Space-Time Prism Model

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SYNONYMS

Space-Time Prism; Bead; Chain of Space-Time Prism; Lifeline Necklace

DEFINITION

During the last decade, the proliferation of location-aware devices, such as GPS and smartphones, has given rise to the collection of huge amounts of moving object data in the form of finite sequences of time-stamped geographical locations. These sequences of space-time points encode (a sample of) the trajectories of moving objects. The location of the moving objects is unknown between these measured space-time points. Several models have been proposed to model the uncertainty of a moving object's location in between sample points. Linear interpolation between the sample points is a first and obvious example.

However, if additional information is available, such as a limit on the possible speed of a moving object, a more informative model is provided by the framework of *space-time prisms*. This model gives, between two consecutive measured locations, the envelope of possible locations of a moving object in space-time space, given an upper bound on the moving object's speed. In the context of space-time prisms, the measured space-time points are usually called *anchor points*. A space-time prism between two anchor points is a geometric object in space-time space that enclosed all locations where the moving object can possibly have been, given the measured locations, the time interval between them and the object's speed bound. The projection of a space-time prism on the spatial component of space-time space is an ellipse which delimits the collection of possible locations that the moving object can have visited.

The collection of space-time prisms between a sequence of anchor points is referred to as a *chain of space-time prisms*. Here, it should be clear that the upper bound on the object's speed may differ from location to location and can be regarded as geographical background information. Figure 1 depicts an example of a space-time prism between two consecutive anchor points and a chain of space-time prisms between a sequence of anchor points with varying local speed bounds.

HISTORICAL BACKGROUND

The model of space-time prisms originates from the area of *time-geography* and started with the work of Hägerstrand in the early 1970s [3]. In the 1990s, the space-time prism model became popular in the area of GIS. Here, it was used, for instance, in the context of the allocation of resources, where questions such as "given a person's available time and speed, where should resources such as restaurants, schools and banks be located?" are of interest. A frequently cited paper in this context is the work of Miller [9]. Whereas, initially, space-time prisms were studied for unconstrained movement in the two-dimensional plane, the 1990s have also seen the first studies of space-time prisms for objects that are constrained to move on, for example, road networks.

Around the year 2000, space-time prisms were re-discovered in the area of *spatial* and *spatio-temporal databases* by Pfoser & Jensen [13] and Egenhofer & Hornsby [2]. It should be noted that, in some of this literature, space-time prisms are called *beads* and chains of space-time prisms are called *lifeline necklaces*. Around the same period, the area of *moving object databases* (MODs) emerged in the field of spatio-temporal databases, starting with, for example, the work of Wolfson [16] and the spatio-temporal object modeling of Chomicki and Revesz [1]. Here,

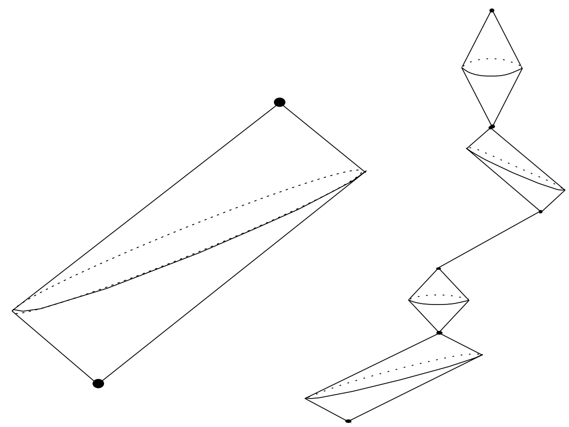

Figure 1: An example of a space-time prism between two consecutive anchor points (left) and a chain of space-time prisms between a sequence of six anchor points with varying speed bounds in between them (right).

the focus was on data about moving objects that is collected, for example, by location-aware devices and that is discrete in nature. For an overview of the field of moving object databases, we refer to the book of Güting and Schneider [5] and for a review on spatio-temporal databases we refer to the book of Revesz [14]. In this field, several models were proposed to account for the moving object's unknown locations in between known, measured space-time points. Linear interpolation and the cylinder model are two such initial attempts to model uncertainty. The use of space-time prisms that use additional information (mainly bound on the object's speed) turned out to be a more precise approach.

In the database literature on space-time prisms, many technical questions were raised and solved. An example of such a technical problem is the *alibi query*, first proposed by Egenhofer & Hornsby [2] and by Miller [10]. This boolean query asks whether two moving objects, that are given by some finite samples of space-time points and speed limitations, could have physically met. This is a query with practical applications in the area of criminal investigation, where an alibi for (not committing) some crime may be needed. This question adds up to deciding whether the chains of space-time prisms of these moving objects intersect or not. This problem was initial solved using approximations of space-time prisms. Later on, an exact and efficient solution, based on techniques from algebraic geometry, was given in the work of Othman et al. [6], who also solved this problem and related problems for movement that is constrained to road networks [7]. We refer the reader, who is interested in algorithmic and mathematical solutions to these type of questions to the PhD thesis of Othman [12].

A more recent extension of the space-time prism model concerns taking acceleration limitations of moving objects into account in addition to bounds on their speed [8].

SCIENTIFIC FUNDAMENTALS

As mentioned before, sample data about a moving object, collected by, for example, location-aware devices, such as GPS and smartphones, is at the basis of the space-time prism model. These data take the form of finite sequences of time-stamped geographical locations and we can model such a sequence of such anchor points as a set $\{(x_0, y_0, t_0), (x_1, y_1, t_1), ..., (x_n, y_n, t_n)\}$, where the couples (x_i, y_i) represent geographical or spatial locations given by their coordinates in the two-dimensional plane $\mathbf{R}^2$ and the t_i represent (real) points in time (for $i = 0, 1, ..., n$). Such a *trajectory sample* has a naturally order, induced by the order $t_0 < t_1 < \cdots < t_n$ on the time component. The elements (x_i, y_i, t_i) can be viewed as points in the real space-time space $\mathbf{R}^3$. In practice, the numbers x_i, y_i and t_i will be rational numbers, since they are produced by measuring devices.

For time points t strictly between t_i and t_{i+1} (for $0 \leq i < n$), the location of the moving object is not measured and thus unknown. In the space-time prism model, it is assumed that an upper bound v_i on the speed of the moving object in the time interval given by t_i and t_{i+1} is known. This speed limit v_i is a non-negative real number, where zero means stationary movement.

This information enables us to bound the set of possible locations (x, y) in $\mathbf{R}^2$ of the moving object for moments in time t with $t_i \leq t \leq t_{i+1}$. The set of possible space-time points (x, y, t) where the moving object can be in this time interval is then given by the following system of three polynomial inequalities in x, y and t:

$$\begin{cases} t_i \leq t \leq t_{i+1} \\ (x - x_i)^2 + (y - y_i)^2 \leq (t - t_i)^2 v_i^2 \\ (x - x_{i+1})^2 + (y - y_{i+1})^2 \leq (t_{i+1} - t)^2 v_i^2. \end{cases}$$

The first inequality expresses that the time point t is in the time interval $[t_i, t_{i+1}]$.

The second polynomial inequality expresses that the location (x, y) is at most $v_i \cdot (t - t_i)$ far from the spatial component (x_i, y_i) of the first anchor point. Indeed, during the elapsed time $t - t_i$ the object can at most have traveled a distance $v_i \cdot (t - t_i)$ away from (x_i, y_i) when the its speed is limited by v_i. In the three-dimensional space-time space (with coordinate system (x, y, t)), the second inequality described an upward pointing cone with top (or apex) at the anchor point (x_i, y_i, t_i). The axis of these cones is parallel to the t-axis. The slope of this cone is determined by the speed limit v_i. This is depicted in the left part of Figure 2.

Similarly, the third polynomial inequality expresses that the location (x, y) is at most $v_i \cdot (t_{i+1} - t)$ far from the spatial component (x_{i+1}, y_{i+1}) of the second anchor point. Again, during the remaining time $t_{i+1} - t$ the object can be at most a distance $v_i \cdot ({}_{i+1}t - t)$ away from the destination (x_{i+1}, y_{i+1}) when the its speed is limited by v_i. In the three-dimensional space-time space, the third inequality described an downward pointing cone with top at the anchor point $(x_{i+1}, y_{i+1}, t_{i+1})$. This is again depicted in the left part of Figure 2.

Finally, the intersection of these three polynomial inequalities, being a time interval and an upward and a downward pointing cone, gives us the *space-time prism* between the anchor points (x_i, y_i, t_i) and $(x_{i+1}, y_{i+1}, t_{i+1})$, given the speed bound v_i. We denote this prism by $\mathcal{P}(x_i, y_i, t_i, x_{i+1}, y_{i+1}, t_{i+1}, v_i)$, which clearly indicates that it is parameterised by the real (or, in practice, rational) numbers $x_i, y_i, t_i, x_{i+1}, y_{i+1}, t_{i+1}$ and v_i. The subset $\mathcal{P}(x_i, y_i, t_i, x_{i+1}, y_{i+1}, t_{i+1}, v_i)$ of $\mathbf{R}^3$ consists of all space-time points that the moving object can have visited when traveling from anchor point (x_{i+1}, y_{i+1}) to anchor point $(x_{i+1}, y_{i+1}, t_{i+1})$ when the its speed is limited by v_i. And example of a space-time prism is given in the right part of Figure 2.

Given a sequence of anchor points $\{(x_0, y_0, t_0), (x_1, y_1, t_1), ..., (x_n, y_n, t_n)\}$ and speed limitations $v_0, ..., v_{n-1}$ between them, the union of the space-time prisms $\mathcal{P}(x_i, y_i, t_i, x_{i+1}, y_{i+1}, t_{i+1}, v_i)$ for $i = 0, 1, ..., n - 1$ forms the *chain of space-time prisms* for these anchor points and speed limitations.

We now discuss some geometric properties of space-time prisms.

The space-time prism $\mathcal{P}(x_i, y_i, t_i, x_{i+1}, y_{i+1}, t_{i+1}, v_i)$ is the intersection of two cones with slope determined by v_i. At each moment t, with $t_i \leq t \leq t_{i+1}$, the intersection of the space-time prism with the plane at moment t, parallel to the (x, y)-plane is a an intersection of two disks, which is a *disk* or a *lens*. At a fixed moment in time t, this disk or lens is given by the constraints

$$\begin{cases} (x - x_i)^2 + (y - y_i)^2 \leq (t - t_i)^2 v_i^2 \\ (x - x_{i+1})^2 + (y - y_{i+1})^2 \leq (t_{i+1} - t)^2 v_i^2. \end{cases}$$

The case of a lens is illustrated in Figure 3.

Given anchor points $(x_i, y_i, t_i), (x_{i+1}, y_{i+1}, t_{i+1})$, with $t_i < t_{i+1}$ and $v_i \geq 0$, the projection of the space-time prism $\mathcal{P}(x_i, y_i, t_i, x_{i+1}, y_{i+1}, t_{i+1}, v_i)$ onto the (x, y)-plane is the area bordered by the ellipse with foci (x_i, y_i) and

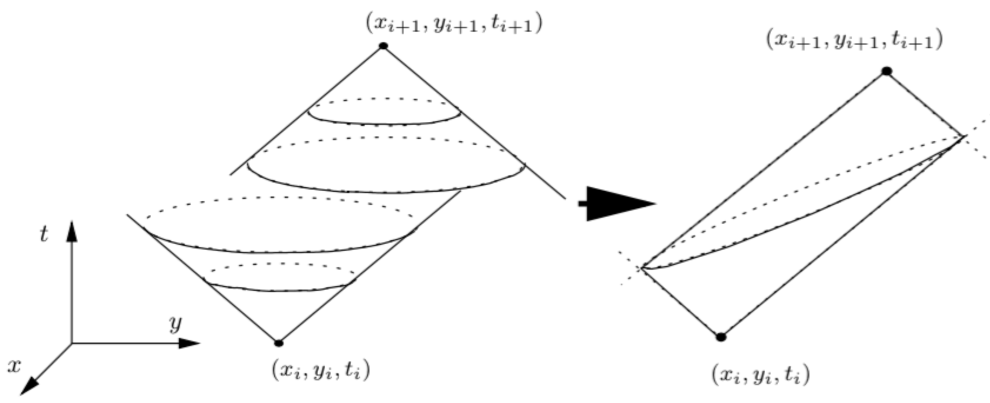

Figure 2: An example of a space-time prism $\mathcal{P}(x_i, y_i, t_i, x_{i+1}, y_{i+1}, t_{i+1}, v_i)$ on the right. On the left side the upward and downward cones are shown.

(x_{i+1}, y_{i+1}) and with long axis $\frac{v_i(t_{i+1}-t_i)}{2}$. The equation of this ellipse is

$$\frac{(2x - x_i - x_{i+1})^2}{v^2(t_{i+1} - t_i)^2} + \frac{(2y - y_i - y_{i+1})^2}{v^2(t_{i+1} - t_i)^2 - (x_i - x_{i+1})^2 - (y_i - y_{i+1})^2} = 1.$$

This ellipse spatially delimits the places which the moving object can have visited when traveling from anchor point (x_{i+1}, y_{i+1}) to anchor point $(x_{i+1}, y_{i+1}, t_{i+1})$ when the its speed is limited by v_i.

We conclude the technical discussion with some results about space-time prisms for movement that is constrained to happen on a road network. A *road network* can be modelled as an embedding in $\mathbf{R}^2$ of a labelled graph given by a finite set of vertices $\mathsf{V} = \{(x_i, y_i) \in \mathbf{R}^2 \mid i = 0, \ldots, n\}$ and a set of edges $\mathsf{E} \subseteq \mathsf{V} \times \mathsf{V}$ that are labelled by a *speed limit* and an associated *time span* (that expresses the time needed to traverse an edge). This graph embedding satisfies the following conditions. Vertices are embedded on themselves and edges are embedded as straight line segments between vertices. We remark that these edge embeddings may intersect (to model bridges and tunnels). If an edge between (x_i, y_i) and (x_j, y_j) is labeled by the speed limit $v_{ij} > 0$, then its time span w_{ij} is $\frac{\sqrt{(x_i-x_j)^2+(y_i-y_j)^2}}{v_{ij}}$, i.e., it is the time needed to get from one side of an edge to another when travelling at the speed limit. Edge embeddings represent roads that can be travelled in both directions.

In practice, when the trajectory of a moving object on a road network is collected by a location-aware device, the recorded space-time points will not always be on the road network and a process called *map matching* maps the recorded points to some "nearest" points on the road network. Once this is done, we can consider trajectories (or samples of trajectories) of moving objects on road networks. Movement on a road network can be viewed as essentially one-dimensional. Figure 4 depicts a trajectory on a road network. The spatial projections of the trajectory and the trajectory sample are also shown.

Without going into details, we illustrated, in Figure 5, what a space-time prism looks like for movement that is constrained to a road network. The road network is depicted in black (and green) in this figure. The two anchor points of the space-time prism are the top and bottom red points and the prism itself is shown in red. The spatial projection of the space-time prism on the road network is shown in green. This figure illustrates that some road

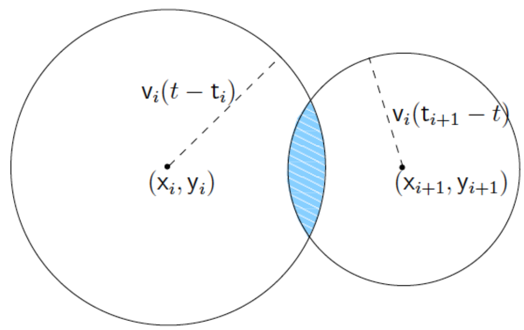

Figure 3: An example of a lens in a space-time prism $\mathcal{P}(x_i, y_i, t_i, x_{i+1}, y_{i+1}, t_{i+1}, v_i)$.

segments may be visited partially and are next traversed in the opposite direction.

KEY APPLICATIONS

Above, we have already mentioned some key motivating applications in this area. Initially, applications concerning the allocation of resources (restaurants, schools, offices, ...) were important. All the important modern applications are situated in the broader area of *moving object data* which now has mainly applications in *location based services* which can include touristic applications (advising restaurants, hotels, ...); (shortest path) route assistance; area evacuation scenarios; spatio-temporal and trajectory data mining and many more. For an overview of this type of applications we refer to the book by Giannotti and Pedreschi [4].

We have already mentioned the alibi query which could come from a criminal investigation application. It asks whether two moving objects, that are given by some finite samples of space-time points and speed limitations, could have physically met. This is an example of an application that has motivated deep technical progress in this field (see [12]).

FUTURE DIRECTIONS

Here we list some open problems or barely addressed topics in the context of space time prisms.

- Currently, it is assumed that anchor points are precisely given. In practice these points are subject to measurement errors and other sources of imprecision. This topic of including uncertainty on the level of the anchor points is largely unaddressed.
- The space-time prism model is based on background knowledge concerning the bounds on the speed of a moving object. The resulting model of chains of space-time prisms allows for unnatural movement (in particular, sudden changes of direction at the anchor points and at the rims of the space-time prisms).

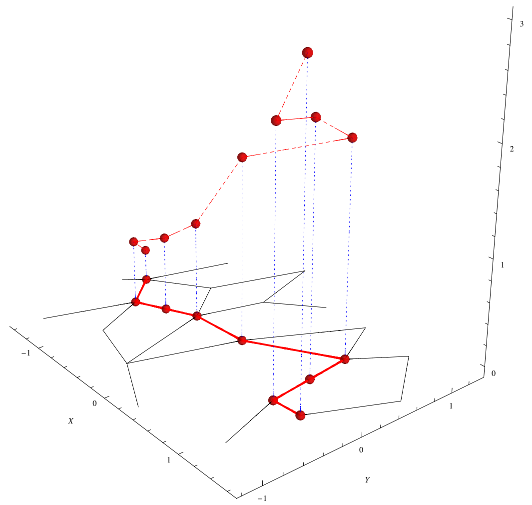

Figure 4: A trajectory on a road network in space-time and its projection on a road network

Such sudden changes would require an infinite local acceleration of the moving object and are therefore not physically realistic. Including a bound on the acceleration of moving objects would lead to more natural models. A first step in this direction can be found in [8].

- There are a number of straightforward generalisations of the space-time prism model that have hardly been investigated. These include generalisations to higher dimensional spaces. For example, the solution to the alibi query in higher dimensions might pose a particular challange.
- There has been a beginning in looking at space-time prisms in spaces which contain obstacles [11]. This area is also large unexplored and also includes the difficulty of in-door movement.
- Another issue is *visiting probability*. Her the question is addressed on how probable it is that certain points or regions within a space-time prism are visited [15].

RECOMMENDED READING

[1] J. Chomicki, P. Z. Revesz. Constraint-based interoperability of spatiotemporal databases, *Geoinformatica*, 3 (3), 211–243, 1999.

[2] M. Egenhofer and K. Hornsby. Modeling moving objects over multiple granularities. *Annals of Mathematics and Artificial Intelligence*, 36(1– 2), 177–194, 2002.

[3] T. Hägerstrand. What about people in regional science? *Papers of the Regional Science Association*, 24, 7–21, 1970.

[4] F. Giannotti, D. Pedreschi (Eds.): *Mobility, Data Mining and Privacy –Geographic Knowledge Discovery*, Springer, 2008.

[5] R. Güting and M. Schneider. *Moving Object Databases*, Morgan Kaufmann, 2005.

[6] B. Kuijpers, R. Grimson, W. Othman. An analytic solution to the alibi query in the space-time prisms model for moving object data, *International Journal of Geographical Information Science*, 25(2), 293–322, 2011.

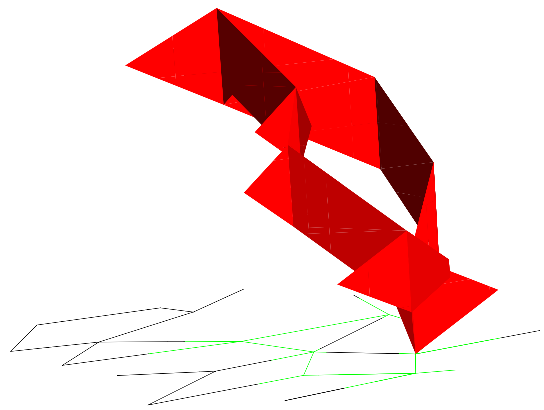

Figure 5: A space-time prism between two anchor points (shown in red) for movement on a road network (shown in black) and the spatial projection of the prism on the road network (shown in green).

[7] B. Kuijpers, W. Othman. Modeling uncertainty of moving objects on road networks via space-time prisms, *International Journal of Geographical Information Science*, 23(9), 1095–1117, 2009.

[8] B. Kuijpers, H.J. Miller, W. Othman. Kinetic space-time prisms. *Proceedings of the ACM GIS Conference 2011*, 162–170, ACM, 2011.

[9] H.J. Miller. Modeling accessibility using space-time prism concepts within geographical information systems, *International Journal of Geographical Information Systems*, 5, 287–301, 1991.

[10] H.J. Miller. A measurement theory for time geography. *Geographical Analysis*, 37(1), 17–45, 2005.

[11] M. Delafontaine, T. Neutens, N. Van de Weghe. Modelling potential movement in constrained travel environments using rough space-time prisms. *International Journal of Geographical Information Science*, 25(9), 1389–1411, 2011.

[12] W. Othman, *Uncertainty management in trajectory databases*, PhD thesis, Hasselt University, 2009.

[13] D. Pfoser and C. S. Jensen. Capturing the uncertainty of moving-object representations, *Advances in Spatial Databases (SSD99)*, volume 1651 of *Lecture Notes in Computer Science*, 111–132, Springer, 1999.

[14] P. Z. Revesz. *Introduction to Databases: From Biological to Spatio-Temporal*, Springer, New York, 2010.

[15] St. Winter, Zhang-Cai Yin. The elements of probabilistic time geography, *GeoInformatica*, 15(3), 417–434, 2011.

[16] O. Wolfson. Moving objects information management: The database challenge, *Proceedings of the 5th International Workshop Next Generation Information Technologies and Systems*, 75–89, Springer, 2002.