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FACULTEIT INDUSTRIËLE INGENIEURSWETENSCHAPPEN
master in de industriële wetenschappen: bouwkunde

Masterproef
Design of primary laminated glass elements

Promotor :
dr. Jose GOUVEIA HENRIQUES

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ing. CLAUDE PIMPURNIAUX

Freya Leurs , Simon Seynaeve

Scriptie ingediend tot het behalen van de graad van master in de industriële wetenschappen: bouwkunde

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universiteit hasselt | KU LEUVEN

ABSTRACT - NEDERLANDS

Structureel glas wordt tegenwoordig steeds vaker gebruikt in de constructiewereld. De toepassing ervan is in recente jaren geëvolueerd van secundaire toepassingen zoals trappen, leuningen,… naar primaire functies zoals het construeren van volledige gebouwgevels. Waar het gebruik van berekeningen bij de secundaire toepassingen beperkt blijft, is dat een absolute noodzaak bij het construeren van veilige, grotere en complexere structuren.

Glas is een vreemd constructiemateriaal in vergelijking met andere (lees: klassieke) bouwmaterialen; omwille van zijn extreem bros breukgedrag moet de nodige aandacht geschonken worden aan de berekening van deze structuren.

Deze thesis behandelt de berekening van de primaire gelamelleerde glaselementen (kolommen, balken, (vloer)panelen en verbindingen) gebaseerd op up-to-date designregels. Op Europees of Belgisch niveau bestaat er echter nog geen uniforme designregelgeving waardoor deze berekeningen berusten op andere nationale regelgevingen betreffende structurele toepassingen van glas. Meer bepaald, de eigenschappen en het gedrag van gelamelleerd glas moeten hiervoor gekend zijn. Het project van het Museum van Europese Geschiedenis in Brussel fungeert in dit geval als case studie ter validatie van de voorgestelde ontwerpmethodes. Tenslotte worden alle elementen gesimuleerd met behulp van eindige-elementen software waarna deze resultaten vergeleken worden met de analytische resultaten. Na validatie van de EE-modellen worden simulaties uitgevoerd om de analytische procedures aan te vullen.

ABSTRACT - ENGLISH

Nowadays, the use of structural glass in construction is increasing. Its application has evolved in recent decades from secondary applications such as stairs, barriers and windows, to primary structural functions such as entire building facades. Where in the first applications the structural calculations are limited, for the latter, it is an absolute necessity for the construction of safer, bigger and more complex structures. Glass is a particular material when compared to any other of the classic building materials, due to its extreme brittle behaviour. Special care is needed to calculate these structures.

This master’s thesis deals with the calculation of primary laminated glass elements (columns, beams, panes and connections) based on the up to date design rules. In Europe no unified design rules exist as for other construction materials therefore, the calculations are based on the available National rules for structural applications of glass. In particular, the properties and behaviour of laminated glass are considered. The project of *the Museum of European History* in Brussels is used as a case study for application of the proposed design methods. Finally, the different glass elements are simulated by means of finite element models. The results of the latter are compared with the analytical calculations. Upon validation of the finite element models, simulations are performed to complement the analytical procedures.

ACKNOWLEDGEMENTS - NEDERLANDS

Het einde van onze opleiding tot bouwkundig ingenieurs is in zicht: we volgden een stage, maakten een bouwkundig project en legden examens af. Het is een druk jaar geweest, want tegelijker tijd hebben we samen dit eindwerk gerealiseerd. Bij het maken van dit eindwerk, onderzochten we voor het eerst een zelfgekozen onderwerp. Het eindwerk dat nu voor u ligt, betreft over het berekenen van structuren in glas meer bepaald het berekenen van primaire structurele element in glas. Onze motivering om te kiezen voor dit onderwerp is gegroeid uit het feit dat het dimensioneren van structuren in glas nog geen deel uitmaakt van onze studie.

Tot slot willen we van deze gelegenheid gebruik maken om onze dank betuigen aan Mr. Claude Pimpurniaux, hoofdcoördinator bij controlebureau SECO, voor het verstrekken van dit onderwerp, de ondersteuning tijdens het academiejaar en de informatie die hij ons gaf. Ook willen we in het bijzonder bedanken Mr. Jose Gouveia Henriques, docent van UHasselt, voor ons te begeleiden naar een goede masterproef. Als laatste nog een klein woord van dank naar onze ouders toe, omdat ze in ons geloofden en ons de kans gegeven hebben een studie aan de universiteit te volgen.

Bedankt!

Freya Leurs & Simon Seynaeve
4 Master Industriële Ingenieurswetenschappen
Universiteit Hasselt & Universiteit Leuven

ACKNOWLEDGEMENTS - ENGLISH

The end of our academic journey to become civil engineers is almost at an end: We did an internship, we worked on an construction engineering project and completed exams. It has definitely been a busy year because in combination with all of the above, we've completed the master's thesis. It was the first we were able to choose our own subject. This thesis, laying here before you, investigates the calculations regarding the design of primary laminated structural glass elements. Our motivation to choose this subject stems from the fact that calculations in glass are not (yet?) a part of our curriculum.

Lastly, we would like to take the opportunity to express our gratitude to the people who helped in the creation of this thesis. First of all, Mr. Claude Pimpurniaux, head coordinator at SECO and our external promotor; for giving us this interesting subject to investigate, the support during the academic year and for the necessary information when needed. Next, we would also like to thank Mr. Jose Gouveia Henriques, our internal promotor, in particular for the kind support and guidance to reach this result. And lastly to our parents, for believing in us and giving us the opportunity to study at Hasselt Univerisity.

Thank you, *obrigado*, *merci*!

Freya Leurs & Simon Seynaeve
4 Master Civil Engineering: Engineering Technology
Hasselt University & Leuven University

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I. GENERAL INTRODUCTION

1.1 Introduction to the subject

Designers and architects are continuously pushing the boundaries of construction engineering, creating more and more complex buildings and structures, often choosing unconventional construction materials. One of those materials is certainly glass.

Glass has been used in construction for hundreds of years mainly for window purposes in building envelopes. It has only been in recent decades that it has found an actual structural function in construction engineering [1] [2]. However, using glass as a structural material poses challenges primarily due to its failure behaviour. Glass is an extremely brittle material, failure occurs without any plastic deformation which can lead to the "instantaneous" collapse of the structure. On the other hand, the primary concern when designing a safe structure is targeting a ductile collapse mechanism.

The question then becomes how glass can be used as a structural material. In its simplest form, floated glass is extremely brittle but different techniques have been developed to improve the performance of glass materials resulting in enhanced glass products. Adhesively bonded glass segments are commonly used in structural applications, overlapping glass segments which are held together using polymeric interlayers. Other products can be reinforced glass elements. Reinforced glass can be described as a glass element that is combined with a stainless steel section which provides the necessary reinforcement in the weakest zone, the tensile zone of the element [3]. ….

In structural applications, the total failure of glass sections are unlikely since structural glass elements are rarely executed using float glass. Laminated glass is a glass product most used is such applications. While single layers in laminated elements are prone to breakage in some cases, e.g. concentrated impact loads, that usually does not result in a total loss of capacity of the section. Moreover, the broken layer(s) have a certain remaining capacity since the polymeric interlayer keeps the shards of glass from falling off [4]. The interlayers influence the crack initiation and development; the initiation of cracks do not correspond to an immediate loss of resistance.

In addition, glass has some advantages compared to other classic building materials. It is probably the most aesthetical material there is, making it the primary reason to choose it. In combination with its versatility and durability, glass then becomes an interesting option. The transparency and translucency of glass give buildings a light feel, both in terms of weight and natural light entering the structure, providing a sensation of openness and space to a building.

On the other hand, glass also has some disadvantages when compared to other construction materials. The most important one is its brittle and unpredictable behaviour. This property of glass requires the designers to incorporate a significant safety factor in order to limit its load capacity, which makes glass inappropriate for many applications. The material possesses microcracks on its surface and in its volume, which act as points where stress concentrations occur. This results in a considerable diminishment of the eventual strength [2]. Presently, designers are also bound by the limited design procedures available for structural applications of glass. These calculations do not yet belong to their repertoire at this moment. Glass is also an expensive material which may result in higher cost to the overall construction.

The current knowledge on the design of structural glass is limited. The corroboration of this fact is the absence of national guidelines in many countries and of course of a harmonized European standard. In Belgium, there is no standard for glass structures, thus the design of these types of structures is grounded in standards from other countries. The most applied standard in Belgium is the German standard. These standards are frequently outdated. Nowadays, most of the calculation methods are still based on experience and tests. It is the scope of the present thesis to contribute to the dissemination of the up to date calculation procedures of structural glass elements by providing a preliminary guidance for the design of primary glass elements.

1.2 Research Approach

1.2.1. Motivation

SECO [5] is an international company offering technical services for construction engineering. As structures are becoming more and more complex specific knowledge is required. Nowadays there is much more focus on different aspects such as structural stability, durability, fire safety, environmental care, efficient erection (time) and maintenance, economics, technological evolutions. SECO can provide quality, meaning a general up to date and highly targeted expertise, regularly implementing innovative solutions and applications going beyond the standards in practice. This includes the application of structural glass components.

Together with concrete, steel, masonry and timber, glass is one of the fundamental building materials found in modern day architecture. And while harmonized European standards exist for the application and the determination of structural strength and stability for these types of structures, which in turn are complemented by national application documents, such a harmonized European standard does not exist for structural glass.

Concrete has its high compressive strength and the flexibility to be moulded in nearly all desired shapes. Steel has its high tensile strength to weight ratio and possesses a certain ductility which allows it to deform before it fails under constant stress thus providing deformation capacity. Masonry systems have a long lifespan and increase a building's thermal mass. Timber combines being the most green building solution with quick erection times.

As contemporary architecture creates more transparent buildings, none of the building materials above are suitable to construct such a structure. Glass then becomes the favourite choice as a load-bearing construction material for structural components like columns, beams and panes. However, due to its brittle nature and unpredictable failure behaviour, glass is considered to be a structurally unsafe material [3]. A harmonized European standard can entail a general yet safe approach to this problem.

1.2.2. Question

One of the main issues in designing glass structures is the behaviour of the material. While for structures using other materials like concrete or steel, where the material behaviour can be predicted with considerable certainty, the variability in the case of glass behaviour is significant. It is known that glass is a brittle material, cracking without any plastic deformation. Furthermore, the impact of micro imperfections reduces its strength considerably, as well as other factors like its chemical composition and the production method.

In the past, the main use of glass in engineering applications was limited to windows where the structural performance was purely secondary. Consequently, calculation methods were limited, as no structural function was needed for these elements. Nowadays, many of the applications of glass have primarily structural requirements. These can be for example glass panels around staircases and/or elevators which have to resist loads induced by people resting against them; building façades, one of the most popular applications because of the aesthetic potential of glass. This condition forces the glass components to bear their self-weight and the wind loading; beams and columns, again for aesthetic reasons, this is an application that has known an increasing interest over the past years. In this case the glass components have to support other members and redistribute forces to the supports. Thus, for these reasons, safety has to be guaranteed continuously in every situation and therefore an up-to-date calculation method is needed.

1.2.3. Objectives

The general aim of this study is to provide guidance to the design of structural glass elements. In order to accomplish this goal, three main objectives have been defined as target.

The first objective is to provide guidance for the design of structural glass components: hereby performing a revision of the standards and rules for the design of structural glass components based on the different national standards which express the best practice on the subject.

Secondly, the application of the design procedure to a case study: application of the design rules to a real case. The case study is a construction surveyed by company SECO, promoter of this thesis.

Finally, the development of finite element models (FEM) to simulate single structural glass components: glass panes, beams, columns and beam-column connections. The validation of the numerical model based on the FEM to simulate the behaviour of the structural glass components against analytical methods and experimental tests available in literature.

1.2.4. Method

To safely assess the capacity of glass as a structural material, an analogous approach to existing European and national standards will be used. An existing document [2] is used as a guidance for the design of structural glass which will be modified and updated according to the most advanced and innovative structural glass-research and experimental evidence. This revision will be an update of the referred document based on the standards of different countries which express the best practice for the design of structural glass, for example the German standard , and the European Guidance for European Structural Design of Glass Components [6]. One of the main questions to be answered is the value of the safety factor. The analysis of the different standards will provide an assessment of the differences between the reference standards.

More and more, the finite element method is in the daily habits of engineers. If handle with care, the method is a powerful tool for the analysis and design of any structure. Thus, this analysis will be complemented by a finite element approach of laminated glass columns and beams, panes and a simple connection in a glass structure. To fully cover the problem, a simplified approach using regular strength of materials design principles will be added as well.

The effectiveness of both the mathematical and finite element method will be confirmed by comparing the results with those acquired during full scale experiments on structural glazing for the Belgian project of *The Museum of European History* in Brussels. This project includes all of the investigated elements referred above.

1.3 Thesis outline

The goal of this thesis is dual in nature. The first is to provide an indication of the capacity of simple, primary glass elements such as beams, columns, panes and connections. This means that glass properties and behaviour of glass need to be defined in order to create a uniform approach to the problem which will be done in chapters I and 0. This will be done based on available literature and both European and foreign national standards. To verify the validity of the equations, they will be tested by means of comparison with an actual case study in chapter I. Lastly, finite-elements models will be created to confirm and ultimately check the theoretical results in chapter 0.

Secondly, glass is rather new structural building with a lot potential and not only aesthetically. The importance of creating uniform and safe design rules are clearly noticeable when seemingly implausible are featured on the news. That all by itself, deserves glass its place in the spotlights.

II. STRUCTURAL GLASS

2.1 Applications

Structural glass is being used more often for a different number of applications. Usually the structural function goes by unnoticed. A perfect example is glass balustrades used in stair cases. Not just any glass can be used for this application since safety measures have to be taken against accidental forces applied on them e.g. people leaning against them or dropping on object on it. Even the steps of stairs are nowadays sometimes executed using glass panels which should be able to carry the load induced by the users. Structural glass thus has to be used.

Two types of glass' structural applications can be distinguished. Either using glass as a primary structural element or using it as a secondary structural element. Simply said, primary elements carry not only their own weight but also other loads induced by other elements. Secondary elements on the other hand, only have to carry their own weight. A more detailed description can be found in section 2.7. An example of both the primary and secondary use of glass elements can be found in Figure 1 and Figure 2.

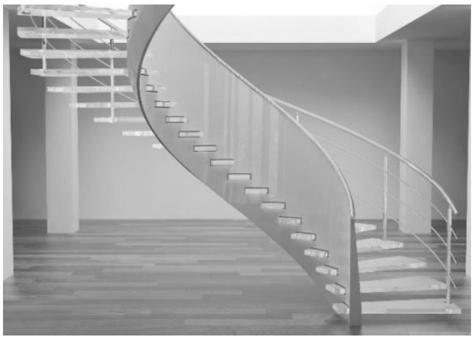


Figure 1: Application of a primary structural glass elements as steps of stairs

Figure 2: Application of a secondary structural glass element as a balustrade

These are small examples of applications where structural glass can be used. Larger structures are built every day surpassing the size of the simple examples. The most important use of structural is its application in entire glass façades, where all primary functions are executed by the glass itself. These are certainly not unknown to many people but the underlying calculation that go with the building of such a structure often are unknown, even to more experienced designers and engineers. A distinction in this case can be made between two types of facades or structures: a metal framework supporting glass panels and façades created entirely out of full load bearing glass elements. It's the latter which are investigated in this thesis.

Figure 3: Combination of a metal framework with glass panels

Figure 4: Construction entirely out of structural glass components

2.2 Glass Products

There are three types of basic glass products: float glass, annealed and toughened glass. Float glass is not suitable for structural purposes such as the ones in this thesis due to its limitations in bending strength. By annealing and/or toughening the glass, a variant on this type of glass has been developed in the form of laminated glass, to increase the bending strength of the glass material. This is due to an increased section without a general increase in imperfections because of the slenderness of the layers. It is in this form that structural component are used most often in structural applications [7].

2.2.1. Toughened or safety glass

Toughened glass (often referred to as safety glass) is one of glass products suited for structural purposes. It is thermically or chemically treated to enhance the properties of the glass and the remove negative effects caused by the production process. This is done by controlling the reheating process up to 600°C and rapidly cooling it down again. Once the glass has gone through its toughening process, the product can no longer be changed, making the toughening the absolute final step in its production process.

Toughened glass has gotten its name of safety glass because of its breakage pattern (Figure 5). When safety glass breaks, it does not break in large sharp pieces like normal glass, but because of the stresses caused by the toughening process breaks into millions of pieces of almost dull glass. This makes it ideally suitable for safety features such as shower doors or glass beams [2].

Figure 5: Breakage pattern toughened glass

2.2.2. Reinforced glass

Glass can also be reinforced either via the introduction of an internal or external metal wiring mesh to the glass product. Or even by the addition of an external metal section connected to the glass element. Glass however, is mostly used because of its aesthetic potential making reinforced glass only suitable in a few particular cases. By using reinforced glass, the designer has most likely chosen strength over transparency [2].

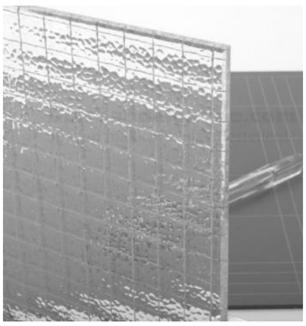


Figure 6: Reinforced glass

2.2.3. Laminated glass

The aim of this thesis is focused on the behaviour of laminated glass. This type of glass consists of multiple layers of (toughened) glass connected by polymeric interlayers. The interlayers are the glue that keep the glass layers together. These wafer-thin foils do not impede the translucency or transparency of the glass element.

This type of element possesses the most optimal properties for structural glass applications. It's the most aesthetically pleasing while performing well under loading, showing significant strength. That is why this is the most applied type of glass in glass façades [2].

Figure 7 shows an example of a laminated glass elements consisting of 2 glass layers and a thin interlayer, which is a common configuration. The elements in the case study will be comprised of up to six glass layers and five interlayers.

Figure 7: Laminated glass

2.3 Material Properties

2.3.1. Theoretical Resistance of Structural Glass

The properties of glass components largely depend on manufacturing, treating and finishing processes of the final product. This makes difficult to assign single, specific, structural material characteristics to it. However, not all types of glass are used for structural purposes making it not a necessity to do so. Annealed and tempered glass are the most suitable variants for structural applications. Therefor only these two types will be discussed in de following sections.

When looking at the stress-strain diagram of glass (Figure 8), it can be seen that glass does not have ductile behaviour as steel. Its elasticity modulus is only a third of the steel but more than twice that of concrete (30,000 N/mm²). All deformations in glass are pure elastic which means that once the load is removed, glass will return to its original state without any residual deformation. This also means that glass does not deform plastically before failing. This results in a brittle fracture behaviour. To prevent the latter, the loads on glass must be limited to cause a stress state well below its ultimate limit strength.

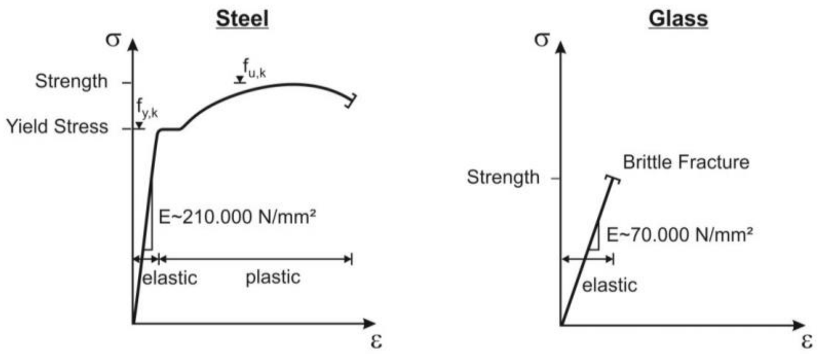


Figure 8: Stress-Strain Diagram Glass & Steel

2.3.1.1 *Compression*

The compression strength of glass is never an issue in both cases of annealed and tempered glass. It far outreaches its tensile strength. A glass element subjected to axial compression will not result in a diminishment of strength over time.

Different values for glass' compressive strength exist according to different consulted literature. None of these values stand out however. As part of the designing process, a reasonable choice must be made by the designer concerning the characteristic compressive strength of glass. This choice should be based on the most adverse value for safety reasons. The following value may be found in the literature [6]. The compressive strength of glass has never been its limiting factor.

$$f_{g,c,k} = 600\ N/mm^2$$

The effects of the annealing process are neglected (the removal of residual stresses caused by the manufacturing process, by reheating the glass to its annealing temperature and slowly cooling it down at a predetermined rate) when calculating the compressive strength. Similar to the case of pre-tensioned concrete sections, the pre-tensioning has a beneficial effect on the tensile forces but does not have any effect on the compressive strength.

2.3.1.2 *Tension*

Theoretically, the tensile strength of glass varies around 5000 to 8000MPa, the strength to break interatomic bonds in the material. However due to structural surface flaws (called Griffith flaws), inherent to glass as a material, the real strength is much lower. This property makes it more difficult to define numerical values for the tensile strength of glass than for its compressive strength because estimating the number and size of these surface flaws is not as evident as it might seem. Since high stress concentrations occur in the cracks and the redistribution of these stresses is not possible because of the lack of ductility, the theoretical bending strength of (annealed) glass reduces in practice to 20-80MPa.

Time is also an important variable to reckon with. Stresses applied during a certain time will result in the growth of the referred surface flaws and will consequently result in a decrease of strength. Therefore, the load duration also needs to be taken into account. This means that for certain loads the duration has to be known in order to correctly calculate the glass element. The relationship between stress and time can be expressed using Equation (1) and can be seen in Figure 9:

$$\sigma^n T = const. \tag{1}$$

where σ is the applied stress, *T* is the duration of the stress and *n* a constant with a value between 12 and 20.

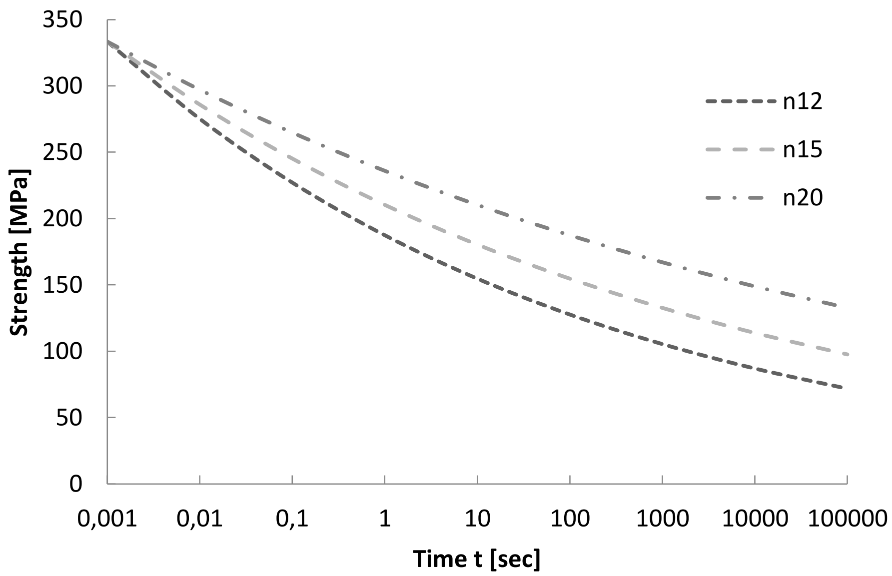


Figure 9: Glass's (tensile) strength diminishes over time

Static fatigue in glass elements is very important to take into account when designing structural elements. Static fatigue results in the failure of an element over time due to a constant load (which also decreases for static fatigue over time) smaller than the critical load. In brittle materials, static fatigue occurs as the slow growth of subcritical cracks to a size at which they will propagate catastrophically resulting in the failure of the section.

The current European prestandard for structural glass (prEN 13474-3) gives the following formula [Equation (2)] to calculate the design tensile strength of annealed glass elements:

$$f_{g,t,d} = \frac{k_{\text{mod}} \cdot k_{sp} \cdot f_{g,t,k}}{\gamma_{M,A}} \qquad (2)$$

With: k_{mod} = modification factor for load duration

k_{sp} = modification factor for the surface profile

$f_{g,t,k}$ = The characteristic bending strength of annealed glass $\left(= 45 N/mm^2\right)$

$\gamma_{M,A}$ = Safety factor for annealed glass

The modification factors and safety factor will be further discussed in chapter III where the design approach for glass elements is given.

2.3.1.3 *Bending*

The bending strength of structural glass elements can be compared to the bending strength of concrete elements. While the material is very strong in compression, it's relatively weak in tension. Therefore, in most cases, a glass element will fail when its maximum tensile strength is exceeded. So, in other words, the tensile strength of glass materials in the case of bending is usually the defining strength.

Depending on the literature, different values can be found for the characteristic bending strength of certain types of glass, ranging from 45N/mm² for annealed glass to 120N/mm² for thermally toughened glass.

The European guidelines [6] currently suggest using a value between 30 to 80N/mm² which is the same for the maximum tensile stress in glass.

2.4 Toughened Glass

The toughening process distributes residual stresses by heating the element up to 650°C and rapidly cooling the surfaces and consequently a retarded cooling of the inner part. Thereby distributing the stresses via a parabolic curve (Figure 10) with the outer layers in compression (effectively closing the Griffith cracks) and the inner layers in tension due to its restrained contraction.

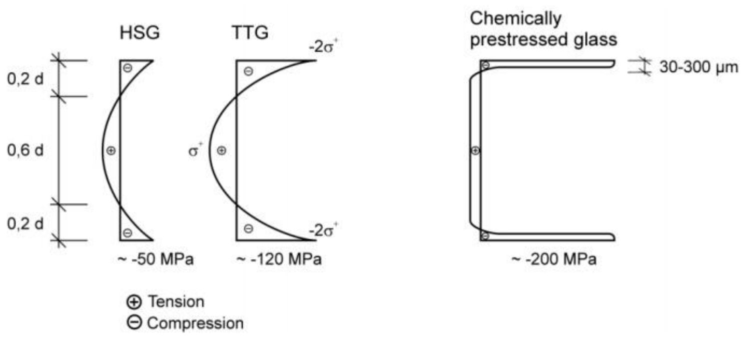


Figure 10: Distribution of Stresses after Toughening

The effects resulting from the toughening process are (Table 1):

- An increased bending strength of the glass compared to regular float glass;
- When thermally toughened, glass will break into small pieces caused by the pre-stress energy. Without tempering, it would break into large shards, increasing the probability of injuries occurring. Therefore, it is also called safety glass.
- The probability of breakage due to accidental impact is also considerably lower.

Table 1: Effect of toughening process on bending strength

	Annealed/Float glass	Heat strengthened glass (HSG)	Thermally toughened glass (TTG)
Characteristic bending strength $f_{b,k}$	40 N/mm²	70 N/mm²	120 N/mm²
Degree of surface pre-stress	≈ 0MPa	≈ 30-50MPa	>90MPa

The toughening process can also be taken into account when calculating the tensile strength. The effect of the hardening process can be superimposed onto the formula found in a previous section resulting in Equation (3):

$$f_{g,t,d} = \frac{k_{\text{mod}} \cdot k_{sp} \cdot f_{g,t,k}}{\gamma_{M,A}} + \frac{k_v \cdot \left(f_{b,k} - f_{g,t,k}\right)}{\gamma_{M,v}} \tag{3}$$

With: k_v = a factor that depends on the type of toughening process (horizontal treatment: 1; vertical treatment: 0,6)

$f_{b,k}$ = the characteristic value of the bending strength due to the toughening process

$\gamma_{M,v}$ = safety factor for the toughening process (can be assumed 1,2)

2.5 Imperfections

The production of glass always results in the introduction of imperfections to the material. Although there are processes which can decrease the amount of imperfections, they can never be fully removed. Moreover, these imperfections are difficult to quantify even if their presence can be determined visually.

Most commonly found voluminal imperfections are microscopic air bubbles, micro-cracks, hard inclusions (such as nickel particles which expand in volume even after treatment). These could lead to a potential failure of glass in future situations.

Contact with foreign materials can also lead to imperfections such as micro-cracks, before (transport), during construction and even after the completion of the structure. These imperfections are called surface flaws.

During inspection, the size of imperfections need to be examined as they significantly decrease the strength of the final product [2].

2.6 Fracture mechanics

Depending on the type, glass can exhibit multiple fracture mechanics, typically as a results of the treatment process or not. The fracture pattern of toughened (or safety) glass has been explained before in section 2.2.1. When it breaks, it does so in tiny glass pieces which are not sharp to the touch.

Floatglass has the most known fracture pattern as it breaks in large chunks of sharp pieces of glass. These could potentially be very dangerous.

Laminated glass has interlayers which act as glue keeping the shards of glass from falling off. Depending on the type of glass different fracture patterns can be found.

Following figures give examples of different fracture patterns [7].

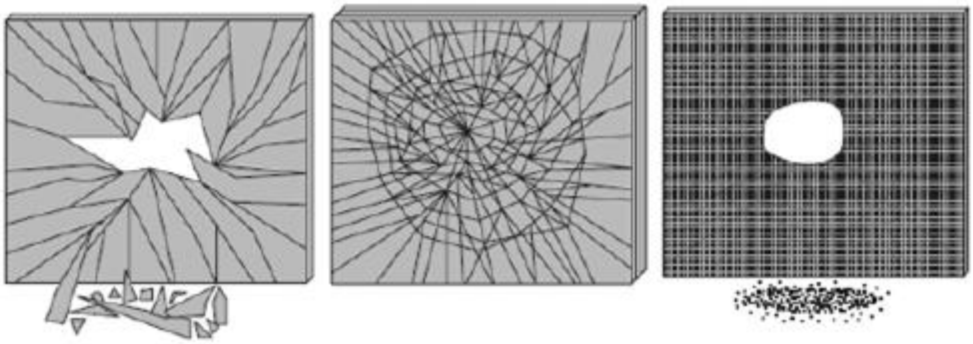


Figure 11: a) Float glass; b) Laminated glass; c) Toughened glass

2.7 Primary & secondary structural glass components

A distinction needs to be made between primary and secondary structural glass components. Primary elements are components which need to be able to resist direct loading and support secondary members. Thus primary components do not only bear their own dead weight and externally imposed loads, they also need to carry the loads coming from other members (primary and secondary). Since the case study only contains primary components, the focus of this thesis is limited to these elements.

As opposed to primary elements, secondary elements are only assumed to carry their own weight and specific loads but do not have to support other members.

No matter the type of element, a calculation method needs to be applied with sufficient care towards the possibility of the further failure of elements after the failure of primary members. This gives another distinction between the two types: the failure of primary element could potentially mean the further failure of other primary and secondary elements and thus the entire structure. The failure of secondary element results in a local failure.

Hereby, it should be noted that a partial failure, meaning one or more glass layers of the section, could also be regarded as a failure of the element.

2.8 Types of connections in glass structures

2.8.1. Metal – Bolted connection

The bolted connection is probably one of the most applied connections in structures. They are even used in glass structures. This requires at least the drilling of one hole in the element. Due to the nature of bolted connections, a concentration of stresses can occur around the openings as experienced in float glass. The bolted connection is therefore only suitable for (thermally) toughened glass. Because of the larger dimensions of the drilled hole compared to the actual bolt, a material is applied, a type of mortar, between the glass and the bolt to avoid direct metal-to-glass contact.

The material has the appearance of glue rather than an actual mortar but its properties are very different [8].

Figure 12: Bolted connection between beam and column

2.8.2. Metal – Clamped connection

The clamped connection is the most applied connection in glass structures. They are used at corners and at edges. The metal 'shoe' is equipped with an elastic lining to avoid damage due to a direct connection between the steel and glass components [8].

Figure 13: Clamped connection at the bottom of a column

2.8.3. Glue – Adhesive connection

A glued connection can be a connection between metal and glass elements by means of an adhesive interlayer like a resin, foil or glue. It can also be between to glass layers but that is typically specified as being an adhesive glass connection [8].

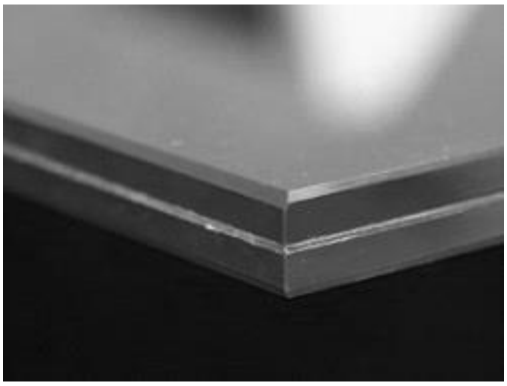


Figure 14: An adhesive glass connection in laminated glass

2.9 Reference standards for the design of structural glass

The amount of actual fully developed practice standard for structural glass component are limited. A general standard does not yet exist. Because of the increased applications of structural glass, wether it is for façades, stairs or bridges, a safe and universal standard should exist for the design of these structures, as it exists for steel, reinforced concrete and wood [EN 1993 et seq., EN1992-1 et seq. and EN 1995-1 et seq.].

Some standards are available; Europe has some preliminary guidelines and Germany, Canada and the United States have extensive standards but are sometimes contradictory. In other European countries, and in particular in Belgium, the design standards for glass structures are very limited. In order to achieve a safer and optimized design, to accomplish the demands and challenges of modern engineering structures, it is required the development of a harmonized European standard for glass.

III. DESIGN OF GLASS COMPONENTS & CONNECTIONS

3.1 Safety factors

Given the unpredictable failure behaviour of glass, which is brittle in nature, and the material imperfections introduced by the manufacturing process, relatively high safety factors need to be considered for the material and the acting loads. This is to guarantee a sufficiently robust and safe structure. Technological advances over the years allowed for a better control of the production process of glass elements reducing the imperfections. However, the exact number of the relative volume or area of imperfections is difficult to quantify, thus meaning that the exact behaviour of the material cannot be accurately predicted.

By choosing the right type of manufacturing process and/or post-process treatment, safety factors can be reduced. This can be done by either changing the cooling process, either the heating process or even both, and in doing so different end results can be expected.

The nature of the behaviour of the material does not change, meaning that it remains brittle. To ensure the material is not subjected to loads near its ultimate capacity limit, significant values for the safety factor need to be kept in place. The European guidelines suggest using a safety factor of 1.8 for annealed glass. For other types glass specific safety factors could not be found.

$$\gamma_{M,A} = 1.8$$

3.2 Modification factor for load duration k_{mod}

The modification factor k_{mod} [2] allows the designer to include the negative effect caused by a decrease of glass strength over time (cfr. timbre); the longer the duration of the load, the smaller this modification factor will become.

The European guidelines describes k_{mod} [6] as a function [Equation (4)] of time [h] and reads:

$$\alpha(t) = k_{\text{mod}} = \left(\frac{t_V(A)}{t_0}\right)^{\frac{1}{n}} = \left(\frac{5}{t}\right)^{\frac{1}{c}} \quad (4)$$

where *t* is the duration of the load in seconds and *c* a constant of corrosion depending on the boundary conditions (Figure 15).

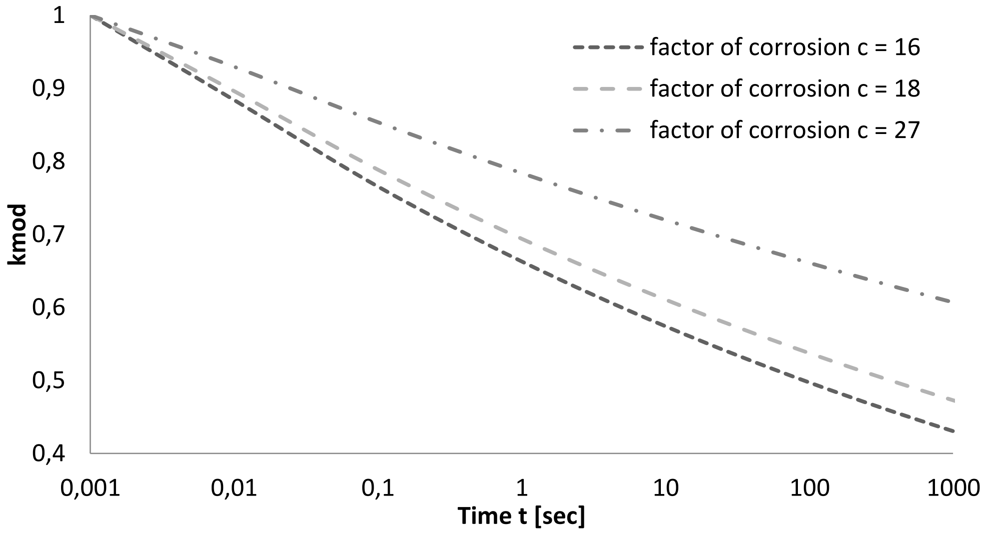


Figure 15: k_{mod} in function of time

The European pre-standard (prEN 13474-3) assumes the following equation [Equation (5)] for k_{mod}:

$$k_{\text{mod}} = 0{,}663 \,.\, t^{-\frac{1}{16}} \tag{5}$$

The maximum value of k_{mod} never exceeds 1 and is always bigger than 0,25. Depending on the country, different functions for k_{mod} are possible. Italy uses a similar function [Equation (6)], but more conservative (-11,8% difference in design strength) (Figure 16).

$$k_{\text{mod}} = 0{,}585 \,.\, t^{-\frac{1}{16}} \tag{6}$$

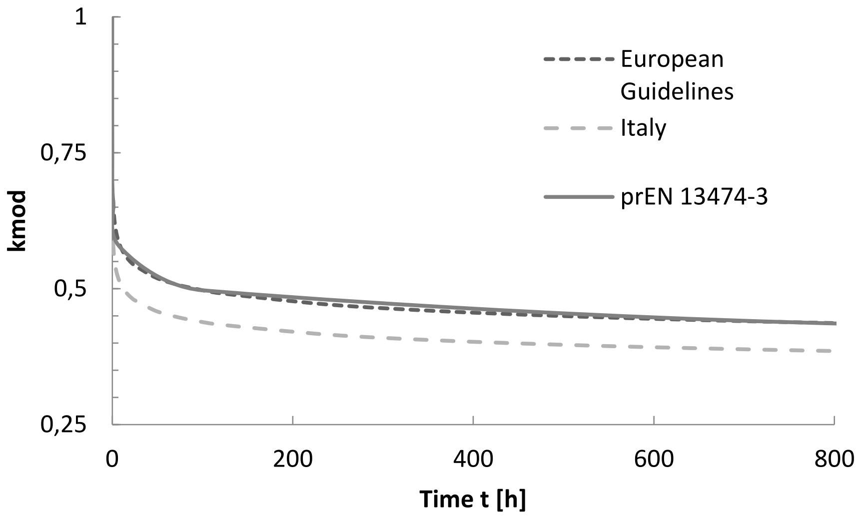


Figure 16:k_{mod} according to the European Guidelines, Italian Norm & European pre-standard

Other national standards, e.g. Germany and Austria, assume certain values (Table 2) depending on the duration of the load rather than being based on a specified equation:

Table 2: kmod-values Germany; Austria

Load Duration	Germany	Austria
Short	0,7	1
Middle	0,4	0,6
Permanent	0,25	0,6

The following table (Table 3) gives some k_{mod}-values for certain specific loads.

Table 3: Examples k_{mod} for specific loads

Load	Duration	k_{mod}
Persons	short and unique	1
Wind	short and multiple	0,74
Snow	intermediate	0,43
Self-weight	permanent	0,29
Average temperature variation (max. 11h)	intermediate	0,57
Annual temperature variation (max. 6 months)	intermediate	0,39
Air pressure variation	intermediate	0,5

The values above, however only reflect the k_{mod}-values of individual loads. In reality, a load rarely acts alone on an element or structure; it will rather be subjected to a combination of different loads with its own unique duration and therefor its own unique time modification factor. Timbre uses the k_{mod} of the shortest acting loads. The question is; is this also the case for glass.

In literature [ref] can be found for glass that k_{mod} is calculated based on each specific combination of loads. This means that for each combination another k_{mod}-factor will need to be used to check the element. This is done by combining the k_{mod}-values of the individual loads to a single $k_{mod,combi}$-value using Equation (7) for the load combination.

$$k_{\mathrm{mod},combi} = \frac{\gamma_G \cdot g_k + \gamma_{Q,1} \cdot q_{k,1} + \sum_i \gamma_{k,i} \cdot \psi_{0,i} \cdot q_{k,i}}{\frac{\gamma_G \cdot g_k}{k_{\mathrm{mod},ind,g}} + \frac{\gamma_{Q,1} \cdot q_{k,1}}{k_{\mathrm{mod},ind,q_1}} + \frac{\sum_i \gamma_{k,i} \cdot \psi_{0,i} \cdot q_{k,i}}{k_{\mathrm{mod},ind,q_i}}} \tag{7}$$

With:

γ_i = safety factors $[-]$

g_k / q_k = charactestic loads [N/m²]

ψ_0 = ponderatio n factor $[-]$

$k_{mod,ind}$ = individual k_{mod} $[-]$

3.3 Modification factor for surface profile k_{sp}

The modification factor for the surface profile of an element assures that the type of glass surface, which depends on the manufacturing process and not the chemical composition, is taken into account. For the most common case, that of float glass, the value for k_{sp} is 1. The table below (Table 4) can be used for other cases.

The lamination of glass elements does not have any effect on the surface profile of the glass layers it is comprised of. Since no surface treatments are used, k_{sp} can also be assumed 1 for laminated section.

Table 4: k_{sp} depending on Type of Glass [7]

Type of Glass	k_{sp}
Float Glass	1,0
Coated Float Glass	1,0
Etched Glass	0,75
Coated Etched Glass	0,75
Polished Reinforced Glass	0,75
Coated Reinforced Glass	0,6

3.4 Beams

3.4.1. Bending

The design of rectangular laminated glass beams is not different from other materials such as steel. A proper analysis of the element can be done by calculating the resistive moment of the beam and comparing it with the acting bending moment. In any case, the resistive moment should be larger than the acting bending moment [see Equation (8)].

$$M_{Rd} \geq M_{sd} \tag{8}$$

Where the resistive moment can be calculated using Equation (9):

$$M_{Rd} = f_{g,d} \cdot W \tag{9}$$

The resistance of the material can be obtained by using the Equations (2) 2.3.1.2and (3) for float glass and toughened glass respectively [2] [10]:

$$f_{g,d} = k_{\text{mod}} \cdot \frac{f_{g,k}}{\gamma_m \cdot k_A} \cdot \gamma_n \tag{10}$$

$$f_{g,t,d} = \frac{k_{\text{mod}} \cdot k_{sp} \cdot f_{g,t,k}}{\gamma_{M,A}} + \frac{k_v \cdot \left(f_{b,k} - f_{g,t,k}\right)}{\gamma_{M,v}} \tag{11}$$

3.4.2. Deflection

The total deflection also has to remain limited. As with other materials, the deflection is checked in SLS in a similar manner. These values can be cross-referenced with a deflection requirement which in most cases is limited to L/300.

3.4.3. Lateral Torsional Buckling

A beam subjected to bending also partially experiences local compressive forces. Because elements subjected to compression are likely to become unstable, lateral torsional buckling needs to be verified. In general for a monolithic material, the following Equation (12) is applied to find the critical bending moment:

$$M_{cr} = C_1 \frac{\pi^2 \cdot E \cdot I_z}{L_{LT}^2} \left[\sqrt{C_2 z_a + \frac{GK \cdot L_{LT}^2}{\pi^2 \cdot E \cdot I_z}} + C_2 z_a \right] \tag{12}$$

C_1 and C_2 are hereby dependent on the type of load, L_{LT} is the length of the beam, G the material shear modulus and K the torsional moment of the section. Factor z_a is the position of the load relatively to the geometric centre of the cross-section [8].

This equation is slightly altered for laminated glass section by substituting the section properties for equivalent or effective section properties. Equation (12) becomes the following Equation (13):

$$M_{cr} = C_1 \frac{\pi^2 \cdot EI_{z,eff}}{L_{LT}^2} \left[\sqrt{C_2 z_a + \frac{GK_{eff} \cdot L_{LT}^2}{\pi^2 \cdot EI_{z,eff}}} + C_2 z_a \right] \tag{13}$$

Where the effective stiffness of the beam $EI_{z,eff}$ can be calculated using Equation (14) [8]:

$$EI_{z,eff} = E \cdot I_s \left(\frac{\alpha \cdot \beta \cdot \pi^2 + \alpha + 1}{1 + \pi^2 \beta} \right) \tag{14}$$

Then, according to the number of glass layers and interlayers, the effective properties of the beam are calculated as described here after [9].

2 glass layers

When the beam is comprised of two glass layers the following method can be used to calculate the critical buckling moment of the beam.

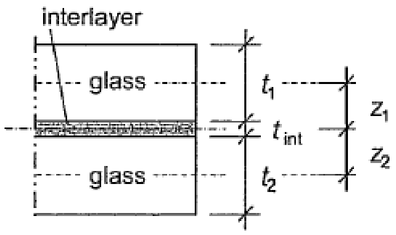


Figure 17: Two-layered laminate

Inertia parameter α and stiffness parameter β can be respectively calculated using Equations (15) and (16):

$$\alpha = \frac{I_1 + I_2}{I_s} \tag{15}$$

$$\beta = \frac{t_{int}}{G_{int} h (z_1 + z_2)^2} \cdot \frac{E \cdot I_s}{L_{LT}^2} \tag{16}$$

Where I_i is calculated around the local bending axis using Equation (17) and G_{int} is the shear modulus of the interlayer. Factors z_1 and z_2 are distances between the respective layers.

$$I_s = h\left(t_1 z_1^2 + t_2 z_2^2\right) \tag{17}$$

Next GK_{comp} of the laminated section can be calculated as [Equation (18)]:

$$GK_{comp} = GI_{s,comp} \left(1 - \frac{2}{\lambda h} \tanh \frac{\lambda h}{2} \right) \tag{18}$$

Where λ becomes [Equation (19)]:

$$\lambda = \sqrt{\frac{G_{int}(t_1 + t_2)}{G(t_{int} t_1 t_2)}} \tag{19}$$

With G is the shear modulus of the glass layers(!).

Furthermore $I_{S,comp}$ is found by using Equation (20):

$$I_{s,comp} = 4\left(\frac{t_1 + t_2}{2} + t_{\text{int}}\right)^2 \frac{t_1 t_2}{t_1 + t_2} h \tag{20}$$

So lastly GK_{eff} becomes Equation (21):

$$GK_{eff} = \begin{cases} GK_{glass1} + GK_{glass2} + GK_{comp} \\ GK_{glass1} + GK_{glass2} + GK_{glass3} + GK_{comp} \end{cases} \tag{21}$$

3 glass layers

When the beam is comprised of three glass layers the following method can be used to calculate the critical buckling moment of the beam.

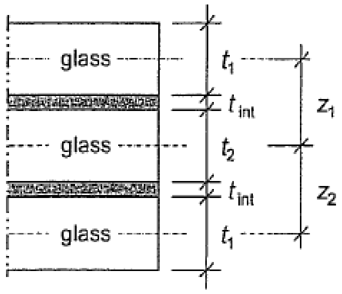


Figure 18: Three-layered laminate

Inertia parameter α and stiffness parameter β become [Equation (22) and (23)] since the first glass layers is used twice:

$$\alpha = \frac{2I_1 + I_2}{I_s} \tag{22}$$

$$\beta = \frac{t_{\text{int}}}{2G_{\text{int}} h z_1^2} \cdot \frac{E \cdot I_s}{L_{LT}^2} \tag{23}$$

Where again I_i is calculated around the local bending axis using Equation (24) and G_{int} is the shear modulus of the interlayer. Factor z_1 is a distances between the respective layers.

$$I_s = 2ht_1 z_1^2 \tag{24}$$

Next GK_{comp} of the three-laminated section can be calculated as [Equation (25)]:

$$GK_{comp} = GI_{s,comp}\left(1 - \frac{2}{\lambda h}\tanh\frac{\lambda h}{2}\right) \tag{25}$$

Where λ [Equation (26)]:

$$\lambda = \sqrt{\frac{G_{\text{int}}(t_1 + t_2)}{G(t_{\text{int}} t_1 t_2)}} \tag{26}$$

With G is the shear modulus of the glass layers(!).

Furthermore $I_{S,comp}$ is found by using [Equation (27)]:

$$I_{s,comp} = 2(t_2 + 2t_1 + t_1)^2 t_1 h \tag{27}$$

And lastly GK_{eff} becomes [Equation (28)], the same as for the two-layered laminate [Equation (21)]:

$$GK_{eff} = \begin{cases} GK_{glass1} + GK_{glass2} + GK_{comp} \\ GK_{glass1} + GK_{glass2} + GK_{glass3} + GK_{comp} \end{cases} \tag{28}$$

3.5 Columns

3.5.1. Compression

Columns in compression can be checked using this simple Equation (29):

$$f_{g,c,d} \geq \frac{N_{sd}}{A} \tag{29}$$

Where A represents the entire glass section of the beam, the influence of the interlayers is neglected.

3.5.2. Buckling

Elements subjected to compression have the tendency to become unstable and buckle. The buckling calculation of a single monolithic glass layer is done using Euler's buckling formula [Equation (30)]:

$$N_{cr} = \frac{\pi^2 \cdot E \cdot I}{L_k{}^2} \tag{30}$$

The value of the critical buckling load is then compared with the actual acting compressive force [2].

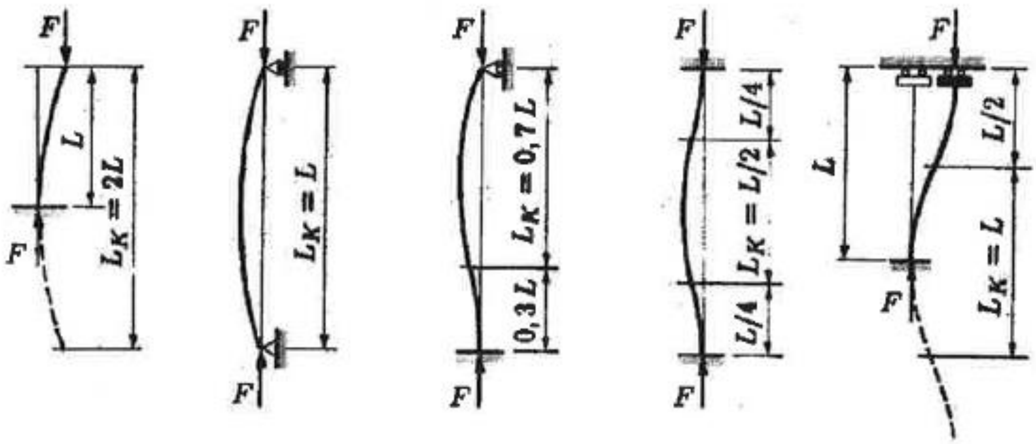


Figure 19: Euler buckling lengths

The design of glass elements subjected to compression are calculated using either one of the two following principles:

- A second order calculation or
- A calculation of the buckling resistance via a reduction coefficient

In the first case, the acting stress is compared to the design stress of the glass. The acting stress can be calculated as [Equation (31)]:

$$\sigma = \frac{N}{A} \pm \frac{N}{W}\left[\frac{e}{\cos\left(L_k \Big/ 2\cdot\sqrt{\frac{N}{E\cdot I}}\right)} + \frac{w_0}{1-\frac{N}{N_{cr,e}}}\right] \leq f_{g,t,d} \tag{31}$$

Where A is the area of cross-section, W is the section modulus, e the eccentricity, w_0 the initial deflection and $N_{cr,e}$ which is Euler's critical load [2].

The buckling resistance of the column using the reduction coefficient can be calculated as [Equation (32)]:

$$N_{rd,b} = \chi_b \cdot A \cdot f_{g,t,d} \tag{32}$$

Where the reduction coefficient is a function of $\bar{\lambda}$ [Equation (33)]:

$$\bar{\lambda} = \sqrt{\frac{f_{g,t,d}}{\sigma_k}} \tag{33}$$

The maximum allowable stress in the glass section is expressed by Equation (34) :

$$\sigma_{\max} = \min\left\{\frac{N}{A} \pm \frac{N}{W}\left[\frac{e}{\cos\left(L_k \Big/ 2\cdot\sqrt{\frac{N}{E\cdot I}}\right)} + \frac{w_0}{1-\frac{N}{N_{cr,e}}}\right]; \frac{N_{rd,b}}{A}\right\} \tag{34}$$

3.5.3. Buckling of n-layers

Elements consisting of multiple glass layers require a different approach. Similarly to the case of lateral torsional buckling, an equivalent stiffness of the section is determined based on the composition of the glass and interlayers.

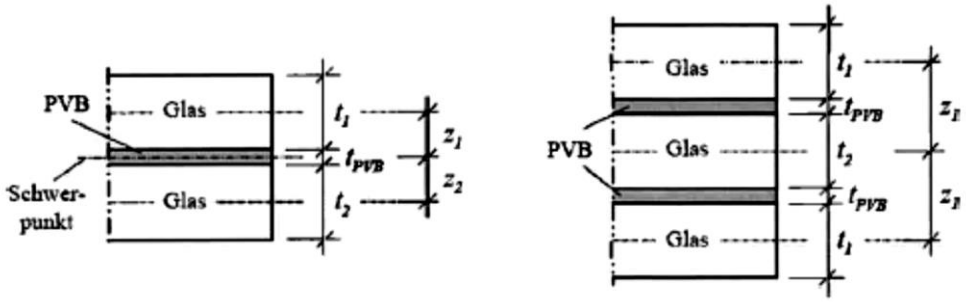


Figure 20: Buckling of n-layers

New factors are introduced: α and β. These account for the laminated effects of the element. This means that the moment of inertia of each individual layer is considered as well as their respective distances to the neutral line. Euler's buckling formula becomes Equation (35):

$$N_{cr} = \frac{\pi^2(1+\alpha+\pi^2\cdot\alpha\cdot\beta)\cdot E\cdot I_S}{(1+\pi^2\cdot\beta)L_k{}^2} \tag{35}$$

Where:

$$\alpha = \frac{\sum_{i=2}^{i=n} I_i}{I_s} \tag{36}$$

$$I_S = b\cdot\sum_{i=2}^{i=n} t_i\cdot z_i^2 \tag{37}$$

$$\beta = \frac{t_{\text{int}}\cdot E\cdot I_S}{G_{\text{int}}\cdot b\cdot\left(\sum_{i=2}^{i=n} z_i\right)\cdot L_k} \tag{38}$$

A different approach can be followed, using Equation (39) to calculate the critical force. This by using the effective section of a layers.

$$t_{eff} = \sqrt[3]{\frac{12\cdot I_S\cdot(1+\alpha+\pi^2\cdot\alpha\cdot\beta)}{b\cdot(1+\pi^2\cdot\beta)}} \tag{39}$$

The same equations as above [Equations (36), (37) and (38)] apply to calculate α and I_S. Euler's buckling equation [Equation (30)] becomes Equation (40) [2]:

$$N_{cr} = \frac{\pi^2\cdot E\cdot I_{eff}}{L_k{}^2} \tag{40}$$

With:

$$I_{eff} = \frac{h\cdot t_{eff}^3}{12} \tag{41}$$

The formulae of the buckling of n-layers and method using the effective thickness [Equation (35) and (40)]] should generate similar results.

3.6 Panes

Although structural glass panes are rarely executed non-laminated, it is worth understanding the behaviour of monolithic panes under different types (or combinations) of loads. Glass panes are glass elements/components mostly used. They can be combined with other glass structural elements, in a glass construction, or with other structural systems using other materials, as steel or aluminium. This means that although the possibility does exist to use glass beams and columns in combination with glass panes, panes can always be found to be applied independently.

When the glass pane dimensions become larger, a different equation other than Kirchhoff and Lagrangres' simple bending of plates theory [9] needs to be used in order to analytically check the stability of glass panes. Kirchhoffs' theory makes certain assumptions that do not apply to a general glass pane, more specifically due the plate's slenderness (total deflection to thickness ratio) as well as the possible occurrence of normal forces and membrane forces and depending on the size of to load, also includes the limitation on the total deflection.

Kirchhoffs' assumptions are the following [10]

1) A homogeneous, isentropic and elastic material
2) A constant plate thickness
3) No membrane forces occur
4) The length and/or width are larger than 5 times the thickness of the plate (slenderness)
5) The deflection is limited to one fifth of the total thickness
6) No normal forces act on the plate

Which leads to Kirchhoff's and Lagrange's general equation [Equation (42)] [11] for the deflection of plates:

$$\frac{\partial^4 w}{\partial x^4} + 2 \cdot \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \tag{42}$$

With D [Equation (43)]:

$$D = \frac{h^3}{12} \cdot \frac{E}{1 - \upsilon^2} \tag{43}$$

This equation expresses the deflection of the pane in function of the position (x,y) on the pane, the size of the load (q) and the flexural rigidity of the pane (D) which describes the stiffness of the plate.

3.6.1. Linear

The assumption of linear behaviour remains the same for the first case, which means the pane is assumed to be deforming purely due to bending moments and normal in-plane forces do not occur. For simply supported rectangular plates, Navier's solution offers important mathematical advantages, since the solution of the governing fourth-order partial differential equation is reduced to the solution of a much more simpler algebraic equation. Navier's solution is presented in [9] for simply supported panes, where the deflection is expressed by the means of a double sine series [Equation (44)]:

$$w(x,y)=\frac{1}{D.\pi^4}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{P_{mn}}{\left[\left(m^2/a^2\right)+\left(n^2/b^2\right)\right]^2}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \quad \text{for } m,n=1,3,5... \tag{44}$$

Where:

$$P_{mn}=\frac{16q}{\pi^2 mn} \tag{45}$$

And:

$$W_{mn}=\frac{P_{mn}}{D\pi^4\left[\left(m^2/a^2\right)+\left(n^2/b^2\right)\right]^2} \tag{46}$$

The substitution of the equation for the deflection[Equation (44)] into the expressions for internal moments, shear and edge forces gives the required values.

Respective bending moments and stresses can then be calculated as [Equation (47) to (52)]:

- **Bending moments**

$$m_x=\pi^2 D\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left[\left(\frac{m}{a}\right)^2+\upsilon.\left(\frac{n}{b}\right)^2\right].W_{mn}.\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \tag{47}$$

$$m_y=\pi^2 D\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left[\left(\frac{n}{b}\right)^2+\upsilon.\left(\frac{m}{a}\right)^2\right].W_{mn}.\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \tag{48}$$

$$m_{xy}=-\pi^2 D(1-\upsilon)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{m.n}{a.b}.W_{mn}.\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right) \tag{49}$$

-

- **Stresses**

$$\sigma_x = -\frac{E.z}{1-\upsilon^2}\left(\frac{\partial^2 w}{\partial x^2} + \upsilon.\frac{\partial^2 w}{\partial y^2}\right) \tag{50}$$

$$\sigma_y = -\frac{E.z}{1-\upsilon^2}\left(\frac{\partial^2 w}{\partial y^2} + \upsilon.\frac{\partial^2 w}{\partial x^2}\right) \tag{51}$$

$$\sigma_{xy} = -\frac{E.z}{1+\upsilon}\left(\frac{\partial^2 w}{\partial x \partial y}\right) \tag{52}$$

It also provides the solution for the deflection in case of a concentrated force. The equation reads [Equation (53)]:

$$w(x,y) = \frac{4.P}{\pi^4 ab D}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\sin(m\pi\xi/a)\sin(n\pi\eta/b)}{\left[\left(m^2/a^2\right)+\left(n^2/b^2\right)\right]^2}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \quad for\, m,n = 1,2,3... \tag{53}$$

Where:

$$P_{mn} = \frac{4P}{ab}\sin\left(\frac{m\pi\xi}{a}\right)\sin\left(\frac{n\pi\eta}{b}\right) \tag{54}$$

The maximum bending moments and stresses can also be calculated using the following equations [Equation (55) to (60)]:

- **Bending moments**

$$m_x = -D\left[\frac{\partial^2 w}{\partial x^2} + \upsilon.\frac{\partial^2 w}{\partial y^2}\right] \tag{55}$$

$$m_y = -D\left[\frac{\partial^2 w}{\partial y^2} + \upsilon.\frac{\partial^2 w}{\partial x^2}\right] \tag{56}$$

$$m_{xy} = -(1-\upsilon)D\left[\frac{\partial^2 w}{\partial x \partial y}\right] \tag{57}$$

- **Stresses**

$$\sigma_x = -\frac{E.z}{1-\upsilon^2}\left(\frac{\partial^2 w}{\partial x^2} + \upsilon.\frac{\partial^2 w}{\partial y^2}\right) \tag{58}$$

$$\sigma_y = -\frac{E.z}{1-\upsilon^2}\left(\frac{\partial^2 w}{\partial y^2} + \upsilon.\frac{\partial^2 w}{\partial x^2}\right) \tag{59}$$

$$\sigma_{xy} = -\frac{E.z}{1+\upsilon}\left(\frac{\partial^2 w}{\partial x \partial y}\right) \tag{60}$$

The issue with this solution for the concentrated force is that the convergence is slow in the vicinity of the concentrated load. The second derivatives of the deflection equation will even diverge at the point where the force is applied meaning the exact deflection in the centre of the pane cannot be determined analytically.

3.6.2. Non-Linear

In reality, due to the slenderness of the pane, the glass will not only be subjected to pure bending but will also develop in-plane axial forces or membrane forces. This is called non-linear behaviour and will cause the pane to react more stiff than what is suspected from being solely subjected to pure bending. These membrane forces need to be calculated as they generate an extra amount of stress on the glass pane.

A solution is suggested by Nishawala in [12] for a simply supported pane. By using the following equations [Equations (61) to (67)]:

$$w(x, y) = \sum \omega_{mn} W_{mn}(x, y) \quad (61)$$

$$W_{mn} = \sin(\alpha_m x)\sin(\gamma_n y) \quad (62)$$

$$\phi(x, y) = \sum \phi_{mn} W_{mn}(x, y) \quad (63)$$

$$\phi_{mn} = \cos(\alpha_m x)\cos(\gamma_n y) \quad (64)$$

Where:

$$\alpha_m = \frac{m\pi}{a}; \qquad \gamma_n = \frac{n\pi}{b} \quad (65)$$

And extracting w_{mn} from following equation:

$$32 \frac{Eh}{D} \frac{\alpha_m^4 \gamma_n^4 \varsigma_{mn}^2}{\left(\alpha_m^2 + \gamma_n^2\right)^4} w_{mn}^3 + w_{mn} = \frac{q_{mn}}{D\left(\alpha_m^2 + \gamma_n^2\right)^2} \quad (66)$$

The deflection due to the non-linear behaviour of the pane can be calculated.

Where:

Table 5: Values of ς_{mn} [Equation (67)] [12]

ς_{mn}					
n/m	**1**	**2**	**3**	**4**	**5**
1	$\frac{4}{3\pi^2}$	0	$\frac{4}{9\pi^2}$	0	$\frac{4}{15\pi^2}$
2	0	0	0	0	0
3	$\frac{4}{9\pi^2}$	0	$\frac{4}{27\pi^2}$	0	$\frac{4}{45\pi^2}$
4	0	0	0	0	0
5	$\frac{4}{15\pi^2}$	0	$\frac{4}{45\pi^2}$	0	$\frac{4}{75\pi^2}$

$$\rightarrow \varsigma_{mn} = \frac{4}{3mn\pi^2} \quad (67)$$

Note that if ς_{mn} is equal to zero, the linear solution is found for that coefficient.

3.6.3. Laminated

The next step in calculating glass panes is taking into consideration the lamination properties of the element. This means the behaviour is no longer only dependent on the behaviour of glass but also on the behaviour of the interlayer between each layer of glass.

Because of the deformability of the interlayer, the stiffness of laminated glass is usually less than those of a monolithic pane with the same total thickness. This is the result of the inability of the interlayer to form a perfect shear coupling. A practical design approach consists of the definition of an effective thickness [4], in other words the thickness of an equivalent monolithic pane that would react identically as its laminated counterpart.

Similar to glass, the response of the viscoelastic interlayer depends on the load duration. To simplify the problem, all materials are considered linearly elastic because a full viscoelastic analysis is rarely done. Moreover, at least in the first order approximation for a preliminary design, geometric non-linearities can be neglected when in-plane loads do not occur.

The suggested approach is called the enhanced effective thickness method which defines an equivalent moment of inertia I_R as the weighted harmonic mean of the moments of inertia corresponding to the layered and monolithic limit.

By defining the flexural rigidity of each glass layer as Equation (68) identical as in the case of the simplified plate theory:

$$D_i = \frac{E \cdot h_i^3}{12 \cdot \left(1 - \upsilon^2\right)} \tag{68}$$

The total flexural rigidity for the monolithic limit for a two-layered laminate equals [Equation (69)]:

$$D_{tot} = D_1 + D_2 + 12 \cdot \frac{D_1 \cdot D_2}{D_1 \cdot h_2^2 + D_2 \cdot h_1^2} \cdot H^2 \tag{69}$$

Then the shape function for the deflection w(x,y) [Equation (70)] can be defined as the elastically deformed surface of monolithic plate as defined in 0 with a constant thickness and identical boundary conditions.

$$w(x,y)_{laminated} = \frac{w(x,y)_{monolithic}}{D_R} \tag{70}$$

Where D_R [Equation (71)] can be determined as:

$$\frac{1}{D_R} = \frac{\eta}{D_{tot}} + \frac{1-\eta}{D_1 + D_2} \tag{71}$$

And η [Equation (72)]:

$$\eta = \frac{1}{1 + \frac{t}{G_{\text{int}}} \cdot \frac{D_1 + D_2}{D_{tot}} \cdot 12 \frac{D_1 \cdot D_2}{D_1 \cdot h_2^2 + D_2 \cdot h_1^2} \cdot \psi} \tag{72}$$

And ψ can be calculated [Equation (73)] and where a and b are the dimensions of the pane to be designed:

$$\psi = \frac{\pi^2 \left(a^2 + b^2\right)}{a^2 \cdot b^2} \tag{73}$$

3.7 Analytical calculation method for bolted connections

The following equation [Equation (74)] can be used to calculated bolted connections in glass elements.

$$\sigma_{\varphi,\max} = \prod_{i=1}^{6} k_i \cdot \left(0{,}40 + 1{,}5 \cdot \frac{a \cdot K_m}{b_m}\right) \cdot \frac{P_d}{a \cdot t} \leq f_{t,d,l} \tag{74}$$

With :
- P_d : The design force [N]
- a : The radius of the drilled hole [mm]
- t : The thickness of a single glass layer [mm]
- b_m : The width of the element [mm]
- K_m : equilibriu m factor
- $f_{t,d,l}$: tensile strength of the drilled hole
- k_i : influence factors

Note that if the width of the element (b_m) becomes smaller than three times the diameter of the drilled hole (d_0), this formula can no longer be used [17].

The values of the design force (P_d), the radius of the drilled hole (a), the thickness of a single pane (t) and the tensile strength of the hole ($f_{d,t,l}$) should be known. All that remains is to calculate the equilibrium factor and the different influence factors.

The influence factors can be found as follows [6]:

- **Influence factor k_1**

The calculation of k_1 depends on the distance between de centre lines of the glass layers defines as e_y.

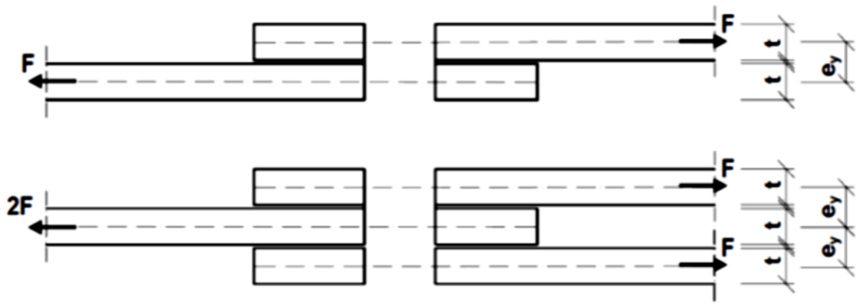


Figure 21: Calculation of influence factor k_1

When this value is known, k_1 can be found using following table (Table 6). Intermediate values can be calculated using interpolation.

Table 6: Calculation of influence factor k_1

e_y [mm]	0	10	15	20	30	45
k_1 [-]	1	3.5	4.8	6.1	8.7	12.6

- **Influence factor k_2**

The second influence factor is calculated based on the width of influence of the drilled hole.

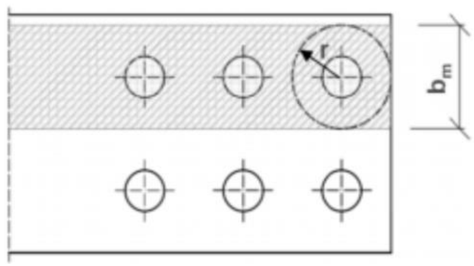


Figure 22: Calculation of influence factor k_2

This values is limited geometrically to the minimum of two times e_1, two times e_2 (the distance of the hole to the edge of the element) or p_2 (the distance between two drilled holes) (see Figure 23 and Equation (75)).

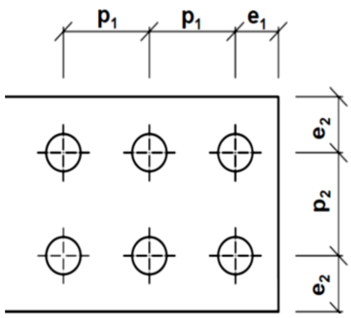


Figure 23: Calculation of influence factor k_2

In other words:

$$b_m = 2 \cdot r = \min\left(\begin{cases} 2.e_1 \\ 2.e_2 \\ p_2 \end{cases}\right) \tag{75}$$

When the value of b_m is known, k_2 can be found using the following table (Table 7).

Table 7: Calculation of influence factor k_2

b_m [mm]	$b_m > 5.d_0$	$3.d_0 > b_m > 5.d_0$
k_2 [-]	1	1.2

- **Influence factor k_3**

The third influence factor also depends on the distance between the drilled holes and the edges of the element (see Table 8), more specifically in relation to the drilled hole diameter. If distances e_1 and e_2 are not equal to one another, k_3 can be assumed 1.

Table 8: Calculation of influence factor k_3

$e_1 = e_2$ [mm]	$1.5d_0$	$2.5d_0$	$3.5d_0$	$4.5d_0$
k_3 [-]	1.21	1.09	1.03	1

- **Influence factor k_4**

Influence factor k4 takes into account the distance between two connections. When the distance between connections is known, the value of k4 can be found in following table ().

Table 9: Calculation of influence factor k_4

p_1 [mm]	$3.d_0$	$5.d_0$	$7.d_0$	$9.d_0$	$>>9.d_0$
k_4 [-]	1.23	1.1	1.06	1.04	1

When there is only one connection, the distance between connection can be seen as much larger than nine times the diameter of the drilled hole and k_4 can be assumed to be 1.

- **Influence factor k_5**

The next influence factor is based on the glass product specifications. When monolithic, k_5 is equal to 1, for two-layered laminated glass k_5 becomes 1.2 (Table 10).

Table 10: Calculation of influence factor k_5

Glass Product	k_5
Monolithic	1
Two-layered laminate	1.2

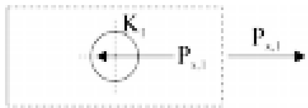
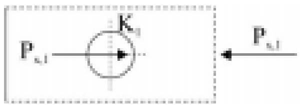
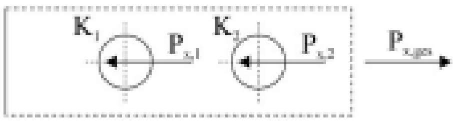
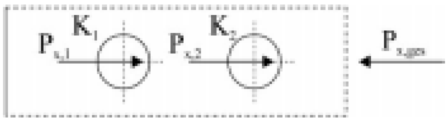
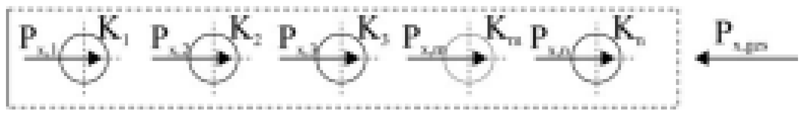
- **Influence factor k_6**

The value for the sixth and final influence factor, three parameters are used: the ratio hole clearance to drilled hole diameter $\Delta s/d_{bolt} < 0.02$, the eccentricity of $e/b_{mortar} \leq 0.4$ and the consideration of all drilling diameters ranging $22mm \leq d_0 \leq 60mm$ with $0.5 \leq d_{bolt}/d_0 \leq 0.77$, for which k_6 equal to 1.5 can be used.

- **Equilibrium factor K_m**

To calculate the equilibrium factor the table below can be used (). This factor takes into account the direction and sense of enacting forces on one or more drilled holes.

Table 11: Calculation of equilibrium factor K_m

Equilibrium System	K_m-value
	$K_1 = 1$
	$K_1 = -1$
	$for\ P_{x,1} = P_{x,2}$ $K_1 = 1$ $K_2 = 3$
	$for\ P_{x,1} = P_{x,2}$ $K_1 = -1$ $K_2 = -3$
	$for\ ambiguous\ holes\ m \leq n$ $K_m = 2.\frac{\sum_{j=1}^{m} \lvert P_{x,j} \rvert}{\lvert P_{x,m} \rvert} - 1$
	$for\ ambiguous\ holes\ m \leq n$ $K_m = 1 - 2.\frac{\sum_{j=1}^{m} \lvert P_{x,j} \rvert}{\lvert P_{x,m} \rvert}$

IV. CASE STUDY: ANALYTICAL CALCULATIONS of GLASS COMPONENTS & CONNECTIONS

4.1 Description of the case study

SECO settled in Brussels, the company which also serves as promotor for this thesis, has kindly provided the case study to which the above described analytical design method could be applied. Specialised in different sectors of construction, they also have a team of engineers dedicated towards the calculation of glass structures.

The construction site which served as this case study is the Museum of European History in Brussels. On the upper floors a new glass façade has been constructed. The simple configuration of beams, columns, panes, bolted and clamped connections to form this glass façades (thereby almost exclusively using primary members in the design) is perfectly suited for this thesis.

Figure 24: Expected result of the glass façades incorporated in the building

Figure 25: Beam-Column-Pane-Connection structure

The beam is hereby considered simply supported and the column clamped at the bottom and simply supported at the top. The panes are considered simply supported as well along its four edges.

4.2 Design of internal structure

4.2.1. Beam – ULS

To calculate this particular beam, the configuration and dimensions of the element are known. The beam is comprised of six glass layers connected by five interlayers. The following dimension were considered for the calculation (Table 12).

Table 12: Beam properties for analytical calculations ULS

Length L [mm]	2350
Width b [mm]	80
Height h [mm]	350
Young's modulus [N/mm²]	70,000
Poisson ratio [-]	0.2
Density [kg/m³]	2500

Note that this is not the only configuration of the beam which is applied in the façades.

The loads acting on the beam are limited to the dead weight of the beam, the dead weight of the roof panel and variable load from the wind. All loads were considered according to the characteristic of the building. An assumption was made for the wind load which after verification did not differ much from the value considered in the actual calculations. A maintenance load is also considered yet based on an assumption.

Table 13: Load case beam analytical calculations

Distributed/Line	Size [kN/m²]	Pond. Factor ψ	Width b [m]	Subtotal [kN/m]
Dead Weight Beam (G)	8.75	1.35	0.08	0.95
Dead Weight Pane (G)	0.5	1.35	1.65	1.11
Wind (Q)	1.3	1.5	1.65	3.22
			Total [kN/m²]	**5.28**
Concentrated	**Size [kN]**	**Pond. Factor ψ**	**Width b [mm]**	**Subtotal [kN]**
Maintenance (Q)	1	1.5	n.a.	1.5
			Total [kN]	**1.5**

To check the acting bending, both the line loads and concentrated force are taken into account. Internal forces on the beam may be calculated as in the case of a simply supported beam according to the structural system referred in 4.1.

$$\boxed{M_{sd} = 4{,}52 kNm}$$

Knowing the bending moment due to the loads, the next step is to calculate the resisting bending moment of the beam and compare. Because the beam is made of laminated glass, the strength of the beam can be calculated using the following formula [Equation (3)] [2]:

$$f_{g,d} = \left(\frac{f_{b,k} - f_{g,k}}{\gamma_v} + k_{\mathrm{mod}} \cdot \frac{f_{g,k}}{\gamma_m \cdot k_A} \right) \cdot \gamma_n$$

$$f_{g,d} = \left(\frac{120 - 45}{1{,}2} + k_{\mathrm{mod}} \cdot \frac{45}{1{,}8} \right) \cdot 1$$

Where k_{mod} is $k_{mod,combi}$, the modification factor for the combination of loads, calculated as [Equation (7)]:

Table 14: kmod-values beam loads

	Duration t	$k_{mod,ind}$
Permanent		
Dead Weight Beam	25 years	0,3
Dead Weight Pane	25 years	0,3
Variable		
Wind	10 min	0,74

$$k_{\mathrm{mod},combi} = \frac{\gamma_G \cdot g_k + \gamma_{Q,1} \cdot q_{k,1} + \sum_i \gamma_{k,i} \cdot \psi_{0,i} \cdot q_{k,i}}{\frac{\gamma_G \cdot g_k}{k_{\mathrm{mod},ind,g}} + \frac{\gamma_{Q,1} \cdot q_{k,1}}{k_{\mathrm{mod},ind,q1}} + \frac{\sum_i \gamma_{k,i} \cdot \psi_{0,i} \cdot q_{k,i}}{k_{\mathrm{mod},ind,qi}}}$$

$$\boxed{k_{\mathrm{mod},combi} = 0{,}326}$$

Thus, the strength of the laminated beam becomes

$$f_{g,d} = \left(\frac{120 - 45}{1,2} + 0,326 \cdot \frac{45}{1,8} \right) \cdot 1$$

$$\boxed{f_{g,d} = 70,65\ N / mm^2}$$

And subsequently, the resistive bending moment of the beam then becomes using Equation (9):

$$M_{Rd} = f_{g,d} \cdot \left(\frac{b \cdot h^2}{6} \right) \tag{76}$$

$$\boxed{M_{Rd} = 115.40 . 10^6\ Nmm = 115.4 kNm \overset{!}{>} 4.52 kNm} \Rightarrow OK$$

The resisting bending moment of the beam is larger than that of acting loads, therefore the cross-section verifies safety.

4.2.2. Beam – SLS

The beam is also verified in service limit state (SLS) to ensure the deflection of the beam is limited. The calculation of the loads in SLS is presented in the table below.

Table 15: Load case beam analytical calculations SLS

Distributed/Line	Size [kN/m²]	Pond. Factor ψ	Width b [m]	Subtotal [kN/m]
Dead Weight Beam (G)	8.75	1	0.08	0.70
Dead Weight Pane (G)	0.5	1	1.65	0.825
Wind (Q)	1.3	1	1.65	2.145
			Total [kN/m²]	**3.67**
Concentrated	**Size [kN]**	**Pond. Factor ψ**	**Width b [mm]**	**Subtotal [kN]**
Maintenance (Q)	1.0	1	n.a.	1.0
			Total [kN]	**1.0**

The total deflection is the result of both the concentrated and distributed load. Both their effects are added together to get the maximum deflection.

$$\boxed{\delta = 0.09 mm}$$

Assuming the deflection is limited to L/300, then it can be found that:

$$\delta_{\max} = \frac{L}{300} = \frac{2350}{300} = 7,83 mm$$

$$\boxed{\delta_{\max} = 7.83 mm > \delta = 0.09 mm} \Rightarrow OK$$

In SLS, for the deflection requirements, the beam also suffices.

4.2.1. Beam – Lateral Torsional Buckling

Because the beam is supported at the top by the panes, it is unlikely that lateral torsional buckling of the beam will occur when the upper part of the beam is subjected to compression.

However, since the possibility exists that the wind load will create an uplift of the panes, it is not unlikely that the bottom of the beam can become unstable, depending on actions of the permanent loads, as well.

For the actual calculation of the lateral torsional, only a finite-element model was considered (see section 5.3.3) because the formulae [Equation (13)] is limited to two- or three-layered laminates.

4.2.2. Column - Compression

The column is different. Although the compressive strength of glass elements is almost never an issue, elements subjected to compression can become unstable, the column can buckle. The column is comprised of six glass layers and five interlayers, identical to the beam.

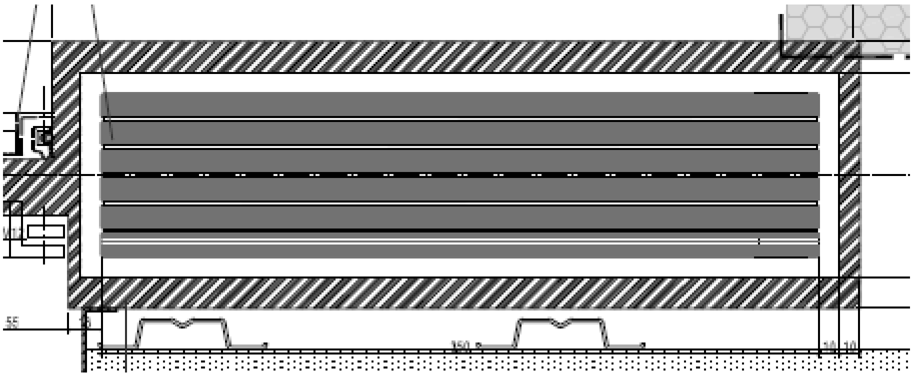


Figure 26: Column section

The loads acting on the column are the following:

Table 16: Load case column analytical calculations compression

Distributed	Size [kN/m³]	Ψ [-]	Width b [m]	Height h [m]	Length [m]	Subtotal [kN]
Dead Weight Column (G)	25	1.35	0.08	0.35	4.75	4.489
Dead Weight Pane Roof (G)	25	1.35	1.65	0.02	2.35	2.617
Dead Weight Beam (G)	25	1.35	0.08	0.35	2.35	2.221
Dead Weight Pane Facade (G)	25	1.35	1.65	0.02	4.75	5.290
Wind (Q) [kN/m²]	1.3	1.5	1.65	n.a.	2.35	7.561
					Total [kN]	**15,979***

*Since the column carries only half the weight of the roof pane, the beam and half of the total force of the wind, these cannot simply be added. The total compressive force on a single column becomes [Equation (77)]:

$$N_{sd} = \frac{DW_{Pane\,roof} + DW_{Beam} + Wind}{2} + DW_{Column} + DW_{Pane\,facade} \quad (77)$$

$$\boxed{N_{sd} = 15{,}98\,kN = 16\,kN}$$

Assuming the glass layers are float glass, the compressive strength then becomes [2]:

$$f_{g,c,d} = \frac{f_{g,c,k}}{\gamma_{M,A}} \tag{78}$$

$$\boxed{f_{g,c,d} = 333{,}33\ N/mm^2}$$

If solely the glass section of the column is considered for the load carrying capacity of the column, the maximum compressive force allowed on the section is:

$$N_{Rd} = f_{g,c,d} \cdot A \tag{79}$$

$$\boxed{N_{Rd} = 8{,}400{,}000\,N = 8{,}400\,kN \overset{!}{>} 16\,kN} \Rightarrow OK$$

4.2.3. Column – Buckling

To calculate the critical buckling load of the column, it is considered to be clamped at both end making the buckling length of the column equal to half its physical length.

$$L_k = \frac{L}{2} = \frac{4750}{2}$$

$$\boxed{L_k = 2375\,mm}$$

As referred before, the column cross-section is comprised of six glass layers and five interlayers. However, formulas in [2] are based on either double-layered or triple-layered elements. So either a calculation can be done combining two times three layers and then two times the newly combined layers or three times two layers followed by combining those three new layers.

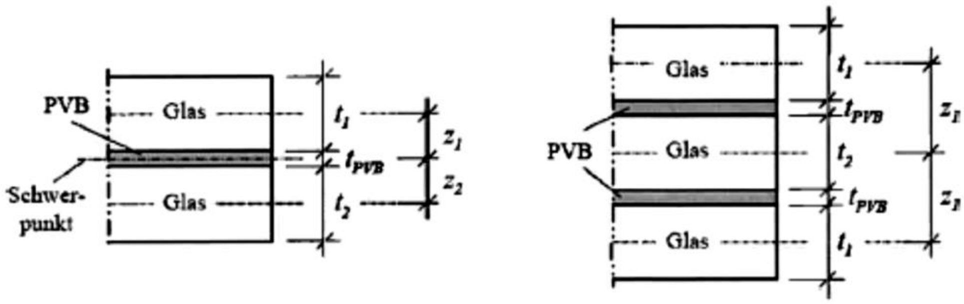


Figure 27: Effective cross section of multi-layered elements

In case of calculating it using the first suggested method via Equation (80):

$$N_{cr} = \frac{\pi^2(1 + \alpha + \pi^2 \cdot \alpha \cdot \beta) \cdot E \cdot I_s}{(1 + \pi^2 \cdot \beta) L_k{}^2} \tag{80}$$

In which:

$$\boxed{I_s = 3.22 \cdot 10^{-6}\,m^4}$$

And:

$$\boxed{\alpha = 0.85}$$

$$\boxed{\beta = 4.62 \cdot 10^{-7}}$$

It can be found that:

$$\boxed{N_{cr} = 729.12\ kN}$$

When verifying using the second suggested method [Equation (39)]:

$$t_{eff} = \sqrt[3]{\frac{12 \cdot I_s \cdot \left(1 + \alpha + \pi^2 \cdot \alpha \cdot \beta\right)}{b \cdot \left(1 + \pi^2 \cdot \beta\right)}}$$

$$\boxed{t_{eff} = 0.059\, m}$$

Where:

$$\boxed{I_{eff} = 5.95\,.\,10^{-6}\, m^4}$$

Thus resulting in:

$$\boxed{N_{euler} = 729.12\ kN}$$

Which gives the same result as the first method. Say the stress needs to be calculated then:

$$\boxed{A_{eff} = 0.124\, m^2}$$

$$\boxed{\sigma = 5.897\ MPa}$$

Using the method of the reduction factor [Equation (81)]:

$$N_{Rd} = \chi_b \cdot A_{eff} \cdot f_{g,d} \qquad (81)$$

Where:

$$\boxed{\lambda = 3{,}447}$$

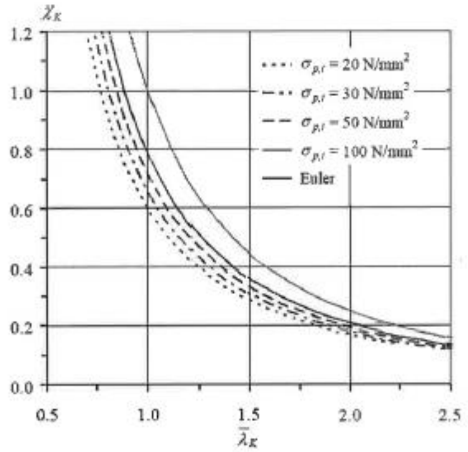


Figure 28: Reduction factor curves

Estimating the value of χ_b as 0.075 with a λ equal to 3.5 then the critical buckling load becomes:

$$N_{Rd} = \chi_b \cdot A_{eff} \cdot f_{g,d}$$

$$\boxed{N_{Rd} = 649{,}912\ kN}$$

Based on this value, the method using the reduction factor gives a plausible value. The buckling resistance of a member (649,9kN) is smaller than the resistance of cross-section (8,400kN).

4.3 Design of Glass Panes

Since all available material of the case study are building design plans, the only exact known properties of the panes are its dimensions. They are comprised of two layers of glass connected by a single interlayer. The other properties are found in different literary works.

Table 17: Pane properties for analytical calculations

Length [mm]	4750
Width [mm]	1650
Thickness [mm]	20
Young's modulus [N/mm²]	70,000
Poisson ratio [-]	0.2
Density [kg/m³]	2500

The load case (Table 18) for the design of the panes is limited to wind forces and the pane's dead weight. The maintenance load which is assumed to be a concentrated force, is considered separately from the pane's dead weight and the wind load (in pressure) which are distributed forces.

Table 18: Load case pane analytical calculations

Distributed	**Size [kN/m²]**	**Pond. Factor ψ**	**Subtotal [kN/m²]**
Dead Weight	0.5	1.35	0.3375
Wind	1.3	1.5	1.95
		Total [kN/m²]	**2.2875**
Concentrated	**Size [kN]**	**Pond. Factor ψ**	**Subtotal [kN]**
Maintenance	1	1.5	1.5
		Total [kN]	**1.5**

Normally, it should also be checked to see if the wind load is actually a pressure instead of an uplifting force but the most negative combination is assumed in this case which is the wind and dead weight acting in the same direction. In the other case the dead weight would have had a beneficial effect on the uplifting force generated by the wind.

To assure that the value of the deflection at the end of the calculation has the unit of [mm], all parameters used in the following equations use units of [N], [mm] or a combination.

4.3.1. Linear – Distributed force

The known parameters for the calculation of the pane under the influence of a distributed the force are given in Table 19 below:

Table 19: Analytical calculation parameters

Length a [mm]	4750
Width b [mm]	1650
Thickness h [mm]	20
Young's modulus E [N/mm²]	70,000
Poisson ratio v [-]	0.2
Distributed force q [N/mm²]	0.0024405

Szilard [9] gives a solution [Equation (44)] for the calculation of the deflection, the equation reads:

$$w(x,y) = \frac{1}{D.\pi^4} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{P_{mn}}{\left[\left(m^2/a^2\right) + \left(n^2/b^2\right)\right]^2} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$$

$$for\, m, n = 1,3,5...$$

Where:

$$P_{mn} = \frac{0.003956}{m.n}$$

And:

$$D = 4.86.10^7$$

The maximum deflection occurs in the middle of the pane so:

$$x = 2,375; \quad y = 825$$

And is equal to (for the first three terms):

$$w(x,y) = 4.51427\ mm$$

Bending moments and stresses can be calculated [Equations (47) to (52)] in the case of simply supported edges which results in:

- **Bending moments:**

$$\Rightarrow m_x = 194.062\, Nmm$$

$$\Rightarrow m_y = 777.585\, Nmm$$

$$\Rightarrow m_{xy} = 0\, Nmm$$

- **Stresses**

$$\Rightarrow \sigma_x = 3.13262\, N/mm^2$$

$$\Rightarrow \sigma_y = 11.7594\ N/mm^2$$

$$\Rightarrow \sigma_{xy} = 0\ N/mm^2$$

Table 20: Analytical solutions linear distributed force

	Deflection [mm]	σ_{xx} [N/mm²]	σ_{yy} [N/mm²]	σ_{xy} [N/mm²]	$M_{x,max}$ [Nmm]	$M_{y,max}$ [Nmm]	$M_{xy,max}$ [Nmm]
Analytical	4.51427	3.13262	11.7594	0	194.062	777.585	0

4.3.2. Linear – Concentrated force

The same properties as in section 4.3.1can be used to calculate different parameters for the pane in the case of a concentrated force. Only this, a concentrated force is used.

Table 21: Concentrated Force analytical calculations

Concentrated force F [N]	1950

$$w(x,y)=\frac{4.P}{\pi^4 ab\,D}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\sin(m\pi\xi/a)\sin(n\pi\eta/b)}{\left[\left(m^2/a^2\right)+\left(n^2/b^2\right)\right]^2}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \quad \text{for}\, m,n=1,2,3... \tag{82}$$

Where:

$$P_{mn}=\frac{26\sin\left(\frac{m\pi}{2}\right)\sin\left(\frac{n\pi}{2}\right)}{26125}$$

$$D=4.86.10^7$$

Thus:

$$w(x,y)=0.702732\ mm$$

The stresses can be calculated the same way as with the distributed force, the bending moments however are calculated differently using Equation (55) to (60):

- **Bending moments**

$$\Rightarrow m_x=20.3138\,Nmm$$

$$\Rightarrow m_y=111.874\,Nmm$$

$$\Rightarrow m_{xy}=0Nmm$$

- **Stresses**

$$\Rightarrow \sigma_x=0.304707\,N/mm^2$$

$$\Rightarrow \sigma_y=1.6781\,N/mm^2$$

$$\Rightarrow \sigma_{xy}=0\,N/mm^2$$

Table 22: Analytical solutions linear concentrated force

	Deflection [mm]	σ_{xx} [N/mm²]	σ_{yy} [N/mm²]	σ_{xy} [N/mm²]	$M_{x,max}$ [Nmm]	$M_{y,max}$ [Nmm]	$M_{xy,max}$ [Nmm]
Analytical	0.702732	0.304707	1.6781	0	20.3138	111.874	0

4.3.3. Non-Linear – Distributed force

In this case, non-linear behaviour is taken into account for the calculations. Every other parameter remains the same. The deflection can be found using the following Equation (61):

$$w(x,y)=\sum \omega_{mn} W_{mn}(x,y)$$

The first step is to extract the value of w_{mn} out of the following Equation (66):

$$32\frac{Eh}{D}\frac{\alpha_m^4\gamma_n^4\varsigma_{mn}^2}{\left(\alpha_m^2+\gamma_n^2\right)^4}w_{mn}^3+w_{mn}=\frac{q_{mn}}{D\left(\alpha_m^2+\gamma_n^2\right)^2}$$

This can most easily be done by doing a substitution e.g.

$$A=32\frac{Eh}{D}\frac{\alpha_m^4\gamma_n^4\varsigma_{mn}^2}{\left(\alpha_m^2+\gamma_n^2\right)^4};\qquad k=w_{mn};\qquad B=\frac{q_{mn}}{D\left(\alpha_m^2+\gamma_n^2\right)^2}$$

So the equation is written as:

$$A.k^3+k=B$$

Solving this equation generates three possibilities, two of which are complex solutions, if the real solution is kept, it can be found that:

$$k=\frac{\sqrt[3]{\frac{2}{3}}}{\sqrt[3]{\left(9A^2B+\sqrt{3}\sqrt{4A^3+27A^4B^2}\right)}}+\frac{\sqrt[3]{\left(9A^2B+\sqrt{3}\sqrt{4A^3+27A^4B^2}\right)}}{\sqrt[3]{2}\,.3^{2/3}A} \qquad (83)$$

By resubstituting A, B and k in Equation (83), w_{mn} can be calculated. It should be noted that q_{mn} can be substituted by the P_{mn} as calculated Equation (45) for the distributed force when linear behaviour was assumed.

Next, all remaining parameters can be calculated:

$$D=4.86.10^7$$

$$\varsigma_{mn}=\frac{4}{3mn\pi^2}$$

$$\alpha_m=\frac{m\pi}{a}=\frac{m\pi}{4750}$$

$$\gamma_n=\frac{n\pi}{b}=\frac{n\pi}{1650}$$

$$W_{mn}=\sin\left(\frac{m\pi}{2}\right)\sin\left(\frac{n\pi}{2}\right)$$

The deflection is calculated in the middle of the pane, so:

$$x = 2375; \qquad y = 825$$

And the deflection can be calculated as:

$$w(x, y) = \sum \omega_{mn} W_{mn}(x, y)$$
$$for\, m, n = 1,2,3...$$

$$\boxed{w(x, y) = 4.16615}$$

Table 23: Analytical solutions non-linear distributed force

	Deflection [mm]
Analytical	4.16615

Note that in this case only the calculation of the deflection is given. During calculations the results of the equations for membrane forces, stresses and bending moment were inconclusive, generating meaningless answers therefore they are not represented.

Equations used to calculate membrane forces, bending moment and stresses generated by distributed forces can be found in [12] for the non-linear calculations.

4.3.4. Laminated

The laminated calculation, which takes into account the layered structure of the panes, should give the best approximation of reality. The monolithic pane is now divided into three layers; two glass layers and a single polymeric interlayer. The interlayer is considered perfectly incompressible and the bond between interlayer and glass layers is considered perfect so no slip should occur between the layers.

Glass layer (2)	
Length a [mm]	4750
Width b [mm]	1650
Thickness h [mm]	10
Young's modulus E [N/mm²]	70,000
Poisson ratio v [-]	0.2
Distributed force q [N/mm²]	0.0024405
PVB-interlayer (1)	
Thickness t [mm]	1.52
Young's modulus E [N/mm²]	300

The flexural rigidity of each glass layer can be calculated as:

$$D_i = 6.08.10^6$$

The total flexural rigidity of the pane can then be calculated as:

$$D_{tot} = 6.05.10^6$$

The shear modulus of the interlayer can be calculated as:

$$G = 100\, N/mm^2$$

The total deflection of a simply supported pane under a distributed load can be calculated using the classic Navier solution for a monolithic pane with flexural rigidity D_R:

$$w(x,y) = \frac{16p}{\pi^6 D_R} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)}{mn\left(\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right)^2}$$

Given that the first term of this approximation is accurate enough, the shape function g(x,y) becomes:

$$g(x,y) = \frac{16p}{\pi^6} \frac{1}{\left(\frac{1}{a^2} + \frac{1}{b^2}\right)^2} \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right)$$

The term η* can be calculated as:

$$\eta^* = 0.995501$$

With:

$$\psi = 4.063.10^{-6}\,mm^{-2}$$

The flexural rigidity of the laminated pane can then be calculated according to:

$$D_R = 5.95.10^7$$

D_R is larger than D_{tot} which means that the laminated pane should have a larger flexural rigidity and will therefore act more stiff compared to its monolithic counterpart. The total deflection should therefore also be smaller. This is seems counter-intuitive however and seems implausible; because of the flexibility of the interlayer, one could expect that laminated glass should not be as stiff as the monolithic cross-section actually resulting in a larger deflection .

Given that the first term of the equation for the deflection of the laminated is sufficiently accurate, it can be found that the maximum deflection in the middle of the pane is equal to:

$$w(x,y) = 4.03065$$

Table 24: Analytical solutions non-linear concentrated force

	Deflection [mm]
Analytical	4.03065

Note that in this case only the calculation of the deflection is given. During calculations the results of the equations for membrane forces, stresses and bending moment were inconclusive, generating meaningless answers therefore they are not represented.

Equations used to calculate membrane forces, bending moment and stresses generated by distributed forces can be found in [4] for the calculations of laminated panes.

4.4 Connections

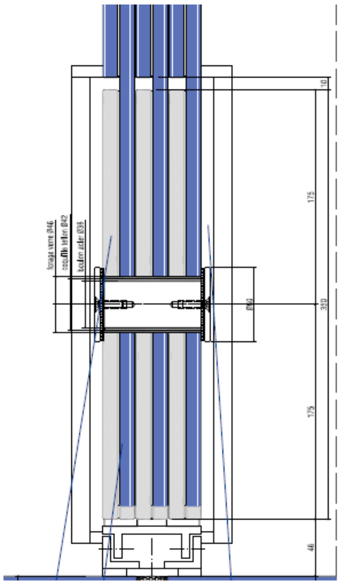


Figure 29: Configuration connection

The case study applies multiple types of connections. In between glass layers, the interlayers form an adhesive or glued connection. De connections between column and floor and beam and wall are clamped connections using special ‘shoes’. The connection however between the beam and column is a bolted connection. Here, only the bolted connection is considered. This can be checked by using the formula given in section 3.7, Equation (74) [17].

$$\sigma_{\varphi,\max} = \prod_{i=1}^{6} k_i \cdot \left(0{,}40 + 1{,}5 \cdot \frac{a \cdot K_m}{b_m}\right) \cdot \frac{P_d}{a \cdot t} \leq f_{t,d,l}$$

Where parameters can be calculated as equal to:

$$P_d = 16\left(/6\right) kN;$$
$$a = 23 mm \left(d_0 = 46 mm\right)$$
$$t = 12 mm$$
$$b_m = 350 mm$$
$$K_m = -1$$
$$f_{t,d,l} = 45 \, N/mm^2$$

Note that the actual calculation is done for a single glass layer at which only a sixth of the entire connection load is applied. The values of k_i and K_m can be calculated given the connection dimensions. The tensile strength of the glass at the hole is considered equal to the tensile stress of the glass before toughening as an extra safety.

- **Influence factor k_1**

Since only a single layer of glass is considered, the calculation of e_y is irrelevant. Or in other words, this value becomes 0. Based on the table given in the previous section, the value of k_1 is equal to one.

$$\boxed{k_1 = 1}$$

- **Influence factor k₂**

Seeing there is only one hole forming the connection, the calculation of k_2 is simple. Factor b_m is:

$$b_m = \min \begin{cases} 2e_1 \\ 2e_2 \\ p_2 \end{cases} = \min \begin{cases} 2 \cdot 175 \\ 2 \cdot 175 = 350mm \\ / \end{cases}$$

And

$$5d_0 = 5 \cdot 46 = 230\,mm$$

Which means:

$$b_m > 5d_0 \Rightarrow \boxed{k_2 = 1}$$

- **Influence factor k_3**

Because of the symmetry in the connection and e_1 and e_2 both being larger than $3{,}5d_0$, k_3 can be found to be:

$$e_1 = e_2 = 175\,mm$$
$$3{,}5d_0 = 3{,}5 \cdot 46 = 161\,mm$$

$$\boxed{k_3 = 1}$$

- **Influence factor k_4**

As said before, k_4 takes into account the spacing between holes. In this connection, there is only one so the distance can be assumed very large. For that reason, the factor becomes:

$$\boxed{k_4 = 1}$$

- **Influence factor k_5**

Considering a monolithic pane, k5 becomes:

$$\boxed{k_5 = 1}$$

- **Influence factor k_6**

Since the assumptions for k_6 being equal to 1.5 do not apply to our case, its value is considered one.

$$\boxed{k_6 = 1}$$

The stress in the connection can be calculated as:

$$\sigma_{\varphi,\max} = \prod_{i=1}^{6} k_i \cdot \left(0{,}40 + 1{,}5 \cdot \frac{a \cdot K_m}{b_m}\right) \cdot \frac{P_d}{a \cdot t} \leq f_{t,d,l}$$

$$\boxed{\sigma_{\varphi,\max} = 2.91\, N/mm^2 \leq 8.15\, N/mm^2} \Rightarrow OK$$

V. FE-SIMULATION of GLASS COMPONENTS & CONNECTIONS

5.1 Description of FE-modelling

All finite-element calculations are done using *Dassault Systèmes 3DS Simulia Abaqus* [13]. This numerical tool provides a complete and flexible solution for a large variety of problems. Abaqus possesses quite the number and variety of finite elements in the ABAQUS library. It offers the possibility of incorporating geometric and material non-linearities into the analysis. Multiple numerical techniques can be used. The software offers both manual and automatic meshing techniques.

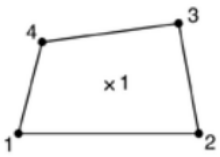
Because Abaqus can be used for a whole variety of engineering issues, a complete and detailed description of the program is outside the scope of this thesis. More detailed information can be found in [13].

Since most numerical problems are not very complex, they can be brought down to two dimensional problems, shell (homogeneous and composite) elements can be used to model beams, columns and panes. The connection was considered a contact problem therefore a solid element approach was used. In most of all the cases, calculations were done taken both geometrical linearities and non-linearities and material linearities into account. One model of the pane was also developed using solid elements.

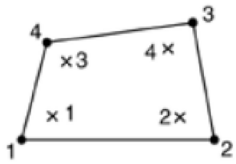
A brief description of the most relevant modelling tools used in the numerical simulations is given below.

- **Finite elements**

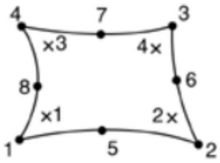
The choice of finite element largely depends on the application and thus on the problem to be solved. The three dimensional conventional stress/displacement 2D shell finite elements of linear and quadratic order are most of interest in this case due to the fact that they are used when the thickness is significantly smaller than the other dimensions. The geometry is hereby defined on a reference surface and the thickness defines through the section properties. For the linear order, reduced and full integration are available. The quadratic order is limited to a reduced integration. The solid elements are interesting because they can be used for complex nonlinear analyses such as contact problems and large deformations. They can be composed of one homogeneous material or can include several layers of different materials for the analysis of laminated composite elements. Table 25 gives a brief overview of the main characteristics of these elements.



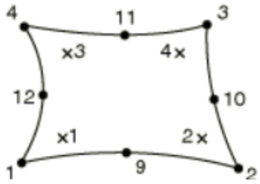
a) **Linear order shell element with 4 nodes, reduced integration**



b) Linear order shell element with 4 nodes, full integration



c) Quadratic order shell element with 8 nodes, reduced integration



d) Quadratic order solid element with 8 nodes, reduced integration

Figure 30: Finite elements selected for the numerical calculations

Table 25: Properties of the finite elements selected for the numerical calculations

Finite elements	Main properties
S4R	4 nodes 1 integration point 6 degrees of freedom per node
S4	4 nodes 4 integration points 6 degrees of freedom per node
S8R	8 nodes 4 integration points 6 degrees of freedom per node
C3D20R	20 nodes 8 integration points 3 degrees of freedom per node

- **Interactions**

Two different types of interactions are considered in the models. First, because no slip is assumed between the interlayer and glass layer of the laminated sections, a rigid connection must be put in place between the contact surfaces of the parts. In ABAQUS is type of connection is defines by a Tie Constraint and is based on a Master-Slave surface interaction. The displacements and stresses in the Master surface dictate the displacements of the Slave surface.

A second type of connection is based on the same type of Master-Slave interaction between two surfaces. Forces coming from the beam must be transmitted to the column which is done via a bolted connection between the two component. In this case a Hard Contact is implied between the glass elements and the bolt. In the normal direction, when in contact, pressure can be transmitted without any or minimal penetration of the two bodies. No contact does also mean no stresses. In the tangential direction, behaviour is assumed frictionless. This way, no shear stress develops and the surfaces are free to slip.

5.2 Simulation of Panes

5.2.1. Convergence Study

In finite element modelling, a finer mesh (i.e. smaller size mesh elements) normally results in a more accurate solution. However, when the mesh density increases (and with it, the amount of degrees of freedom) so does the computational time and the economic cost of the overall calculation.

In this case, the components are very straightforward and do not have complex shapes which means the computational time should not be determinative. Nonetheless, the accuracy of the modelling is needed to a certain degree because these results will be compared with the analytical calculations.

The first step in calculating the different components therefor consists of executing a convergence study.

The panes are estimated to be 4750mm long and 1650mm wide and have a thickness of 20mm. For the convergence study the panes are considered monolithic. They are considered to be simply supported along all of its four edges.

The total load applied to the pane is calculated under the assumption that the only loads applying to the surface are the pane's dead weight (at a rate of 2,500kg/m³ with a thickness of 20mm) and the downward pressure generated by the wind, meaning that for this pane distributed forces are applied. To also have an indication of the total deflection of and the stress in the pane due to a concentrated force, a point load is also applied in a later stage (in case that the pane is accessible for maintenance e.g. cleaning) but not taken into account for the convergence study. Table 26 gives an overview of the calculation of the total distributed load.

Table 26: Applied load for Convergence Study

	Size (kN/m²)	Pond. Factor ψ	Subtotal [kN/m²]
Dead Weight	0,5	1,35	0,3375
Wind	1,3	1,5	1,95
		Total [kN/m²]	**2,2875**
Maintenance	1	1,5	1,5

The convergence study (CS) has been done for two important factors for glass panes, its maximum deflection w [mm] and maximum (tensile) stress σ [N/mm²]. Both linear (reduced - S4R and full integration - S4) and non-linear (quadratic) (reduced integration – S8R) approaches are executed for various mesh densities. The results can be found in following tables and graphs (Table 27 & Figure 31).

Table 27: Results CS Pane Deflections w [mm]

Number of Nodes (hxb)	Pane - I (Linear - RI)	Pane - I (Linear - FI)	Pane - II (Quadratic - RI)
	Deflection w [mm]	Deflection w [mm]	Deflection w [mm]
11x4 (44)	4,34585	4,33858	4,51772
18x8 (144)	4,47936	4,47703	4,51347
22x12 (264)	4,4974	4,49596	4,51347
28x18 (504)	4,50474	4,5039	4,51347
35x22 (770)	4,50683	4,5063	4,51347
40x25 (1000)	4,50794	4,50753	4,51347

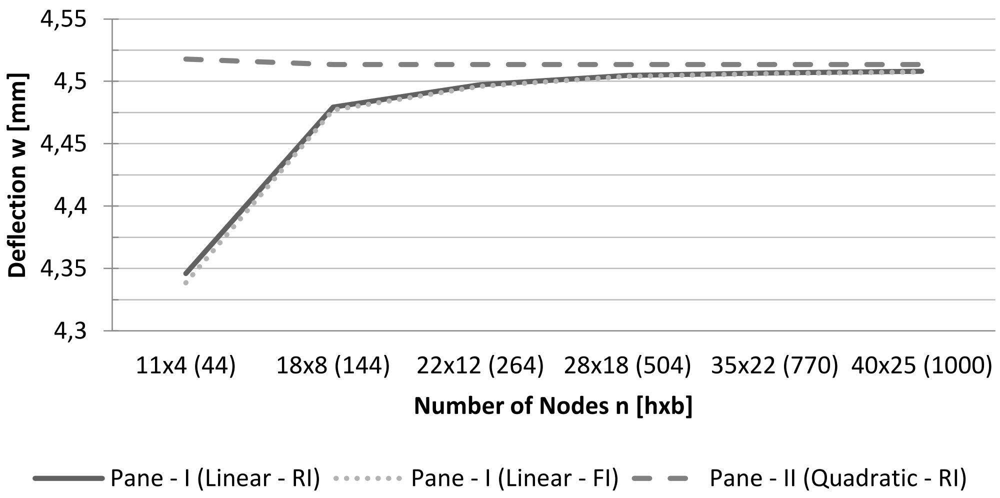


Figure 31: Results CS Pane Deflections w [mm]

For the deflections it can already be seen that a good accuracy can be become even when not using the smallest mesh. Using 504 nodes (element size of approximately 50x50mm) for this specific part results in reasonably accurate values for the deflection of the pane.

Following table and graph show the results for the stresses σ (Table 28 & Figure 32).

Table 28: Results CS Maximum Pane Stresses σ [N/mm²]

Number of Nodes (hxb)	Pane - I (Linear - RI)	Pane - I (Linear - FI)	Pane - II (Quadratic - RI)
	Stress σ [N/mm²]	Stress σ [N/mm²]	Stress σ [N/mm²]
11x4 (44)	10,0211	10,0264	10,1763
18x8 (144)	10,4336	10,4373	10,4737
22x12 (264)	10,4899	10,4928	10,5143
28x18 (504)	10,5138	10,5156	10,5299
35x22 (770)	10,521	10,5222	10,5335
40x25 (1000)	10,5246	10,5254	10,5351

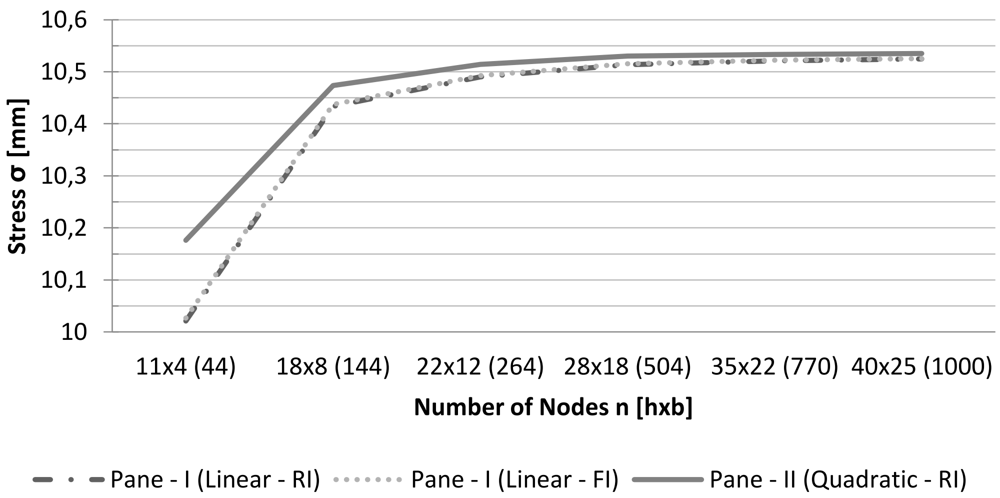


Figure 32: Results CS Maximum Pane Stresses σ [N/mm²]

The same can be said about the convergence for the maximum stresses. Both parameters converge fairly quickly to a very similar result proving the adequacy of the convergence study. For the deflections the value becomes 4,50mm approximately and for the stresses circa 10,53N/mm². In order to make a compromise between accuracy and computational speed and to achieve a similar accuracy when comparing results, for further calculations a mesh size of maximum 50x50mm is used.

To simulate the behaviour of glass panes using finite element modelling, different approaches were considered. Since a monolithic pane can be considered as a simple 2D problem, Shell-Homogeneous elements are suitable to use to model the pane effectively. These elements are used for all pane simulations, the laminated pane excluded. For the laminated simulation, Shell-Composite and Solid elements are used.

5.2.2. Linear– Distributed Force

The first approach of interest is to simplify the laminated glass pane to a monolithic pane simply supported along four of its edges with the same dimensions as the laminated panes handled in the case study. The deformation behaviour of the pane is assumed to be linear which means in plane forces are neglected and the pane is therefore subjected to pure bending.

As with the convergence study the three integration types are used to be able to compare the results. Following table (Table 30) gives the results from the modelling for each specific type of integration. The applied load is a distributed force due to the pane's dead weight and a wind pressure according to Table 26 of the convergence study.

The properties taken into account to model the pane were limited to its dimensions, its Young's modulus and the Poisson ratio and the density of glass. Only a quarter of the pane needs to be modelled due to reasons of symmetry and in doing so computational time can be spared because of the smaller total amount of nodes.

Table 29: Properties of linearly modelled pane

Length [mm]	2375
Width [mm]	825
Thickness [mm]	20
Young's modulus [N/mm²]	70,000
Poisson ratio [-]	0.2
Density [kg/m³]	2500

Table 30: Results Linear Modelling Monolithic Pane Distributed Force

	Deflection [mm]	σ_{xx} [N/mm²]	σ_{yy} [N/mm²]	σ_{xy} [N/mm²]	$M_{x,max}$ [Nmm]	$M_{y,max}$ [Nmm]	$M_{xy,max}$ [Nmm]
S4R	4,50474	2,93482	11,6693	0,00194259	195,655	777,955	0,129506
S4	4,50390	2,93704	11,6719	0	195,803	778,124	0
S8R	4,51347	2,94171	11,7063	0	196,114	780,422	0

The deflection of the pane along its longest dimension according to the FE-model can be seen in following Figure 33 and Figure 34. It gives a very similar outcome as what can be expected for a beam under the influence of a line load. It can also be noted that result is only slightly affected by the choice of integration type. The deflection of the pane largely revolves around 4,50mm according to the linear FE-model which in this case is about a quarter of the thickness of the actual pane.

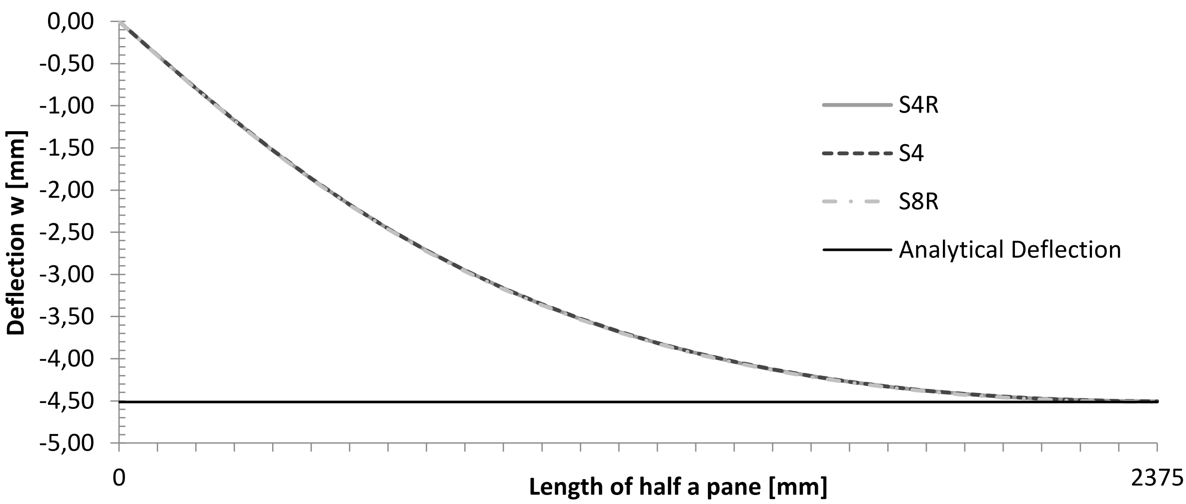


Figure 33: Linear deflection of a monolithic pane under a distributed load

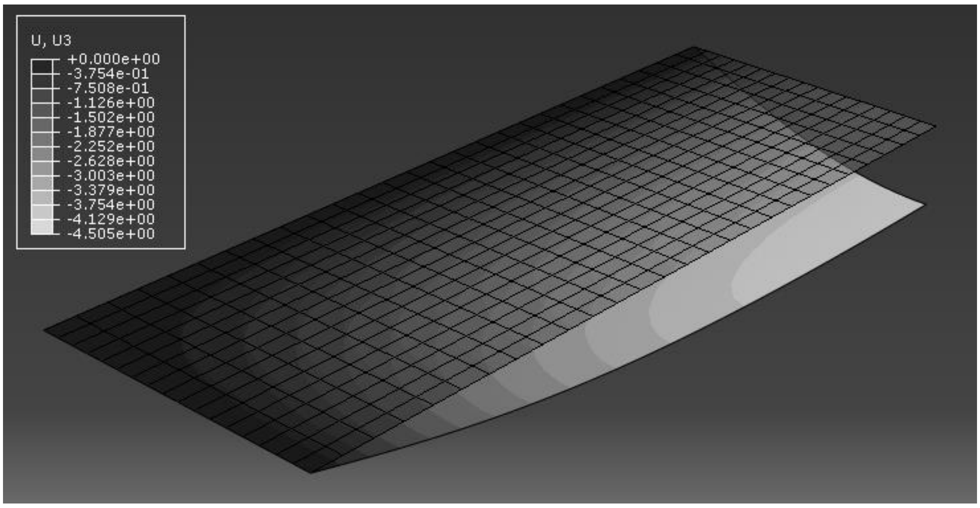


Figure 34: Linear deflection of a monolithic pane under a distributed load (Abaqus model)

5.2.3. Linear - Concentrated Force

A second approach replaces the previously applied distributed force by a concentrated force in de middle of the pane; the other conditions remain identical to the preceding approach. The results from de FE-model can be found in following table (Table 31) and figures.

Table 31: Results Linear Modelling Monolithic Pane Concentrated Force

	Deflection [mm]	σ_{xx} [N/mm²]	σ_{yy} [N/mm²]	σ_{xy} [N/mm²]	$M_{x,max}$ [Nmm]	$M_{y,max}$ [Nmm]	$M_{xy,max}$ [Nmm]
S4R [N368]	0,70455	0,41468	1,87508	1,12203	27,6453	125,006	74,8022
S4 [N368]	0,70430	0,414521	1,68924	1,1262	27,6348	112,616	75,08
S8R [N368]	0,70471	0,301265	1,68576	1,12796	20,0844	112,384	75,1976

Note that the specific parameters are not calculated in the centre of the pane but rather in a node away from the centre. This is because the equation to calculate the analytical value in a previous section 28 diverges at the point where the concentrated is applied. This means that the value for the deflection in the centre of the cannot be analytically found when the concentrated force is also applied in the centre of the pane. Thus to check the validity of the equations, a point other than the centre must be compared or in this case at node 368. At node 368 according the model, the deflection is equal to 0.70mm. The model in Abaqus gives a total deflection at the centre due to a concentrated force approximately equal to 1.85mm, almost a tenth of the thickness of the pane.

Table 32: Maximum Linear Deflection Concentrated Force

	Deflection w [mm]
S4R	1,85318
S4	1,85531
S8R	1,85842

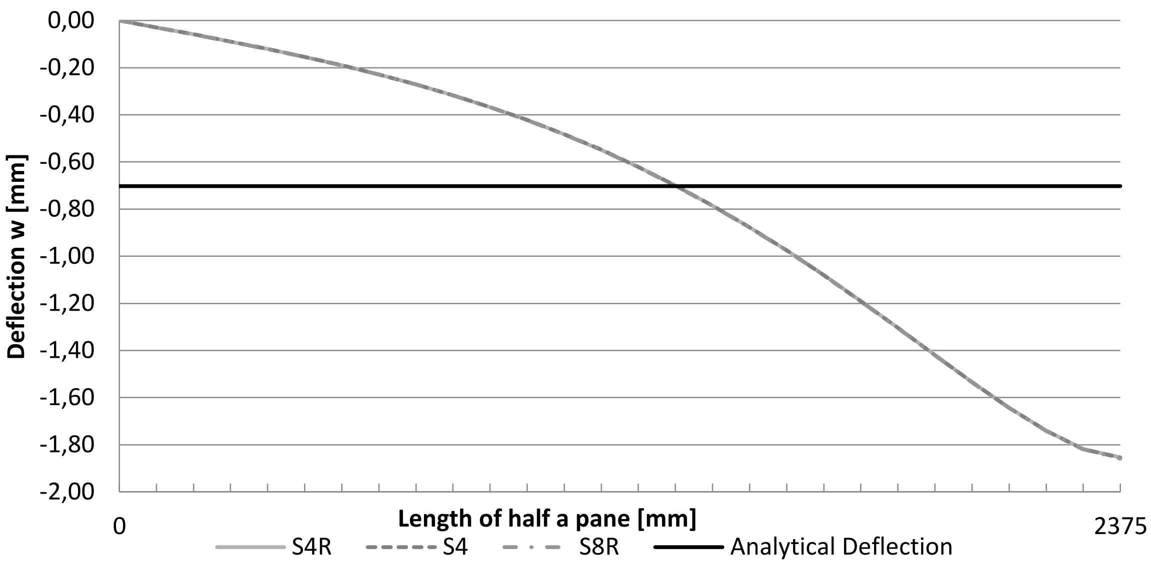


Figure 35: Linear Deflection of pane under a concentrated force

The value calculated in a point off centre is very similar, proving the accuracy of the given equations in case of the linear deflection under a concentrated load.

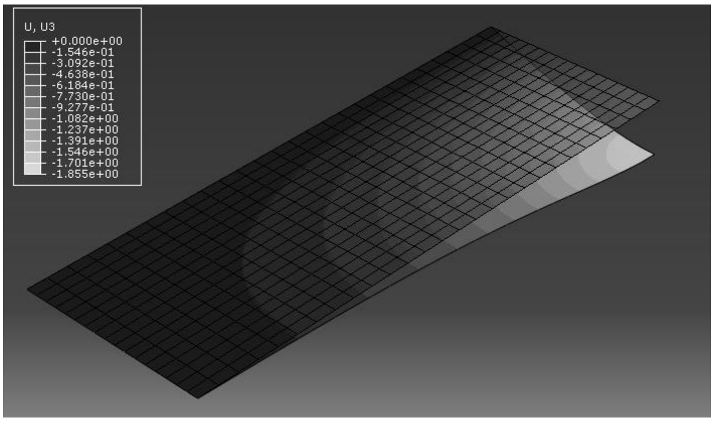


Figure 36: Linear deflection of a monolithic pane under a concentrated force (Abaqus model)

5.2.4. Non-linear – Distributed Force

The next model considers non-linear effects in the pane. Due to the slenderness of the element, the pane is no longer purely subjected to bending moment. The forces acting on the element result in membrane forces occurring in the pane. This causes the model to act more stiff than its linear counterpart. This can be seen in the results of the modelling taking the non-linear effects into account (Table 33).

Table 33: Results Non-Linear modelling monolithic pane under a distributed force

	Deflection [mm]	σ_{xx} [N/mm²]	σ_{yy} [N/mm²]	σ_{xy} [N/mm²]	$M_{x,max}$ [Nmm]	$M_{y,max}$ [Nmm]	$M_{xy,max}$ [Nmm]
S4R (NL)	4,09605	2,84831	11,6868	0,00141102	171,514	704,447	0,0991581
S4 (NL)	4,09537	2,85037	11,689	0	171,639	704,573	0
S8R (NL)	4,10109	2,8529	11,7161	0	171,742	706,141	0

The deflection is smaller than the deflection found for the linearly modelled pane. This confirms the assumption that the pane acts more stiff. Which in turn means that the strains, stresses and bending moments are smaller than what can be expected for pane purely subjected to bending moments.

However, the analytical result show an slight overestimation of the maximum deflection. This could be due to a safe approach.

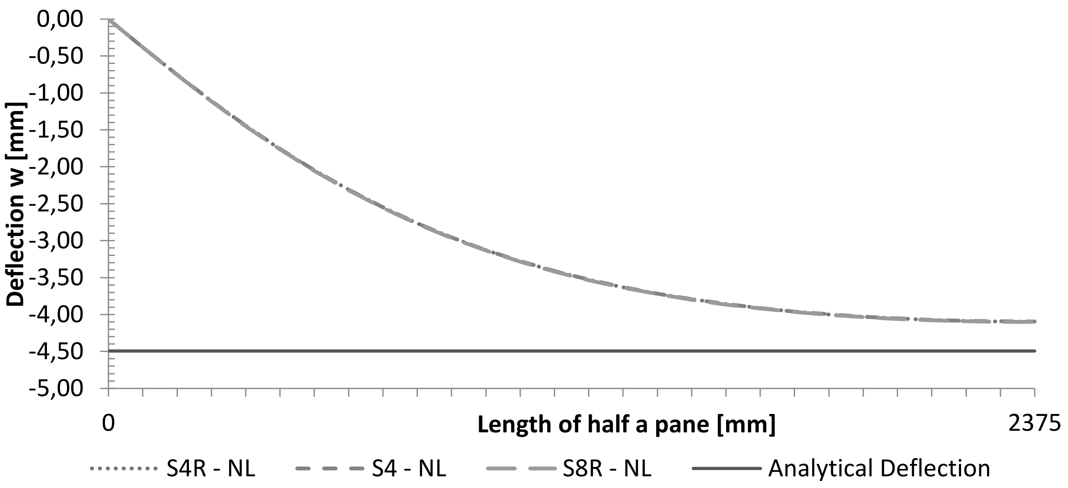


Figure 37: Non-linear deflection of a monolithic pane under a distributed force

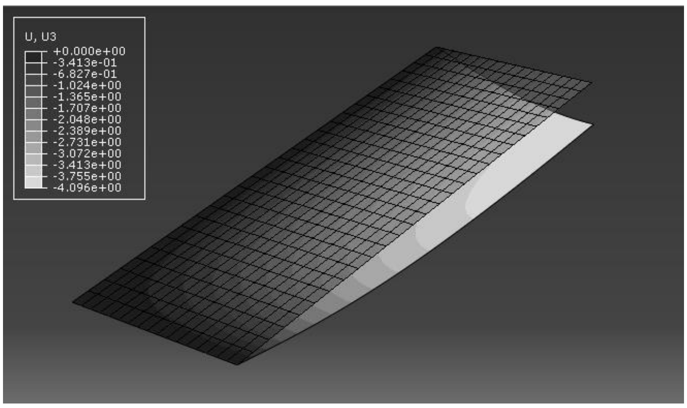


Figure 38: Non-linear deflection of a monolithic pane under a distributed force (Abaqus model)

5.2.5. Non-linear – Concentrated Force

The fourth monolithic model takes into account the non-linear effects for the modelling of the pane subjected to a concentrated force. The results can be found the next table (Table 34). These results are again for a node other than the node where the concentrated force was applied, which again was in the centre of the pane.

Table 34: Results non-linear modelling monolithic pane under a concentrated force

	Deflection [mm]	σ_{xx} [N/mm²]	σ_{yy} [N/mm²]	σ_{xy} [N/mm²]	$M_{x,max}$ [Nmm]	$M_{y,max}$ [Nmm]	$M_{xy,max}$ [Nmm]
S4R NL [N368]	0.69662	0.437185	1.95551	1.13056	27.1487	123.596	74.1707
S4 NL [N368]	0.69637	0.436651	1.76329	1.13412	27.1574	111.253	74.4166
S8R NL [N368]	0.69666	0.323264	1.75698	1.13586	19.6757	110.999	74.5213

The change in deflection is significantly smaller than that of the pane under a distributed force. However, there is a difference which means membranes forces cannot simply be neglected in the case of a concentrated force. The maximum deflection calculated in Abaqus can found in Table 35 below.

Table 35: Maximum non-linear deflection concentrated force

	Deflection w [mm]
S4R - NL	1.83552
S4 - NL	1.83765
S8R - NL	1.84049

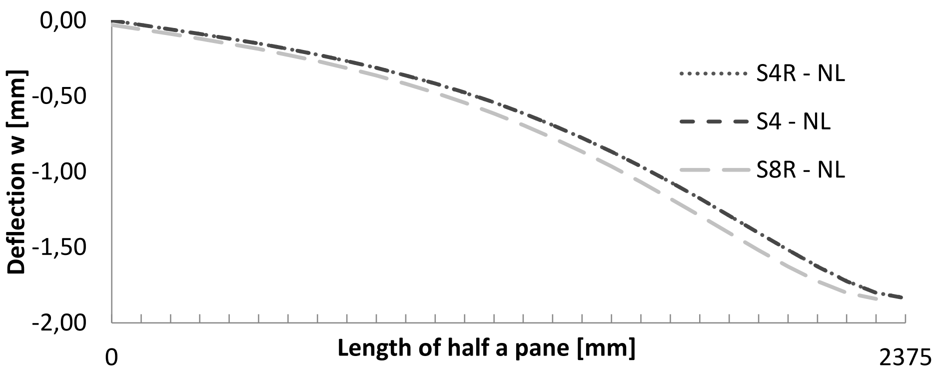


Figure 39: Non-linear deflection of a monolithic pane under a concentrated force

Since no formulae were found to calculate the effect of a concentrated force on a thin plate, analytical results could not be generated.

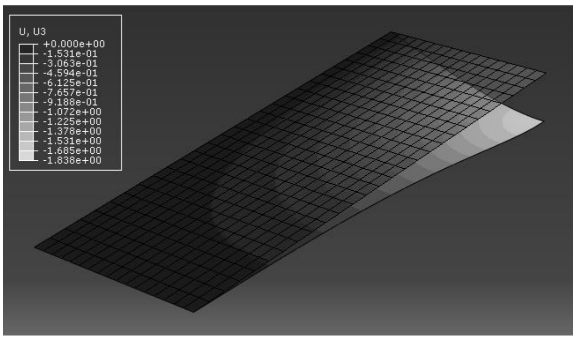


Figure 40: Non-linear deflection of a monolithic pane under a concentrated force (Abaqus model)

5.2.6. Laminated

In reality, the pane is comprised of multiple glass layers, in this case two glass layers one polymeric interlayer. The assumption of a monolithic pane in previous section is thus a simplification of reality. In order to accurately model the laminated pane and to compare the difference, Shell-Composite and Solid elements are used. They allow the individual modelling of layers as well as interaction between those layers.

As a simplification the interaction between the interlayer and glass layers is assumed to be perfect, meaning no slip of the interlayer will occur at the glass surface. The interlayer has an important influence on the general behaviour of the pane making it less stiff. Again, a distinction is made between models with and without non-linearities and between a pane simply supported along two and along four edges, in this case only with a distributed load.

Table 36: Maximum deflection laminated glass pane under distributed force

	Deflection w [mm]
S8R	3.73324
S8R - NL	3.51641
C3D20D	3.70472
C3D20D - NL	3.64687

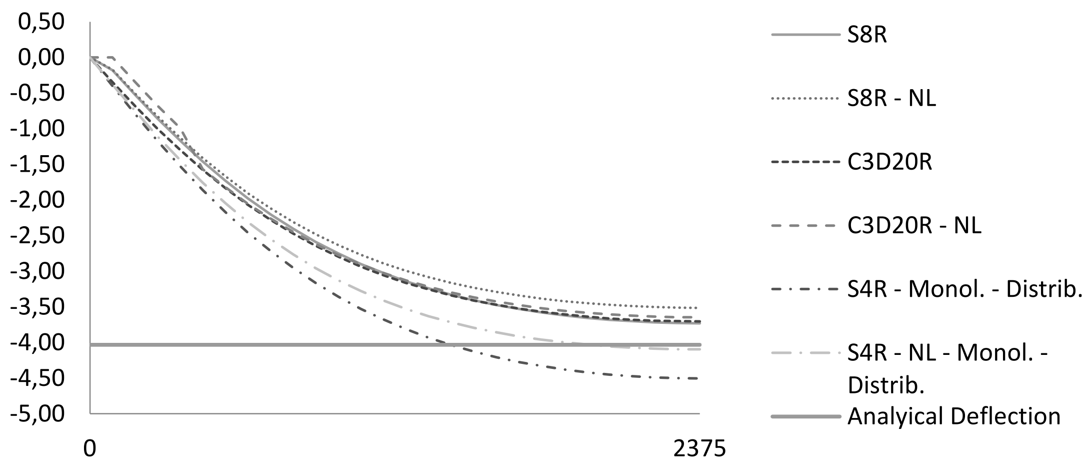


Figure 41: (Non-)linear of a laminated pane under a distributed force

The results from both the modelling and analytical calculations generate a deflection which is smaller than what could be assumed if the pane is actually less stiff than the monolithic pane.

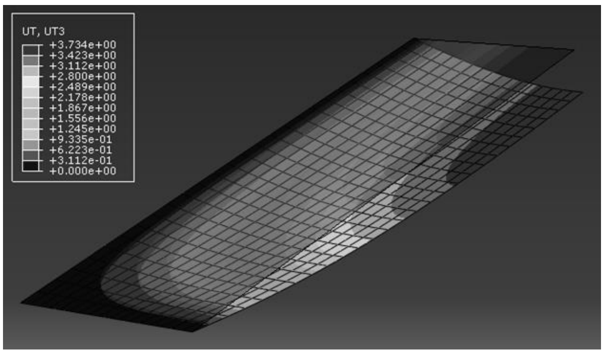


Figure 42: Linear deflection of a Shell-Composite laminated pane (Abaqus model)

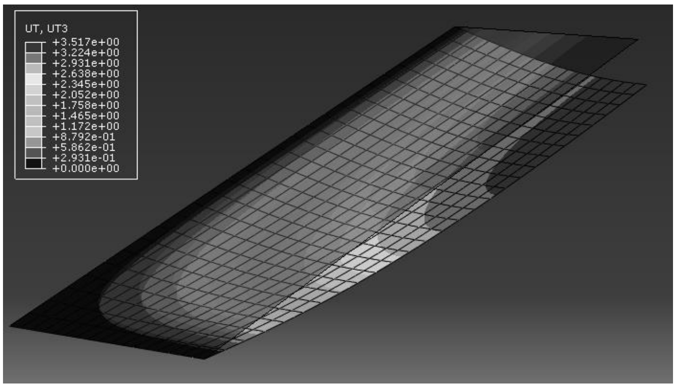


Figure 43: Non-linear deflection of a Shell-Composite laminated pane (Abaqus model)

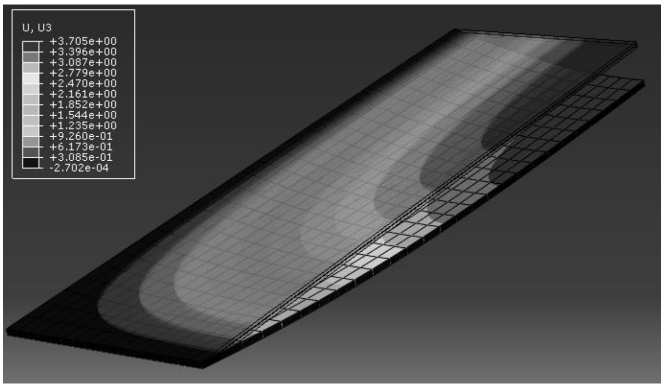


Figure 44: Linear deflection of a Solid laminated pane (Abaqus model)

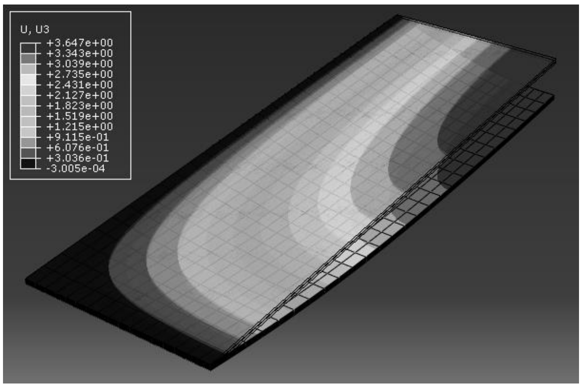


Figure 45: Non-linear deflection of a Solid laminated pane (Abaqus model)

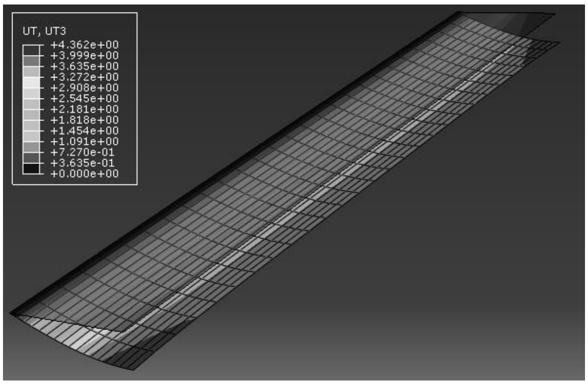


Figure 46: Linear deflection of a Shell-Composite laminated (2 edges) (Abaqus model)

5.3 Simulation of a Beam

5.3.1. The test beam

The simulation of a beam was done in two steps. The first step was to reproduce the results of an actual physical four-point-bending test on a two-layered laminate beam from [14]. This in order to verify and compare the results generated by that test to the actual finite-element model created in Abaqus.

The beam for the test was comprised of two layers of glass (10mm) with a Sentryglass interlayer (1.52mm) in between. The total height of the beam was 30 centimetres. Two concentrated forces were applied 40cm of the middle of the beam over an area of 21.52x20mm². Rigid supports alongside the beam were used to stabilise the beam (prevent lateral torsional buckling).

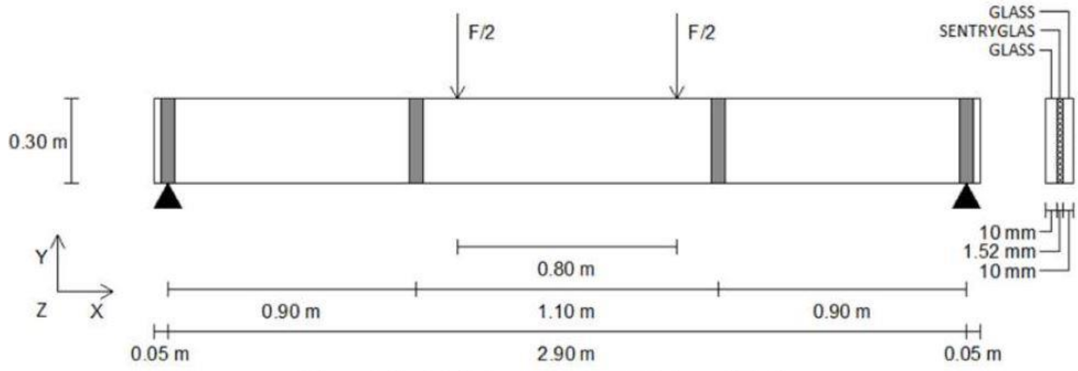


Figure 47: Test beam dimensions [14]

The properties of the beam were also given.

Table 37: Test glass beam properties

Material	E [MPa]	ν [-]	f_y [MPa]	$f_{u,t}$ [MPa]	$F_{u,c}$ [MPa]
Glass	70000	0.22	-	45	1000
Sentryglas	300	0.5	23	-	-

By implementing a beam in exactly the same conditions, with identical dimensions and properties as in the test, the FE-model was created.

Figure 48: Test beam FE-model

Stiffeners are implemented as boundary conditions and forces are applied as 'distributed' loads rather than actual concentrated forces. Multiple element-types were used to model the beam: Shell-Homogeneous, Shell-Composite and Solids. Since the result depend on the choice of element-type, not all models fully represent reality. They need to be checked relatively.

In the actual experiment strain gauges and displacement transducers were used to extract data from the test beam at both sides.

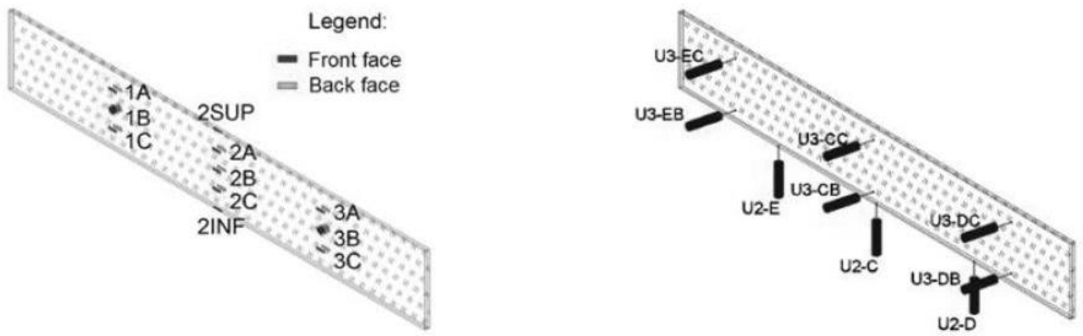


Figure 49: Placement of strain gauges (L) and displacement transducers (R) [14]

The results of the experiment are represented in the following graph.

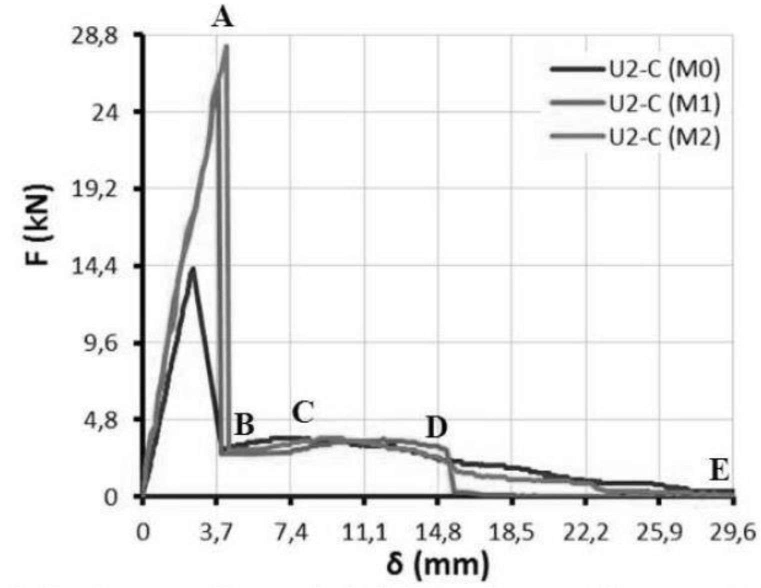


Figure 50: Test beam results [14]

To compare the results of the FE-model with the results of the experiment, the model should generate a deflection of about 3.7mm when a total force of 28.8kN is applied. This deflection is measured in the middle and at the bottom of the beam.

Since all units in Abaqus need to be the same to get a meaningful result, these forces were changed to the appropriate unit. For shell-elements this meant transforming the force into a line load:

$$q = \frac{F/2}{s} = 720 N/mm$$

For the solid-elements this becomes a pressure:

$$\sigma = \frac{F/2}{A} = 33{,}46 N/mm$$

Next, the models are compared with the test beam. These are the result. The first model, the shell-homogeneous considers the beam to be monolithic. The monolithic beam will react more stiff than the actual laminated beam so the deflection should be smaller.

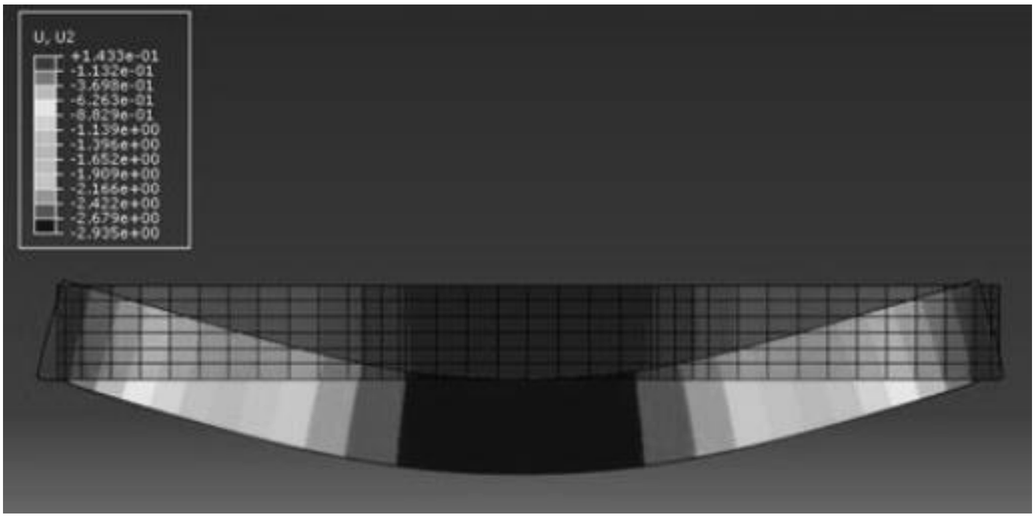


Figure 51: Shell-Homogeneous FE-model test beam

When the maximum deflection of the beam is extracted this is confirmed. As expected, the total deflection remains limited to 2.935mm, which is about 0.8mm difference.

By introducing multiple layers to the system, the FE-model, reality is more correctly approached. This is done by using Shell-Composite elements. This allows us to create a section comprised of multiple materials with each their own characteristics and specifications. The interaction between these layers are modelled to be perfect in case of Shell-Composite elements.

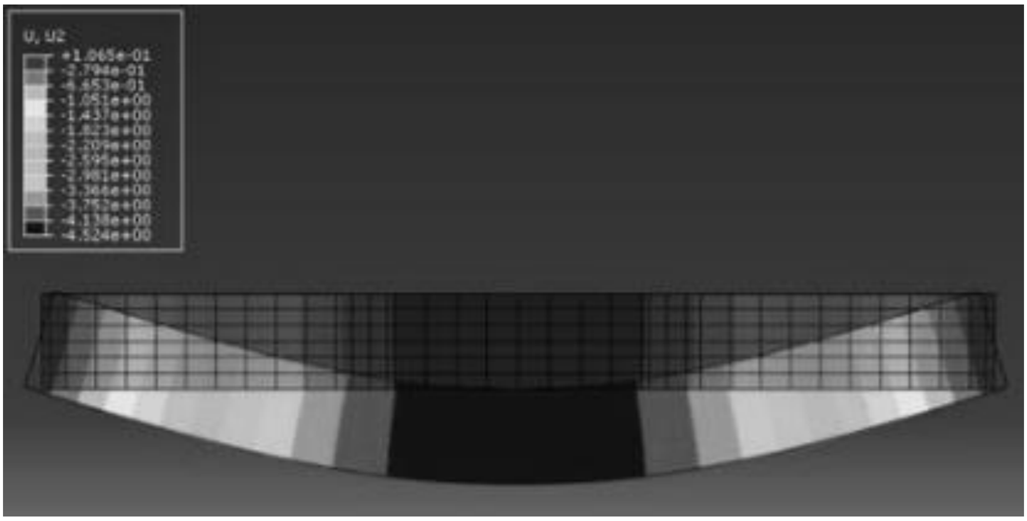


Figure 52: Shell-Composite FE-model test beam

The deflection does increase to 4.524mm but is now larger than the deflection measured during the experiment which means that the introduction of an interlayer lessens the stiffness of the beam.

Finally, the most real model with the best approximation should be given by the Solid element-type. It allows for a 3D stress situation where Shell-elements do not. Here, both materials are modelled separately and combined to an assembly of layers. In this case the actual behaviour between the layers can be altered. The choice however, was to assume a perfect interaction where no slip of the interlayer occurs.

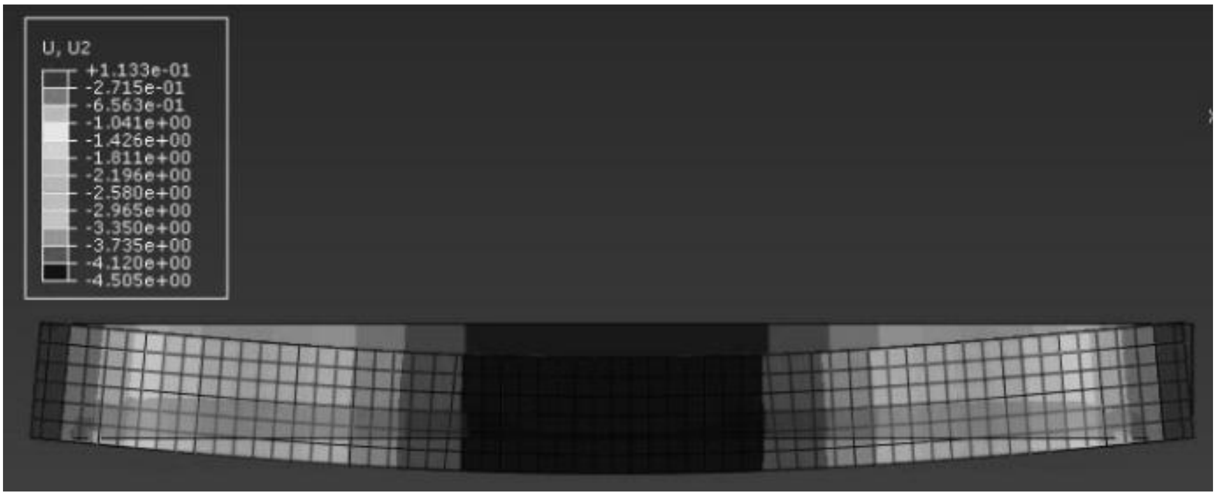


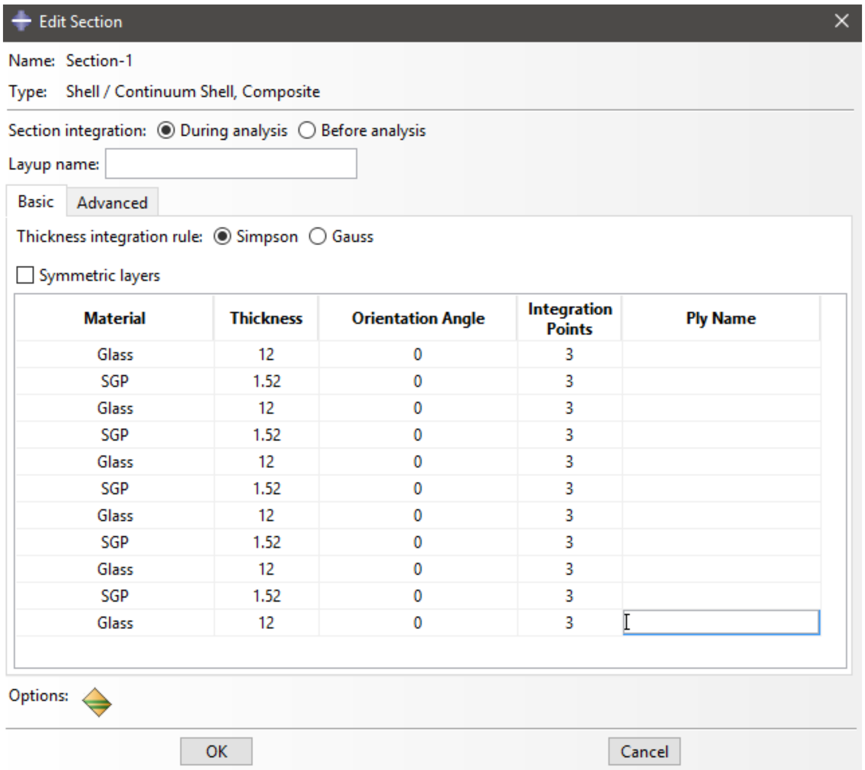
Figure 53: Solid FE-model test beam

The deflection (4.505mm) in this case is not much different, though slightly smaller than that of the Shell-Composite element. This is probably because of the assumption of the perfect interaction.

5.3.2. The case study beam

The overall accuracy is sufficient enough to continue with the modelling of the beam of the case study, an overestimation of the actual deflection should not be any problem but provides an extra safety. By comparison of the stresses in and maximum deflection of the beam, the analytical values are validated.

The beam is modelled using Shell-Composite elements. This 2D model should provide a result that is accurate enough. The beam in the case study is comprised of six glass layers and five interlayers. Since the exact thickness and properties of the interlayer are not known, they are assumed to be identical to those used in the test case. The glass layer properties are known.


Edit Section

Name: Section-1

Type: Shell / Continuum Shell, Composite

Section integration: (•) During analysis ( ) Before analysis

Layup name:

Basic | Advanced

Thickness integration rule: (•) Simpson ( ) Gauss

[ ] Symmetric layers

Material	Thickness	Orientation Angle	Integration Points	Ply Name
Glass	12	0	3	
SGP	1.52	0	3	
Glass	12	0	3	
SGP	1.52	0	3	
Glass	12	0	3	
SGP	1.52	0	3	
Glass	12	0	3	
SGP	1.52	0	3	
Glass	12	0	3	
SGP	1.52	0	3	
Glass	12	0	3	

Options:

OK Cancel

Figure 54: Case study beam layer composition

The total deflection of the beam according to the conditions of the case study are found to be:

$$\delta_{\max} = 0.11mm$$

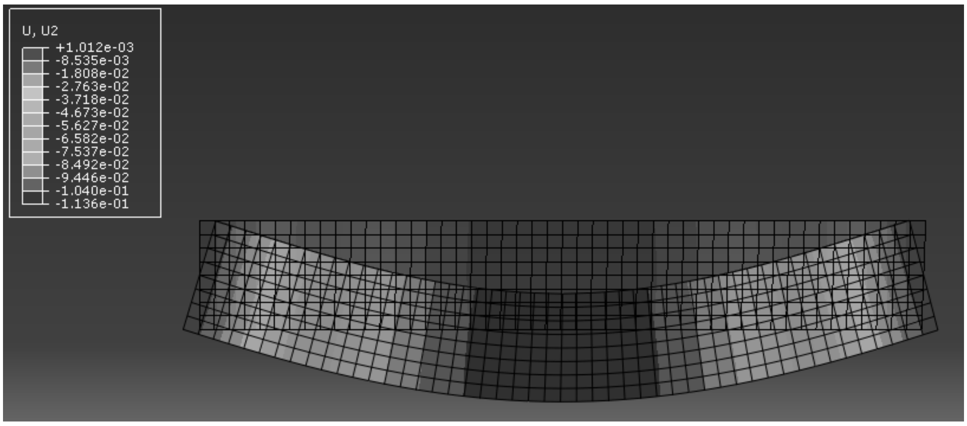


Figure 55: Case study beam deflection

The tensile stress in the beam is:

$$\sigma_{xx} = 3.32\, N/mm^2$$

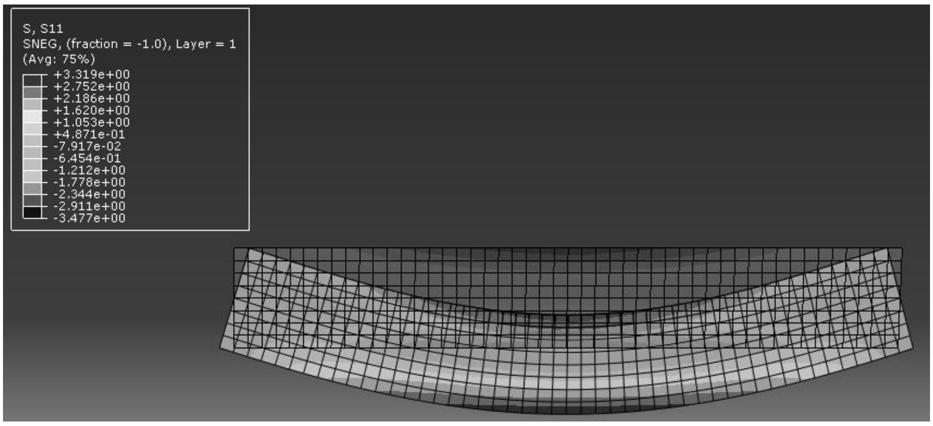


Figure 56: Case study beam (tensile) stress

Table 38: Comparison Deflection-Stress Analytical-Numerical

	w [mm]	σ [N/mm²]
Analytical	0.09	2.77
Numerical	0.11	3.32

These values are not 100% accurate but are in the same order. They are however rather small but this can be explained by the large cross-section of the beam and the relatively small load acting on the beam. The differences could be founded in the choice to use shell-composite elements to model the beam.

5.3.3. Lateral Torsional Buckling Analysis

An LBA is also done for single glass layer, the monolithic and the test beam and finally the beam of the case study.

For the monolithic glass layer, the properties of a single layer of the beam were assumed. For a single glass layer, the general equation applies to calculate the critical buckling moment. This calculation was done for both a linear load as a concentrated load. The finite-element results from Abaqus can be found in the following table where 'constant', 'parabolic' and 'triangular' define the shape of the bending moment diagram.

Table 39: Abaqus Critical bending moment buckling single monolithic glass layer of a beam

	C_1 [-]	C_2 [-]	Num. M_{cr} [kNm]	Analyt. M_{cr} [kNm]
Parabolic (q)	1.13	0.46	**6.548**	6.4
Triangular (Q)	1.36	0.55	**7.626**	7.5

These values are very similar indicating that both approaches could be used to calculate the critical buckling moment of a monolithic pane. The equations provided can be used in the case of a monolithic pane to calculate the maximum lateral torsional buckling moment.

The results for the test beam can be found in the next table.

Table 40: Abaqus Critical bending moment buckling test beam [2+1 (inter)layers]

	C_1 [-]	C_2 [-]	Num. M_{cr} [kNm]	Analyt. M_{cr} [kNm]
Constant (M)	1	0	**20.26**	20.22
Parabolic (q)	1.13	0.46	**20.98**	21.41
Triangular (Q)	1.36	0.55	**24.52**	25.45

Also in this case a good approximation is obtained. It can be concluded that the formulas can be used to accurately calculate a two-layered laminated beam.

Finally, the result of the beam from the case study can be found in [] below. The difficulty to calculate multi-layered laminated beams (more than three) becomes apparent when trying to calculate it analytically. The equations only provide solutions for two- and three-layered beams.

Table 41: Comparison two- & six-layered beam

Numerical	M_{cr} 2 layers [kNm]	M_{cr} 6 layers [kNm]
Constant (M)	20.26	/
Parabolic (q)	20.98	555.70
Triangular (Q)	24.52	558.62

The increase when simulating a six-layered beam however is more difficult to explain and seems implausible. At this point, the difference between these values cannot be explained with certainty.

5.4 Simulation of a Column

The column is solely modelled for a buckling analysis using shell elements. This to find the critical buckling load and to be able to later on compare it with the analytical value. According to Abaqus, the critical buckling moment for the column is 650.11kN which is very similar to

Table 42: Comparison critical buckling force column numerical-analytical

	Num. N_{cr} [kN]	N_{cr} Method I [kN]	N_{cr} Method II [kN]
Column	650.11	729.12	729.12

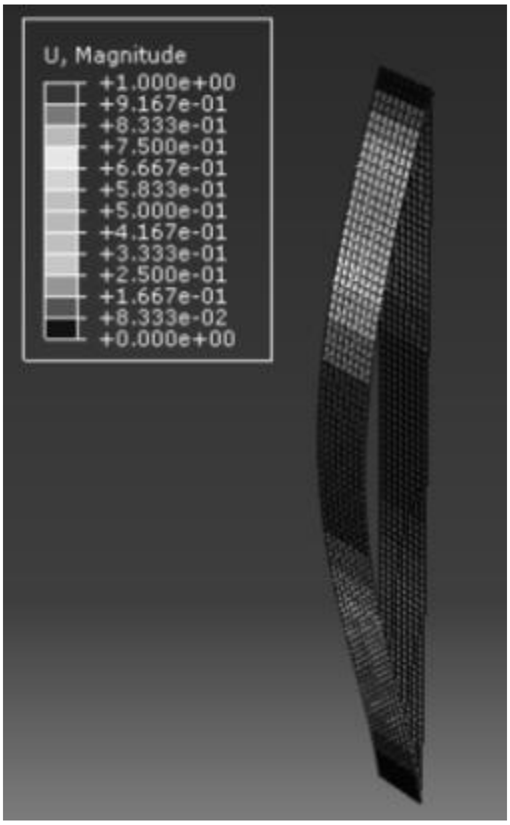


Figure 57: Buckling of the column

5.5 Simulation of a Connection

The connection between beam and column is a vital part in the system since it has to transfer to loads from the beam to the column. Because a 3D stress state is to be expected, solid element are used. A single glass layer is modelled with a sixth of the entire load acting on the pane. By modelling the connection, the maximum tensile stress can be found. According to Abaqus, the maximum tensile stress occurring at the joint is 1.234N/mm², which is the stress component in the same direction as the applied force.

Figure 58: Configuration connection with force

The values for the comparison between the analytical and numerical results of the stresses occurring in the connection can be found in. The tensile forces at the connection should remain smaller than the allowable stress in a glass layer.

Table 43: Comparison maximum stress connection numerical-analytical

	Num. σ [N/mm²]	Analyt. σ [N/mm²]
Connection	**1.234**	**2.49**

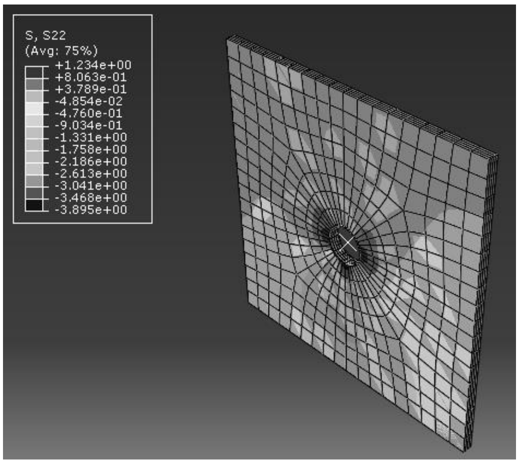


Figure 59: Model of the connection (single layer)

The stresses differ quite a lot when compared. Both results however are smaller than the allowable stress, even when not taking into account the benefits of the toughening process.

This begs the question if the approach made for the analytical calculation are too conservative to overcome the problems that may occur e.g. the drilling of the holes creating microcracks. This is something which needs to be further investigated in future research.

VI. CONCLUSION & FUTURE WORK

Some general conclusions can made regarding the results generated by this thesis:

- The first remark should be the limitations in this work. It barely scratches the surface of glass behaviour and design. More extensive design of glass elements could and should be done before the actual application of the elements. Glass can be used in many more applications with a variety of loads types and combinations on glass elements most likely generating different results.

  The theoretical knowledge does exist to calculate the deflections, stresses etc. of glass elements, preliminary results presented in this thesis show a good approximation. However they are not yet combined in a single reference document for the calculation of glass structures. This makes it both difficult and time consuming (and therefore expensive) when designing entire glass façade or other glass structures for that matter. There is thus a need for a glass standard.

- Theoretical calculations often include complex formulae to calculate elements by hand. This could mean that the inclusion of finite-element modelling in the design approach of glass structure could be a necessity. A thorough understanding of finite-element modelling is thereby of vital importance to assess the validity of the results.

  However, in simple cases, the regular design approach used for other materials seem also valid up to certain point for glass elements (but are limited e.g. 2 or 3-layered laminates).

Future work in glass design can be:

- The inclusion of design rules for element comprised of more than 2 or 3 layers seeing in this case, the elements were comprised of 6 layers and 5 interlayers;
- A further investigation of glass-glass connections and connections with possible other materials such as aluminium or steel.
- Even the entire modelling of glass structures in finite-element software;
- A more extensive research on the applied safety factors for glass. This way, the confidence in glass as a structural material can grow;

Whatever the case is, there is definitely a need for a more detailed approach to the design of glass structures. Glass design will always have a certain amount of uncertainty to it due to the irregularities in the material itself. By creating a uniform design approach, these limitations of glass as a structural material could eventually be reduced.

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Design of primary laminated glass elements

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Voor akkoord,

Leurs, Freya **Seynaeve, Simon**

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