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# A Simplified Car-Following Model Based on the Artificial Potential Field

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## Abstract

Car-following models, which describe the interaction between successive vehicles in the same lane, have been studied for decades. A group of models are derived from the stimulus-response pattern concentrating on the effect of diverse stimulus type. This study presents a potential-based car-following model using the concept of the artificial potential field, which aims for the precise and fast interactive operations in an evolving environment. Spacing headway is divided into two parts according to the potential influence region. The variation rate of the spacing headway generates the control force within the potential influence region, while the difference of desired and current velocity takes control out of the influence range. Calibration and validation of the simplified model are conducted using NGSIM data. Statistical tests show that the proposed model can reproduce the car-following process very well.

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## 1. Introduction

Since the day of the first vehicle invented, complex traffic situation emerged along with world-wide attention. Nowadays, many works have been performed with different perspectives to investigate the various aspects of traffic phenomena. Traffic flow, however, is the very basic and comprehensive theory, which is widely studied for decades, and forms a comparatively complete system. To understand the complicated traffic behavior, many mathematical

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models have been proposed using both macroscopic and microscopic methods to discuss the relations between variables and densities of vehicles in the traffic stream with congestion, such as car-following models, cellular automation models, gas kinetic models and hydrodynamic models.

As one of the most significant microscopic models, car-following model has been studied for half a century. Based on the hypothesis that the acceleration was proportional to relative speed, Chandler et al. [3] put forward the first prototype car-following model known as GM model in 1958. Much more development in microscopic calibration and validation was followed by Herman et al. [7], Treiterer et al. [16] and Aron [1]. However, due to the contradictory aspects as to the values of  $m$  and  $l$ , GM model was used less frequently since late seventies. On account of GM model, Helly [6] presented a new linear car-following model which took into account the effect of the driver's acceleration by the braking process of one or two vehicles ahead. Michaels [13] came up with the concept that drivers would take action once the visual angle exceeded the threshold and Kikuchi et al. [10] proposed the fuzzy logic-based model which divided inputs into some 'fuzzy sets' describing how sufficiently a variable fits the description of a 'term'. Instead of concentrating on the stimulus-response type, Kometani et al. [11] paid more attention to the safe following distance and proposed the collision avoidance model. Gipps [4] developed CA model to make it widespread used in simulations, such as PROMETHEUS program, INTRAS and CARSIM. Despite the fact that the above models are used very less now, the concepts of diverse theories have made a profound difference studying the driving behavior. Bando et al. [2] proposed the well-known optimal velocity model which was widely accepted and used since then. Different from the stimulus type of GM model, the stimulus of OV model is the difference of desired speed and current speed. Due to more accurate description of the characteristics of real traffic flow, many researchers have developed the OV model taking into account more variables by Helbing et al. [5], Jiang et al. [8] and Xie et al. [17]. Other than the above models, a few researchers have adopted the concept of the changing energy to better depict the interaction between adjacent vehicles in the traffic stream influenced by the evolving environment. According to the artificial potential field approaches, Leonard et al. [12] combined virtual leaders and artificial potentials to develop a framework for coordinated control of multiple autonomous vehicles. Tao et al. [15] presented a car-following model by abstractly taking efficiency as the attractive force and safety as the repulsive force. The results show that the validation of the new model appeared higher accuracy than that of the GM model. Previous studies have explored microscopic driving behaviors with diverse perspectives and indicated that more and more researchers are inclined to apply potential theory. However, all existing models are of complex format in both calibration and validation, and some of them are not of high accurate. In this study, we apply the artificial potential field to provide a very simple format model to depict the interactive forces put on vehicles in one dimension, and lay basic foundation for further investigation of forces when vehicles are at intersections.

## 2. Artificial potential based car-following model

### 2.1. Artificial potential field method

The artificial potential field is one of the methods to control the navigation of mobile robots in the dynamic environment and widely used in the real-time obstacle avoidance since firstly developed by Khatib [9]. Robots are assumed to move in a field of forces in which the destination draws the attractive force and the obstacles generate the repulsive force, the interaction between attractive and repulsive forces is the major drive that successfully guides robots to the goal position without hitting any barriers. The expression of the artificial potential in path planning by Khatib is,

$$U_{arr}(x) = U_{x_d}(x) + U_o(x) \quad (1)$$

Where,  $U_{arr}(x)$  is the artificial potential field,  $U_{x_d}(x)$  is the attractive potential field,  $U_o(x)$  is the repulsive potential field.  $U_{x_d}(x)$  and  $U_o(x)$  are of the form as follows, respectively,

$$U_{x_d}(x) = \frac{1}{2}k_p(x - x_d)^2 \quad (2)$$

$$U_o(x) = \begin{cases} \frac{1}{2}\beta \left( \frac{1}{f(x)} - \frac{1}{f(x_o)} \right)^2 & \text{if } f(x) \leq f(x_o) \\ 0 & \text{if } f(x) > f(x_o) \end{cases} \quad (3)$$

Where,  $x$  is the position of the robot,  $x_d$  is the goal position,  $k_p$  is the position gain,  $f(x)$  is the potential function,  $x_o$  is a given point in the vicinity of the obstacle and  $\beta$  is a constant gain. The region of influence of this potential field is bounded by the surfaces  $f(x) = 0$  and  $f(x) = f(x_o)$ .

However, based on the fact that no precise destination exists when driving in the traffic stream, how to formulate the attractive force is the priority. Leonard et al. [12] divided the distance between successive vehicles into three phases: repulsive force in the first phase, attractive force in the second and non-irrelevant in the last. The potential can be given as,

$$V = \begin{cases} \alpha \left( \ln(r_{ij}) + \frac{d_0}{r_{ij}} \right), 0 < r_{ij} < d_1 \\ \alpha \left( \ln(d_1) + \frac{d_0}{d_1} \right), r_{ij} \geq d_1 \end{cases} \quad (4)$$

Where,  $V$  is the potential field,  $\alpha$  is the scalar control gain,  $r_{ij}$  represents the distance between successive vehicles,  $d_0$  and  $d_1$  are scalar constants. The interaction force that derives from an artificial potential is,

$$f = \begin{cases} \nabla_{r_{ij}} V, 0 < r_{ij} < d_1 \\ 0, r_{ij} \geq d_1 \end{cases} \quad (5)$$

A controlled dissipative force  $f_v = \varepsilon(v_i - v_d)$  is also applied to the vehicle.  $v_i$  is the current velocity. The dissipative force is designed to be zero when the vehicle is moving at a desired velocity  $v_d$ . So the control force of the vehicle is given as,

$$u = -f + f_v \quad (6)$$

This model is similar with the social force model in format. However, it is the headway that generates both attractive and repulsive forces in the potential field model, while the difference between desired velocity and current velocity draws the attractive force in the social force model.

## 2.2. A simplified car-following model

Based on the theory of the artificial potential field method, spacing headway is the main drive for the vehicles to move along the lane and avoid rear at the same time. The safety distance and the potential influence region (we only consider one dimension situation) are taken into account to define the boundary of two different forces. Fig.1 shows the safety distance between two adjacent vehicles. We should note that the safety distance is not an absolute value, but varies with time according to the present velocity, maximum deceleration etc.

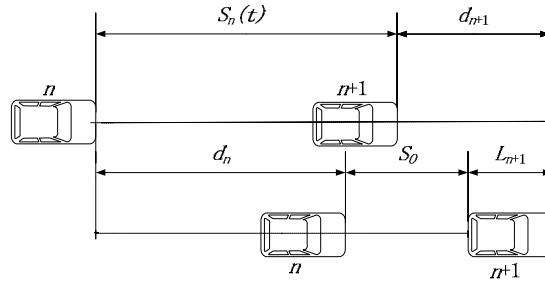


Fig. 1. The safety distance between two adjacent vehicles

Shao et al. (2015) investigated three kinds of safety distance under different situations. We adopt the absolute safety distance and critical safety distance in our study. The absolute safety distance is given as,

$$S_n(t) = (d_n + S_0 + L_{n+1}) - d_{n+1} = S_0 + L_{n+1} + v_n(t) \times T + \frac{v_n^2(t)}{2a_{nd \max}} - \frac{v_{n+1}^2(t)}{2a_{(n+1)d \max}} \quad (7)$$

Where,  $d_n$ ,  $d_{n+1}$  are the braking distance of the  $n$  th and  $(n+1)$  th vehicle respectively,  $S_0$  is the minimum distance when both vehicles are stationary,  $T$  is the reaction time of drivers,  $L_{n+1}$  is the length of the preceding vehicle,  $v_n(t)$ ,  $v_{n+1}(t)$  are the velocity of the  $n$  th and  $(n+1)$  th vehicle at time  $t$ ,  $a_{nd \max}$ ,  $a_{(n+1)d \max}$  are the maximum deceleration of  $n$  th and  $(n+1)$  th vehicle.

However, if  $v_n(t) \ll v_{n+1}(t)$ , the absolute safety distance could be smaller than  $S_0$ , even less than 0. Therefore, the critical safety distance is needed as follows,

$$S_c = S_0 + L_{n+1} \quad (8)$$

Combining different situations, the safety distance can be expressed as,

$$S_n(t) = \max \left\{ S_0 + L_{n+1} + v_n(t) \times T + \frac{v_n^2(t)}{2a_{nd \max}} - \frac{v_{n+1}^2(t)}{2a_{(n+1)d \max}}, S_0 + L_{n+1} \right\} \quad (9)$$

In GM model, the velocity difference of successive vehicles is considered as the stimulus, which is hard to be perceived by drivers when the velocity difference fluctuates in a small range. While the spacing headway and safety distance caused by the difference of speed are easier to be measured and captured. Fig.2 shows the transfer of the attractive force and repulsive force. When  $0 < \Delta x_n(t) < S_n(t)$ , the following vehicle tends to decelerate under the repulsive force, when spacing headway is between the safety distance and the potential influence distance  $x_d$ , the distance between two vehicles act as the stimulus to attract the following vehicle to accelerate, when  $\Delta x_n(t) > x_d$ , it is the difference of desired and current velocity generates attractive force instead of the spacing headway.

Hence, we can derive the potential-based model as,

$$a_n(t+T) = \begin{cases} \lambda \times \ln\left(\frac{\Delta x_n(t)}{S_n(t)}\right), 0 < \Delta x_n(t) < x_d \\ \eta \times (v_d - v_n(t)), \Delta x_n(t) \geq x_d \end{cases} \quad (10)$$

Where,  $a_n(t+T)$  is the acceleration (or deceleration) of the  $n$  th vehicle,  $\lambda$  and  $\eta$  are scalar control gain,  $\Delta x_n(t) = x_{n+1}(t) - x_n(t)$  indicates the spacing headway between  $n$  th and  $(n+1)$  th vehicle,  $x_d$  represents the potential influence distance,  $v_d$  is the desired velocity. Fig.2 shows the transfer of the attractive force and repulsive force.

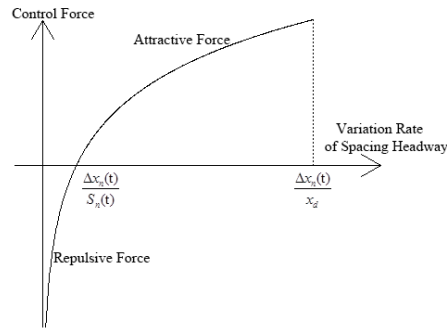


Fig. 2 The transfer of control force according to the variation rate of spacing headway

### 3. Calibration and validation of the simplified car-following model

#### 3.1. Model calibration

We use the Next Generation Simulation (NGSIM) data to calibrate and validate the model. NGSIM database consists of 11,779 vehicle trajectories including detailed parameters such as vehicle ID, frame ID, global time, velocity, acceleration, spacing headway and time headway etc., and the data is widely used in many studies to perform calibration and validation.

To further calibrate the model, we first set some parameters:  $T \approx 1s$ ,  $a_{nd\ max} = a_{(n+1)d\ max} \approx 3.5m/s^2$ ,  $S_0 \approx 1m$ ,  $v_d \approx 22m/s$ ,  $x_d = 50m$ . In this study, acceleration and deceleration will be discussed separately. When  $\Delta x_n(t) \geq x_d$ , the response of drivers comes from the difference of desired and current velocity, and we assume that vehicles will accelerate in this case.

10 complete sets of two-vehicle-following data are chosen, in which each set of data consists of over 1000 group data including vehicle ID, Frame ID, velocity and acceleration of two adjacent vehicles, spacing headway etc. Table 1 shows the calibration results of the model under different conditions.

Table 1. Calibration of the simplified car-following model.

| Driving State | Number of Samples | $\lambda$ | $\eta$ | Correlation coefficient |
|---------------|-------------------|-----------|--------|-------------------------|
| Acceleration  | 1371              | 1.827     | 0      | 0.715                   |
|               | 1146              | 0         | 0.241  | 0.859                   |
| Deceleration  | 1105              | -5.033    | 0      | 0.658                   |

It can be concluded from Table 1 that the ratio of spacing headway and safety distance is an important stimulus to vehicles when they are within the potential influence distance. Regardless of the errors and noise existing in the data, the correlation coefficients of both cases are comparatively low. The safety distance we formulate in this study is relatively large, however, the actual safety distance perceived by drivers is not similar with the ones we give. In fact, differences even exist in the acceleration and deceleration phase, drivers are intended to follow the preceding vehicle closer in the acceleration case to improve the driving efficiency, while they prefer large distance to avoid any risks of crashing under deceleration. Table 1 also indicates that the difference of desired and current velocity plays a significant role in the acceleration control when vehicles are beyond the potential influence distance. The

correlation coefficient of this case is higher than that of distance-stimulus case, which implies that once vehicles are far from each other, the distance barely affect drivers.

### 3.2. Model validation

15 groups of data are chosen for acceleration and deceleration cases to carry out the validation. The driving state of following vehicles which is calculated with the data will be compared with the real driving situation to check out the deviation of the model. Mean absolute error and root mean square error are applied to verify the difference between calculated values and measured values. Error of the simplified car-following model is revealed in Table 2.

Table 2. Error of the simplified car-following model.

| Driving State                                    | Process | Number of samples | Acceleration |       |
|--|---------|-------------------|--------------|-------|
|  |         |                   | MAE          | RMSE  |
| Acceleration (within potential influence region) | 1       | 224               | 0.78         | 1.01  |
|  | 2       | 220               | 0.57         | 1.15  |
|  | 3       | 228               | 1.28         | 1.47  |
|  | 4       | 207               | 1.22         | 1.50  |
|  | 5       | 196               | 0.96         | 1.12  |
| Acceleration (beyond potential influence region) | 6       | 234               | 0.95         | 0.658 |
|  | 7       | 161               | 0.86         | 1.01  |
|  | 8       | 232               | 1.24         | 1.40  |
|  | 9       | 160               | 1.08         | 1.26  |
|  | 10      | 190               | 1.20         | 1.37  |
| Deceleration                                     | 11      | 170               | 1.41         | 1.67  |
|  | 12      | 186               | 1.44         | 1.67  |
|  | 13      | 256               | 0.84         | 1.01  |
|  | 14      | 210               | 1.48         | 1.67  |
|  | 15      | 207               | 0.87         | 1.07  |

It can be concluded from Table 2 that all mean absolute error and root mean square error stay in a reasonable range, while the deviation in the acceleration case is smaller than that of the deceleration case. Fig. 3 illustrates the car-following behaviours under different situations (process 2, 7, 13 are chosen).

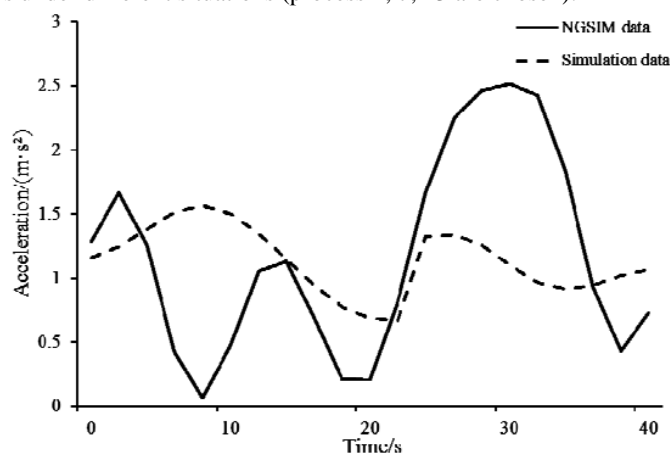


Fig. 3. (a) Acceleration case within potential influence region;

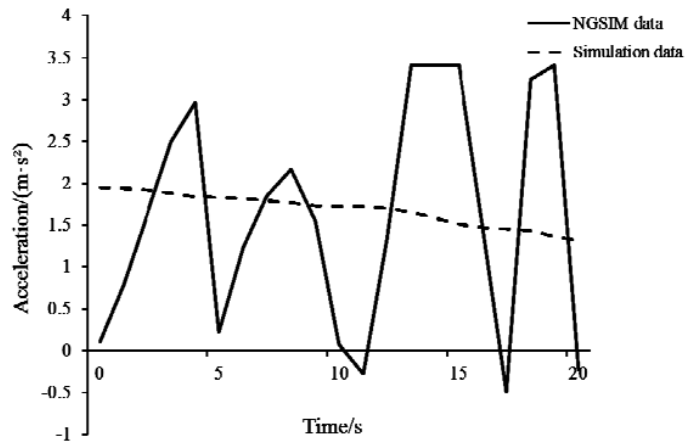


Fig. 3. (b) Acceleration case beyond potential influence region;

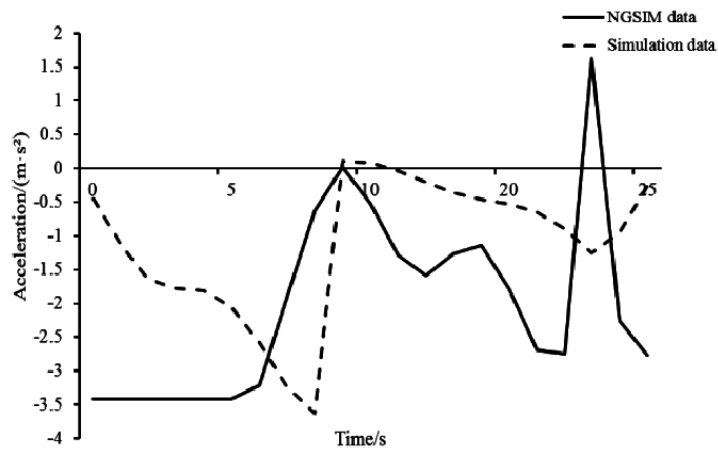


Fig. 3. (c) Deceleration case.

Combining results from Table 2 and Fig. 3, we come to a conclusion that when the spacing headway of the following vehicle doesn't exceed the potential influence region, the calculated values of simplified model fit well with the NGSIM data. On the other hand, though the correlation coefficient of out-of-range acceleration case is better than that of the other two situations, Fig. 3 (b) indicates low correlation between measured and calculated data. As we supposed that vehicles will accelerate when the preceding vehicle is far apart, an approximately linear force is drawn by the difference of desired and current velocity. However, small fluctuation of velocity can lead to dramatic change of acceleration, which results in the nonlinear tendency.

#### 4. Conclusion

In this study, the simplified car-following model is developed based on the artificial potential field to describe the evolving traffic environment using the concept of changing energy. Three different cases are taken into account, critical points are defined according to the interrelationship between different form of control force and stimulus. When  $0 < \Delta x_n(t) < S_n(t)$ , the following vehicle tends to decelerate under the repulsive force, when  $S_n(t) < \Delta x_n(t) < x_d$ , attractive force is drawn by the increasing spacing headway, when  $\Delta x_n(t) > x_d$ , it is the

difference of desired and current velocity generates attractive force instead of the spacing headway. Parameters of the model under different situations are calibrated with NGSIM data. Validation is carried out using mean absolute error and root mean square error, and statistical tests show that the simplified model can reproduce the car-following process very well. In future research, we will extend the artificial potential field to explore driving behaviors at intersections and divide the potential influence zone into several sections in which vehicles are driven by different forces. This study is the basic theoretical foundation for ITS application to avoid any conflicts at intersections.

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