Chapter 4
Fuzzy Data Envelopment Analysis in Composite Indicator Construction

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Abstract Data envelopment analysis (DEA) as a performance evaluation methodology has lately received considerable attention in the construction of composite indicators (CIs) due to its prominent advantages over other traditional methods. In this chapter, we present the extension of the basic DEA-based CI model by incorporating fuzzy ranking approach for modeling qualitative data. By interpreting the qualitative indicator data as fuzzy numerical values, a fuzzy DEA-based CI model is developed, and it is applied to construct a composite alcohol performance indicator for road safety evaluation of a set of European countries. Comparisons of the results with the ones from the imprecise DEA-based CI model show the effectiveness of the proposed model in capturing the uncertainties associated with human thinking, and further imply the reliability of using this approach for modeling both quantitative and qualitative data in the context of CI construction.

Keywords Alcohol performance index · Composite indicators · Data envelopment analysis · Fuzzy ranking approach · Qualitative data · Road safety

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1 Introduction

As a performance evaluation methodology, data envelopment analysis (DEA) is traditionally used to measure the so-called relative efficiency of a homogeneous set of decision making units (DMUs) by allowing direct peer comparisons on the basis of multiple inputs and multiple outputs [1]. However, as noted by Adolphson et al. [2], it is possible to adopt a broader perspective, in which DEA is also appropriate for comparing any set of homogeneous units on multiple dimensions. Based on this perspective, DEA has been introduced to the field of composite indicators (CIs), which is to aggregate a set of individual indicators that measure multi-dimensional concepts but usually have no common units of measurement [3]. The most attractive feature of DEA, relative to the other methods in developing a CI, such as regression analysis (RA), principal components analysis (PCA), factor analysis (FA), analytic hierarchy process (AHP), and the technique for order preference by similarity to ideal solution (TOPSIS) (see also Saisana and Tarantola [4], OECD [3], and Bao et al. [5]) is that, each DMU obtains its own best possible indicator weights, i.e., the weights resulting in the highest index score for a DMU. This implies that dimensions on which the DMU performs relatively well get a higher weight. It is thereby also called 'benefit of the doubt' (BOD) approach [6]. In this way, policymakers could not complain about unfair weighting, because each DMU is put in the most favorable light, and any other weighting scheme would generate a lower composite score. In other words, if a country turns out to be underperforming based on the most favorable set of weights, its poor performance cannot be traced back to an inappropriate evaluation process [7]. Due to the aforementioned characteristic, the DEA-based CI construction has been widely explored in several recent studies such as environmental performance index [8], human development index [9], macro-economic performance index [10], sustainable energy index [11], technology achievement index [12], and road safety performance index [13, 14].

However, as a 'data-oriented' technique, the applicability of DEA in the construction of CIs relies mostly on the quality of information about the indicators. In other words, obtainment of measurable and quantitative indicators is commonly the prerequisite of the evaluation. Under many conditions, however, quantitative data are inadequate or inappropriate to model real world situations due to the complexity and uncertainty of the reality. Therefore, it is essential to take into account the presence of qualitative indicators when making a decision on the performance of a DMU. Very often it is the case that an indicator can, at most, be specified with either ordinal measures, from best to worst, or with the help of experts' subjective judgments, such as 'high', 'medium' and 'low'. Under these circumstances, the basic DEA models are not capable of deriving a satisfactory solution. Generally, two strategies have appeared in the literature to the treatment of qualitative data within the DEA framework. One is to reflect the rank position of each DMU with respect to each ordinal indicator by setting corresponding constraints, which results in the so-called imprecise DEA (IDEA) (see e.g., Cooper et al. [15]; Cook and Zhu [16]). The other is to deal with the natural uncertainty inherent to some production processes by means of fuzzy mathematical programming, such as the tolerance approach developed by Sengupta [17] and Kahraman and Talgo [18], the a-level based approach introduced by Meada et al. [19], the defuzzification and the possibility approach proposed by Lertworasirikul [20] and Lertworasirikul et al. [21], and the fuzzy ranking approach developed by Guo and Tanaka [22]. All of them are collectively named as fuzzy DEA (FDEA).

In this Chapter, we investigate FDEA, and more specifically, the fuzzy ranking approach, to model qualitative data in the construction of CIs. Based on a brief review of the basic DEA model and the DEA-based CI model in Sect. 2, we elaborate the development of a FDEA-based CI model in Sect. 3. In Sect. 4, the proposed model is illustrated by constructing a composite alcohol performance index for road safety evaluation of a set of European countries, and the results are compared with the ones from the IDEA model. The chapter ends with conclusions in Sect. 5.

2 DEA-based CI Model

Data envelopment analysis initially developed by Charnes et al. [1] is a non-parametric optimization technique which employs linear programming tools to obtain the empirical estimates of multiple inputs and multiple outputs related to a set of DMUs. During the last decades, a number of different formulations have been proposed in the DEA context, the best-known of which is probably the Charnes–Cooper–Rhodes (CCR) model, and its multiplier form is presented as follows.

$$
\begin{aligned}
& E_0 = \max \sum_{r=1}^{s} u_r y_{r0} \\
& s.t. \qquad \sum_{i=1}^{m} v_i x_{i0} = 1, \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \; j = 1, \cdots, n \\
& u_r, v_i \geq \varepsilon, \; r = 1, \cdots, s, \; i = 1, \cdots, m
\end{aligned}
\tag{1}
$$

The above linear program is computed separately for each DMU, and the subscript, 0, refers to the DMU whose relative efficiency is to be evaluated. y_{rj} and x_{ij} are the rth output and ith input respectively of the jth DMU. u_r is the weight given to the rth output, v_i is the weight given to the ith input, and ε is a small non-Archimedean number [23] for preventing the model to assign a weight of zero to unfavorable factors.

To use DEA for CI construction, i.e., aggregating a set of individual indicators into one overall index, however, only inputs or outputs of the DMUs will be taken into account in the model. Mathematically, the DEA-based CI model (DEA-CI) can be realized by converting the DEA model in (1) into the following constrained optimization problem, which is also known as the CCR model with constant inputs.

$$
\begin{aligned}
& CI_0 = \max \sum_{r=1}^{s} u_r y_{r0} \\
& s.t. \quad \sum_{r=1}^{s} u_r y_{rj} \leq 1,\ j = 1, \cdots, n \\
& u_r \geq \varepsilon,\ r = 1, \cdots, s
\end{aligned}
\tag{2}
$$

The n DMUs are now to be evaluated by combining s different outputs (or indicators) with higher values indicating better performance, while the inputs of each DMU in model (1) are all assigned with a value of unity. This linear program is run n times to identify the optimal index score for all DMUs by selecting their best possible indicator weights separately. In other words, the weights in the objective function are chosen automatically with the purpose of maximizing the value of DMU_0's index score and also respect the less than unity constraint for all the DMUs. Meanwhile, all the weights are required to be positive. In general, a DMU is considered to be best-performing if it obtains an index score of one in (2), whereas a score less than one implies that it is underperforming.

3 Fuzzy DEA-CI Model

In model (2), the performance evaluation is generally assumed to be based upon a set of quantitative data. However, in situations where some indicators might better be represented in either ordinal measures or the help of expert subjective judgments, the standard DEA-CI model cannot be used directly, because ordinal (or qualitative) data cannot be simply treated as numerical ones for which a score of 2 is twice as large as a score of 1. The most that can be judged is that the former one is preferred to or more important than the latter in a maximization context. In recent years, fuzzy set theory [24] has been proposed as a valuable way to quantify imprecision and vagueness in DEA framework, and a number of different fuzzy DEA models has been developed (see e.g., Hatami-Marbini et al. [25]). In CI construction, by interpreting the qualitative indicator data as fuzzy numerical values which can be represented by means of fuzzy numbers or fuzzy intervals, the basic DEA-CI model (2) can also be naturally extended to the following fuzzy one:

$$
\begin{aligned}
& CI_0 = \max \sum_{r=1}^{s} u_r \tilde{y}_{r0} \\
& s.t. \quad \sum_{r=1}^{s} u_r \tilde{y}_{rj} \precsim 1,\ j = 1, \cdots, n \\
& u_r \geq \varepsilon,\ r = 1, \cdots, s
\end{aligned}
\tag{3}
$$

where $\tilde{y}_{rj}$ denotes the rth fuzzy indicator value of the jth DMU.

The resulted fuzzy DEA-based CI model (FDEA-CI) takes the form of a fuzzy linear programming problem with fuzzy coefficients in the objective function and also the constraints. Therefore, to compute the final index score for each DMU, some fuzzy operations including 'maximizing a fuzzy variable' and 'fuzzy inequality' are required. In what follows, we simply recall how to perform the basic operations of arithmetics and the comparison of fuzzy intervals for ranking purposes. To be more precise, we deal with *LR*-fuzzy numbers whose definition is as follows.

Definition 1 [26] A fuzzy number $\tilde{M}$ is an *LR*-fuzzy number, $\tilde{M} = (m^L, m^R, \alpha^L, \alpha^R)_{L,R}$, if its membership function has the following form:

$$\mu_{\tilde{M}}(r) = \begin{cases} L\left(\frac{m^L - r}{\alpha^L}\right), & r \le m^L \\ 1, & m^L \le r \le m^R \\ R\left(\frac{r - m^R}{\alpha^R}\right), & r \ge m^R \end{cases} \quad (4)$$

where the subset $[m^L, m^R]$ consists of the real numbers with the highest chance of realization, α^L is the left spread, α^R is the right spread, and L and R are reference functions defining the left and the right shapes of the fuzzy number, respectively, which should satisfy the following conditions:

$$L, R : 0, \infty \rightarrow 0, 1,$$
$$L(x) = L(-x), R(x) = R(-x),$$
$$L(0) = 1, R(0) = 1, \text{ and}$$

$L(x)$ and $R(x)$ are strictly decreasing and upper semi-continuous on $supp(\tilde{M}) = \{r : \mu_{\tilde{M}}(r) > 0\}$.

In addition, an *LR* fuzzy number becomes an *LL* fuzzy number when $L(x) = R(x)$, an *LL* fuzzy number with $L(x) = \max(0, 1 - |x|)$ is known as a triangular fuzzy number, and a symmetrical *LL* fuzzy number is for the case of $\alpha^L = \alpha^R$.

Let us now recall the definition of the maximum of two fuzzy numbers.

Definition 2 [27] Let $\tilde{M}$ and $\tilde{N}$ be two fuzzy numbers and h a real number, $h \in [0, 1]$. Then $\tilde{M} \gtrsim^h \tilde{N}$ if and only if, $\forall k \in [h, 1]$, the following two statements hold:

$$\begin{aligned} inf\{s : \mu_{\tilde{M}}(s) \ge k\} \ge inf\{t : \mu_{\tilde{N}}(t) \ge k\} \\ sup\{s : \mu_{\tilde{M}}(s) \ge k\} \ge sup\{t : \mu_{\tilde{N}}(t) \ge k\} \end{aligned} \quad (5)$$

where *inf* stands for infimum (lower bound or minimum), and *sup* stands for supremum (upper bound or maximum).

Hence, for *LR*-fuzzy numbers with bounded support, and using this ranking method, at a given possibility level h, expression (5) becomes

$$\begin{aligned} m^L - L^*(k)\alpha^L \geq n^L - L'^*(k)\beta^L \; \forall k \in [h, 1] \\ m^R + R^*(k)\alpha^R \geq n^R + R'^*(k)\beta^R \; \forall k \in [h, 1] \end{aligned} \tag{6}$$

Therefore, using *LR* fuzzy numbers in the FDEA-CI model (3), i.e., $\tilde{y}_{rj} = (y_{lrj}, y_{urj}, a_{rj}, b_{rj})$, the constraint $\sum_{r=1}^{s} u_r \tilde{y}_{rj} \tilde{\leq} 1$ can then be considered as inequalities between an *LR* fuzzy number and a real number, and the use of an ordering relation in (6) allows us to convert this fuzzy constraint into a crisp inequality as: $\sum_{r=1}^{s} u_r \left(y_{urj} + b_{rj} R^*(h) \right) \leq 1$.[1]

Concerning 'maximizing a fuzzy variable', i.e., $\max \sum_{r=1}^{s} u_r \tilde{y}_{r0}$, still using the ordering relation in (6), this objective function can then be decomposed into two crisp relations as: $\max \sum_{r=1}^{s} u_r \left(y_{lr0} - a_{r0} L^*_{r0}(h) \right)$ and $\max \sum_{r=1}^{s} u_r \left(y_{ur0} + b_{r0} R^*_{r0}(h) \right)$, $h \in [0, 1]$, which should be maximized simultaneously. To this end, a weighted function $\lambda_1 \sum_{r=1}^{s} u_r \left(y_{lr0} - a_{r0} L^*_{r0}(h) \right) + \lambda_2 \sum_{r=1}^{s} u_r \left(y_{ur0} + b_{r0} R^*_{r0}(h) \right)$ with $\lambda_1 \geq 0$, $\lambda_2 \geq 0$, and $\lambda_1 + \lambda_2 = 1$ is used to obtain the compromise solution. Three situations are usually considered, which are optimistic if $\lambda_2 = 1$, pessimistic if $\lambda_1 = 1$, and indifferent if $\lambda_1 = \lambda_2$.

Thus, the FDEA-CI model (3) can now be transformed in the following crisp linear programming problem:

$$\begin{aligned} & CI_0 = \max \lambda_1 \sum_{r=1}^{s} u_r \left(y_{lr0} - a_{r0} L^*_{r0}(h) \right) + \lambda_2 \sum_{r=1}^{s} u_r \left(y_{ur0} + b_{r0} R^*_{r0}(h) \right) \\ & s.t. \quad \sum_{r=1}^{s} u_r \left(y_{urj} + b_{rj} R^*_{rj}(h) \right) \leq 1, \; j = 1, \cdots, n \\ & u_r \geq \varepsilon, \;\; r = 1, \cdots, s \end{aligned} \tag{7}$$

Definition 3 DMU_0 is called fuzzy best performing if and only if it obtains a fuzzy index score of one at least at one possibility level h. Otherwise, it is fuzzy underperforming.

Definition 4 DMU_0 is called fuzzy non-dominated best performing if and only if it obtains a fuzzy index score of one at all possibility levels h.

In particular, if indicators $\tilde{y}_{rj}$ are assumed to be symmetrical triangular fuzzy numbers, which are often used to represent the uncertainty of information for

[1] $\sum_{r=1}^{s} u_r \left(y_{lrj} - a_{rj} L^*(h) \right) \leq 1$ is always satisfied when $\sum_{r=1}^{s} u_r \left(y_{urj} + b_{rj} R^*(h) \right) \leq 1$.

simplification, they can then be denoted by the pairs consisting of the corresponding centers and spreads, $\tilde{y}_{rj} = (y_{rj}, \alpha_{rj}),\ r = 1, \cdots, s,\ j = 1, \cdots, n$, and the model (7) can be substantially simplified as follows:

$$\begin{aligned} & CI_0 = \max\ \lambda_1 \sum_{r=1}^{s} u_r(y_{r0} - (1-h)\alpha_{r0}) + \lambda_2 \sum_{r=1}^{s} u_r(y_{r0} + (1-h)\alpha_{r0}) \\ & s.t. \quad \sum_{r=1}^{s} u_r\left(y_{rj} + (1-h)\alpha_{rj}\right) \le 1,\ j = 1, \cdots, n \\ & u_r \ge \varepsilon,\ r = 1, \cdots, s \end{aligned} \tag{8}$$

Note that for triangular fuzzy numbers, $L^*_{rj}(h) = R^*_{rj}(h) = 1 - h$, $0 \le h \le 1$, $r = 1, \cdots s$. The fuzzy index score of DMU_0 can then be defined as $\{\sum_{r=1}^{s} u^*_r(y_{r0} - (1-h)\alpha_{r0}), \sum_{r=1}^{s} u^*_r y_{r0}, \sum_{r=1}^{s} u^*_r(y_{r0} + (1-h)\alpha_{r0})\}$, which represents the pessimistic, indifferent, and optimistic situation, respectively.

4 Application and Discussion

To illustrate the use of the proposed FDEA-CI model, we apply it to construct an alcohol performance index for a set of European countries based on both quantitative and qualitative indicators. In road safety context, driving under the influence of alcohol is believed to increase the risk and severity of road crashes more than most other traffic law violations [28]. Therefore, it is valuable to compare the situation of drinking and driving between countries for the sake of better understanding of this risk factor in each country. In doing so, several relevant indicators can be considered. First, *the percentage of road fatalities attributed to alcohol*, which represents the consequence of drinking and driving from the view of the final outcome level, is commonly used as a representative alcohol indicator for cross-country comparison. Moreover, at the intermediate outcome level, an alcohol performance indicator is also developed, which is *the percentage of drivers above the legal blood alcohol concentration (BAC) limit in roadside checks*. In addition to the above two quantitative indicators, one more indicator related to policy output, i.e., *the effectiveness of overall enforcement against drinking and driving*, is also suggested to supplement the alcohol performance of a country. Such a policy performance indicator, derived from the Global Status Report on Road Safety prepared by the World Health Organization [29], in which the respondents were asked to reach a consensus on their assessment of the enforcement in the country, is qualitative in nature, and can only take the form of ordered classes rated on a 0–10 scale (with 0 represents the worst drink driving enforcement while 10 the best) rather than numerical values for the purpose of description, comparison and evaluation of this risk factor for various countries.

Table 1 Normalized numerical data and ordinal data on three alcohol indicators for 28 European countries

	Alcohol indicators		
	% of alcohol-related fatalities	% of drivers above legal alcohol limit in roadside checks	Effectiveness of overall enforcement on drinking and driving
AT	0.463	0.116	9
BE	0.654	0.068	3
BG	0.855	0.123	7
CY	0.182	0.137	4
CZ	0.675	0.145	9
DK	0.143	0.301	8
EE	0.080	0.860	8
FI	0.136	0.593	8
FR	0.123	0.263	4
DE	0.306	0.093	4
EL	0.432	0.273	7
HU	0.283	0.279	5
IE	0.119	0.237	5
IT	0.992	0.098	7
LV	0.175	0.218	7
LT	0.321	0.555	6
LU	0.248	0.102	5
NL	1.000	0.081	9
NO	0.159	0.142	4
PL	0.438	0.091	7
PT	0.610	0.137	8
RO	0.423	0.070	8
SK	0.607	0.067	9
SI	0.078	0.122	6
ES	0.402	0.398	7
SE	0.357	1.000	6
CH	0.230	0.141	6
UK	0.228	0.051	5

Data on these three indicators for the 28 European countries[2] are presented in Table 1, in which the first two quantitative indicators are normalized using the distance to a reference approach [3] so as to ensure that they are expressed in the same direction with respect to their expected road safety impact, i.e., a high indicator value should always correspond to a low crash/injury risk. Taking the percentage of alcohol-related fatalities as an example, the Netherlands performs the best (1.000) while Slovenia worst (0.078), and all other countries' values lie within this interval.

[2] Missing data are imputed by using Multiple Imputation in SPSS 20.0 [30].

Table 2 Representation of symmetrical triangular fuzzy numbers for the ordinal indicator values

Ordinal data ($\tilde{y}_{rj}$)	Symmetrical triangular fuzzy numbers (y_{rj}, α_{rj})	Ordinal data ($\tilde{y}_{rj}$)	Symmetrical triangular fuzzy numbers (y_{rj}, α_{rj})
0	$\left(0, \frac{1}{10}\right)$	1	$\left(\frac{1}{10}, \frac{1}{10}\right)$
2	$\left(\frac{2}{10}, \frac{1}{10}\right)$	3	$\left(\frac{3}{10}, \frac{1}{10}\right)$
4	$\left(\frac{4}{10}, \frac{1}{10}\right)$	5	$\left(\frac{5}{10}, \frac{1}{10}\right)$
6	$\left(\frac{6}{10}, \frac{1}{10}\right)$	7	$\left(\frac{7}{10}, \frac{1}{10}\right)$
8	$\left(\frac{8}{10}, \frac{1}{10}\right)$	9	$\left(\frac{9}{10}, \frac{1}{10}\right)$
10	$\left(1, \frac{1}{10}\right)$		

To combine these three alcohol indicators into one index score, symmetrical triangular fuzzy numbers are first used for the ordinal data in this study, which are defined as in Table 2.

In addition, to guarantee that all the three indicators will be used to some extent by the models, the share of each of these three indicators in the final index score is restricted to lie within the interval [0.1, 0.5], yet is rather broad to allow a high level of flexibility, and the ε value is chosen as 0.0001.

The alcohol performance index score of the 28 European countries can now be computed by applying the FDEA-CI model (8). The results are shown in Table 3, together with the ones from the IDEA-CI model. For more information on this model, we refer to Shen et al. [31].

By using the FDEA-CI model, fuzzy index scores are obtained based on different possibility levels of h. In practice, the given possibility degree by decision makers reflects their attitude on uncertainty. When $h = 1$, the ordinal data are actually treated as numerical ones and the same index scores are obtained for each country, no matter whether the decision makers are in a pessimistic, indifferent, or optimistic consideration. When the given value of h becomes lower, it means the decision makers are more cautious. As a consequence, a wider range of index scores will be derived. In such a way, the uncertainties associated with human thinking are effectively interpreted. Taking Belgium as an example, which was assigned the lowest value of 3 for this ordinal indicator among all the 28 European countries, it obtains an index score of 0.392 when $h = 1$. That is, decision makers have no doubt about this value in representing the true performance of Belgium with respect to this indicator, which is half of the value of 6 and one third of 9. When h decreases to 0.5, this implies that decision makers are no longer fully sure about the relation between 3 and 6, and the other numbers. In other words, the value of 6 could be more (or less) than twice as large as the value of 3, and the most that can be judged is that the former one is preferred to or more important than the latter. As a result, an interval index score is obtained for Belgium, which is between 0.359 (pessimistic) and 0.401 (optimistic), with a medium value of 0.382 (indifferent). The widest interval is derived when $h = 0$, which is {0.318, 0.373, 0.409}. Among all the 28 European countries, Sweden is the only non-dominated best-performing country since it obtains the fuzzy index score of one at

Table 3 Composite alcohol performance index scores of 28 European countries based on the FDEA-CI model and the IDEA-CI model

	FDEA-CI			IDEA-CI	
	$h = 0$	$h = 0.5$	$h = 1$		
SE	{0.872, 0.947, 1.000}	{0.940, 0.973, 1.000}	{1.000, 1.000, 1.000}	SE	1.000
CZ	{0.768, 0.792, 0.812}	{0.795, 0.806, 0.816}	{0.820, 0.820, 0.820}	CZ	0.880
ES	{0.684, 0.733, 0.775}	{0.729, 0.752, 0.774}	{0.773, 0.773, 0.773}	ES	0.847
LT	{0.670, 0.727, 0.778}	{0.721, 0.750, 0.776}	{0.774, 0.774, 0.774}	FI	0.833
PT	{0.694, 0.727, 0.749}	{0.726, 0.740, 0.752}	{0.755, 0.755, 0.755}	PT	0.826
FI	{0.686, 0.720, 0.750}	{0.719, 0.735, 0.751}	{0.752, 0.752, 0.752}	LT	0.803
BG	{0.672, 0.703, 0.729}	{0.703, 0.717, 0.730}	{0.732, 0.732, 0.732}	EL	0.780
EL	{0.634, 0.679, 0.717}	{0.674, 0.696, 0.715}	{0.713, 0.713, 0.713}	BG	0.776
AT	{0.624, 0.642, 0.658}	{0.645, 0.654, 0.662}	{0.666, 0.666, 0.666}	AT	0.711
IT	{0.598, 0.623, 0.643}	{0.623, 0.634, 0.644}	{0.646, 0.646, 0.646}	IT	0.679
NL	{0.566, 0.579, 0.590}	{0.581, 0.587, 0.592}	{0.594, 0.594, 0.594}	NL	0.678
EE	{0.535, 0.556, 0.574}	{0.554, 0.564, 0.573}	{0.572, 0.572, 0.572}	DK	0.626
HU	{0.468, 0.518, 0.558}	{0.509, 0.532, 0.553}	{0.547, 0.547, 0.547}	HU	0.623
PL	{0.505, 0.523, 0.537}	{0.523, 0.531, 0.538}	{0.539, 0.539, 0.539}	EE	0.589
DK	{0.496, 0.513, 0.526}	{0.513, 0.521, 0.528}	{0.530, 0.530, 0.530}	PL	0.567
SK	{0.467, 0.475, 0.482}	{0.476, 0.480, 0.484}	{0.486, 0.486, 0.486}	SK	0.563
LV	{0.440, 0.459, 0.474}	{0.458, 0.466, 0.474}	{0.474, 0.474, 0.474}	RO	0.562
RO	{0.446, 0.456, 0.466}	{0.457, 0.462, 0.467}	{0.469, 0.469, 0.469}	LV	0.500
CH	{0.402, 0.427, 0.448}	{0.424, 0.435, 0.446}	{0.443, 0.443, 0.443}	DE	0.488
LU	{0.357, 0.389, 0.414}	{0.382, 0.397, 0.410}	{0.405, 0.405, 0.405}	CH	0.474
DE	{0.339, 0.384, 0.423}	{0.371, 0.394, 0.414}	{0.404, 0.404, 0.404}	BE	0.466
IE	{0.360, 0.386, 0.405}	{0.382, 0.393, 0.403}	{0.401, 0.401, 0.401}	LU	0.464
FR	{0.340, 0.380, 0.408}	{0.371, 0.389, 0.404}	{0.399, 0.399, 0.399}	FR	0.450
BE	{0.318, 0.373, 0.409}	{0.359, 0.382, 0.401}	{0.392, 0.392, 0.392}	IE	0.425
CY	{0.300, 0.336, 0.362}	{0.327, 0.343, 0.357}	{0.351, 0.351, 0.351}	CY	0.415
NO	{0.291, 0.324, 0.347}	{0.316, 0.330, 0.343}	{0.337, 0.337, 0.337}	NO	0.393
UK	{0.290, 0.304, 0.315}	{0.302, 0.309, 0.314}	{0.314, 0.314, 0.314}	UK	0.324
SI	{0.250, 0.258, 0.264}	{0.257, 0.261, 0.264}	{0.264, 0.264, 0.264}	SI	0.268

all possibility levels h. Whereas for other countries, their ranking could be slightly changed when different possibility level and consideration are taken into account.

Moreover, by comparing the alcohol performance index scores of the 28 European countries with the ones from the IDEA-CI model, in which a crisp index score is achieved, we find that the FDEA-CI score is lower than the one from the IDEA-CI model, even in the optimistic situation with the lowest possibility level of h. This can be partly explained by the fact that a relatively small and constant value of ε is used in the IDEA-CI model to reflect the minimum allowable gap between the two ranking positions in terms of the indicator value, which results in an extreme index score for each country. In other words, based on the same ε value, the index score from the FDEA-CI model would not exceed the one from the IDEA-CI model. Nevertheless, a high correlation coefficient (0.989) is deduced between the IDEA-CI score and the FDEA-CI score (taking $h = 0.5$ and the indifferent situation as an example). This not only demonstrates the robustness of their ranking results, but also implies the reliability of using fuzzy ranking approach for modeling qualitative data.

5 Conclusions

In this chapter, we investigated the usage of fuzzy ranking approach in the DEA framework for modeling both quantitative and qualitative data in the context of composite indicator construction. By interpreting the qualitative indicator data as fuzzy numerical values, a fuzzy DEA-based CI model was developed, and it was further transformed into a crisp linear programming problem. The model was demonstrated by combining three alcohol indicators (two quantitative and one qualitative) into an alcohol performance index score for the 28 European countries. The analysis of the results showed that fuzzy index scores obtained based on different possibility levels were powerful in capturing the uncertainties associated with human thinking, which was therefore superior over the imprecise DEA-based CI model that only resulted in a crisp index score. However, the high similarity of the ranking result based on these two models verified its robustness and also implied the reliability of using the fuzzy ranking approach for modeling qualitative data. In the future, exploration on the dual envelopment formulation of this model and on the usage of other fuzzy techniques such as the a-level based approach and the possibility approach, are worthwhile.

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