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Estimating Nonlinear Parameters Present In OFDM-based System Using Non-Linear Least Squares

Absar-Ul-Haque Ahmar[a] , Ansar-Ul-Haque Yasar[b]

Electronics and IT Dept, Vrije Universiteit Brussel, Belgium[a]

Transportation Research Institute (IMOB), Hasselt University, Belgium[b]

Abstract

Wireless communication systems are omnipresent in our everyday life and high standards regarding capacity, reliability, speed and power are naturally expected. In order to satisfy these requirements, more sophisticated and efficient technology is needed. This is especially true for the integrated amplifiers at the transmitter side, which are responsible for the majority of the power consumption. In order to obtain high gain at the low power levels, these amplifiers are operated into their nonlinear region. However, many communication schemes cannot handle the nonlinear distortion generated by such amplifiers. The nonlinear distortions generated at the transmitter side can be compensated through the use of digital predistortion. This method compensates the non-linear effects by first estimating a non-linear model for the amplifier, and then applying the inverse model onto the data during transmission such that the nonlinearities are canceled. The main advantage of this approach is that it allows to use the classical linear communication scheme once the precompensation is performed.

In this work, Special attention goes to the estimation of the nonlinear model used for the predistortion. Most nonlinear models are not only nonlinear in behavior but also nonlinear in their parametrization, which requires special care during estimation of the model parameters. Performance of the simple linear least square estimator and nonlinear least square estimator is investigated using a National Instrument student setup.

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Keywords: Error Rates;Fast Fourier transform;Intersymbol Interference;Nonlinear Least Squares;National Instruments;Peak to Average Power ratio;Non-linearity;Orthogonal Frequency Division Multiplexing;Signal to Noise Ratio.

1. Linear and Non-linear System

In communication a channel is a medium used to convey an information signal.In wireless communication the channel can be modeled by calculating the reflection from every object in the environment, attenuation

*Corresponding author. Tel.: +32488968303; Email: ahahmar@gmail.com.

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(known as fading) of a transmitted signal, followed by additive noise. These various models (Linear and nonlinear) at transmitter and channel may reflect following signal impairments like:

- AWGN (additive white gaussian noise) channel, a linear continous memoryless model

- Interference model, for example cross-talk and ISI etc.

- Distortion from amplification stage at transmitter can be modeled as nonlinear system. and the list goes on and on.

So, due to spectral regrowth and interference in adjacent channels(ICI), as shown in Figure 1 where red line is the spectrum of the input signal and blue is the spectrum of the output signal.

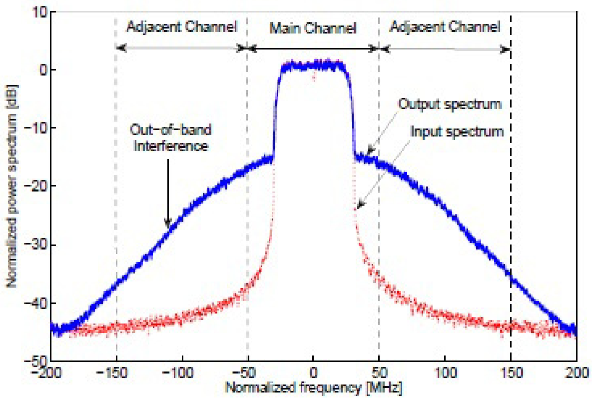

Figure 1. Input-Output Spectrum due to spectral regrowth

1.1. System NonLinearity

The rapid evolution of global information technology demands high data-rate transmissions in the presence of available bandwidth, which, in turn, requires spectrally efficient modulation techniques. In this case, multi - level modulation techniques (M - ary) are the favorable candidates. Among all the existing M - ary modulation techniques, M-QAM modulation offers the maximum spectral efficiency and appears to be a potentially attractive modulation scheme for communications[2,5].

In every communication system, power amplifiers are used to amplify the signal before they are send out. since when the input signal approaches the rated value of power amplifier it starts to saturate, resulting in gain compression or reduction in available gain, the input signal also experiences phase variations near the compression region and cause the skewing and compression of signal constellation. So eventually this causes the timing synchronization errors at the receiver. Figure 2 shows the distortion of a signal resulting in phase rotation and amplitude compression when it pass through amplification stage.

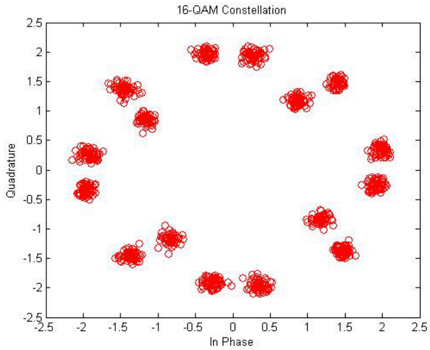

Figure 2. 16QAM signal at the output of a nonlinear amplifier

The (AM/AM) conversion for a nonlinear system is the relation between the amplitude of the system output and the amplitude of the system input. The (AM/PM) conversion for a nonlinear system is the relation between the phase change of the system input and output, and the amplitude of the input signal.

When we operate near the maximum power region, these amplifiers cause non-linear distortions which are modeled in terms of amplitude-to-amplitude (AM/AM), and amplitude-to- phase (AM/PM) conversions.Figure 3 shows that the operating point of the amplifier moves close to the saturation region, hence the non-linear distortions increase.

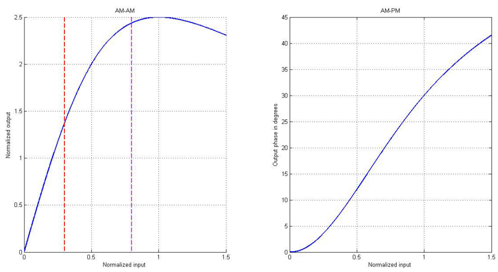

Figure 3. AM/AM and AM/PM Conversions

2. Selected Nonlinear Model

We implemented Saleh model that mimic the nonlinear behavior of an amplifier when operating at maximum power region, The reason why we selected this model is:

- Less complex to implement.
- Nonlinear distortions can easily be modeled in terms of amplitude and phase.
- Widely used model in OFDM systems.

An amplifier is driven to its nonlinear region while operating at high power. Thus, generate nonlinearities in the system under study.The model we used to mimic the nonlinear behaviour of an amplifier is the most widely used saleh model that introduces more prominent AM - AM and AM - PM distortions than any other model available. In saleh model, the nonlinearity is determined by four parameters e.g: $\alpha_a, \alpha_\varphi, \beta_a, \beta_\varphi$. AM - AM , AM - PM characteristics of a Saleh model can be modelled as:

$$A(r) = \frac{\alpha_a r}{1 + \beta_a r^2} \qquad \phi(r) = \frac{\alpha_\varphi r^2}{1 + \beta_\varphi r^2} \tag{1}$$

Figure 4 shows the distortions caused by PA(Power amplifier) on 16QAM constellation and the normalized AM - AM , AM - PM conversions,by taking the true parameters of my model as : $\alpha_a = 4, \alpha_\varphi = \frac{\pi}{3}, \beta_a = 1, \beta_\varphi = 1$.where, A(r),represents the output amplitude and $\phi(r)$ represents the difference in phase between the input and output. Both A(r) and $\phi(r)$ depend on r which is amplitude of the input. Observe that in the limit case for large r the amplitude become proportional with $1/r$ and the phase becomes constant[1,3].

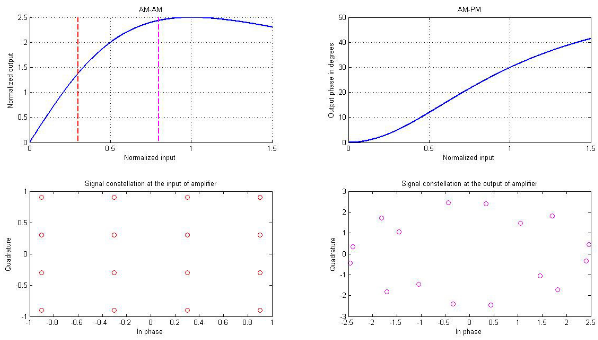

Figure 4. 16QAM Input/Output constellation

3. Estimating parameter of our NonLinear model Using Nonlinear Least Squares

The output from nonlinear model can be defined as: $b_y = A[r]e^{j(\alpha_x + \phi[r])}$where symbol stream (r) is going to nonlinear block(i.e amplifier) that is modeled using saleh model using the true parameters as explained earlier and α_x is the angle computed for r symbols, b_y is the distorted output as shown in figure 4.Starting with the two equations in 1 of saleh model we can rewrite the equations for A(r) and $\phi[r]$ as:

$$A + A\beta_a r^2 = \alpha_a r \qquad \phi + \phi\beta_\phi r^2 = \alpha_\phi r^2 \tag{2}$$

Rearranging the equations we get.

$$\alpha_a r - (Ar^2)\beta_a = A \qquad \alpha_\phi r^2 - (\phi r^2)\beta_\phi = \phi \tag{3}$$

These equations can be written in matrix form as follows.

$$\begin{pmatrix} r & -Ar^2 \end{pmatrix} \begin{pmatrix} \alpha_a \\ \beta a \end{pmatrix} = \begin{pmatrix} A \end{pmatrix} \qquad \begin{pmatrix} r^2 & -\phi r^2 \end{pmatrix} \begin{pmatrix} \alpha_\phi \\ \beta\phi \end{pmatrix} = \begin{pmatrix} \phi \end{pmatrix} \tag{4}$$

Both matrices are of the form of $K\theta = Y$ where $[r \quad -Ar^2] = K_1$ and $[r^2 \quad -\phi r^2] = K_3$. Introducing K2=[0 0] with the same dimensions as K1 and K3, the equations in 4 can be combined into one single matrix equation 5.

$$\begin{pmatrix} K_1 & K_2 \\ K_2 & K_3 \end{pmatrix} \begin{pmatrix} \alpha_a \\ \beta a \\ \alpha_\phi \\ \beta\phi \end{pmatrix} = \begin{pmatrix} A \\ \phi \end{pmatrix} \tag{5}$$

Therefore, by solving the set of matrices we get our estimated parameters $\alpha_a, \alpha_\varphi, \beta_a, \beta_\varphi$. In Matlab I computed θ by using a backslash operator $\theta = K \setminus Y$. I applied linear least square on my nonlinear model to get the best initial starting values to estimate parameter.There was no analytical solution so I solved it numerically using a solver in Matlab using lsqcurvefit function, that uses behind the scene Levenberg-Marguardt algorithm[4].The important thing here is to have good initial conditions to initiate the nonlinear estimation and that good initial guess we get from linear least square and then we improve on it. The function lsqcurvefit behaves as follows: X = lsqcurvefit(@fun,X_o ,xdata,ydata). Where;

- xdata,ydata are the input and the output data to be matched.
- X_o is the starting value that we derived from the linear least square estimation, to fit the coefficient X to best fit the nonlinear function.
- @fun is the function which gives the amplitude and phase of our model.

4. Test Results

All the results in this section are computed by running the test for 10000 realizations and plot the average result of 10000 realizations with SNR value of 20dB and pick the average, maximum and minimum values.Figure 5 shows the AM - AM , AM - PM curve fitting plots computed on the 16QAM signal for linear and nonlinear least squares. We can already see from the results that the estimated parameters are good enough when we applied a linear least squares on our nonlinear model. So these parameters are itself good enough to be used as the initial conditions to initiate nonlinear least squares estimation. We used these as initial conditions for the nonlinear least squares estimation.

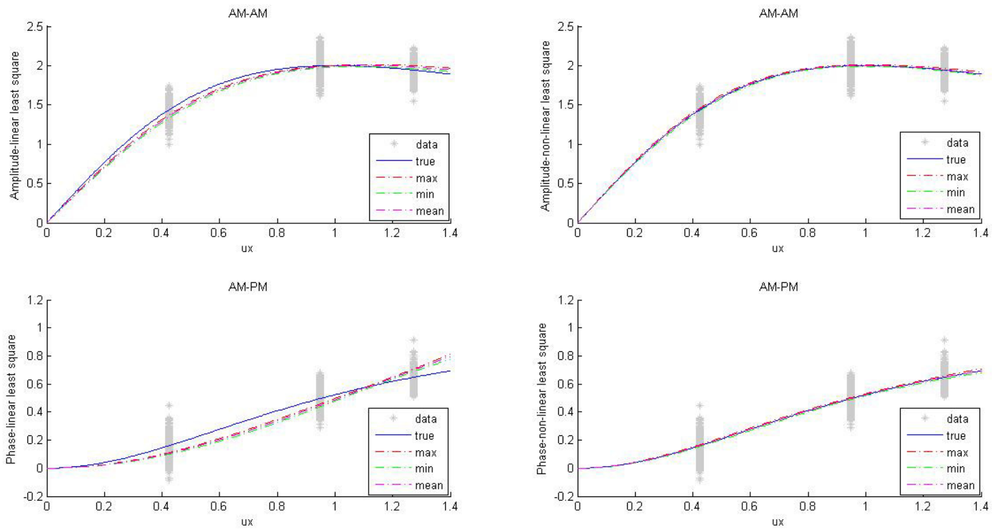

Figure 5. Comparing AM-AM, AM-PM curve fitting of nonlinear model by implementing Linear and Nonlinear least square Estimation

Relative Bias and Standard deviation of our model for linear and nonlinear least square cases has also been calculated to provide the better overview in terms of spread of the estimated values, and the difference between average computed from 10000 iterations and true value is a bias.

In a perfect case bias should be zero. If the bias is zero it means that the estimator on average finds the true values of the parameters. The bias and variance for the linear and nonlinear least squares are computed by.

Computing Bias

$$Bias_\theta[\hat{\theta}] = E_\theta[\hat{\theta}] - \theta = E_\theta[\hat{\theta} - \theta] \tag{6}$$

where, $\hat{\theta}$ is an estimator of θ based on the parameter values , E_θ is the expected value over the distribution. We computed the relative bias shown in the table below as.

$$Bias_{\theta_k} = \frac{|mean(\hat{\theta}_k) - \theta_k|}{\theta_k.100)} \tag{7}$$

Computing Standard Deviation

The variance is computed as the square of the difference of average and the actual value .

$$std_{\theta_k} = (\frac{std(\hat{\theta}_k)}{\theta_k.100}) \tag{8}$$

So the lower the variance and the bias values are the better the estimated model will be, as can easily be visualized by looking at the boxplots in Figure 6, we can see that the estimated values are close enough to the true/expected values and centered around zero when we estimated our model using nonlinear least squares as compared to linear least squares .

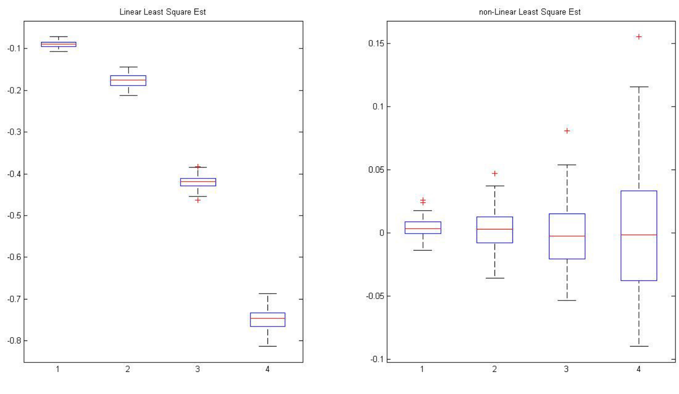

Figure 6. Boxplot for the linear and nonlinear least square estimation

The resulted estimated parameter values for linear and nonlinear least squares estimation are shown below from taking average of 10000 iterations.

```
True Values
  aa--->4  ap--->1.0472  ba--->1  bp--->1
*********************************************
Linear least square estimation
  aa--->3.6782  ap--->0.851  ba--->0.5945  bp--->0.229
Relative bias
  aa--->9  ap--->17  ba--->42  bp--->60
Relative standard deviation
  aa--->1  ap--->2  ba--->2  bp--->3
*********************************************
Nonlinear least square estimation
  aa--->4.0129  ap--->1.012  ba--->1.0184  bp--->0.9457
Relative bias
  aa--->0  ap--->0  ba--->1  bp--->1
Relative standard deviation
  aa--->1  ap--->1  ba--->3  bp--->4
```

Figure 7. Comparing linear and nonlinear least square estimation

We can already see in the table that the estimated parameters of our model has been greatly improved using the nonlinear least squares that are close enough to the true parameters of our model. The bias and variance are also much lower in case of nonlinear least squares(computed in percentage).

The distribution is centered around with little variance around it due to the presence of noise, α_a, α_ϕ are unbiased.

Conclusion

The aim of the research is to study the Estimation of the nonlinearities present in OFDM based system due to amplification stage at the transmitter side, these nonlinear distortions greatly deteriorates the overall performance of our system which need to be taken into account. The focus here was on estimating the parameters of our nonlinear model(Saleh model in our case) such that they are close enough to the true parameters of our nonlinear model by applying nonlinear least squares method on our data, so at later stages we can use these estimated values to construct a nonlinear predistorter at transmitter side inorder to compensate the signal output before it is send into the medium.We focused our self on modeling and estimating nonlinearities that mimic the nonlinearities caused by amplification stage using USRP-2920 device only.

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