x}_{ij}^t \boldsymbol{\beta} + b_i) + \epsilon_{ij}$$

with h the inverse of the logit and log function, respectively. A first order Taylor expansion around the estimated fixed effect and posterior mode of the random effect and further re-arrangements (Tuerlinck et al., 2006) lead to

$$Y_{ij} \approx \hat{\mu}_{ij} + v(\hat{\mu}_{ij})\mathbf{x}_{ij}^t(\boldsymbol{\beta} - \hat{\boldsymbol{\beta}}) + v(\hat{\mu}_{ij})(b_i - \hat{b}_i) + \epsilon_{ij} \tag{7}$$

with $v(\hat{\mu}_{ij})$ the approximate variance of the error term. It can be shown that (7) can be rewritten as a linear mixed model for pseudo data $\mathbf{Y}_i^*$ with fixed effects $\boldsymbol{\beta}$, random effects $\mathbf{b}_i$ and error terms ϵ_i^*. Therefore, estimation of $\boldsymbol{\beta}$, the fixed effect parameters, and the variance of b_i, can be obtained by iterating between updating the pseudo response and fitting the linear mixed model to the pseudo-data. This approach is referred to as penalized quasi-likelihood (PQL). The advantage of using such pseudo-likelihood approach is that it becomes possible to fit generalized linear mixed models without random effects and to take a marginal view with only residual assocation effects. We will further label this model as the marginalized GLMM. In contrast to the constrained random intercept models (3) and (5) for example, this marginalized GLMM allows -

similar to model (2) for Gaussian outcomes - to model negatively correlated non-Gaussian outcomes. Alternatively, one may take a marginal view on the random intercept models (3) and (5) and give up the constraint $\tau > 0$ ('the unconstrained' random intercept approach). By doing so, one can allow for a negatively correlated outcomes as well. In practice, it turns out that this is possible when the Laplace-approximation is used, but not under the Gaussian quadrature approximation (Pryseley et al., 2011).

3 Generalized Estimating Equations

Generalized estimating equations, as introduced by Liang and Zeger (1986), can be considered as an extension of the generalized linear model (GLM) that accommodates correlated outcome data too. It provides a general framework for the analysis of correlated continuous, ordinal, dichotomous, or count dependent data. GEE is often referred to as a marginal (or population-averaged) approach as opposed to the conditional approach exploited by multilevel models (Diggle, Heagerty, Liang, & Zeger, 2002). Whereas multilevel models explicitly specify the joint distribution of the outcomes, focus on modeling the dyad-specific expectation as a function of explanatory values, and allow one to disentangle the variability at the different levels, GEE is a moment-based method and only models the marginal expectations as a function of explanatory variables.

The GEE fitting algorithm can be described in 4 different steps (Ghisletta & Spini, 2004).

1. A GLM is fitted assuming independence between observations. This GLM requires the specification of a link function that describes the linear relationship between the expected outcome and its predictors (for example the identity link for Gaussian data, the logit link for dichotomous or ordinal data and the log link for count data) and of the relationship between the mean μ and the variance, denoted $v(\mu)$.

2. Standardized residuals, contrasting the observed and expected (model-based) outcome, are calculated. Based on an assumed structure of the correlation matrix (such as independence or unstructured), a "working" correlation matrix C that characterizes the correlations among observations within dyads is computed using these standardized residuals. We suggest to use the unstructured working correlation structure here. [7]

3. An estimate of the covariance parameters is obtained from the assumed mean-variance association $v(\mu)$ and the working correlation matrix C.

4. Given the covariance estimate obtained in step 3, a set of estimating equations for the regression coefficients is solved [8].

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[7] An unstructured covariance matrix is no guarantee for a correct specification since the covariance structure may further depend, for example, on certain covariates. Assuming independent observations within dyads on the other hand, and hence the choice for an independence working correlation matrix, may lead to some small gain in efficiency in estimating the actor- and partner-effects provided the independence assumption truly holds.

[8] The ith dyad contributes a three-way product involving the partial derivative of μ_i with respect to the regression parameter, times the inverse of the dyad's variance-covariance matrix, times the difference between the dyad's responses and their mean.

5. The steps 2 to 4 result in an iterative scheme that switches between estimating the regression coefficients for fixed values of the covariance parameters, and estimating the covariance given the regression coefficients, and is continued until convergence occurs.

This scheme yields consistent estimators for the regression coefficients even if the correlational structure in step 2 was misspecified (but provided the linear relationship is correctly specified). These estimators are asymptotically multivariate normally distributed with a covariance matrix that can be consistently estimated (also in case the correlational structure was misspecified) by a so called sandwich estimate (resulting in the "robust standard errors").

As the GLMM, the GEE-approach can easily deal with a wide range of outcome types like binary, categorical, count, or interval data. Unlike the constrained GLMM with a random intercept though, the correlation of outcomes within a dyad is not restricted to be positive. The GEE-approach does not make full distributional assumptions (only the mean-variance relationship), and no likelihood-based methods as in the GLMM can be used for testing actor and partner effects for example. Instead, parameter testing can be based on Wald statistics constructed with the asymptotic normality of the estimators together with their estimated covariance matrix. A criticism often made is that the sandwich variance estimate of GEE may underestimate the variability in the parameter estimates when the number of clusters (dyads in this particular case) is small (McCaffrey & Bell, 2006), resulting in tests that have greater than nominal type 1 error rates. Rotnitzky and Jewell (1990) describe an alternative procedure for testing effects of predictors, the so-called 'score test'. The test statistic for this score test is based on the generalized estimating 'score-like' equations [9] that are solved to produce parameter estimates for the GEE model. Finally, while GLMMs explicitly specify the correlation, the 'unstructured' working correlation (as suggested in step 2) in the GEE-approach is only a device to support estimation of the regression parameters, and no standard errors are given along these working correlations. The resulting correlations should therefore only be interpreted informally (Molenberghs & Verbeke, 2005)[10].

4 Simulations

In this section we compare the performance of 5 different approaches to the estimation of actor and partner effects in the APIM-model and the estimation of the within-dyad correlation for Bernoulli or Poisson dyadic outcomes, which are either positively or negatively correlated, with GEEs or GLMMs:

(1) a GEE-approach with p-values for tests of fixed effect parameters based on a robust Wald test, and using an unstructured working correlation matrix;

(2) the same GEE-approach as in (1) but with p-values based on the score

[9] Loosely speaking these score like equations are of similar form as the score equations derived for GLM, and the principle of the GEE score test is the same as the likelihood-based score test.

[10] When the association structure is of primary interest, one should turn to some extensions of GEE. Examples of the latter are second-order extensions of GEE (GEE2) that include the marginal pairwise association as well, or alternating logistic regressions that use conditional probability ideas (Molenberghs & Verbeke, 2005).

test;

(3) a GLMM with a random intercept (RI), a constrained variance component ($\tau > 0$), and computation based on adaptive Gaussian quadrature;

(4) a GLMM with a random intercept (RI), an unconstrained variance component, and computation based on Laplace approximation;

(5) a marginalized GLMM, and computation based on linearized approximation (pseudo likelihood methods).

All simulations were performed in SAS version 9.2 and used the GENMOD procedure for (1) and (2) (with TYPE3 option in the MODEL statement for the latter), the NLMIXED procedure for (3) (with default method= adaptive gaussian adaptive quadrature), and the GLIMMIX procedure for (4) and (5) (with method=LAPLACE and option NOBOUND for (4), and method=RSPL for (5)). A literature review of studies using the APIM revealed that sample sizes typically ranged from 30 to 300 dyads (first quartile=60, median=100 and third quartile=150) [11], but even dyadic sample sizes as small as 12 were recently reported (Tambling, Johnson, & Johnson, 2012). We therefore considered number of dyads equal to 10, 30, 60, 100, 150 or 300 in the simulation study. Results from each simulation settting are based on 2000 repetitions. It should be noted though that in case of convergence issues for a particular estimation method, estimates were not included for that approach. Such non-convergence occurred

[11] Special thanks to Robert Wickham from the university of Houston for sharing his database on the use of the APIM.

in about 15% of the cases for the marginalized GLMM when the ICC was positive or negative, and for the constrained GLMM with RI when the ICC was negative (both for small and large samples).

4.1 Correlated Bernoulli outcomes

For the simulation settings with a positive intra-cluster correlation, responses Y_{ij} were generated from a Bernoulli distribution with probability p_{ij} following the APIM with fixed effects for the actor and partner's predictor and a distinguishing variable, and a random intercept

$$\text{logit}(p_{ij} \mid b_i) = \beta_0 + \beta_{act}x_{act,ij} + \beta_{par}x_{par,ij} + \beta_{dis}x_{dis,ij} + b_i, \quad j = 1, 2, \quad (8)$$

with $x_{dis,ij}$ coded as 1 if $j = 1$ and -1 if $j = 2$, actor and partner predictors x_{act} and x_{par} generated from a standard bivariate normal distribution with correlation 0.50 [12]. We set $\beta_0, \beta_{act}, \beta_{par}$ and β_{dis} equal to zero, while values of τ were chosen such that the intra-cluster correlations (ICCs) were approximately equal to 0.30, 0.15, or 0.05.

For the simulation setting with a negative intra-cluster correlation, we rely on Leisch, Weingessel and Hornik (1988) who show how to simulate multivariate binary distributions with a given correlation structure from a multivariate normal distribution. By dichotomizing the normal variates and the appropriate choice of the correlation between normal variates, one can obtain the required marginal and pairwise probabilities. We generated binary dyad data with marginal prob-

[12] Smaller correlations between predictors were considered in this setting as well in all settings described further, but did not reveal major differences from the results presented.

abilities 0.5 and 0.5 (i.e., no actor and partner effect of the standard bivariate normal distributed x_{act} and x_{par} with correlation 0.50, and no effect of the distinguishing variable x_{dis}) for Y_{i1} and Y_{i2} and joint probability 0.175, 0.2125 and 0.2375, leading to ICCs of -0.3, -0.15 and -0.05, respectively.

For the GEE-approaches and marginalized GLMM-approach, the following working model is assumed:

$$\text{logit}(p_{ij}) = \beta_0 + \beta_{act}x_{act,ij} + \beta_{par}x_{par,ij} + \beta_{dis}x_{dis,ij}, \quad j = 1, 2, \tag{9}$$

while for the GLMM-approaches with random intercept, model (8) is assumed.

To ensure equal marginal and conditional parameter effects, data were first generated under the null hypothesis of no actor and no partner effect. By doing so, the size of the test of $\beta_{act} = 0$ (β_{par}=0, respectively) at the nominal 5% level under each of the 5 approaches can easily be assessed (note that with 2000 simulations, the standard error on the estimated size is about 0.5%, and empirical type 1 errors for appropriate tests are therefor expected to lie between 4% and 6%). Empirical type 1 errors for the test of no actor effect are presented in figure 3 (results for the test of no partner effect were very similar). While under the GEE-approach, the robust Wald test tends to be too liberal when the numbers of dyads is extremely small, the performance of the score test is satisfactory under all settings considered (slightly conservative for small number of dyads in some cases). Both the constrained and unconstrained random-intercept model yield a too conservative test under positive ICC settings when the number of dyads is small. With increasing negative values of the ICC, the constrained random intercept model (which will then typically force the random intercept variance to be zero) yields a much too conservative test. The unconstrained random-intercept approach jumps from too conservative tests for small samples to too liberal tests for larger samples when the ICC is negative. The marginalized GLMM-approach performs relatively well in terms of type 1 error, both under positive and negative ICC scenarios, except when the number of dyads is small. Overall, we conclude that under the null the marginal approaches perform better than the conditional approaches, and proceed for now with the former only to explore the performance in estimating the residual ICC. The upper panel of figure 5 shows the median of the estimated ICC under both marginal approaches for the 6 values we considered for the ICC. Although the 'standard' GEE-approach does not formally aim to estimate the ICC, its estimate obtained from the unstructured working correlation is very informative and recovering the ICC well. In contrast, there is - regardless of the sample size - indication of a serious negative bias for the ICC estimate from the marginalized GLMM for increasing absolute values of the ICC.

Next data were generated following model (8) assuming an effect of x_{act} and x_{par} ($\beta_1 = \log 1.5 \approx 0.405$ and $\beta_2 = \log 0.75 \approx -0.287$, respectively). Note that the marginal effect of x_{act} and x_{par} have no longer the same value, but can be approximated by $\beta_1/\sqrt{c^2\tau + 1}$ and $\beta_2/\sqrt{c^2\tau + 1}$ respectively, with $c = 16\sqrt{3}/(15\pi)$ (Molenberghs & Verbeke, 2005). Because of the approximation techniques that are used for GLMMs, estimates of non-zero fixed effect in the GLMMs are known to be frequently biased [13]. The upper left panel of figure 6

[13] Breslow and Lin (1995) studied the 'worst case' scenario of binary responses in a matched-

presents the mean of the estimated actor and partner effects for the scenario where the ICC equals 0.15. We found no evidence of severe bias for either the marginal or conditional effects using the 5 approaches, except when the sample size is extremely small. The upper middle panel of figure 6 shows the power to detect the actor effect at the nominal 5% significance level. Not surpisingly we find the GEE-Wald test to have highest power at lower sample sizes (as the test was seen to be too liberal). The robust score test from the GEE-approach is performing well as compared to the multilevel approaches. Finally, to shed some light on the performance of the approximation formula (4) for the ICC in the random intercept model, the estimated within-dyad correlations under the 5 approaches are presented in the upper right panel of figure 6, illustrating once more the excellent performance of the GEE-approach in recovering the ICC.

4.2 Correlated Poisson outcomes

For the simulation settings with a positive ICC, responses Y_{ij} were generated from a Poisson distribution with mean μ_{ij} following the APIM-model with random-intercept

$$\log(\mu_{ij} \mid b_i) = \beta_0 + \beta_{act}x_{act,ij} + \beta_{par}x_{par,ij} + \beta_{dis}x_{dis,ij} + b_i, \quad j = 1, 2 \quad (10)$$

with $\beta_0 = \beta_{act} = \beta_{par} = \beta_{dis} = 0$, $b_i \sim N(0, \tau)$ and x_{act}, x_{par} and x_{dis} as before. The number of dyads i considered was again 10, 30, 60, 100, 150 or 300.

pairs design and found the asymptotic bias in the pseudo-likelihood estimator of β to be of the order of $\mid \tau \mid$. The bias for the Laplace estimator is of smaller order, while adaptive quadrature leads to nearly unbiased estimated (Pinheiro and Chao, 2006).

Values of τ were chosen such that ICCs approximately equal to 0.30, 0.15, or 0.05 were obtained.

For the simulation setting with a negative ICC, we extend the approach of Leisch et al. (1988) and show how to simulate multivariate Poisson distributions with a given correlation structure. We first generate samples from a bivariate standard normal distribution with correlation ρ_N. Whereas before the Gaussian random variables where dichotomized to yield binary events of 0 or 1, they will now be discretized into M different states to yield counts of $0, 1, 2, \ldots, M$. Precisely, we want to generate counts Y_{ij} that have count probabilities $\Pr(Y_{ij} = k) = p_{ijk}$. Samples are generated by discretizing a 2-dimensional normal random variable U by setting $Y_{ij} = k$ if $\gamma_{ij,k} < U_{ij} \leq \gamma_{ij,k+1}$, with $\gamma_{ij,k} = \Phi^{-1}(\Pr(Y_{ij} < k))$ for each $k = 1, 2, \ldots, M$. It can be shown that the value of ρ_N is uniquely determined by the value of the desired correlation between Poisson outcomes. We generated bivariate Poisson outcomes with marginal means equal to 2 (i.e., no effect of x_{act}, x_{par} and x_{dis}) for Y_{i1} and Y_{i2} and pairwise correlation equal to -0.30, -0.15 and -0.05, respectively.

For the GEE-approaches and marginalized GLMM-approach, the following working model was assumed

$$\log(\mu_{ij}) = \beta_0 + \beta_{act} x_{act,ij} + \beta_{par} x_{par,ij} + \beta_{dis} x_{dis,ij}, \quad j = 1, 2, \tag{11}$$

while for the GLMM-approaches with random intercept, model (10) was assumed.

As data were generated first under the null hypothesis of no effect of X here too, we can again assess the size of the test of $\beta_{act} = 0$ ($\beta_{par} = 0$, respectively) at the nominal 5% level. The empirical sizes for the test of no actor effect are presented in figure 4. While under the GEE-approach, the robust Wald test is far too liberal when the numbers of dyads is small, even more pronounced than in the Bernoulli setting, the performance of the score test is satisfactory under all settings considered, except for some conservatism in very small samples. The constrained (and to a smaller extent the unconstrained) random-intercept model yield a too conservative test under positive ICC settings when the number of dyads is small. With increasing negative values of the ICC, the constrained random-intercept model, which will then typically force the random intercept variance to be zero, yields a way too liberal test. The unconstrained random-intercept approach again jumps from too conservative tests for small samples to too liberal tests for larger samples when the ICC is negative. The marginalized GLMM-approach tends to perform well in all settings (except for extremely small sample sizes). Interestingly when comparing the marginal approaches in their performance to estimate the ICC, we observe similar findings as for the Bernoulli outcomes (lower panel of figure 5).

Next data were generated following model (10) assuming an effect of x_{act} and x_{par} ($\beta_1 = \log 1.25 \approx 0.223$ and $\beta_2 = \log 0.85 \approx -0.163$, respectively). As derived before, the marginal effect of x_{act} and x_{par} are the same as the conditional effects for this setting. The lower left panel of figure 6 presents the mean of the estimated actor and partner effects for the scenario where the ICC equals 0.30. We found no evidence of any bias for any of the 5 approaches, except when the sample size is extremely small. The lower middle panel of figure 6 shows the power to detect the actor effect at the nominal 5% significance level. Not surpisingly we find again the GEE-Wald test to have highest power at lower sample sizes (as the test was seen to be too liberal). In contrast to the setting with Bernoulli outcomes, the robust score test from the GEE-approach is performing slightly worse now in terms of power as compared to the multilevel approaches. The ICC is again well recovered from the working correlation in the GEE-approach (the lower right panel of figure 6), better than by the approximation (6) or the marginalized GLMM.

5 Examples

The two studies presented below are sub-samples of the Interdisciplinary Project for the Optimization of Separation Trajectories conducted in Flanders (IPOS; www.scheidingsonderzoek.be), which is a cooperation of psychologists, lawyers, and economists from Ghent University and the University of Leuven. This research project carried out a large-scale recruitment of formerly married partners. All couples who divorced between March 2008 and March 2009 in four major courts in Flanders were systematically approached in the waiting room to participate in a study on divorce (N = 8896). The individual respondents (i.e., not both ex-partners) willing to participate (N = 3921; response rate = 44.1%) were subsequently contacted for an interview in view of a computerized survey. To reduce the survey's length and lessen the burden on the respondents, the survey was divided into a basic intake assessment assigned to each respondent, and three different questionnaire packages (measuring emotions, parent-child relationships, or ex-partner relationships) which were randomly distributed among the participants. As the recruitment strategy did not directly target the ex-partners simultaneously, only dyadic data from about 30 ex-couples were of part of the same sample for each of the questionnaire packages. Therefore results presented below should merely be seen as an illustration of the different approaches.

5.1 Correlated binary data: forcing behavior or not during negotiations in ex-couples

The first example explores the effect of feeling guilty on negotation behavior. Negotiation behavior was assessed with the Dutch Test for Conflict Handling (DUTCH, De Dreu, Evers, Beersma, Kluwer, & Nauta, 2001). One of the subsscales of the DUTCH measures forcing behavior (e.g., "I fight for a good outcome for myself."), measured with 4 items to be answered on a five-point Likert scale from totally disagree (1) to totally agree (5). For illustration purposes, participants with an average score higher than 3 were artificially classified here as showing forcing behavior (denoted as $Y = 1$). Out of the 29 ex-couples in total, there was 1 couple where both ex-partners showed forcing behavior, 6 cases where only the male partner showed forcing behavior, 9 cases where only the female partner showed forcing behavior and 13 couples were none of the partners showed forcing behavior. Guilt was assessed with the Guilt in Separation Scale (Wietzker, Buysse, Loeys, & Brondeel, 2012) and is computed as the mean of 10 items (e.g. "I am responsible for his/her misery."), measured on a seven-point Likert scale (from 1=never to 7=always). Throughout the analysis we will use the mean values of guilt, with the person's own score denoted as GUILTA and his or her partner's score as GUILTP. In addition, we use gender as the distinguishing variable in the couple, denoted as SEX and effect coded as 1 for men and -1 for women.

We used both marginal approaches and conditional approaches to explore the impact of feeling guilty on forcing behavior. For the GEE-approach and the marginalized multilevel approach, we specify the following linear relation on the logit scale between showing forcing behavior and feeling guilty (as there was no evidence of gender-specific actor or partner effects no additional interactions are considered):

$$\text{logit}[E(Y_{ij})] = \beta_0 + \beta_1 * GUILTA_{ij} + \beta_2 * GUILTP_{ij} + \beta_3 * SEX_{ij}. \quad (12)$$

For the conditional multilevel approach, we consider the following random-intercepts model

$$\text{logit}[E(Y_{ij} \mid b_i)] = \beta_0 + \beta_1 * GUILTA_{ij} + \beta_2 * GUILTP_{ij} + \beta_3 * SEX_{ij} + b_i, \quad (13)$$

with $b_i \sim N(0, \tau)$ and τ either constrained (using adaptive Gaussian quadrature for optimization) or not (using Laplace approximation for optimization). Results are presented in Table 2 (corresponding SAS-code can be found in Appendix A1). The following trends are observed: feeling guilty is associated with a decrease of the forcing behavior, while guilt emotions of the partner have a reverse effect. Overall, males show less forcing behavior than females. We repeat the different interpretation of the conditional and marginal models here. For example, from the constrained random intercept approach, we estimate that within a dyad, a one-unit increase in the guilty score of the partner, corresponds to an increase of exp(0.69) of the odds of showing forcing behavior. Marginally, we estimate with the GEE-approach that such increase is associated with an increase of exp(0.58) of that odds in the sample of ex-couples. It's worth noting here that the estimated actor and partner effects under the marginalized multilevel approach are substantially different from the marginal effects under the GEE-approach, as are the significance of the effects. This might be attributed to poor convergence of the GLMM for these particular data. The estimated intra-dyad correlation from the working GEE-correlation matrix equals -0.18. Given this indication of negative correlation in forcing behavior between ex-partners, it is therefore not surprising that the constrained random-intercept model (13) resulted in an estimated zero random effect variance, and some conservatism in the estimated standard errors of the predictors.

5.2 Correlated count data: the number of unwanted pursuit behaviors in ex-couples

In the second example, we focus on a sample of 33 ex-couples who responded to an adapted version of the Relational Pursuit-Pursuer Short Form (RP-PSF; Cupach & Spitzberg, 2004) used to assess the extent of UPB-perpetrations towards the ex-partner since the break-up. The total of 28 RP-PSF items (ranging from 'leaving unwanted gifts' to 'threatening to hurt yourself'), each measured on a

5-point Likert scale (from 0=never to 4=over 5 times), was used as an overall index of perpetration (with higher scores indicating higher levels of perpetrations). A participant who answered 'never' to all these 28 UPB-items will have an UPB-outcome equal to 0; a participant who answered 'over 5 times' to 'leaving unwanted gifts' and 'never' to all other items will have an UPB-total equal to 4 for example; while a participant who answered 'over 5 times' to all items will have the maximum score of 140. While many predictors for the UPB-outcome were measured, we limit our attention here to the impact of the actor's and partner's level of anxious attachment in their relationship with their ex-partner before the break-up, which was measured using a total of five anxious attachment items (e.g., 'My desire to be very close sometimes scared my ex-partner away') from an adapted Experience in Close Relationships Scale-Short form (ECR-S; Wei, Russell, Mallinckrodt, & Vogel, 2007). Throughout the analysis we will use the mean-centered values of anxious attachment, with the person's own score denoted as ANXA and his or her partner's score as ANXP.

Figure 2 shows the right-skewed distribution of the observed number of UPB-perpetrations. Such count data are frequently modeled using the Poisson distribution, but the corresponding predicted frequencies in Figure 2 clearly reveal lack-of-fit here. The negative binomial distribution, relaxing the Poisson-assumption of equality of the mean and the variance, yields a much better fit, and will further be assumed.

As in example 1, we used both marginal and conditional approaches to explore the impact of anxious attachment on the number of UPBs. For the GEE-approach and the marginalized multilevel-approach, we specify the following linear relation on the logarithmic scale between the expected number of UPBs and its predictors:

$$\begin{aligned}\log[E(UPB_{ij})] &= \beta_0 + \beta_1 * ANXA_{ij} + \beta_2 * ANXP_{ij} + \beta_3 * SEX_{ij} \\ &\quad + \beta_4 * ANXP_{ij} * SEX_{ij} + \beta_5 * ANXP_{ij} * SEX_{ij}, \end{aligned} \tag{14}$$

while for the conditional multilevel-approach, we consider the following a random intercept model

$$\begin{aligned}\log[E(UPB_{ij} \mid b_i)] &= \beta_0 + \beta_1 * ANXA_{ij} + \beta_2 * ANXP_{ij} + \beta_3 * SEX_{ij} \quad (15) \\ &\quad + \beta_4 * ANXP_{ij} * SEX_{ij} + \beta_5 * ANXP_{ij} * SEX_{ij} + b_i, \end{aligned}$$

with $b_i \sim N(0, \tau)$. Estimated parameters under different estimation methods are presented in Table 3 (the corresponding SAS code can be found in Appendix A2).The estimated ICC from the working correlation in GEE equals 0.07. Because of the linearity of the random effect on the log-scale, the conditional and marginal approaches lead to the same interpretation of parameters. Differences between the 5 approaches are smaller than in example 1 (except again for the marginalized multilevel approach). Both for male and female actors, we observe an increase in the expected number of UPBs for increasing levels of the anxious attachment level of the actor. In contrast, while increasing anxious attachment levels of the male partner before the break-up is associated with an increase the number of UPBs in females, the reverse trend is observed in men.

6 Discussion

While linear mixed models have frequently been used to model Gaussian dyadic outcomes, we have shown in this paper that generalized linear mixed models might not be the best option to model non-Gaussian dyadic outcomes. This becomes especially true when the correlation between outcomes in a dyad is negative and/or the sample size is small. We explored the performance of different estimation techniques within the GLMM-framework, along with their potential to allow for negative ICCs, but found none of these to be overall satisfactory. While the marginalized GLMM performed relatively well with respect to estimating actor and partner effects in settings with negative and positive within-dyad correlation, the latter is poorly estimated under such approach. The GEE-approach, which is relatively unused within the social sciences, offers an interesting alternative in this context. The robust Wald test of GEE turned out to perform well, except for (extremely) small samples where the score test can be used instead. Although the GEE-approach treats the within-dyad correlation as nuisance, we found that its estimate from the unstructured working correlation can still be informative. If formal inference about the ICC is needed though, GEE-extensions allowing for this are available (Molenberghs & Verbeke, 2005), but these are less commonly available in standard software packages.

Besides the LMM-framework, the structural equation modeling (SEM) framework is frequently used to analyze Gaussian dyadic data too (Newsom, 2002). In contrast to the LMM-framework the SEM-framework easily allows for formal tests of goodness of fit [14], for mediational models (see Ledermann, Macho, & Kenny, 2011, for mediation in dyadic data) and for latent variables. When dealing with binary or ordinal response scales, SEM assumes that these data represent categorizations of underlying continuous variables. The relationships of these underlying continuous variables are captured in a polychoric correlation matrix, and (robust) weighted least square estimation could be used for the parameters of the marginal model matching the GEE-model. Such approach will be theoretically reasonable only in some cases. While for attitude items, the researcher may be more interested in the relationships among the continuous underlying latent variables than in the relationship beween the observed 'agree' and 'strongly agree' responses on the items; it may be difficult for other variables like current drug user ('yes' or 'no') to conceive them as realizations of an underlying continuous variable. Moreover for count outcomes such approach does not work either. Interestingly, Muthén, du Toit and Spisic (1997) compared the performance of robust WLS and (the second order extension of) GEE for binary outcomes in a longitudinal simulation setting, and found superior behavior of GEE in settings with 200 or 400 observation units, especially when the prevalence of the outcome is small, but more comparable behavior in larger samples that are less frequently seen in dyadic context though.

On the other hand the flexibility of SEM to deal with latent variables should not be neglected. Within the dyadic modeling world this might not only be an

[14] It should be noted that using SEM to estimate the APIM with distinguishable dyads allowing for 'gender-specific' actor and 'gender-specific' partner effects is a saturated model and so it has zero degrees of freedom and no measures of fit can be computed.

important asset for the Actor Partner Interdependence Model discussed here, but even more so for estimation in the Common Fate Model (right panel of figure 1). The latter is indeed most easily seen as a latent variable dyadic model, and SEM the most natural framework to disentangle between variability at the dyadic and at the subject level. While Gonzalez and Griffin (2002) showed how the CFM with distinguishable dyads can be casted within the multilevel framework too with the common-fate variables conceptualized as random intercepts, the CFM can not be tackled with the marginal approach taken by GEE. Still, given its ease of implementation for a large range of outcome types, this paper has shown the merits of the GEE approach in the wide range of typicale dyadic sample sizes. Indeed, by expanding the types of data that can easily be analyzed with the APIM and its straightforward allowance for both positive and negative within-dyad correlations, GEE can add significantly to the toolbox of relationship researchers everywhere.

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Appendix A1.

```
* GEE with binary outcome FORCING2*;

proc genmod data=couple1 descending;

class GENDER ID;

model FORCING2=GUILT_A GUILT_P SEX/D=binomial link=logit type3;

repeated subject=ID/type=un withinsubject=GENDER corrw;

run;


* constrained random intercept *;

proc nlmixed data=couple1;

parms beta0=-1.01 beta1=-0.61 beta2=0.59 beta3=-0.85 s2u=1;

eta=beta0+beta1*GUILT_A+beta2*GUILT_P+beta3*SEX+u;

mu=exp(eta)/(1+exp(eta));

model FORCING2~binary(mu);

random u~normal(0,s2u) subject=ID;

run;


* unconstrained random intercept *;

proc glimmix data=couple1 method=laplace nobound;

model FORCING2=GUILT_A GUILT_P SEX/dist=bin link=logit s;

random intercept/subject=ID;

run;


* marginalized multilevel model *;

proc glimmix data=couple1 method=RSPL;

model FORCING2=GUILT_A GUILT_P SEX/dist=bin link=logit s;

random _residual_/subject=ID type=un VCORR;

run;
```

Appendix A2.

```
* GEE with count outcome UPB *;
proc genmod data=couple2;
class ID GENDER;
model UPB=ANX_A ANX_P SEX SEX*ANX_A SEX*ANX_P/D=nb link=log TYPE3;
REPEATED SUBJECT=ID/type=UN withinsubject=GENDER corrw;
run;

* constrained random intercept *;
proc nlmixed data=couple2;
      parms b0=0, b1=0, b2=0, b3=0, b4=0, b5=0, k=4,s2u=0.1;
      linp =b0+b1*ANX_A+b2*ANX_P+b3*SEX+b4*SEX*ANX_A+b5*SEX*ANX_P+u;
      mu = exp(linp);
      p  = 1/(1+mu*k);
      model UPB ~ negbin(1/k,p);
      random u~normal(0,s2u) subject=ID;
run;

* unconstrained random intercept *;
proc glimmix data=couple2 method=laplace nobound;
     model UPB = ANX_A ANX_P SEX SEX*ANX_A SEX*ANX_P / dist=negbin s;
 random intercept/subject=ID;
run;

* marginalized multilevel model *;
proc glimmix data=couple2 method=RSPL;
     model UPB = ANX_A ANX_P SEX SEX*ANX_A SEX*ANX_P / dist=negbin s;
     random _residual_/subject=ID type=un VCORR;
run;
```

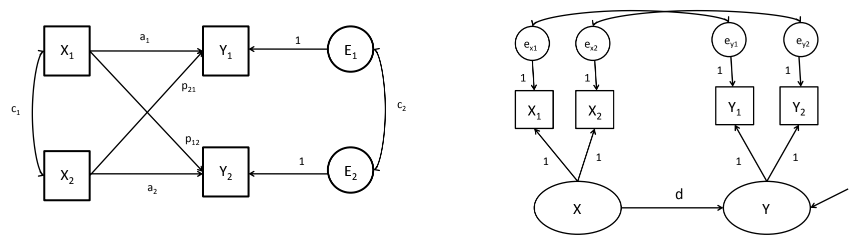

Figure 1: Left panel: The Actor Partner Interdependence Model for distinguishable dyads where a is the actor effect and p is the partner effect. Right panel: The Common Fate Model where d is direct effect.

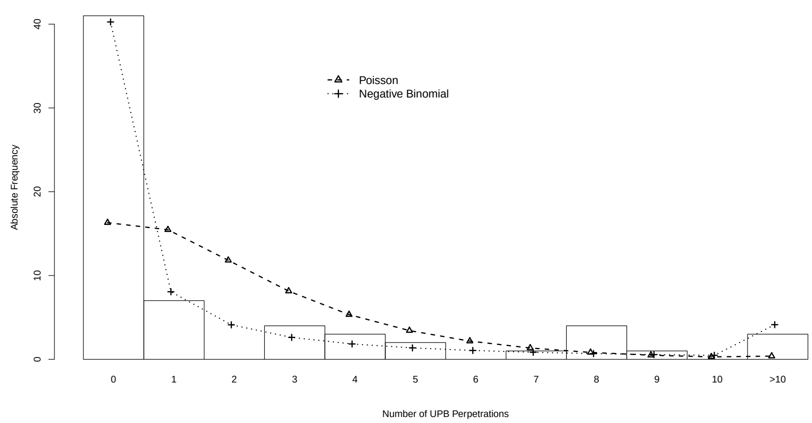

Figure 2: The observed distribution of the number of unwanted pursuit behaviors in the 33 ex-couples (with the expected distribution under a Poisson and negative binomial distribution).

outcome	Gaussian	Bernoulli	Poisson
link	identitiy	logistic	logarithm
$E(Y_{ij})$	$\mathbf{x}_{ij}^t\boldsymbol{\beta}$	$E\left[\frac{\exp(\mathbf{x}_{ij}^t\boldsymbol{\beta}+b_i)}{1+\exp(\mathbf{x}_{ij}^t\boldsymbol{\beta}+b_i)}\right]$ $\neq \frac{\exp(\mathbf{x}_{ij}^t\boldsymbol{\beta})}{1+\exp(\mathbf{x}_{ij}^t\boldsymbol{\beta})}$	$\exp(\mathbf{x}_{ij}^t\boldsymbol{\beta}+\tau/2)$
$\mathrm{covar}(Y_{i1},Y_{i2})$	τ	$\mathrm{cov}\left[\mathrm{expit}(\mathbf{x}_{i1}^t\boldsymbol{\beta}+b_i), \mathrm{expit}(\mathbf{x}_{i2}^t\boldsymbol{\beta}+b_i)\right]$	$\exp(\mathbf{x}_{i1}^t+\mathbf{x}_{i2}^t)\boldsymbol{\beta}\ \left[\exp(\tau)\left(\exp(\tau)-1\right)\right]$

Table 1: Random intercept models (with random intercept $b_i \sim N(0,\tau)$): marginal mean and covariance.

	GEE		Multilevel		
	WALD	SCORE	RI CONSTR.	RI UNCONSTR.	MARGINALIZED
GUIA	-0.61 (0.38)		-0.74 (0.38)	-0.53 (0.30)	-2.20 (1.02)
	p=0.103	p=0.043	p=0.058	p=0.093	p=0.671
GUIP	0.58 (0.27)		0.69 (0.38)	0.73 (0.30)	0.82 (0.44)
	p=0.030	p=0.032	p=0.076	p=0.022	p=0.074
SEX	-0.85 (0.41)		-1.11 (0.65)	-0.67 (0.40)	-1.65 (3.20)
	p=0.039	p=0.034	p=0.100	p=0.104	p=0.609

Table 2: Example 1: the effect of feeling guilty on forcing behavior: comparison of 5 estimation/modeling methods. Estimated parameters (with standard errors) and corresponding p-values are presented

	GEE		Multilevel		
	WALD	SCORE	RI CONSTR.	RI UNCONSTR.	MARGINALIZED
ANXA	0.122 (0.021)		0.123 (0.042)	0.125 (0.039)	0.181 (0.032)
	$p \leq 0.001$	p=0.005	p=0.007	p=0.004	$p \leq 0.001$
ANXP	-0.035 (0.022)		-0.042 (0.044)	-0.039 (0.041)	-0.085 (0.032)
	p=0.120	p=0.298	p=0.350	p=0.343	p=0.012
SEX	-0.510 (0.226)		-0.540 (0.293)	-0.549 (0.275)	-0.706 (0.218)
	p=0.024	p=0.062	p=0.075	p=0.055	p=0.003
SEX*ANXA	0.041 (0.018)		0.052 (0.043)	0.036 (0.039)	0.093 (0.033)
	p=0.027	p=0.121	p=0.234	p=0.368	p=0.008
SEX*ANXP	-0.074 (0.024)		-0.074 (0.042)	-0.075 (0.041)	-0.135 (0.033)
	p=0.002	p=0.048	p=0.091	p=0.076	$p \leq 0.001$

Table 3: Example 2: the effect of anxious attachment on unwanted pursuit behavior: comparison of 5 estimation/modeling methods. Estimated parameters (with standard errors) and corresponding p-values are presented.

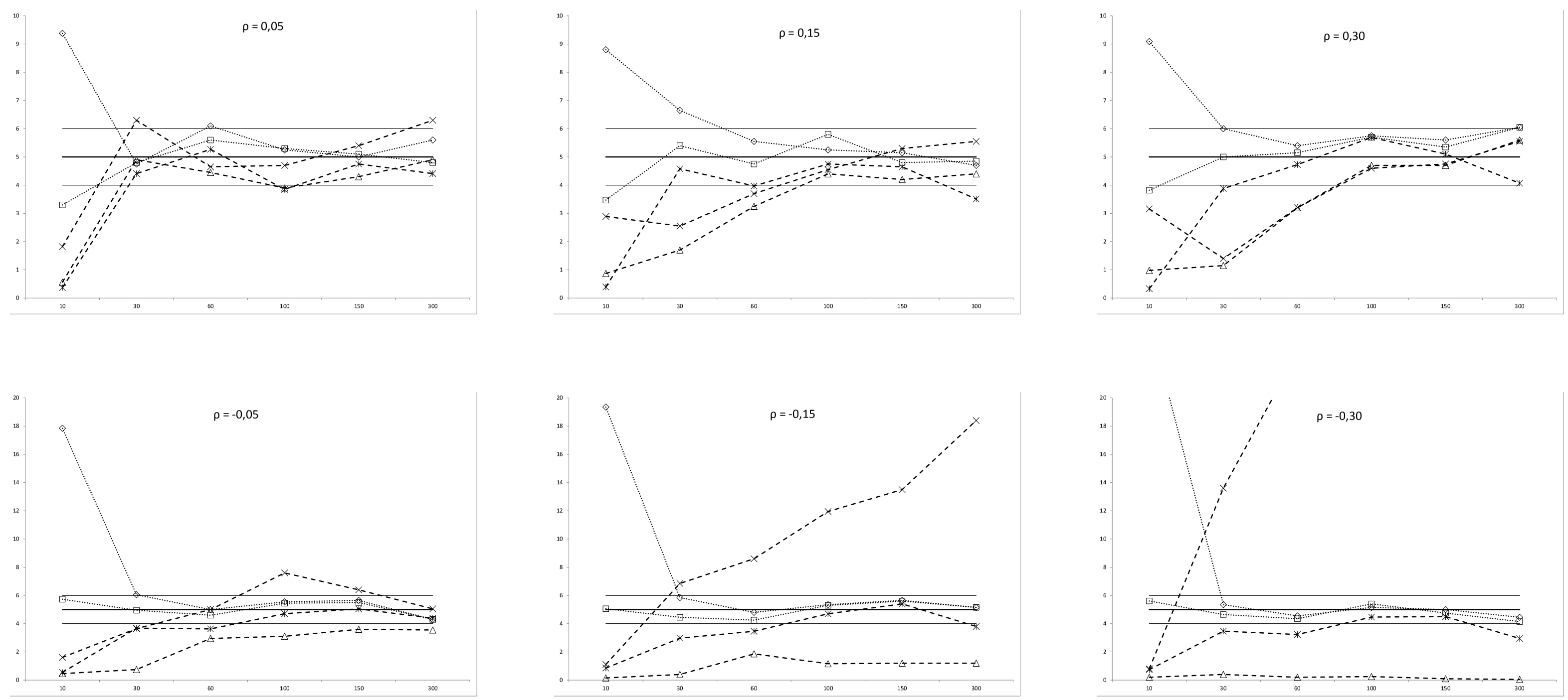

Figure 3: Performance under the APIM-model for binary outcome under the null: empirical type 1 error of the test for an actor effect at the nominal 5% level of the GEE-Wald ($\diamond$), GEE-Score ($\square$), Multilevel Constrained Random Intercept ($\triangle$), Multilevel Unconstrained Random Intercept ($\times$) and Multilevel Marginalized ($*$).

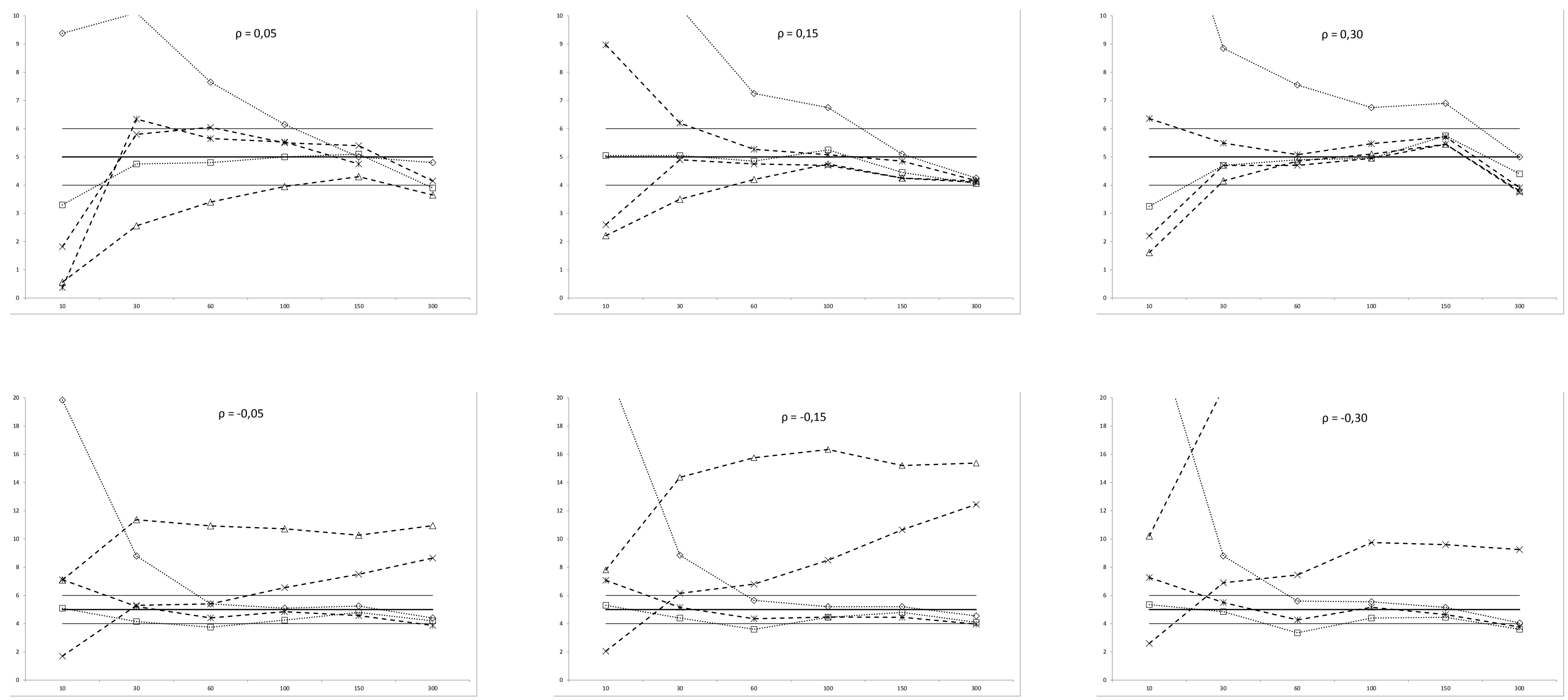

Figure 4: Performance under the APIM-model for count outcome under the null: empirical type 1 error of the test for an actor effect at the nominal 5% level of the GEE-Wald ($\diamond$), GEE-Score ($\square$), Multilevel Constrained Random Intercept ($\triangle$), Multilevel Unconstrained Random Intercept ($\times$) and Multilevel Marginalized ($*$).

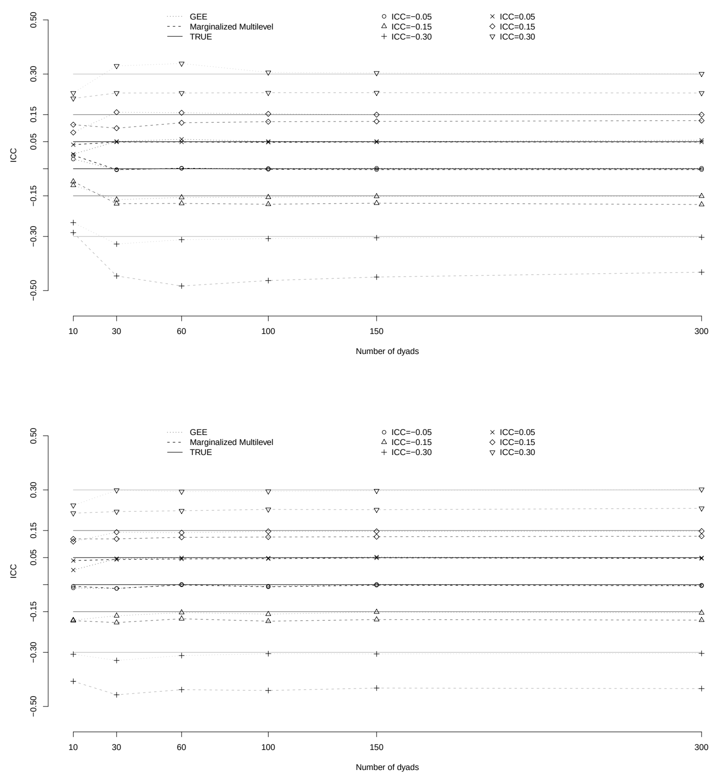

Figure 5: Estimation of the ICC (true ICC=solid line) in the APIM based on the working correlation under the GEE-approach (dotted line) and on the correlation of the pseudo-residuals under the marginalized multilevel approach (dashed line).

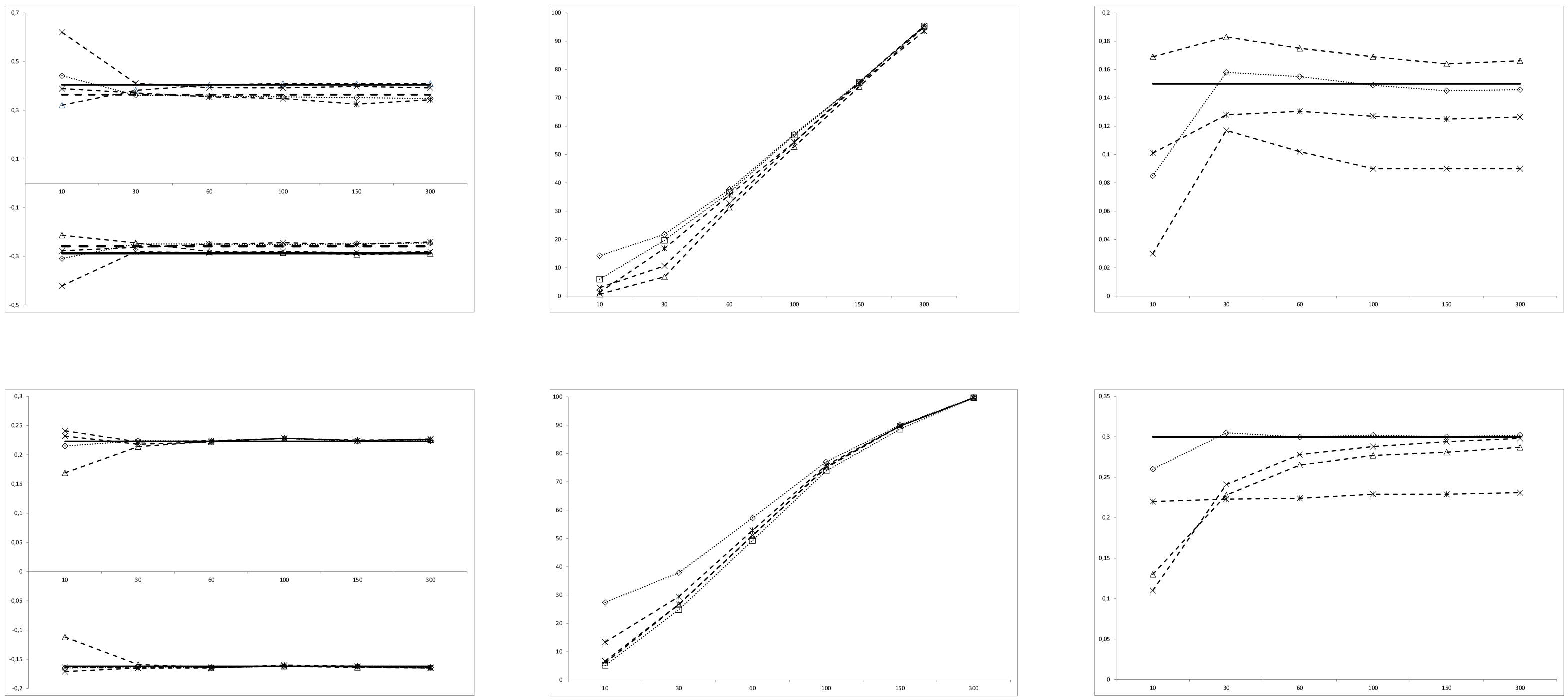

Figure 6: Performance under the APIM-model for Bernoulli outcome (upper panel) and count outcome (lower panel) in the presence of an actor and partner effect for the GEE-Wald ($\diamond$), GEE-Score ($\square$), Multilevel Constrained Random Intercept ($\triangle$), Multilevel Unconstrained Random Intercept ($\times$) and Multilevel Marginalized ($\ast$) approach. The left panel shows the mean estimated actor (positive) and partner (negative) effect (solid black represents the true marginal effect, while the dotted line represents the conditional effect). The middle panel presents the empirical power for the test of no actor effect as a function of sample size. The right panel presents the estimated ICC (the black solid line corresponds to the true ICC).