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INTEGRATED INVENTORY-TRANSPORTATION PROBLEMS WITH COMMON CARRIER FREIGHT CHARGES

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ABSTRACT

Inventory management models aim to find an optimal solution in terms of minimal costs by proposing decisions on delivery quantities using data on order quantity purchasing costs and holding costs. Goods have to be transported from the supplier to the customer and the transport (freight charge) cost can be included in various ways. Some production companies make use of a private carrier, offering transport services only for one or a few companies. Most companies make use of the services of a common carrier, which offers its transport service to whatever customer in need for transporting goods. A common carrier offers its transport services mostly in terms of less-than-truckload tariffs. Those tariffs depend on shipment sizes, but also on commodity class (representing ease of handling or risk of shipping) and on shipment distance. These additional costs make the integration of ordering and shipment decision problem more complex. In this paper, after the literature review, the optimal solutions are investigated for four different inventory model types with or without backlogging using various types of cost functions in combination with a number of freight charge models. For most combinations no analytical results are available but the optimal solutions can be easily found using spreadsheets based on our formulas. In this way the models are kept accessible to practitioners without the need for a complex software.

INTRODUCTION AND LITERATURE REVIEW

The service objectives, as put forward by a company, are influenced by several logistics strategies: an inventory strategy, a transport strategy and a location strategy. Decisions with respect to those three strategies many times are taken independently of each other, while it has been considered that interactions between the strategies may play an important role. Transportation many times is the most important single element in the logistics costs in many companies. The movement of freight might absorb between one-third and two-thirds of total logistics costs (Ballou, 2004, chapter 6).

In this study a closer look is taken how inventory and transportation decisions might interact. The link between transport and inventory in literature many times is related to the choice of transport mode. In their frequently cited article Baumol and Vinod (1970) try to make a trade-off between speed of transport and cost by introducing, what is later known in literature as the inventory-theoretic approach. When lead times are uncertain, maybe due to uncertainty in travel times, decisions on modes of transport or transportation quantities interplay with order size and safety stocks. Tyworth (1991, 1992) has developed some models including uncertainties both in sales and in lead times.

Inventory decisions made without taking into account transportation costs would fail to take advantage of the economies of scale in shipping. Since the work by Baumol and Vinod (1970) more research has been done in determining inventory policies when transportation costs are included (e.g. Langley (1980), Larson (1988), Tyworth and Zeng (1998)).

Transportation costs or freight charges are brought into decision models in various ways, depending on the organisation of transport by the company and on the structure of the freight charges. With respect to the structure, freight charges may be related to shipment size (weight), shipment distance (rate basis), and commodity type (class). With respect to the organisation, a shipper may provide his own transportation as a private carrier. The alternative is that a shipper hires a common carrier. A common carrier transports goods for any person or company and is responsible for any loss of goods during transport. A common carrier provides two basic types of service: less-than-truckload (LTL), and full truckload (TL). An LTL service is rendered for shipments which are small enough to require consolidation with other shipments at the carrier's terminal facility. A TL service is rendered for shipments which are large enough to be shipped directly from origin to destination by the same truck.

MODEL FORMULATION

This section introduces a number of models, which belong to a class of stationary lot sizing problems, like in Muth and Spremann (1978). They all make use of the following assumptions:

- there is only a single product,
- the demand rate is constant over time,
- the production capacity is constant in time,
- the production cost, or alternatively the ordering cost is an affine linear function of the quantity,
- the inventory cost per unit time is proportional to the amount of inventory,
- there are no restrictions on inventory capacity,
- the backlog cost is either proportional to the backlog or proportional to both the backlog and the time delay caused by it.

Before constructing a mathematical formulation, we first define notations used in this paper as follows:

q is the production lot size,
k is the setup (ordering) cost per order,
d is the demand rate,
m is the marginal production cost,
h is the inventory carrying unit cost per unit time,
$AC(.)$ is the average total cost per item for a model type,
P is the production rate,
s is the amount of backlog,
p is the penalty cost per unit of backlog,
c_t is the freight charge units per weight unit,
F is cost of shipping in currency units per shipment,
V is shipment size in weight units,
n is the number of shipments per period,
f is a function of basis and class of commodity.

This paper considers four different inventory models which have been widely studied in literature:

Model A: Instantaneous production without backlogging,

Model B: Uniform production without backlogging,

Model C: Instantaneous production with backlogging (penalty cost is proportional to backlog),

Model D: Instantaneous production with backlogging (penalty cost is proportional to backlog and time delay).

Let, for a production or replenishment lot size q, the production/ replenishment cost be written as
$$C_p(q) = k + m.q$$
and the production lot size and transport lot size connected by
$$q = n.V$$

The average costs for the various types of models are obtained as follows (in which the freight charge per item c_t is not yet modeled in further detail). The average costs for Models A, B, C, and D are written as:

for model type A
$$AC(q,n) = \frac{k}{q} + m + \frac{hq}{2d}\left(1 - \frac{1}{n}\right) + c_t$$

for model type B
$$AC(q,n) = \frac{k}{q} + m + \frac{hq}{2d}\left(1 - \frac{1}{n}\right) - \frac{hq}{2P}\left(1 - \frac{2}{n}\right) + c_t$$

for model type C
$$AC(q,s,n) = \frac{k}{q+s} + m + \frac{hq^2}{2d(q+s)}\left(1 - \frac{q+s}{n}\right) + \frac{ps}{q+s} + c_t$$

for model type D
$$AC(q,s,n) = \frac{k}{q+s} + m + \frac{hq}{2d}\left(\frac{q}{q+s} - \frac{1}{n}\right) + \frac{ps}{2d}\left(\frac{s}{q+s} + \frac{1}{n}\right) + c_t$$

In order to integrate transportation and inventory decisions, transportation charge models have to be formulated. Those models are based on freight rates offered by common carriers.

Each item which is transported is a commodity. As it is impossible to offer tariffs for each individual commodity, a classification system has been introduced. Commodities of a similar nature are grouped in commodity classes based on characteristics like the commodity's density, load ability, value, and susceptibility to damage. Less-than-truckload tariffs, in reality, consist of tables. By commodity class and by rate basis, rates exist for a range of shipment sizes. Five different freight charge models are considered. First they are formulated while an explanation towards reality follows afterwards.

Model 1: $c_t = $ constant A

Model 2: $c_t = \dfrac{A}{V^\alpha} = A\left(\dfrac{n}{q}\right)^\alpha$

Model 3: $c_t = \dfrac{f(basis, class)}{V^\alpha}$

Model 4: $c_t = a^V \dfrac{A}{V^\alpha} = Aa^V\left(\dfrac{n}{q}\right)^\alpha$

Model 5: $c_t = B + \dfrac{A}{V} = B + A\left(\dfrac{n}{q}\right)$

The simplest charge model, Model 1, relates to contract or private carriage in which a fixed cost F for a shipment is charged for one type of product from an origin to a destination. A common carrier may consolidate shipments to reduce the transportation cost per unit weight. This phenomenon is illustrated in Model 2. The value of α may be interpreted as a scale economy factor. If α > 0, the transportation cost declines with shipment size, so scale economies exist.

In practice, LTL tariffs consist of a series of tables. Each table has rates organized by commodity class and rate basis. This combination is expressed in the function f appearing in Model 3. A *commodity class* is an index number given to a commodity based upon various factors like weight per volume unit, ease in handling, or risk of shipping. The classifications are in the US published by the National Motor Freight Classification. A *rate basis* is an estimate of the shipment distance. It is calculated from the principal tonnage points within areas of about 40 square miles. The rate basis numbers are published in tables similar to distance tables in a road map.

In total 20 models may be considered which form the combination of a cost of inventory model and a freight charge model, for example Model A_1 combines Model A with Model 1.

The objective of these models is to find the optimal order quantity q^*, number of shipping n^*, and amount of backlog s^* (if any) by minimizing average total cost $AC(.)$ for each model.

RESULTS AND DISCUSSION

Specific analytical findings

In *Model A_1*, the freight charge is constant. Then it can be easily shown that $AC(q,n)$ is increasing in n. When the value of n is fixed, the optimal order quantity can be written as:

$$q^* = \sqrt{\frac{2kd}{h\left(1-\frac{1}{n}\right)}}$$

When $n = 1$, the optimal lot size q^* approaches to ∞. On the other hand, when n is large, the optimal lot size approaches to

$$q^* = \sqrt{\frac{2kd}{h}}$$

which is the optimal solution in the basic EOQ model.

In *Model B_1*, we have

$$q^* = \sqrt{\frac{2kd}{h}}\sqrt{\frac{P}{P\left(1-\frac{1}{n}\right)-d\left(1-\frac{2}{n}\right)}}$$

In *Model C_1*, it can be shown that $s^* = 0$. Thus, average total cost $AC^*(q,s,n)$ does not depend on the shortage cost, p. In addition, the optimal solutions in this model reduce to Model A_1.

In *Model D_1*, the optimal solutions are

$$q^* = \begin{cases} \sqrt{\dfrac{2kd}{h\left(1-\dfrac{1}{n}\right)}} & \text{for } p > h(2n-1) \\[3ex] \sqrt{\dfrac{2kd}{(h+p)(4n^2hp-(h+p)^2)}\left(\dfrac{2np+p+h}{2nh-h-p}\right)} & \text{for } p \le h(2n-1) \end{cases}$$

$$s^* = \begin{cases} 0 & \text{for } p > h(2n-1) \\[3ex] \sqrt{\dfrac{2kd}{(h+p)(4n^2hp-(h+p)^2)}} & \text{for } p \le h(2n-1) \end{cases}$$

When n approaches ∞, the optimal solution becomes

$$q^* = \sqrt{\frac{2kd}{h}}\sqrt{\frac{p}{p+h}}, \quad s^* = \sqrt{\frac{2kd}{h}}\sqrt{\frac{h}{p+h}}$$

The optimal solutions for the other model combinations cannot be obtained explicitly.

Numerical simulation of other models

Numerical examples are considered in order to investigate how the optimal order quantity q^*, optimal shipment size n^*, and optimal backlog s^* change for different sets of parameters. We consider seven different scenarios. The parameters for each scenario are as shown in Table 1.

Scenario	k	m	h	d	A
1	100	1	20	10,000	1,000
2	10	1	2	10,000	1,000
3	10	1	2	10,000	4.66
4	10	1	2	10,000	11.7
5	10	1	1	10,000	11.7
6	1000	1	20	10,000	1,000
7	10	1	20	10,000	1,000

Table 1: Parameter values for scenario simulation

The value of α is varied between 0.4 and 3 and $a = \{0.5, 1, 1.5, 2, 2.5, 2.7, 3, 3.5\}$. The production rate, P, is varied between 15,000 and 25,000 in Models B. The penalty cost, p, is varied between 40 and 480 in Models C and D. The conclusions of the simulation are shown in Table 2.

Also some graphical results are shown for specific models with limited comments. They include optimal cost versus n for model B_1, optimal lot size versus n for models B_2, and optimal backlog versus n for model A_2.

Model	AC(n)	q*(n)	s*(n)	Model	AC(n)	q*(n)	s*(n)
A_1	↑ in n	↓ in n	-	B_1	↓ in n (for P<2d) ↑ in n (for P>2d)	↑ in n (for P<2d) ↓ in n (for P>2d)	-
A_2	↑ in n	see fig in n	-	B_2	convex or ↑ in n	convex or ↑ in n	-
A_4	convex in n	↑ in n	-	B_4	↑ or ↓ in n	convex or ↑ in n	-
A_5	↑ in n	see fig in n	-	B_5	↑ in n	↑ in n	-
Model	AC(n)	q*(n)	s*(n)	Model	AC(n)	q*(n)	s*(n)
C_1	↑ in n	↓ in n	0	D_1	↑ in n	↓ in n	non-decreasing in n
C_2	↑ in n	↑ in n	0	D_2	↑ in n	see Fig. in n	↑ in n
C_4	↑ or ↓ in n	↑ in n	0	D_4	convex or ↑ in n (a>1)	see Fig. in n (a<1) ↑ in n (a>1)	↑ in n (a<1) see Fig. (a>1)
C_5	↑ in n	↑ in n	0	D_5	↑ in n	↑ in n	non-decreasing in n

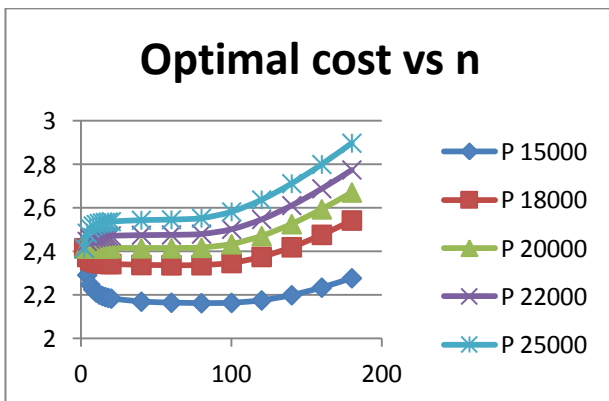Table 2: Relation between the cost and optimal parameter values versus n

Figure 1: Optimal cost versus n for model B_1

The behavior of optimal costs changes in the number of shipments n depending on the model. The costs may increase, decrease, or be convex. It can be seen in Figure 1

showing the results for model B_1. When $P > 2d$, the optimal cost increases in n. On the other hand, when $P < 2d$, the optimal cost decreases in n. In model B_2, the optimal cost behavior depends on the value of α. When α is low, the optimal cost increases in n and when α is high, the optimal cost is convex in n.

Similar results can be obtained for the optimal lot size, q^*. It can increase, decrease, or first decrease and then increase. Examples are given in Figure 2 for model B_2.
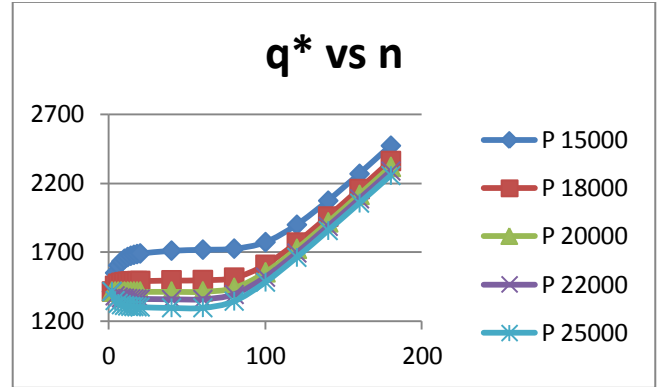

Figure 2: Optimal lot size versus n for model B_2

Also the optimal value of allowed backlog s^* shows some relationship with n which is not monotone and also depends on the parameter α. An example is shown in Figure 3.
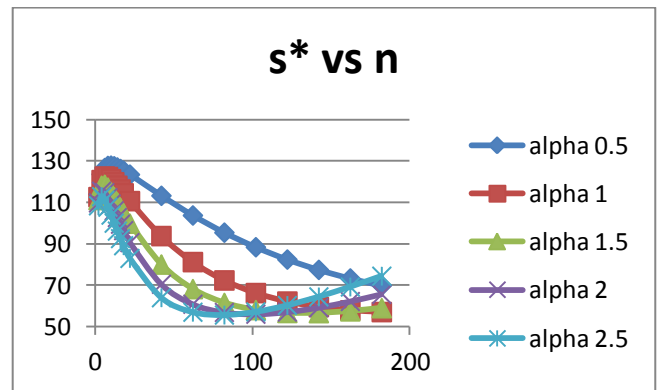

Figure 3: Optimal backlog versus n for model A_2

4. CONCLUSIONS

It is well-considered in the business world that placing an order for a product is one decision but negotiation about the number of shipments from the production plant to the warehouse is another story as it may greatly influence the total logistics cost. The study of the transportation cost per unit in practice many times is not an easy mathematical relationship which leads to standard nice formulae. The paper has shown that in most cases no analytical form of the optimal order size can be obtained, so numerical simulation is required. Furthermore it has been shown that the behavior of the cost versus the number of shipments highly depends on the parameters of the cost function of the transportation cost part. By this, no general conclusions can

be formulated as each case, depending on the cost parameters should be carefully investigated.

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BIOGRAPHY

Rawee Suwandechochai holds a Ph.D. in Industrial and Systems Engineering from Virginia Tech, U.S.A. Het Ph.D. topic dealt with capacity investment, flexibility, and product substitution/complimentarity under demand uncertainty. Currently she is lecturer at the Department of Mathematics in the Faculty of Science at Mahidol University, Bangkok, Thailand. Her research interests include Operations Management and Operations Research.

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