Journal of Applied Operational Research (2014) **6**(2), 111–123

ISSN 1735-8523 (Print), ISSN 1927-0089 (Online)

Optimal policies for demand forecasting and inventory management of goods with intermittent demand

Katrien Ramaekers * and Gerrit K. Janssens

Research Group Logistics, Hasselt University, Diepenbeek, Belgium

Abstract. Demand forecasting is one of the most crucial aspects of inventory management. For intermittent demand, i.e. demand peaks follow several periods of zero or low demands, accurate forecasting is difficult. A framework is developed to support inventory management decision making for intermittent demand. Several forecasting methods and inventory management policies are compared in this framework. Because mathematical models cannot accurately describe the complex system, a simulation model is built. To determine the best combination of a demand forecasting method and an inventory management policy, a continuous tabu search metaheuristic is developed to optimize the simulation model. Depending on the experimental environment, two options for optimal strategies can be distinguished for a reliable supplier. In addition, the robustness of the results to uncertainty on the supply side is investigated.

Keywords: simulation-optimization; forecasting; inventory management; intermittent demand

Received December 2013. Accepted February 2014

Introduction

Inventory systems have to cope with uncertainty in demand. The inventory control literature mostly makes use of the Normal or Gamma distribution for describing the demand during the lead-time. The Poisson distribution has been found to provide a reasonable fit when demand is very low (only a few pieces per year). Less attention has been paid to irregular demand. This type of demand is characterized by a high level of variability, but may also be of the intermittent type, i.e. demand peaks follow several periods of zero or low demands. Syntetos *et al.* (2005) propose a theoretically coherent scheme for categorizing demand into smooth, erratic, intermittent or lumpy. In this categorization scheme, the average inter-demand interval (ADI) and the squared coefficient of variation (CV^2) of demand are compared with cutoffs of 1.32 for ADI and 0.49 for CV^2, as follows:

- Smooth demand: ADI < 1.32, $CV^2 < 0.49$;
- Erratic demand: ADI < 1.32, $CV^2 > 0.49$;
- Intermittent demand: ADI > 1.32, $CV^2 < 0.49$;
- Lumpy demand: ADI > 1.32, $CV^2 > 0.49$.

* *Correspondence*: Katrien Ramaekers, Research Group Logistics, Hasselt University, 3590 Diepenbeek, Belgium.
E-mail: katrien.ramaekers@uhasselt.be

Bartezzaghi *et al*. (1999) consider five characteristics that cause demand to be of the intermittent type: the numerousness of potential customers, the heterogeneity of customers, the frequency of customers requests, the variety of customers requests and the correlation between customers requests. In practice, items with intermittent demand include service or spare parts and high-priced capital goods. A common example of such goods are spare parts for an airline fleet. The intermittent character of demand makes accurate forecasting difficult. However, the high cost of a modern aircraft and the expense of such repairable spares constitute a large part of the total investment of many airline operators. These parts are critical to operations and their unavailability can lead to excessive downtime costs.

Croston's method is considered to be a common forecasting method for intermittent demand items (Croston, 1972). This method builds estimates taking into account both demand size and the interval between demand occurrences. Despite the theoretical superiority of such an estimation procedure, empirical evidence suggests modest gains in performance when compared with simpler forecasting techniques like exponential smoothing and (weighted) moving averages (Syntetos and Boylan, 2001). In preliminary research, the presence of interaction between demand forecasting and inventory decision making for intermittent demand has been demonstrated in a small experimental setup (Ramaekers and Janssens, 2004). In this paper, in order to generalize the results of the previous work, a framework is developed for inventory management decision support for intermittent demand. The main contributions of this paper are threefold. First, several forecasting methods and inventory management policies for intermittent demand are compared in a formal framework in case no disruptions at the supply side occur. Because mathematical models cannot accurately describe the complex system, a simulation model is built. Second, the model is optimized using a tabu search to find a best strategy in combining inventory decision making and demand forecasting for intermittent demand. Third, the robustness of the best strategy to uncertainty in supply is investigated. Both qualitative and quantitative factors need to be optimized in this model and the quantitative factors depend on the choice of the qualitative factors. For example, if exponential smoothing is used as a forecasting method, the smoothing parameter α needs to be optimized. If weighted moving averages is used as a forecasting method, the weights need to be optimized. A continuous tabu search metaheuristic is developed for the optimization of the quantitative parameters for each combination of qualitative parameters (forecasting method and inventory policy). In a next step, the best combination of qualitative factors is determined.

The organization of the paper is as follows: section 2 provides an overview of related literature; in section 3 the simulation model and optimization method are described; the experimental environment is described in section 4; section 5 discusses the results; in section 6 the robustness of the results towards uncertainty in supply is investigated and in section 7 conclusions are formulated.

Literature review

Intermittent demand is difficult to predict, and errors in prediction may be costly in terms of obsolescent stock or unmet demand (Syntetos and Boylan, 2005). It is not only the variability of the demand size, but also the variability of the demand pattern that make intermittent demand so difficult to forecast. Furthermore, selecting the right periodic inventory system for low and intermittent demand is a problem facing many organizations (Sani and Kingsman, 1997). Syntetos and Boylan (2001) state that intermittent demand creates significant problems in the manufacturing and supply environment as far as forecasting and inventory control are concerned.

The majority of the literature on intermittent demand focuses on forecasting intermittent demand accurately (e.g. Croston, 1972; Willemain *et al*., 1994; Johnston and Boylan (1996; Syntetos and Boylan, 2001; Syntetos and Boylan, 2005; Gutierrez *et al*., 2008; Teunter and Sani, 2009; Teunter and Duncan, 2009) and on methods used to compare the performance of the forecasting methods (e.g. Syntetos and Boylan, 2005; Boylan and Syntetos, 2007; Mukhopadhyay *et al*., 2012). Croston (1972) proposes forecasting intermittent demand by estimating demand sizes (when demand occurs) and inter-demand intervals separately. Syntetos and Boylan (2001) show that Croston's method is biased. Syntetos and Boylan (2005) propose a modification to Croston's estimator and demonstrate empirically the improved forecast accuracy achieved. This method is referred to as the approximation method by Syntetos and Boylan. Despite the theoretical superiority of such an estimation procedure, empirical evidence suggests modest gains in performance when compared with simpler forecasting techniques (Syntetos and Boylan, 2001). However, commonly used forecasting methods are often inappropriate when faced with intermittent demand. For example, exponential smoothing places most weight on the more recent data, giving estimates that are highest just after a demand and lowest just before a demand.

Most papers that address inventory control for intermittent demand items assume that an appropriate forecasting method is in place to estimate future demand requirements. They focus on the improvement of stock control rules and assume an accurate forecast can be provided (e.g. Silver, 1970; Ward, 1978; Schultz, 1987; Watson, 1987; Dunsmuir and Snyder, 1989; Segerstedt, 1994). Relatively simple techniques for inventory management for intermittent demand are recommended in the literature. A periodic review system is recommended for application in an intermittent demand context (Sani, 1995).

The combined performance of the system is investigated in only a few studies (e.g. Sani and Kingsman, 1997; Eaves and Kingsman, 2004; Syntetos and Boylan, 2006; Solis *et al*., 2012). These studies mostly make use of real data and predefined control parameter values. Simulation models are used. For example, Sani and Kingsman (1997) carry out a simulation using real data, requiring a single run for each data series. Syntetos and Boylan (2006) set up a simulation experiment to quantify the inventory implications of the forecasting methods (exponential smoothing, Croston, approximation method, simple moving averages), given a periodic order-up-to-level inventory model. The control parameter values of the simulation experiment are predefined. However, no attempt to find optimal values of the control parameters is made. Neither the influence of uncontrollable factors, e.g. lead time, on the optimal policy is investigated.

Simulation model and optimization method

Because of the uncertainty present in the inventory system, often mathematical models cannot accurately describe the system. Therefore, a simulation model is used in this paper. The main advantage of simulation is that most complex, real-world systems which cannot be accurately described by a mathematical model can be evaluated analytically (Law and Kelton, 1991). Since the aim in this paper is to decide on the optimal combination of forecasting method and inventory management policy, a simulation-optimization method is used. In the following subsections, the simulation model, the experimental design and the optimization method are described.

Simulation model

This paper focuses on a single-product inventory system facing demand of the intermittent type. The implementation of such an inventory control system in practice requires the following steps: (1) choosing the inventory policy; (2) choosing the performance criterion; (3) estimating the stochastic features of the system; and (4) determining the control parameter values for the inventory policies.

As *inventory policies*, two periodic review models are considered: the (R, s, S) policy and the (R, s, Q) policy. In the (R, s, S) policy, inventory is checked every R units of time. If the inventory level is at or below the reorder point s, a sufficient quantity is ordered to raise it to S. In the (R, s, Q) policy also every R units of time an order decision is made. If the inventory level is at or below the reorder point s, a fixed quantity Q is ordered such that the inventory level is raised to a value between s and $s + Q$. A deterministic lead-time of length L is assumed. The simulation starts with an initial inventory level I_0.

As *performance criterion* a cost function is defined. This means that the total costs (including shortage costs) are to be minimized. The following costs are considered: unit holding cost per period C_h, ordering cost C_O and unit shortage cost per period C_s. In the system the demand is the only *stochastic* element. To generate intermittent demand, demand occurrence and demand size are separately generated. The demand occurrence is generated according to a first-order Markov process with transition matrix

$$\boldsymbol{P} = \begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix}$$

where p_{00} is the probability of no order in the next period when there has been no order in the current period and p_{10} is the probability of no order in the next period when there has been an order in the current period. Individual order sizes are generated using a Gamma distribution with shape parameter γ and scale parameter β.

The simulation model aims to find the optimal inventory control policy, the optimal values of s, S and/or Q depending on the inventory control policy and also the optimal method to forecast the unknown demand. The standard forecasting method for intermittent demand items is considered to be Croston's method. However, in practice, single exponential smoothing and simple moving averages are often used to deal with intermittent demand. These three forecasting methods are compared. The single exponential smoothing (ES) method is easy to apply because only three pieces of data are needed to forecast the future: the most recent forecast, the most recent actual demand and a smoothing constant α. The smoothing constant determines the weight given to the most recent past observations and therefore controls the rate of smoothing or averaging. It is commonly constrained to be in the range of zero to one. The equation for ES is:

$F_t = \alpha X_{t-1} + (1 - \alpha)F_{t-1}$

where X_{t-1} is the most recent actual demand, F_t is the exponentially smoothed forecast for period t and F_{t-1} the exponentially smoothed forecast of the prior period.

The assumption of the moving average (MA) forecasting method is that a future value will equal an average of past values. The number of past values used to calculate the forecast can vary. The simple n-period moving average forecast is calculated as:

$F_t = (X_{t-n} + X_{t-(n-1)} + \ldots + X_{t-2} + X_{t-1})/n.$

Croston's method (Croston 1972) has been developed to provide a more accurate forecast of the mean demand per period. Croston's method (CR) applies exponential smoothing separately to the intervals between nonzero demands and their sizes. Let I_t be the smoothed estimate of the mean interval between nonzero demands, and let D_t be the smoothed estimate of the mean size of a nonzero demand. Let q be the time interval since the last nonzero demand. Croston's method works as follows:

if $X_t = 0$ then

$D_t = D_{t-1}$; $I_t = I_{t-1}$; $q = q+1$

else

$D_t = \alpha X_t + (1-\alpha)D_{t-1}$; $I_t = \alpha q + (1-\alpha)I_{t-1}$; $q = 1$

where α is the smoothing parameter. Combining the estimates of demand size and interval provides the forecast:

$F_t = D_t/I_t.$

These estimates are only updated when a demand occurs. When a demand occurs every period, Croston's method is identical to single exponential smoothing. The simulation model is developed in Microsoft Excel spreadsheets making use of VBA. The simulation model starts by generating intermittent demand. Next, the inventory system is simulated for 52 periods. At each review-time, a demand forecast and an order decision are made. The total costs of the inventory system are determined. Ten replications are made for each simulation run.

Experimental design

The parameters of the inventory system relate both to qualitative and to quantitative factors. The experimental design includes two qualitative factors: the forecasting method and the inventory control policy. Three forecasting methods are considered: Croston's method, single exponential smoothing and 4-period simple moving averages. Two inventory control policies are considered: the (R, s, S) policy and the (R, s, Q) policy. In addition, depending on the choice of the qualitative factors, a set of quantitative factors are part of the experimental design.

If the (R, s, Q) inventory control policy is used, the reorder point s and order quantity Q are the parameters to optimize. If the (R, s, S) inventory control policy is used, the reorder point s and order-up-to-level S are the optimizing parameters. A deterministic review period R is considered. The reorder point s can take any discrete value greater than or equal to 0. The order-up-to-level S and the order quantity Q can take any discrete positive value.

In case single exponential smoothing or Croston's method is used as a forecasting method, the smoothing parameter α is the optimizing parameter and, if moving averages is used as a forecasting method, the weights of the past values are the parameters to optimize. The smoothing parameter α can take any continuous value between 0 and 1. The weights of the past values of the moving averages forecasting method can take any continuous value between 0 and 1, under the condition that the sum of the weights equals 1.

Optimization method: Continuous tabu search

Because the quantitative factors depend on the choice of the qualitative factors, a specific optimization method is developed and described in this section. For every combination of forecasting method and inventory management policy, the optimal values of the quantitative factors are determined using a continuous tabu search metaheuristic (Glover, 1989; Dengiz and Alabas, 2000). The total costs of the inventory system are minimized: the unit holding cost, the ordering cost and the unit shortage cost. The specifics of tabu search in the case of continuous variables is shortly described below. Once the near-optimal values of the quantitative factors are found for every combination of the qualitative factors, the best combination of forecasting method and inventory management policy is chosen.

In previous research (Ramaekers, 2009), several simulation-optimization methods are compared: Design of Experiments (Taguchi), Response Surface Methodology and meta-heuristics (tabu search). Tabu search leads to better and more accurate results. Tabu search is a heuristic optimization technique which is developed specifically for combinatorial optimization problems. Few works deal with the application to the global minimization of functions depending on continuous variables. Hu (1992) is the first to adapt tabu search to continuous optimization. However, the algorithm of Hu (1992) is rather far from original tabu search. Siarry and Berthiau (1997) propose an adaptation of tabu search to the optimization of continuous functions where the purpose is to keep as close as possible to original simple tabu search. As neighborhood of the current solution, they perform a partition of the space around the current solution using a set of concentric balls. Inside each ball, a random neighbor is selected. The tabu list contains *m* balls, corresponding to the immediate neighborhoods of the *m* last retained solutions. Chelouah and Siarry (2000) improve the algorithm of Siarry and Berthiau (1997) and propose an Enhanced Continuous Tabu Search for the global optimization of continuous functions. They replace the balls by hyper-rectangles for the partition of the current solution neighborhood and add diversification and intensification concepts to the algorithm. Our method is based on Siarry and Berthiau (1997) and Chelouah and Siarry (2000). In Figure 1, a general flow-chart of the TS algorithm is shown. Two issues have to be examined: the generation of current solution neighbors and the elaboration of the tabu list.

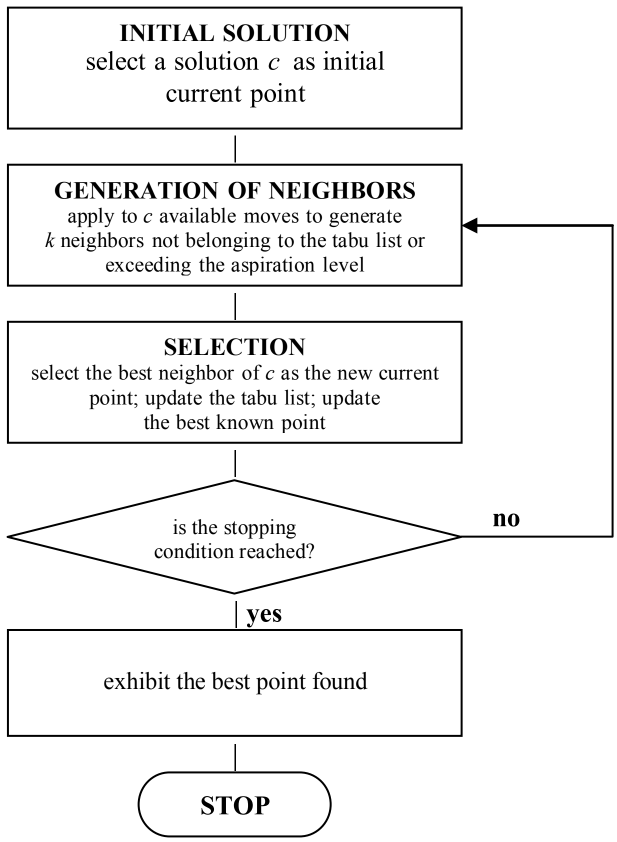

Fig. 1. General flow chart of tabu search

To define a neighborhood of the current solution *c*, a set of hyper-rectangles is used for the partition of the current solution neighborhood. The *k* neighbors of the current solution are obtained by selecting, at random, a single point inside each hyper-rectangular zone. In Figure 2, a two-dimensional example of such a partition for $k = 4$ neighbors of the current solution is shown.

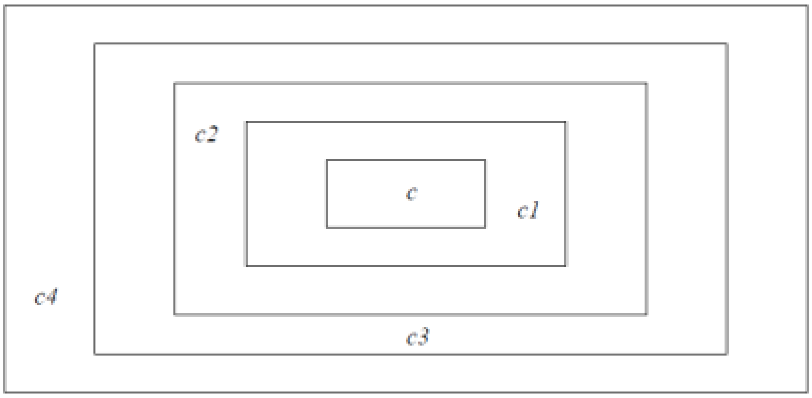

Fig. 2. Partition of current solution neighborhood

Once a new current solution is determined, the immediate neighborhood of the previous solution is added to the tabu list. This immediate neighborhood is also a hyper-rectangle. The tabu list contains *m* hyper-rectangles corresponding to the *m* last retained solutions. A solution belonging to the tabu list can lose its tabu status if its objective value is higher than the aspiration level.

The tabu search algorithm optimizes the quantitative parameters α, s, S and Q, depending on the forecasting method and inventory policy. A neighborhood consists of five neighbors and the tabu list contains five tabu areas. Ten simulation runs are made for each experimental choice. The tabu search is stopped after 200 iterations. The robustness of the results of the tabu search has been tested by repeating the algorithm several times. Each replication led to the same result. The continuous tabu search algorithm was implemented in VBA and runs for about 30 minutes on an Intel Core 2.3 GHz laptop with 4 GB RAM.

Experimental environment

The experimental environment contains the uncontrollable factors of the inventory system: the costs of the inventory system and the parameters for generating intermittent demand. These factors can have an effect on the results. The optimization method described above, is executed using a single combination of the costs of the inventory system and demand. A fractional factorial design of 16 experimental points is set up for these uncontrollable factors and the optimization phase is repeated for each experimental point. A fractional factorial design is chosen to get good estimates of the effects of the uncontrollable factors at a fraction of the computational effort required for a full factorial design.

Demand occurrence is generated using a first-order Markov process with transition matrices:

$$\boldsymbol{P_1} = \begin{bmatrix} 0.7875 & 0.2125 \\ 0.85 & 0.15 \end{bmatrix}$$

or

$$\boldsymbol{P_2} = \begin{bmatrix} 0.5667 & 0.4333 \\ 0.65 & 0.35 \end{bmatrix}$$

The matrices correspond to a probability of 0.2 that a demand occurs in a certain period for the matrix $\boldsymbol{P_1}$ and to a probability of 0.4 that a demand occurs in a certain period for the matrix $\boldsymbol{P_2}$. The *size of demand* is generated using a Gamma distribution with 4 different combinations of the scale parameter γ and the shape parameter β. The values of the combinations are shown in Table 1. Levels 1 and 3 lead to an expected order size of 6. Levels 2 and 4 lead to an expected order size of 12. The levels of the *costs* of the inventory system are given in Table 2. The initial inventory level $I_0 = 5$. The deterministic lead time $L = 1$.

Table 1. Parameters of the Gamma distribution

Level	γ	β
1	6	1
2	12	1
3	3	2
4	24	0.5

Table 2. Levels for the costs of the inventory system

Level	C_o	C_h	C_s
1	100	2	5
2	200	4	10

The fractional factorial design is shown in Table 3. This fractional factorial design makes it possible to determine the impact of the uncontrollable factors as the cost structure and the demand on the optimal strategy in inventory decision making and demand forecasting for intermittent demand. Although only 16 experimental points are investigated in this paper, the results can be generalized to draw conclusions with respect to an optimal strategy in combining inventory decision making and demand forecasting for intermittent demand since the levels of costs can be seen in relation to each other rather than as absolute values.

Table 3. Experimental design for uncontrollable factors

Experiment	C_o	C_h	C_s	*Freq*	γ	β
1	200	4	10	0.4	12	1
2	100	4	5	0.4	12	1
3	200	2	5	0.4	24	0.5
4	100	2	10	0.4	24	0.5
5	200	2	5	0.4	3	2
6	100	2	10	0.4	3	2
7	200	4	10	0.4	6	1
8	100	4	5	0.4	6	1
9	200	2	10	0.2	12	1
10	100	2	5	0.2	12	1
11	200	4	5	0.2	24	0.5
12	100	4	10	0.2	24	0.5
13	200	4	5	0.2	3	2
14	100	4	10	0.2	3	2
15	200	2	10	0.2	6	1
16	100	2	5	0.2	6	1

Discussion of the results for a reliable supplier

Each run of a single experiment from the fractional factorial design leads to a best inventory policy, together with its set of near-optimal parameter values, and to a best forecasting method, together with its set of near-optimal parameter values. This section discusses the results of the reliable supplier. The results can be found in Table 4. This section aims to investigate which design factors have an influence on the choice of inventory policy and forecasting method. The levels of the design factors can be found in the fractional factorial design in Table 3. In this section, the results of Table 4 are analyzed to determine the influence of the design factors in Table 3. For example, for the first eight experimental points, the demand frequency is high and for the last eight experimental points, the demand frequency is low. The results in Table 4 are studied in order to find an effect of this difference in demand frequency. An attempt is made to simplify and structure these findings in a limited set of rules, which are generated by a classification tree.

Table 4. Optimal results based on tabu search for a reliable supplier

Experiment	*Best strategy*		
1	MA/FOQ	*ROP=0*	*Q=25*
2	ES/OUL	*s=0*	*S=1*
3	MA/OUL	*s=0*	*S=30*
4	MA/OUL	*s=0*	*S=25*
5	MA/FOQ	*s=0*	*Q=20*
6	MA/OUL	*s=0*	*S=15*
7	MA/FOQ	*s=0*	*Q=15*
8	ES/OUL	*s=0*	*S=1*
9	MA/FOQ	*s=0*	*Q=20*
10	ES/OUL	*s=0*	*S=1*
11	CR/OUL	*s=0*	*S=1*
12	ES/OUL	*s=0*	*S=1*
13	MA/OUL	*s=0*	*S=1*
14	MA/OUL	*s=0*	*S=1*
15	MA/OUL	*s=0*	*S=15*
16	ES/OUL	*s=0*	*S=1*

Eight experimental points have the order-up-to-level inventory management (OUL) policy with S = 1 as best strategy but with various best forecasting methods. For the other eight experimental points, the best strategy is an OUL-inventory management policy with $S \geq 15$ or a fixed order quantity (FOQ) model with $Q \geq 15$. An FOQ-inventory management policy as best goes together with the moving averages (MA) method as best forecasting method. Also in case the OUL-inventory management policy with $S \geq 15$ is best, MA shows to be the best forecasting method. In case the OUL-inventory management policy with S=1 is best, no specific forecasting method is preferred. The results also indicate that the parameters of the forecasting method have no significant impact on the results. In the next paragraphs, the influence of the uncontrollable factors on the results is examined in further detail.

To investigate the impact of the *demand frequency*, the results are compared for both levels of the demand frequency (column Freq in Table 3). When the demand frequency is generated using matrix $\boldsymbol{P_1}$, corresponding to a probability of 20% of having a non-zero demand in a certain period, an order-up-to-level S of 1 unit is near-optimal. When the demand frequency is generated using matrix $\boldsymbol{P_2}$, which corresponds to a probability of 40% of having demand in a certain period, the order-up-to level S or fixed order quantity Q has a value between 15 and 30. This can be explained because the intermittent character of demand is more outspoken with a low probability of non-zero demand, leading to a near-optimal order-up-to level S = 1 unit. When the intermittent character of demand is less outspoken (40%), it is better to order a quantity of at least 15 units. The only exception to this order-up-to-level S = 1 unit for a demand probability of 20% occurs when both the ordering cost and the unit shortage cost are high and the unit holding cost is low. In these circumstances it is better to order a larger quantity because it is less costly to hold inventory than to have a stock-out or to order a small quantity every time. Inversely, when a demand probability of 40% is used, it is better to use an order-up-to-level S = 1 unit when both the ordering cost and the unit shortage cost are low and the unit holding cost is high. The same reasoning as above can be made here. It can also be noted that when the demand frequency is doubled, Croston's method becomes less useful as forecasting method. When the results are compared for changing the parameters of the *demand size*, no significant impact of these changes can be detected. This means the only impact of demand is due to the demand frequency, and not on the size of demand (when it occurs).

Changes in the cost structure of the inventory system have a significant impact on the results. When the *ordering cost* is at its low level, an OUL-inventory management policy is used with the order-up-to-level S = 1, except when the unit holding cost is low, the unit shortage cost is high and the demand probability of a certain period equals 40%. The latter factor combination favors holding more units in inventory. It changes the best policy to a policy with an order-up-to level S or fixed order quantity between 15 and 30. When the ordering cost is at its high level, the order-up-to-level S or fixed order quantity Q is between 15 and 30, except when the unit holding cost is high, the unit shortage cost is low and the probability of a non-zero demand in a certain period equals 20%. In a certain period, as an explanation, the opposite reasoning of above can be used.

When the *unit holding cost* is at its low level, an order-up-to-level S or fixed order quantity Q between 15 and 30 is best, unless both the ordering cost and the unit shortage cost are low and the probability of a non-zero demand in a certain period equals 20%. With this combination of factor levels, an inventory policy with an order-up-to-level S = 1 is better used because this combination favors lower inventory levels. When the unit holding cost is at its high level, an order-up-to-level S = 1 is the best choice, unless the ordering cost and unit shortage cost are high and the probability of a non-zero demand in a certain period is 40%. This combination of factor levels favors a higher inventory level and, by this, an order-up-to-level Q or fixed order quantity S between 15 and 30 is better used.

A *unit shortage cost* at its low level implies an order-up-to-level S = 1 unit, except when the unit holding cost is low and the probability of a non-zero demand in a certain period equals 40%. When the shortage cost is low, it is not necessary to keep a high level of inventory. Therefore, an order-up-to-level S = 1 is the best policy. However, if the holding cost is low and the intermittent character of demand is not so outspoken, it is better to have more units in inventory. A unit shortage cost at its high level leads to an order-up-to-level S or a fixed order quantity Q between 15 and 30, except when the unit holding cost is high and the probability of a non-zero demand in a certain period equals to 20%. The same reasoning as before can be used to explain this exception.

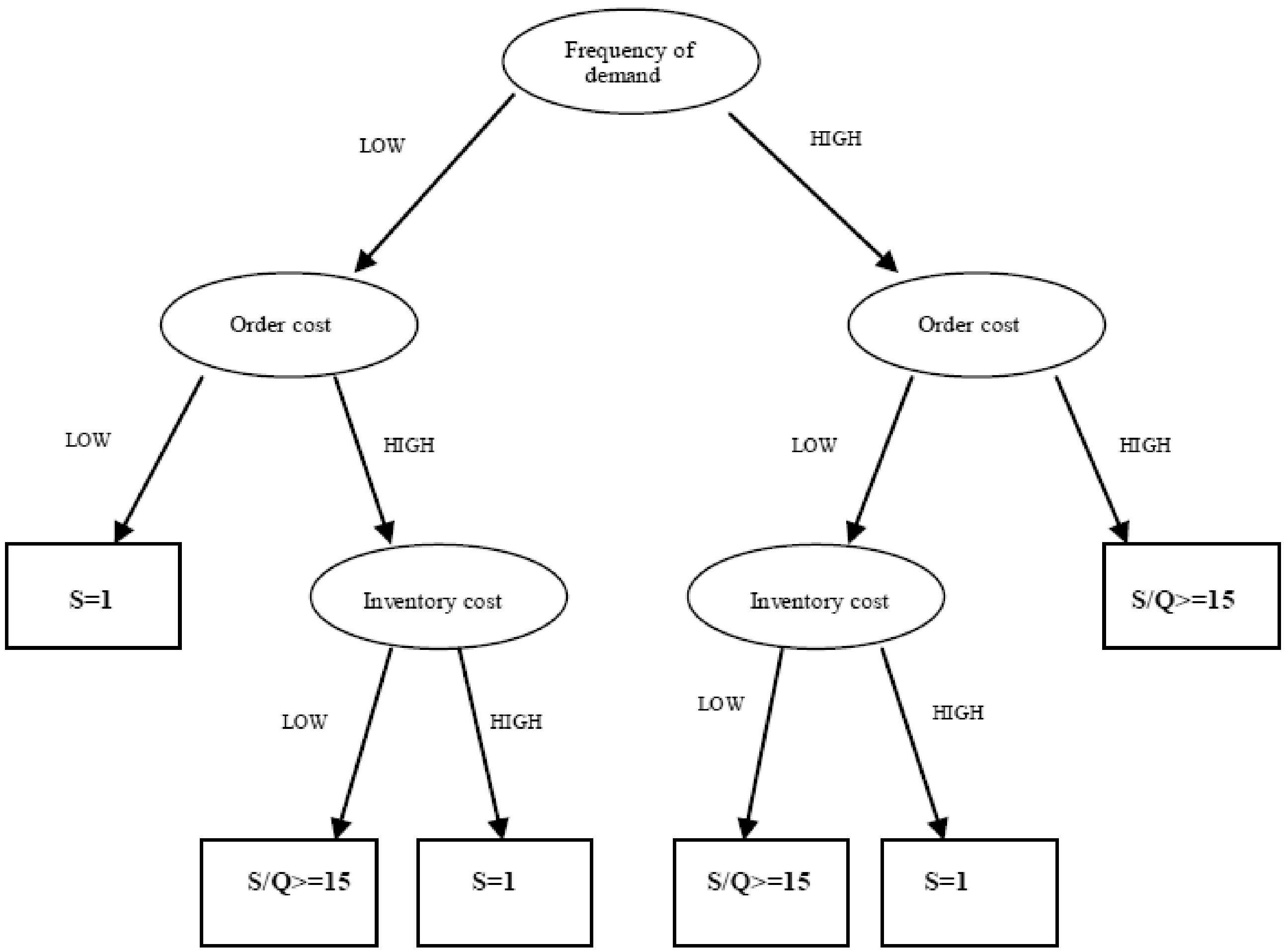

Fig. 3. Classification tree

Overall, it can be concluded that the uncontrollable factors have an impact on the best strategy on the combination of an inventory management policy and a demand forecasting method for intermittent demand. Furthermore, some interactions between the uncontrollable factors have been observed. To study this interaction in more detail, a *classification tree* is constructed using the C4.5 algorithm, a well-known algorithm in data mining (Quinlan (1993)). The classification tree, generated by the algorithm with the use of the simulation data, is represented in Figure 3. Using this tree, it can be decided which of both strategies is best: an order-up-to-level inventory management policy with $S = 1$ or an order-up-to-level inventory management policy with $S \geq 15$ or a fixed order quantity model with $Q \geq 15$. Three factors are required to determine the best strategy in combining inventory decision making and demand forecasting: the frequency of demand, the order cost and the inventory cost. If one of these three factors is unknown, the knowledge of the stock-out cost can be used to make a classification. Summarizing, it can be stated that if three factors out of four factors (frequency of demand, order cost, inventory cost and stock-out cost) are fixed, the best strategy is formulated.

A good classification is necessary because there is a considerable increase in the costs of the inventory system when using an alternative strategy. When a fixed order quantity inventory management policy with $Q \geq 15$ is used instead of an order-up-to-level inventory management policy with $S = 1$, total costs are on average 20% higher. In the opposite case, when an order-up-to-level inventory management policy with $S = 1$ is used instead of an order-up-to level inventory management policy with $S \geq 15$ or a fixed order quantity model with $Q \geq 15$, total costs increase with more than 40% on average.

Discussion of the results for an unreliable supplier

Studies in literature mainly focus on uncertainty in demand. In many inventory models the continuous availability of supply at any time in the future is an implicit assumption. However, uncertainty may be present at the supply side too. This type of uncertainty occurs in delivery time, in interruption of delivery during a certain period, or in mismatches in order and delivery in order and delivery in terms of quality or quantity. In this section, the focus is on uncertainty in availability. The supplier alternates randomly between an available and an unavailable state. When the supplier is available, the order is delivered after the usual lead time. When the supplier is unavailable, the order is executed when the supplier turns available again. In the simulation model, uncertainty in supply is randomly generated. In every period, there is 20% chance that the supplier is unavailable. If the supplier is unavailable, the order is delivered one lead time after the supplier becomes available again.

First, the strategies found in Table 4 are also used to determine output measures for the inventory system with an unreliable supplier. Table 5 contains 95% confidence intervals (CI) for the difference in total costs between a reliable and an unreliable supplier. For eight of the experimental points, the difference in costs between the reliable and the unreliable case is significant. The alternative with the unreliable supplier has higher total costs than the one with the reliable supplier. The experimental points with a significant difference in costs are those experimental points for which the best strategy for a reliable supplier has an order-up-to-level $S = 1$. For the other experimental points, the best strategy for a reliable supplier has an order-up-to-level or fixed order quantity $Q \geq 15$.

Table 5. Confidence intervals for comparing costs of a reliable and an unreliable supplier

Experiment	*Confidence interval*	
1	-150.68	149.88
2	-516.98	-248.38
3	-74.65	97.89
4	-135.08	-5.64
5	-25.81	144.886
6	-65.95	74.03
7	-103.07	139.35
8	-369.92	-144.44
9	-117.40	131.08

Experiment	*Confidence interval*	
10	-359.40	-124.36
11	-679.87	-279.44
12	-597.74	-290.24
13	-459.96	-108.36
14	-507.49	-263.19
15	-100.16	91.3
16	-232.57	-33.19

Next, a new best combination of forecasting method and inventory management policy is determined for the inventory management system with intermittent demand and an unreliable supplier. Furthermore, the near-optimal settings for the safety stock and the fixed order quantity or order-up-to-level are determined.

Table 6 shows the best strategy for the inventory system with an unreliable supplier. For these eight experimental points, the best strategy for the inventory system with intermittent demand and no uncertainty in supply is an order-up-to-level inventory management policy with $S = 1$. When the results in Table 6 are compared to the results in Table 4, the experimental points can be divided in two categories. For the experimental points 2, 8, 13 and 16, the best strategy is an inventory management policy with a fixed order quantity $Q \geq 10$. When a fixed order quantity inventory management policy is used, the moving averages method is always the best forecasting method. For the experimental points 10, 11, 12 and 14, the best strategy is an order-up-to-level inventory management policy with $S = 1$, which is the same strategy as found for the inventory system with a reliable supplier. When the order-up-to-level $S = 1$ is the best inventory management policy, no specific forecasting method is preferred.

Table 6. Results of tabu search for an unreliable supplier

Experiment	*Best strategy*		
2	MA/FOQ	*s=0*	*Q=18*
8	MA/FOQ	*s=0*	*Q=12*
10	ES/OUL	*s=0*	*S=1*
11	CR/OUL	*s=0*	*S=1*
12	MA/OUL	*s=0*	*S=1*
13	MA/FOQ	*s=0*	*Q=10*
14	ES/OUL	*s=0*	*S=1*
16	MA/FOQ	*s=0*	*Q=10*

Conclusions

When demand is of the intermittent type, a framework is developed for inventory management decision support. A best strategy in combining inventory decision making and demand forecasting is proposed, using a simulation model. An experimental design is set up to determine the impact of the cost structure and the demand. Depending on the experimental environment, two options for best strategies can be distinguished: an order-up-to level inventory management policy with an order-up-to level equal to 1 and a reorder point equal to 0 or an inventory management policy with a fixed order quantity $Q > 1$ or an order-up-to level $S > 1$ and a reorder point equal to 0. Four factors of the experimental environment have an influence on which of the two strategies is best chosen: the frequency of demand, the inventory holding cost, the order cost and the stock-out cost. When the level of three factors out of these four are fixed, it is possible to determine the best strategy.

It is important to know which of both strategies is best because there is a significant increase in total costs of the inventory system if the wrong strategy is chosen. Although only an incomplete number of experimental points are investigated in this paper, the results can be generalized to draw conclusions with respect to a best strategy in combining inventory decision making and demand forecasting for intermittent demand since the levels of costs can be seen in relation to each other rather than as absolute values. The robustness of the results to uncertainty in the supply side is investigated. One specific type of uncertainty in supply is considered: uncertainty in availability. This uncertainty in availability causes a significant difference in performance measures compared to the reliable situation. For intermittent demand, uncertainty in availability leads to significantly higher total costs for those experimental points that led to the optimal policy of an order-up-to level equal to 1 without uncertainty in availability. A new best strategy in combining inventory decision making and demand forecasting is determined for these points: a fixed order quantity inventory management policy with a fixed order quantity $Q > 1$ and a reorder point equal to 0.

References

Bartezzaghi, E., Verganti, R., Zotteri, G., 1999. A simulation framework for forecasting uncertain lumpy demand. International Journal of Production Economics 59, 499–510

Boylan, J.E., Syntetos, A.A., 2007. The accuracy of a modified Croston procedure, International Journal of Production Economics, 107, 511-517.

Chelouah, R., Siarry, P., 2000. Tabu search applied to global optimization. European Journal of Operational Research 123, 256–270.

Croston, J.D., 1972. Forecasting and stock control for intermittent demands, Operational Research Quarterly, 23, 289-303.

Dengiz, B., Alabas, C., 2000. Simulation optimization using tabu search. Proceedings of the 2000 Winter Simulation Conference, 805–810.

Dunsmuir, W.T.M., Snyder, R.D., 1989. Control of inventories with intermittent demand, European Journal of Operational Research, 40, 16-21.

Eaves, A.H.C., Kingsman, B.G., 2004. Forecasting for the ordering and stock-holding of spare parts, The Journal of the Operational Research Society, 55(4), 431-437.

Glover, F., 1989. Tabu search - part I. ORSA Journal on Computing 1 (3), 190–206.

Gutierrez, R.S., Solis, A.O., Mukhopadhyay, S., 2008. Lumpy demand forecasting using neural networks, International Journal of Production Economics, 111: 409-420.

Hu, N., 1992. Tabu search method with random moves for globally optimal design. International Journal for Numerical Methods in Engineering 35, 1055–1070.

Johnston, F.R., Boylan, J.E., 1996. Forecasting for items with intermittent demand, The Journal of the Operational Research Society, 47(1), 113-121.

Law, A.M., Kelton, W.D., 1991. Simulation Modeling and Analysis. McGraw-Hill International Editions, Singapore.

Mukhopadhyay, S., Solis, A.O., Gutierrez, R.S., 2012. The accuracy of non-traditional versus traditional methods of forecasting lumpy demand, Journal of Forecasting, 31, 721-735.

Ramaekers, K., Janssens, G.K., 2004. The interaction between forecasting methods and inventory management policies for intermittent demand. In: Bruzzone, A., Frydman, C., Giambiasi, N., Mosca, R. (Eds.), Proceedings of the International Mediterranean Modelling Multiconference, Bergeggi, Italy, pp. 370–375.

Ramaekers, K., 2009. A simulation-optimisation approach for inventory man¬agement decision support based on incomplete information. 4OR: A quarterly Journal of Operations Research 7(1), 93-96.

Sani, B., 1995. Periodic inventory control systems and demand forecasting methods for low demand items. Ph.D. thesis, Lancaster University, UK.

Sani, B., Kingsman, B.G., 1997. Selecting the best periodic inventory control and demand forecasting methods for low demand items, The Journal of the Operational Research Society, 48(7), 700-713.

Schultz, C.R., 1987. Forecasting and inventory control for sporadic demand under periodic review, Journal of the Operational Research Society, 38, 453-458.

Segerstedt, A., 1994. Inventory control with variation in lead times, especially when demand is intermittent, International Journal of Production Economics, 35, 365-372.

Siarry, P., Berthiau, G., 1997. Fitting of tabu search to optimize functions of continuous variables. International Journal of Numerical Methods in Engi¬neering 40, 2449–2457.

Silver, E.A., 1970. Some ideas related to the inventory control of items having erratic demand patters, Canadian Operational Research Society Journal, 8, 87-100.

Solis, A.O., Nicoletti, L., Mukhopadhyay, S., Agosteo, L., Delfino, A., Sartiano, M., 2012. Intermittent demand forecasting and stock control: an empirical study, Proceedings of the International Conference on Modelling and Applied Simulation, 367-374.

Syntetos, A.A., Boylan, J.E., 2001. On the bias of intermittent demand estimates, International Journal of Production Economics, 71, 457-466.

Syntetos, A.A., Boylan, J.E., 2005. The accuracy of intermittent demand estimates, International Journal of Forecasting, 21, 303-314.

Syntetos, A.A., Boylan, J.E., 2006. On the stock control performance of intermittent demand estimators, International Journal of Production Economics, 103, 36-47.

Syntetos, A.A., Boylan, J.E., Croston, J.D., 2005. On the categorization of demand patterns, The Journal of the Operational Research Society, 56(5), 495-503.

Syntetos, A.A., Babai, M.A., Dallery, Y., Teunter, R., 2009. Periodic control of intermittent demand items: theory ad empirical analysis, Journal of the Operational Research Society, 60(5), 611-618.

Teunter, R., Sani, B., 2009. On the bias of Croston's forecasting method, European Journal of Operational Research, 194, 177-183.
Teunter, R., Duncan, L., 2009. Forecasting intermittent demand: a comparative study, Journal of the Operational Research Society, 60, 321-329.
Ward, J.B., 1978. Determining re-order points when demand is lumpy, Management Science, 24, 623-632.
Watson, R.B., 1987. The effects of demand-forecast fluctuations on customer service and inventory cost when demand is lumpy, Journal of the Operational Research Society, 38, 75-82.
Willemain, T.R., Smart, C.N., Schwarz, H.F., 2004. A new approach to forecasting intermittent demand service parts inventories, International Journal of forecasting, 20, 375-387.