The capacitated vehicle routing problem with loading constraints
Peer-reviewed author version

common extension of the basic CVRP is the VRP with backhauls (VRPB) in which again pickups and deliveries may be done in one tour, but first all delivery requests need to be performed, and afterwards the empty vehicle may pick up goods at customer locations. (Toth and Vigo 2002)

Many solution methods already exist for the 'classic' VRPs mentioned in the previous paragraph. In real-life, companies are facing several additional constraints that greatly increase the complexity of the problem. Examples of such complicating constraints are maximum route length and duration, time-dependent routing, incompatibilities between goods and vehicles and loading constraints. 'Rich' VRPs are routing problems that take into account some of these additional realistic constraints (Battarra, Monaci and Vigo, 2009). This paper focuses on routing problems combined with loading constraints.

A survey conducted by the authors among several Belgian logistics service providers pointed out that they are faced with important loading problems in their route planning. The most common loading problems that are encountered by distributors are multi-dimensional packing constraints, unloading sequence constraints, stability constraints, load-bearing strength constraints and axle weight constraints. Multi-dimensional packing constraints include that items cannot overlap and should be completely enclosed by the vehicle. In a three-dimensional problem, the three dimensions (length, width and height) of the vehicle are taken into account to check this constraint. In a one dimensional or two dimensional problem respectively a single or two dimensions of the vehicle are taken into account. The unloading sequence constraint ensures that when arriving at a customer, no items belonging to customers served later on the route block the removal of the items of the current customer. In a one dimensional problem this constraint can be referred to as a Last-In-First-Out (LIFO) constraint. Stability constraints guarantee vertical as well as horizontal stability of the cargo in the vehicle. When items are stacked on top of each other in the vehicle, the items have to be supported by other items or by the floor to ensure the vertical stability of the cargo. Vertical stability constraints specify the minimum supporting area of each item (for example as a percentage of the base area of the item). Horizontal

ABSTRACT

Distributors of goods have to take loading constraints into account to make a realistic planning for their delivery vehicles, while current planning tools generally do not include these constraints. The most common loading problems encountered in the distribution of goods are multi-dimensional packing constraints, unloading sequence constraints, stability constraints and axle weight constraints. This paper combines vehicle routing problems with loading problems. First, an overview of the relevant literature is provided. Second, the paper will shed light on axle weight limits since, to our knowledge, VRPs with axle weight constraints have not yet been considered in literature. A two dimensional VRP with sequence based loading is formulated. This model is used to perform computational experiments on a small network. For each computed vehicle route, the weight on the axles is calculated and compared with legal limits.

Keywords: vehicle routing problem, loading constraints, axle weight, two dimensional VRP

1. INTRODUCTION

The Vehicle Routing Problem (VRP) concerns the distribution of goods between depots and customers (Toth and Vigo 2002). It is the most common studied combinatorial optimization problem in transport and logistics. The goal of the vehicle routing problem is to find a set of routes for a fleet of vehicles where the objective function (e.g. total distance, routing costs) is optimized. Every demand needs to be fulfilled and vehicle capacities need to be respected. The basic version of the VRP is the Capacitated VRP (CVRP). The CVRP considers a homogeneous vehicle fleet with a fixed capacity (in terms of weight or number of items) which delivers goods from a single depot to customer locations. Split deliveries are not allowed. The CVRP can be extended to VRP with time windows (VRPTW) by specifying time windows in which deliveries need to take place. Another variant is the VRP with Pickups and Deliveries (VRPDP) in which orders may be picked up and delivered at customer places. For each order, an origin location (pickup location) and a destination (delivery location) is specified. It is possible to have both deliveries and pickups at a given location. A third

stability of the cargo refers to the support of the lateral sides of the items in the vehicle to avoid the items from moving around in the vehicle. The load-bearing strength of an item is the maximum pressure that can be applied on this item (Junqueira et al. 2012). This is taken into account to prevent items to be damaged because of the pressure of other items that are placed above them. Fragile items can be defined as items that cannot bear much pressure. This may imply that no item can be placed upon fragile items or that only other fragile items can be placed upon them. Not only the total weight of the load inside the vehicle is of importance, but also the distribution of the weight of that load over the axles of the vehicle is an important issue. Axle weight limits impose a great challenge for transportation companies. Transporters face high fines when violating these limits, while current planning programs do not incorporate axle weight constraints. Legislation about axle weight limits varies by country (for an overview of the axle weight limits in Europe, the reader is referred to the International Transport Forum). The axle weight is the total weight that is being placed on the axles of a truck or a trailer. This is illustrated in figure 1. When item j is placed onto a vehicle, the weight of the item is divided over the axles of the truck and the axles of the trailer. F_K^j represents the weight of the items of customer j placed on the axles of the truck. F_A^j represents the weight of the items of customer j on the axles of the trailer.
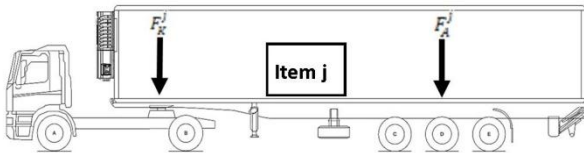

Figure 1: Axle weight truck and trailer

The main goal of the paper is to demonstrate it is necessary to take axle weight constraints into account in a VRP model. In section 2, an overview of the literature concerning VRPs combined with loading constraints is provided. A two dimensional CVRP is formulated in section 3. In section 4, the model is used to perform computational experiments on a small network. For each computed vehicle route, the weight on the axles is calculated and compared with legal limits. In the last section, conclusions and future research opportunities are discussed.

2. LITERATURE STUDY
The combination of VRP and loading problems is a fairly recent domain of research. The two problems are separately already NP hard and very difficult to solve (Iori and Martello 2010). Combining these problems is therefore very challenging but leads to a better overall logistic solution. Papers dealing with VRP with loading problems may be placed in following categories based on the type of routing problem and the loading characteristics that are dealt with: VRP and 2D loading, VRP and 3D loading, multiple pile VRP, multi-

compartments VRP and PDPs with loading constraints. We will focus in this article on the general VRPs in which all items are picked up at the depot and afterwards delivered to the customers. We will therefore not go into detail about the PDPs. For a detailed overview of literature on this topic up to 2010 the reader is referred to Iori and Martello (2010).

2.1. VRP with 2D loading constraints
In the two-dimensional VRP, the demand of customers and the vehicle measurements are expressed in two dimensions.

Iori, Salazar-Gonzales and Vigo (2007) are the first to address a two dimensional vehicle routing problem. They develop an exact method to solve the problem that takes into account sequence based loading, fixed orientation constraints and a maximum weight capacity. Total routing costs are minimized with a branch-and-bound method. Gendreau et al. (2008) develop a Tabu Search (TS) method to solve the same problem heuristically. Fuellerer et al. (2009) employ an Ant Colony Optimisation (ACO) method for a similar problem, with a small alteration in the loading constraints. The items are allowed to rotate 90° on the horizontal plane in their model. Zachariadis, Tarantilis and Kiranoudis (2009) develop a Guided Tabu Search method which is a combination of Guided Local Search and TS to solve the 2L-CVRP formulated by Iori et al. (2007). Duhamel et al. (2011) address the 2L-CVRP without sequence based loading by using a Greedy Randomized Adaptive Search Procedure (GRASP) in an Evolutionary Local Search (ELS) framework. Leung et al. (2013) develop a Simulated Annealing (SA) model to solve the 2L-CVRP with heterogeneous fleet. The packing constraints that are considered in this model are the same as in Iori et al. (2007). The vehicles have different weight capacities and different measurements. Currently, this is the only multi-dimensional CVRP in which a heterogeneous fleet is considered.

2.2. VRP with 3D loading constraints
In the 3L-CVRP, the three dimensions of the vehicle are taken into account and customer demands consist of three-dimensional items as well. Since the height dimension is considered, additional loading constraints concerning load bearing strength and stability of the cargo can be specified.

Gendreau et al. (2006) are the first to tackle the three-dimensional Capacitated Vehicle Routing Problem (3L-CVRP). They develop a TS method to solve the problem with the objective to minimize total route length. Their model takes into account sequence-based loading, the weight capacity of the vehicle, fragility and stability constraints and a fixed vertical orientation of the items in the vehicles (it is allowed to rotate the items 90° on the width-length plane).

Several papers take into account the same loading constraints as Gendreau et al. (2006) to solve the 3L-

CVRP (e.g. Ren, Tian, and Sawaragi 2011; Ruan et al. 2013; Bortfeldt and Homberger 2013).

Moura (2008) develops a multi-objective genetic algorithm to solve the 3L-CVRP with time windows (3L-VRPTW). The model takes into account sequence based loading, orientation constraints and stability constraints. The presented problem has three objectives: minimisation of the number vehicles, minimisation of the total distance travelled and maximisation of the volume utilisation. In 2009, Moura and Oliveira develop a sequential and a hierarchical approach to solve the 3L-VRPTW. The objective is to minimise the number of vehicles and total route time. The hierarchical approach, takes into account sequence based loading, orientation constraints and stability constraints. In the sequential approach, container loading and vehicle routes are planned simultaneously. The unloading sequence constraint is relaxed in this solution approach.

Massen, Deville and Van Hentenryck (2012) develop a column generation method for vehicle routing problems with black box feasibility (VRPBB). In the VRPBB, the routes of the basic VRP need to satisfy a number of unknown constraints. A black box algorithm is used to verify the feasibility of a route. Their approach is tested on the 3L-CVRP as well as on the multiple pile-VRP.

Junqueira, Oliveira and Morabito (2012) are the first to propose an exact method to solve the 3L-CVRP. They take into account sequence based loading, orientation constraints and stability constraints. The authors introduce multi-drop constraints that take into account the unloading pattern. By specifying a reach length of the worker or forklift, they avoid that items that are placed on top of other items cannot be reached. They propose an integer linear programming model to solve problems of moderate size.

2.3. Multiple pile VRP
The multiple pile vehicle routing problem (MP-VRP) is introduced by Doerner et al. (2007). They develop a TS method and an ACO heuristic to solve a real-world transportation problem regarding the transport of wooden chipboards. A distinction is made between small chipboards and large chipboards. For every order, chipboards of the same type are grouped into a unique item, which is placed onto a single pallet. The vehicle is divided into three piles in which the pallets can be stacked. The pallets containing large chipboards can extend over multiple piles. The other pallets can be placed into a single pile. An example of a loading plan of a multiple-pile vehicle can be found in figure 2.

Because of this specific configuration of pallets placed into multiple piles, the original three dimensional problem can be reduced to a one dimensional one. In the model of Doerner et al. (2007), sequence based loading is taken into account and a homogeneous vehicle fleet is assumed.
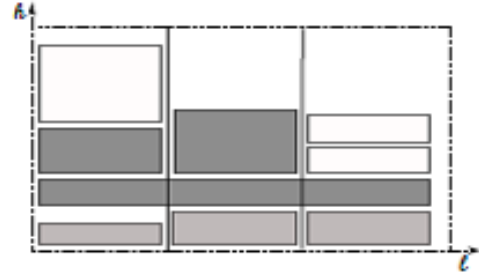

Figure 2: Example of a Multiple Pile Vehicle (Massen, et al., 2012)

Tricoire et al. (2011) develop a combination of Variable Neighbourhood search and branch-and-cut to solve the MP-VRP exactly for small-size instances and heuristically for large-size instances. In both solutions, sequence-based loading is taken into account. Massen et al. (2012) test a column generation method for vehicle routing problems with black box feasibility (VRPBB) on the MP-VRP. They take into account sequence based loading and assume a homogeneous fleet.

2.4. Multi-compartments VRP
The multi-compartments VRP is related to the multiple pile VRP. Vehicles with multiple compartments allow the transport of inhomogeneous products in the same vehicle, but in different compartments. The compartments are not always compatible with every type of product and certain product pairs cannot be loaded together into the same compartments. Vehicle routing problems with compartments are encountered in several industries like the distribution of petrol (e.g. Brown and Graves 1981; Cornillier et al. 2012), the distribution of food (e.g. Chajakis and Guignard 2003) and waste collection (e.g. Muyldermans and Pang 2010).

3. PROBLEM FORMULATION
We start with an investigation of a classic two dimensional CVRP with sequence based loading, whereas in further research the conclusions may be tested in more complex VRPs. Multiple homogenous vehicles are considered. Vehicle capacity is expressed both in weight capacity (Q^k) (maximum 30 tonnes) as in maximum length L_i and width W_i of the loading space (10 m and 2 m respectively). The demand of the customers consists of pallets of 1x1 meters. Pallets cannot be placed on top of each other, but can be placed beside each other in the vehicle. In total 20 pallets can be placed inside each vehicle.

Consider a complete directed graph $G = (V, E)$, where V is the set of vertices corresponding to the begin depot (node *1*), end depot (node *n*) and the customers (node *2 … n-1*). E is the set of edges where each edge has an associated transport cost c_{ij}, for $(i, j) \epsilon E$. Let K be the set of identical vehicles. Each customer i has a demand of m_i pallets. The total weight in kilograms of the pallets ordered by each customer i is expressed as w_i. Stages are incorporated into the model in order to know in which sequence the locations are visited. This

way, we can calculate the axle weights in a next step. This notation is also used by Junqueira et al. (2012). The routing decision variables d_{ij}^{kt} of the model are defined as:

$$d_{ij}^{kt}$$
$$= \begin{cases} 1, & \text{if vehicle } k \text{ goes directly from node } i \\ & \text{to node } j \text{ in stage } t; \\ 0, & \text{otherwise .} \end{cases}$$

The vehicle routing model may be formulated as follows:

$$Min \ \sum_{k \in K} \sum_{(i,j) \in E} \sum_{t \in T} c_{ij} * d_{ij}^{kt} \qquad (1)$$

Subject to

$$\sum_{k \in K} \sum_{i \in V} \sum_{t \in T} d_{ij}^{kt} = 1 \quad \forall j \in V \text{ with } 1 < j < n \ (2)$$

$$\sum_{k \in K} \sum_{j \in V} \sum_{t \in T} d_{ij}^{kt} = 1 \quad \forall i \in V \text{ with } 1 < i < n \ (3)$$

$$\sum_{i \in V} \sum_{j \in V} d_{ij}^{kt} \leq 1 \quad \forall k \in K, t \in T \qquad (4)$$

$$\sum_{i \in V} \sum_{j \in V} d_{ij}^{kt} \geq \sum_{i \in V} \sum_{j \in V} d_{ij}^{k,t+1} \quad \forall k \in K, t \in T \quad (5)$$

$$\sum_{i \in V} \sum_{j \in V} \sum_{t \in T} (d_{ij}^{kt} * w_i) \leq Q^k \quad \forall k \in K \qquad (6)$$

$$\sum_{i \in V} \sum_{j \in V} \sum_{t \in T} (d_{ij}^{kt} * m_i) \leq 2 * L_k \quad \forall k \in K \qquad (7)$$

$$\sum_{j \in V} d_{ji}^{kt} = \sum_{j \in V} d_{ij}^{kt+1} \quad \forall k \in K, \forall t \in T, \forall i \in V, 1 < i < n \qquad (8)$$

$$d_{ij}^{kt} \in \{0,1\} \quad \forall i, j \in V, k \in K, t \in T \qquad (9)$$

The objective function (1) aims to minimize the total cost for the vehicles to visit all customers. Constraints (2) and (3) ensure that each customer is visited exactly once. Constraint (4) ensures that each vehicle can only visit a single arc per stage while constraint (5) does not allow a vehicle to visit a customer in stage t when the vehicle has not visited a customer in stage $t-1$. Constraint (6) prevents the total weight of the load to exceed the weight capacity of the vehicle. Constraint (7) limits the amount of pallets per vehicle to the length of the vehicle multiplied by two, since two pallets can be placed beside each other in the vehicle. Constraint (8) eliminates the possibility of subtours.

4. COMPUTATIONAL RESULTS
The model formulated in the previous section is used to perform computational experiments on a small network. This network consists of ten nodes: the begin depot (node 1), the customers (nodes 2 to 9) and the end depot (node 10). The demand of the customers (node 2 to 9) is D_C = *(8, 4, 2, 6, 4, 6, 8, 10)*. To define the bound of the amount of stages, we computed the amount of

stages required when the customers with the smallest amount of items would be carried in the same vehicle. Maximum four customers can be serviced by the same vehicle, namely customers 2, 3, 4 and 5. The total amount of pallets of these customers is 16, which is below the maximum capacity of 20 pallets. This means that a maximum bound of 6 stages (the begin and end depot are also taken into account) can be set. The model is solved with Cplex 12.5 on a personal laptop.

The model solution contains three vehicle routes. For each vehicle route, the axle weight of the pallets of customer j on the trailer (F_A^j) and on the truck (F_K^j) is calculated with equations (10) and (11) respectively. In figure 3, the parameters in the equations are graphically represented. Parameter f represents the distance from the beginning of the trailer to the center of gravity of the item. Parameter c denotes the distance from the beginning of the trailer to the coupling (which is the link between the truck and the trailer). The final parameter, d, represents the distance between the coupling and the middle axes of the trailer. For our calculations we use 1,67 m for parameter c and 7,8 m for parameter d.

$$F_A^j = \frac{(f - c) * w_j}{d} \qquad \forall j \in V \qquad (10)$$

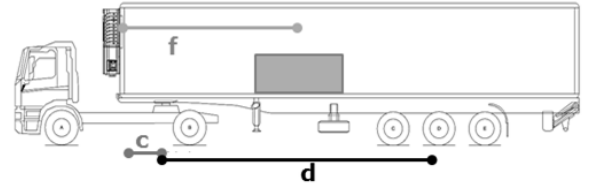$$F_K^j = w_j - F_A^j \quad \forall j \in V \qquad (11)$$

Figure 3: Truck (with two axles) and trailer (with tridem axles)

According to Belgian legislation, the maximum total weight on the two axles of this truck is 22 tonnes. The empty truck weighs 8 tonnes, therefore the maximum weight of the load on the axles of the truck is 14 tonnes (or 14000 kg), as stated in constraint (12).

$$\sum_{i \in V} \sum_{j \in V} \sum_{t \in T} (d_{ij}^{kt} * F_K^j) \leq 14000 \quad \forall k \in K \qquad (12)$$

Belgian legislation limits the maximum weight of the trailer with tridem axles to 30 tonnes. The empty trailer weighs 14 tonnes. The weight of the load on the axles of the trailer is therefore limited to 16 tonnes (or 16000 kg), as stated in constraint (13).

$$\sum_{i \in V} \sum_{j \in V} \sum_{t \in T} (d_{ij}^{kt} * F_A^j) \leq 16000 \quad \forall k \in K \qquad (13)$$

The weight capacity of the vehicle is 30 tonnes. This equals the sum of the axle weight limits of the truck and trailer. We can therefore conclude that if the loading is optimally divided over the axles of the vehicle, the weight of the load can amount to 30 tonnes.

Tables 1, 2 and 3 present the calculation of the axle weight on the truck and the trailer of the vehicle from respectively the first, second and third route. In the first column, the customer nodes that are visited in the route are identified. The second column presents the customer demand divided by two, which equals the total loading meters required by customer j. To ensure ease of computation, we assumed that the customer demand is even. This constraint can easily be relaxed. In the third column f_j, the distance from the beginning of the trailer to the center of gravity of the pallets of customer j, is presented. It is assumed that the center of gravity of each pallet is in the middle of the pallet. In the next column the weight of the pallets in kilograms of each customer j is presented. In the fifth column the weight of the items of customer j on the axles of the trailer is calculated. In the last column the weight of the items of customer j on the axles of the truck is presented. In the last row the sum of the loading meters, weight of the pallets, axle weight on trailer and axle weight on the truck for each route is presented.

From table 1 can be concluded that while the total weight in the vehicle (18 tonnes) is still far below vehicle capacity (30 tonnes) and while length capacity of the vehicle is also respected (<10 m), there is a violation of the axle weight constraint of the truck since this exceeds 14 tonnes. Similar results can be drawn from tables 2 and 3. In this example the axle weight limit of the truck is in each vehicle violated. However – although not illustrated in this example – it is also possible that the axle weight limit of the trailer is violated, or (in the ideal case) that both limits are respected.

Route 1 (1-5-8-3-10)					
Node (j)	$m_j/2$	f_j	w_j	F_A^j	F_K^j
5	3	1,5	9200	-200	9400
8	4	5	8500	3628	4871
3	2	8	5156	5288	-132
Total	7		17700	3428	**14271**

Table 1: Axle weight calculation trailer (F_A) and truck (F_K) vehicle route 1

Route 2 (1-9-7-10)					
Node (j)	$m_j/2$	f_j	w_j	F_A^j	F_K^j
9	5	2,5	19150	2038	17112
7	3	6,5	10250	8542	1708
Total	8		29400	10579	**18821**

Table 2: Axle weight calculation trailer (F_A) and truck (F_K) vehicle route 2

Route 3 (1-2-6-4-10)					
Node (j)	$m_j/2$	f_j	w_j	F_A^j	F_K^j
2	4	2	5000	211	4788
6	2	5	14020	5985	8034
4	1	6,5	9000	7500	1500
Total	7		28020	13697	**14323**

Table 3: Axle weight calculation trailer (F_A) and truck (F_K) vehicle route 3

CONCLUSIONS AND FUTURE RESEARCH
This paper shows the need of integrating axle weight constraints into VRP models. While research has been done about VRP combined with loading constraints, the literature is still silent about axle weight limits. Axle weight has become however an increasingly important issue for transportation companies. Transporters are faced with high fines when violating these limits, while current planning programs do not incorporate these constraints. Future research can integrate axle weight constraints into the model. This can include taking into account axle weight constraints when the vehicle is fully loaded and axle weight increases when items are dropped off at customer places. Subsequently, a heuristic method can be developed to solve the integrated problem in an acceptable time frame. Lastly, other realistic constraints such as time windows and three dimensional loading can be added to the model.

REFERENCES
Aprile, D., Egeblad, J., Garavelli, A.C., Lisi, S., Pisinger, D, 2007. Logistics optimization: vehicle routing with loading constraints. *Proceeding of 19th International Conference on Production Research*. July 29 - August 2, 2007, Chile.
Battarra, M., Monaci, M., Vigo, D., 2009. An adaptive guidance approach for the heuristic solution of a minimum multiple trip vehicle routing problem. *Computers & Operations Research*, 36(11): 3041 – 3050.
Bortfeldt, A., Homberger, J., 2013. Packing first, routing second a heuristic for the vehicle routing and loading problem. *Computers & Operations Research*, 40(3), 873–885.
Brown, G.G., Graves, G.W., 1981. Real-time dispatch of petroleum tank trucks. *Management Science*, 27(1):19-32.
Chajakis, E.D., Guignard, M., 2003. Scheduling deliveries in vehicles with multiple compartments. *Journal of Global Optimization*, 26(1):43-78.
Cornillier, F., Boctor, F., Renaud, J., 2012. Heuristics for the multi-depot petrol station replenishment problem with time windows. *European Journal of*

Operational Research, 220(2):361-369.

Doerner, K., Fuellerer, G., Hartl, R., Gronalt, M., Iori, M., 2007. Metaheuristics for the vehicle routing problem with loading constraints. *Networks*, 49(4): 294 – 307.

Duhamel, C., Lacomme, P., Quilliot, A., Toussaint, H., 2011. A multi-start evolutionary local search for the two-dimensional loading capacitated vehicle routing problem. *Computers & Operations Research*, 38(3): 617 – 640.

Fuellerer, G., Doerner, K.F., Hartl, R.F., and Iori, M., 2009. Ant colony optimization for the two-dimensional loading vehicle routing problem. *Computers & Operations Research*, 36(3):655 – 673.

Gendreau, M., Iori, M., Laporte,G., Martello, S., 2006. A tabu search algorithm for a routing and container loading problem. *Transportation Science*, 40(3): 342 – 350.

Gendreau, M., Iori, M., Laporte,G., Martello, S., 2008. A tabu search heuristic for the vehicle routing problem with two-dimensional loading constraints. *Networks*, 51(1):4 – 18.

Iori, M., Martello, S., 2010. Routing problems with loading constraints. *TOP*, 18(1):4 – 27.

Iori, M., Salazar-Gonzalez, J.S., Vigo, D., 2007. An exact approach for the vehicle routing problem with two-dimensional loading constraints. *Transportation Science*, 41(2):253 – 264.

Junqueira, L., Oliveira, J.F., Carravilla, M.A., Morabito, R., 2012. An optimization model for the vehicle routing problem with practical three-dimensional loading constraints. *International Transactions in Operational Research,* forthcoming (DOI: 10.1111/j.1475-3995.2012.00872.x).

Leung, S.C., Zhang, Z., Zhang, D., Hua, X., Lim, M.K., 2013. A meta-heuristic algorithm for heterogeneous fleet vehicle routing problems with two-dimensional loading constraints. *European Journal of Operational Research*, 225(2):199 – 210.

Massen, F., Deville, Y., Van Hentenryck, P., 2012. Pheromone-based Heuristic Column Generation for Vehicle Routing Problems with Black Box Feasibility. *Proceeding of the International Conference on Integration of AI and OR Techniques in Constraint Programming for Combinatorial Optimization Problems (CPAIOR)*, 260 – 274. May 28 – June 1, Nantes, France.

Moura, A., 2008. A multi-objective genetic algorithm for the vehicle routing with time windows and loading problem. In Bortfeldt, A., Homberger, J., Kopfer, H., Pankratz, G., Strangmeier, R., eds. *Intelligent Decision Support, Current Challenges and Approaches.* Wiesbaden, Germany. Gabler, 187-201.

Moura, A., Oliveira, J., 2009. An integrated approach to the vehicle routing and container loading problems. *OR Spectrum*, 31(4):775 – 800.

Muyldermans, L., Pang, G.,2010. A guided local search procedure for the multi compartment capacitated arc routing problem. *Computers & Operations Research*, 37(9):1662-1673.

Ren, J., Tian, Y., Sawaragi, T., 2011. A relaxation method for the three dimensional loading capacitated vehicle routing problem. *Proceeding of the IEEE/SICE International Symposium on System Integration (SII)*, 750 – 755. December 20-22, 2011, Kyoto.

Ruan, Q., Zhang, Z., Miao, L., Shen, H., 2013. A hybrid approach for the vehicle routing problem with three-dimensional loading constraints. *Computers & Operations Research*, 40(6), 1579-1589.

Toth, P., Vigo, D., 2002. *The vehicle routing problem*. SIAM Monographs on Discrete Mathematics and Applications, Philadelphia, PA.

Tricoire, F., Doerner, K., Hartl, R., Iori, M., 2011. Heuristic and exact algorithms for the multi-pile vehicle routing problem. *OR Spectrum*, 33(4): 931 – 959.

Zachariadis, E.E., Tarantilis, C.D., Kiranoudis, C.T., 2009. A guided tabu search for the vehicle routing problem with two-dimensional loading constraints. *European Journal of Operational Research*, 195(3):729 – 743.