

Spatial small area smoothing models for handling survey data with nonresponse
Supplementary material

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Spatial small area smoothing models for handling survey data with nonresponse

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Supplementary Materials

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Supplementary Tables

Description	Missingness probabilities $q_{i(j)k}^m$																	
Age Group	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
<i>No missing Data</i>	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
<i>MAR (1)</i>	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50	0.45	0.40	0.35	0.30	0.25	0.20	0.15	0.10
<i>MAR (2)</i>	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
<i>MAR (3)</i>	0.90	0.80	0.75	0.70	0.65	0.60	0.55	0.50	0.55	0.65	0.55	0.40	0.35	0.30	0.25	0.20	0.15	0.10
<i>MAR (4)</i>	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.90	0.80	0.70	0.60	0.50	0.40	0.30	0.20	0.10
<i>MAR (5)</i>	0.90	0.80	0.70	0.60	0.50	0.40	0.30	0.20	0.10	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90

Table 1: *Definition of the separate missingness probability weight for five different MAR scenarios. MAR (1) and MAR (2) coincide with simulation scenario (S4) and (S6) respectively. Their corresponding spatial counterparts are constructed by the addition of the same random effect u_k described in Section 3.*

	UNW	HT	AN	PL	ES	MB1 (RW1)	MB1 (SP)	MB3 (RW1)	MB3 (SP)	MB3 (SP + OD)
20% Missingness										
<i>MAR (1)</i>	10.96	0.48	0.36	0.24	0.29	0.21	0.27	0.11	0.12	0.17
<i>MAR (2)</i>	25.36	1.63	1.18	0.64	0.78	0.69	0.78	1.13	1.14	1.21
<i>MAR (3)</i>	12.34	0.62	0.43	0.27	0.33	0.26	0.30	0.13	0.12	0.17
<i>MAR (4)</i>	17.84	0.64	0.50	0.28	0.37	0.34	0.41	0.20	0.18	0.26
<i>MAR (5)</i>	15.83	1.14	0.77	0.44	0.50	0.36	0.42	0.43	0.43	0.49
<i>S-MAR (1)</i>	11.27	0.54	0.40	0.26	0.32	0.23	0.30	0.24	0.29	0.33
<i>S-MAR (2)</i>	25.13	1.54	1.10	0.62	0.79	0.66	0.76	1.30	0.79	0.88
<i>S-MAR (3)</i>	12.31	0.58	0.39	0.25	0.30	0.25	0.28	0.22	0.28	0.31
<i>S-MAR (4)</i>	17.94	0.70	0.49	0.28	0.37	0.33	0.40	0.15	0.24	0.32
<i>S-MAR (5)</i>	15.86	1.19	0.77	0.45	0.54	0.37	0.43	0.68	0.64	0.67
40% Missingness										
<i>MAR (1)</i>	4.77	0.22	0.21	0.17	0.22	0.21	0.26	0.19	0.21	0.20
<i>MAR (2)</i>	39.09	4.69	3.49	2.21	2.41	2.79	2.94	4.35	3.15	3.32
<i>MAR (3)</i>	7.03	0.38	0.30	0.22	0.27	0.27	0.32	0.26	0.29	0.28
<i>MAR (4)</i>	19.50	0.68	0.61	0.33	0.44	0.44	0.53	0.13	0.18	0.29
<i>MAR (5)</i>	14.59	1.98	1.37	0.92	1.01	1.01	1.09	1.86	1.71	1.63
<i>S-MAR (1)</i>	5.11	0.24	0.24	0.20	0.22	0.25	0.32	0.19	0.24	0.23
<i>S-MAR (2)</i>	38.41	4.22	3.19	2.07	2.39	2.73	2.87	4.00	3.00	3.15
<i>S-MAR (3)</i>	7.02	0.34	0.29	0.21	0.24	0.23	0.30	0.27	0.25	0.25
<i>S-MAR (4)</i>	19.79	0.89	0.63	0.33	0.46	0.46	0.54	0.13	0.18	0.30
<i>S-MAR (5)</i>	14.43	1.92	1.37	0.90	1.01	0.97	1.06	1.83	1.67	1.59
60% Missingness										
<i>MAR (1)</i>	0.38	0.18	0.09	0.11	0.14	0.18	0.20	0.10	0.08	0.08
<i>MAR (2)</i>	61.97	14.37	11.55	9.03	7.72	11.69	12.02	14.06	12.59	12.68
<i>MAR (3)</i>	1.99	0.10	0.11	0.13	0.15	0.20	0.23	0.15	0.12	0.11
<i>MAR (4)</i>	22.18	0.57	0.57	0.22	0.35	0.35	0.46	0.10	0.11	0.16
<i>MAR (5)</i>	12.87	3.49	2.67	2.16	2.19	3.48	3.68	4.45	4.33	4.12
<i>S-MAR (1)</i>	0.60	0.15	0.10	0.12	0.13	0.20	0.23	0.13	0.10	0.09
<i>S-MAR (2)</i>	59.53	12.57	10.45	8.18	6.77	11.12	11.45	13.79	11.78	11.88
<i>S-MAR (3)</i>	2.13	0.13	0.13	0.13	0.15	0.19	0.23	0.17	0.14	0.12
<i>S-MAR (4)</i>	22.65	1.01	0.72	0.34	0.52	0.45	0.54	0.11	0.14	0.24
<i>S-MAR (5)</i>	13.10	3.26	2.69	2.14	2.03	3.37	3.62	4.35	4.36	4.12

Table 2: Summary statistics of squared bias for the MAR and S-MAR settings using adjusted weights, analysed under the M1 simulation mechanism ($\times 10^3$)

	UNW	HT	AN	PL	ES	MB1 (RW1)	MB1 (SP)	MB3 (RW1)	MB3 (SP)	MB3 (SP + OD)
20% Missingness										
<i>MAR (1)</i>	15.80	7.04	1.47	0.56	0.60	1.03	0.74	0.98	0.84	0.80
<i>MAR (2)</i>	29.96	9.13	2.89	1.27	1.56	1.38	1.41	1.93	1.83	1.97
<i>MAR (3)</i>	17.03	7.21	1.58	0.60	0.65	0.96	0.76	0.87	0.73	0.75
<i>MAR (4)</i>	22.46	7.82	1.86	0.72	0.82	1.08	0.94	0.97	0.80	0.89
<i>MAR (5)</i>	20.58	8.07	2.13	0.84	1.02	0.93	0.92	1.03	0.93	1.02
<i>S-MAR (1)</i>	16.07	6.85	1.51	0.58	0.63	0.94	0.75	1.18	1.04	1.26
<i>S-MAR (2)</i>	29.57	9.05	2.79	1.28	1.49	1.34	1.38	2.16	1.53	1.84
<i>S-MAR (3)</i>	17.10	7.18	1.59	0.60	0.64	1.00	0.75	1.06	0.95	1.26
<i>S-MAR (4)</i>	22.52	7.69	1.85	0.75	0.84	1.01	0.92	0.83	0.82	1.23
<i>S-MAR (5)</i>	20.53	7.94	2.09	0.89	0.98	0.96	0.95	1.33	1.16	1.51
40% Missingness										
<i>MAR (1)</i>	11.75	9.00	1.39	0.52	0.52	1.20	0.70	1.28	1.12	1.25
<i>MAR (2)</i>	44.72	15.53	6.22	3.65	4.17	3.71	3.79	5.44	4.25	4.51
<i>MAR (3)</i>	13.92	9.11	1.51	0.57	0.58	1.05	0.79	1.19	1.06	1.31
<i>MAR (4)</i>	25.76	10.80	2.46	1.05	1.20	1.26	1.15	0.96	0.91	1.36
<i>MAR (5)</i>	20.91	10.89	2.83	1.44	1.50	1.58	1.61	2.50	2.27	2.49
<i>S-MAR (1)</i>	11.62	8.26	1.41	0.51	0.59	1.14	0.74	1.32	1.13	1.24
<i>S-MAR (2)</i>	43.97	15.01	5.91	3.50	4.08	3.61	3.71	5.14	4.09	4.35
<i>S-MAR (3)</i>	13.80	8.63	1.56	0.55	0.64	0.97	0.74	1.23	1.03	1.31
<i>S-MAR (4)</i>	26.10	10.87	2.37	1.00	1.11	1.22	1.15	0.88	0.88	1.36
<i>S-MAR (5)</i>	21.04	11.02	2.93	1.46	1.55	1.58	1.58	2.47	2.25	2.47
60% Missingness										
<i>MAR (1)</i>	11.44	13.01	1.56	0.53	0.62	1.04	0.71	1.36	1.13	1.24
<i>MAR (2)</i>	68.88	31.06	16.07	12.38	12.39	12.92	13.15	15.38	14.05	14.19
<i>MAR (3)</i>	12.98	12.85	1.41	0.54	0.48	0.99	0.72	1.27	1.08	1.34
<i>MAR (4)</i>	31.58	16.30	3.18	1.55	1.69	1.47	1.43	1.31	1.28	1.71
<i>MAR (5)</i>	23.16	17.06	4.37	2.87	2.78	4.11	4.25	5.12	5.08	5.06
<i>S-MAR (1)</i>	11.92	12.91	1.34	0.52	0.47	1.07	0.74	1.38	1.17	1.30
<i>S-MAR (2)</i>	67.00	29.75	14.76	11.29	11.18	12.32	12.55	15.15	13.22	13.37
<i>S-MAR (3)</i>	13.05	12.66	1.60	0.57	0.65	0.96	0.73	1.30	1.11	1.38
<i>S-MAR (4)</i>	31.85	16.32	3.26	1.62	1.81	1.53	1.44	1.34	1.26	1.71
<i>S-MAR (5)</i>	23.20	16.84	4.67	2.93	2.98	4.04	4.24	5.06	5.11	5.08

Table 3: Summary statistics of MSE for the MAR and S-MAR settings using adjusted weights, analysed under the M1 simulation mechanism ($\times 10^3$)

	UNW	HT	AN	PL	ES	MB1 (RW1)	MB1 (SP)	MB3 (RW1)	MB3 (SP)	MB3 (SP + OD)
20% Missingness										
<i>MAR (1)</i>	0.52	0.95	0.97	0.98	0.98	1.00	0.99	0.99	1.00	1.00
<i>MAR (2)</i>	0.27	0.92	0.87	0.88	0.84	0.96	0.94	0.96	0.98	1.00
<i>MAR (3)</i>	0.49	0.95	0.97	0.99	0.98	1.00	1.00	0.99	0.99	1.00
<i>MAR (4)</i>	0.38	0.94	0.96	0.98	0.98	1.00	0.99	1.00	1.00	1.00
<i>MAR (5)</i>	0.41	0.93	0.94	0.96	0.94	0.99	0.99	0.99	0.99	1.00
<i>S-MAR (1)</i>	0.51	0.95	0.97	0.98	0.98	1.00	0.99	0.98	0.99	1.00
<i>S-MAR (2)</i>	0.28	0.91	0.87	0.87	0.84	0.96	0.95	0.96	0.98	1.00
<i>S-MAR (3)</i>	0.49	0.95	0.97	0.98	0.98	1.00	1.00	0.99	1.00	1.00
<i>S-MAR (4)</i>	0.37	0.95	0.96	0.99	0.98	1.00	1.00	1.00	1.00	1.00
<i>S-MAR (5)</i>	0.41	0.93	0.94	0.96	0.94	0.99	0.99	0.99	0.99	1.00
40% Missingness										
<i>MAR (1)</i>	0.79	0.92	0.92	0.88	0.88	0.96	0.95	0.99	1.00	1.00
<i>MAR (2)</i>	0.22	0.83	0.67	0.63	0.57	0.81	0.77	0.85	0.89	0.97
<i>MAR (3)</i>	0.71	0.93	0.96	0.95	0.96	0.99	0.98	0.99	1.00	1.00
<i>MAR (4)</i>	0.42	0.93	0.95	0.97	0.96	1.00	0.99	1.00	1.00	1.00
<i>MAR (5)</i>	0.52	0.91	0.92	0.92	0.91	0.97	0.96	0.96	0.96	1.00
<i>S-MAR (1)</i>	0.78	0.92	0.94	0.89	0.89	0.96	0.96	0.99	1.00	1.00
<i>S-MAR (2)</i>	0.22	0.84	0.69	0.63	0.59	0.82	0.78	0.87	0.90	0.98
<i>S-MAR (3)</i>	0.70	0.94	0.96	0.95	0.94	0.99	0.99	0.99	1.00	1.00
<i>S-MAR (4)</i>	0.41	0.93	0.95	0.98	0.97	1.00	0.99	1.00	1.00	1.00
<i>S-MAR (5)</i>	0.52	0.91	0.91	0.92	0.90	0.97	0.96	0.96	0.96	0.99
60% Missingness										
<i>MAR (1)</i>	0.93	0.85	0.83	0.65	0.64	0.81	0.78	1.00	1.00	1.00
<i>MAR (2)</i>	0.18	0.70	0.43	0.40	0.34	0.50	0.43	0.56	0.61	0.74
<i>MAR (3)</i>	0.89	0.89	0.91	0.82	0.82	0.94	0.92	1.00	1.00	1.00
<i>MAR (4)</i>	0.48	0.91	0.95	0.94	0.93	0.99	0.99	1.00	1.00	1.00
<i>MAR (5)</i>	0.64	0.88	0.87	0.84	0.80	0.89	0.84	0.85	0.82	0.95
<i>S-MAR (1)</i>	0.92	0.86	0.83	0.67	0.67	0.83	0.79	1.00	1.00	1.00
<i>S-MAR (2)</i>	0.19	0.71	0.43	0.41	0.33	0.52	0.45	0.56	0.64	0.77
<i>S-MAR (3)</i>	0.89	0.90	0.91	0.83	0.82	0.95	0.93	0.99	1.00	1.00
<i>S-MAR (4)</i>	0.47	0.91	0.93	0.95	0.92	0.99	0.99	1.00	1.00	1.00
<i>S-MAR (5)</i>	0.63	0.87	0.87	0.84	0.78	0.89	0.84	0.86	0.83	0.96

Table 4: Nominal coverage probabilities for the MAR and S-MAR settings using adjusted weights, analysed under the M1 simulation mechanism

	UNW	HT	AN	PL	ES	MB1 (RW1)	MB1 (SP)	MB3 (RW1)	MB3 (SP)	MB3 (SP + OD)
20% Missingness										
<i>MAR (1)</i>	0.04	0.05	1.37	0.46	2.01	0.47	0.48	0.54	0.47	0.45
<i>MAR (2)</i>	0.05	0.07	1.46	0.53	2.61	0.50	0.51	0.50	0.51	0.49
<i>MAR (3)</i>	0.02	0.03	1.60	0.44	2.79	0.47	0.47	0.45	0.47	0.45
<i>MAR (4)</i>	0.04	0.05	1.51	0.49	2.52	0.48	0.48	0.47	0.48	0.47
<i>MAR (5)</i>	0.05	0.05	1.99	0.48	3.10	0.48	0.48	0.47	0.48	0.46
<i>S-MAR (1)</i>	0.04	0.04	1.49	0.50	2.40	0.50	0.51	0.48	0.50	0.49
<i>S-MAR (2)</i>	0.04	0.05	1.64	0.46	2.36	0.46	0.46	0.65	0.46	0.45
<i>S-MAR (3)</i>	0.04	0.05	1.77	0.46	3.03	0.47	0.48	0.60	0.47	0.46
<i>S-MAR (4)</i>	0.05	0.05	1.84	0.47	2.74	0.49	0.50	0.49	0.50	0.48
<i>S-MAR (5)</i>	0.04	0.05	1.66	0.52	2.51	0.52	0.52	0.51	0.52	0.50
40% Missingness										
<i>MAR (1)</i>	0.04	0.06	2.83	0.78	4.32	0.78	0.78	0.90	0.78	0.76
<i>MAR (2)</i>	0.06	0.10	2.69	0.80	4.65	0.80	0.81	1.06	0.81	0.78
<i>MAR (3)</i>	0.02	0.03	2.67	0.69	4.21	0.75	0.75	0.82	0.74	0.73
<i>MAR (4)</i>	0.05	0.05	2.47	0.71	5.01	0.73	0.73	0.84	0.73	0.72
<i>MAR (5)</i>	0.06	0.06	3.43	0.79	4.94	0.79	0.79	0.95	0.78	0.76
<i>S-MAR (1)</i>	0.06	0.06	2.82	0.76	4.62	0.79	0.79	0.78	0.79	0.78
<i>S-MAR (2)</i>	0.05	0.08	3.05	0.78	5.14	0.81	0.80	0.82	0.80	0.80
<i>S-MAR (3)</i>	0.04	0.05	3.09	0.76	5.12	0.80	0.79	1.01	0.80	0.77
<i>S-MAR (4)</i>	0.05	0.06	3.03	0.70	4.99	0.75	0.75	0.87	0.75	0.73
<i>S-MAR (5)</i>	0.06	0.08	2.65	0.86	4.99	0.84	0.84	0.96	0.84	0.83
60% Missingness										
<i>MAR (1)</i>	0.08	0.11	6.17	1.54	10.58	1.54	1.53	1.53	1.52	1.50
<i>MAR (2)</i>	0.08	0.08	5.68	1.43	10.26	1.55	1.54	1.59	1.52	1.51
<i>MAR (3)</i>	0.04	0.05	5.11	1.51	8.41	1.60	1.59	1.63	1.58	1.56
<i>MAR (4)</i>	0.07	0.06	6.17	1.30	10.61	1.40	1.39	1.70	1.38	1.35
<i>MAR (5)</i>	0.10	0.14	6.26	1.66	9.13	1.64	1.63	1.97	1.62	1.59
<i>S-MAR (1)</i>	0.07	0.07	6.05	1.48	10.42	1.56	1.56	1.54	1.54	1.51
<i>S-MAR (2)</i>	0.05	0.09	5.45	1.49	11.05	1.60	1.61	1.75	1.60	1.55
<i>S-MAR (3)</i>	0.05	0.08	5.20	1.60	8.47	1.67	1.66	1.83	1.65	1.61
<i>S-MAR (4)</i>	0.07	0.08	5.56	1.17	9.77	1.39	1.41	1.52	1.38	1.37
<i>S-MAR (5)</i>	0.07	0.10	6.05	1.65	11.32	1.71	1.71	1.76	1.69	1.64

Table 5: Summary statistics of squared bias for the MAR and S-MAR settings using adjusted weights, analysed under the M2 simulation mechanism ($\times 10^3$)

	UNW	HT	AN	PL	ES	MB1 (RW1)	MB1 (SP)	MB3 (RW1)	MB3 (SP)	MB3 (SP + OD)
20% Missingness										
<i>MAR (1)</i>	2.98	4.26	7.60	3.96	7.81	2.87	2.90	3.45	2.89	2.88
<i>MAR (2)</i>	3.12	4.81	8.61	4.49	11.46	3.03	3.05	3.10	3.06	3.05
<i>MAR (3)</i>	3.00	4.16	8.23	3.89	11.68	2.91	2.93	2.95	2.93	2.92
<i>MAR (4)</i>	3.13	4.74	8.13	4.40	10.19	3.00	3.03	3.06	3.03	3.03
<i>MAR (5)</i>	3.09	4.36	9.03	4.06	11.83	2.96	2.98	3.00	2.99	2.98
<i>S-MAR (1)</i>	2.92	4.16	7.87	3.92	9.71	2.87	2.89	2.90	2.88	2.87
<i>S-MAR (2)</i>	3.08	4.81	8.47	4.41	8.73	2.95	2.97	3.88	2.96	2.96
<i>S-MAR (3)</i>	3.01	4.25	8.21	3.95	11.84	2.90	2.92	3.58	2.92	2.91
<i>S-MAR (4)</i>	3.01	4.56	8.93	4.24	10.03	2.93	2.97	2.99	2.96	2.95
<i>S-MAR (5)</i>	3.04	4.41	8.17	4.16	9.31	2.98	3.00	3.03	3.00	2.99
40% Missingness										
<i>MAR (1)</i>	3.94	5.38	11.10	5.18	12.78	3.97	3.96	4.47	3.96	3.90
<i>MAR (2)</i>	4.09	6.50	11.72	6.14	15.11	4.09	4.11	4.87	4.13	4.08
<i>MAR (3)</i>	4.05	5.33	11.08	5.10	13.37	4.03	4.04	4.19	4.03	4.00
<i>MAR (4)</i>	4.09	6.34	11.66	5.93	17.39	4.03	4.03	4.47	4.02	3.96
<i>MAR (5)</i>	4.24	5.84	12.49	5.57	14.06	4.19	4.20	4.74	4.20	4.15
<i>S-MAR (1)</i>	3.87	5.24	11.36	5.04	14.67	3.91	3.92	3.87	3.92	3.87
<i>S-MAR (2)</i>	4.09	6.78	11.79	6.33	15.82	4.11	4.12	4.08	4.13	4.08
<i>S-MAR (3)</i>	4.01	5.43	11.08	5.17	15.39	4.01	4.04	4.71	4.04	3.99
<i>S-MAR (4)</i>	4.02	6.32	12.27	5.91	15.41	4.00	4.03	4.64	4.03	3.99
<i>S-MAR (5)</i>	4.10	5.76	11.32	5.59	16.42	4.18	4.19	4.53	4.17	4.15
60% Missingness										
<i>MAR (1)</i>	6.25	8.03	17.50	7.44	25.10	6.04	6.02	6.09	6.02	6.02
<i>MAR (2)</i>	5.97	10.30	18.08	9.28	24.83	5.86	5.93	5.93	5.90	5.92
<i>MAR (3)</i>	5.95	7.52	15.39	7.14	20.43	5.95	5.92	6.01	5.93	5.87
<i>MAR (4)</i>	6.10	9.84	18.67	8.75	24.82	5.81	5.84	7.01	5.85	5.87
<i>MAR (5)</i>	6.48	8.54	18.24	8.02	21.30	6.34	6.37	7.37	6.36	6.35
<i>S-MAR (1)</i>	6.26	7.83	17.51	7.35	25.19	6.12	6.09	6.15	6.10	6.03
<i>S-MAR (2)</i>	6.26	10.93	17.53	9.77	27.63	6.08	6.15	6.77	6.18	6.21
<i>S-MAR (3)</i>	6.14	7.97	15.76	7.48	20.52	6.09	6.11	6.72	6.10	6.07
<i>S-MAR (4)</i>	5.94	9.41	17.56	8.40	23.93	5.70	5.73	6.24	5.73	5.73
<i>S-MAR (5)</i>	6.07	8.11	17.62	7.69	27.35	6.02	6.04	6.35	6.05	6.00

Table 6: Summary statistics of MSE for the MAR and S-MAR settings using adjusted weights, analysed under the M2 simulation mechanism ($\times 10^3$)

	UNW	HT	AN	PL	ES	MB1 (RW1)	MB1 (SP)	MB3 (RW1)	MB3 (SP)	MB3 (SP + OD)
20% Missingness										
<i>MAR (1)</i>	0.91	0.87	0.88	0.90	0.83	0.95	0.95	0.94	0.95	0.95
<i>MAR (2)</i>	0.91	0.87	0.86	0.89	0.81	0.94	0.94	0.94	0.94	0.94
<i>MAR (3)</i>	0.91	0.87	0.87	0.90	0.82	0.95	0.94	0.95	0.94	0.95
<i>MAR (4)</i>	0.91	0.87	0.87	0.90	0.84	0.95	0.95	0.94	0.94	0.95
<i>MAR (5)</i>	0.90	0.87	0.86	0.90	0.80	0.95	0.95	0.95	0.95	0.95
<i>S-MAR (1)</i>	0.91	0.87	0.88	0.91	0.84	0.95	0.95	0.95	0.95	0.95
<i>S-MAR (2)</i>	0.91	0.86	0.85	0.89	0.81	0.94	0.94	0.92	0.95	0.94
<i>S-MAR (3)</i>	0.91	0.88	0.87	0.91	0.80	0.95	0.95	0.93	0.95	0.95
<i>S-MAR (4)</i>	0.91	0.87	0.87	0.89	0.82	0.95	0.95	0.94	0.95	0.95
<i>S-MAR (5)</i>	0.91	0.87	0.87	0.90	0.83	0.95	0.95	0.94	0.95	0.95
40% Missingness										
<i>MAR (1)</i>	0.89	0.86	0.87	0.91	0.81	0.95	0.95	0.94	0.95	0.95
<i>MAR (2)</i>	0.89	0.85	0.84	0.88	0.74	0.94	0.94	0.93	0.95	0.95
<i>MAR (3)</i>	0.89	0.86	0.88	0.91	0.82	0.94	0.94	0.94	0.94	0.95
<i>MAR (4)</i>	0.89	0.84	0.85	0.89	0.76	0.94	0.94	0.93	0.94	0.94
<i>MAR (5)</i>	0.89	0.85	0.86	0.90	0.80	0.95	0.95	0.94	0.95	0.95
<i>S-MAR (1)</i>	0.89	0.86	0.86	0.91	0.80	0.95	0.95	0.95	0.95	0.95
<i>S-MAR (2)</i>	0.89	0.84	0.84	0.88	0.77	0.94	0.94	0.94	0.94	0.94
<i>S-MAR (3)</i>	0.89	0.86	0.85	0.91	0.76	0.95	0.95	0.93	0.95	0.95
<i>S-MAR (4)</i>	0.89	0.84	0.85	0.89	0.77	0.95	0.95	0.94	0.95	0.95
<i>S-MAR (5)</i>	0.89	0.86	0.86	0.90	0.77	0.94	0.94	0.94	0.94	0.94
60% Missingness										
<i>MAR (1)</i>	0.86	0.82	0.83	0.91	0.70	0.95	0.95	0.95	0.95	0.95
<i>MAR (2)</i>	0.87	0.80	0.81	0.87	0.69	0.94	0.95	0.94	0.95	0.95
<i>MAR (3)</i>	0.87	0.83	0.83	0.92	0.71	0.95	0.95	0.95	0.95	0.95
<i>MAR (4)</i>	0.85	0.79	0.80	0.87	0.67	0.94	0.95	0.93	0.95	0.95
<i>MAR (5)</i>	0.86	0.82	0.83	0.91	0.71	0.94	0.94	0.93	0.94	0.95
<i>S-MAR (1)</i>	0.86	0.82	0.81	0.91	0.70	0.94	0.94	0.94	0.94	0.94
<i>S-MAR (2)</i>	0.86	0.79	0.79	0.87	0.67	0.94	0.94	0.93	0.94	0.95
<i>S-MAR (3)</i>	0.86	0.83	0.83	0.91	0.73	0.95	0.95	0.94	0.95	0.95
<i>S-MAR (4)</i>	0.85	0.79	0.80	0.88	0.67	0.95	0.95	0.94	0.95	0.95
<i>S-MAR (5)</i>	0.86	0.82	0.83	0.91	0.69	0.94	0.95	0.94	0.95	0.95

Table 7: Nominal coverage probabilities for the MAR and S-MAR settings using adjusted weights, analysed under the M2 simulation mechanism

Supplementary Figures

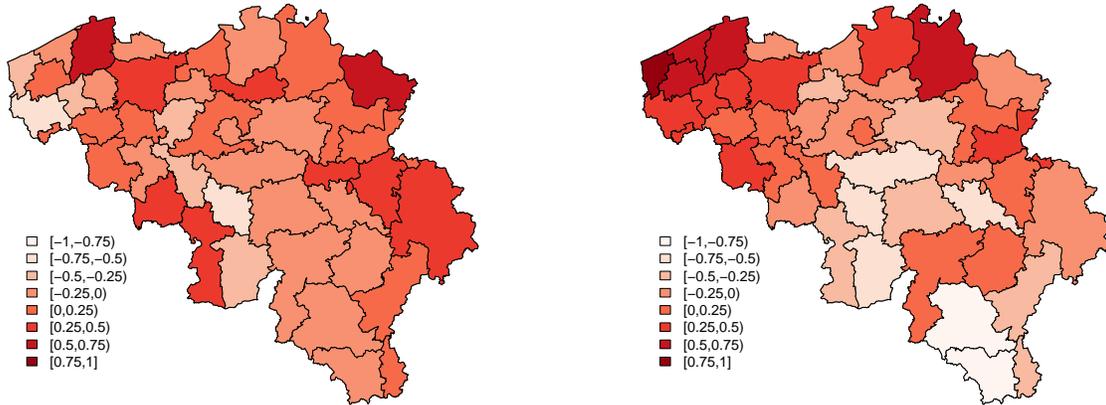


Figure 1: *Geographic distribution of the simulated non-spatial (left) and spatial random effect (right) used in prevalence model M2.*

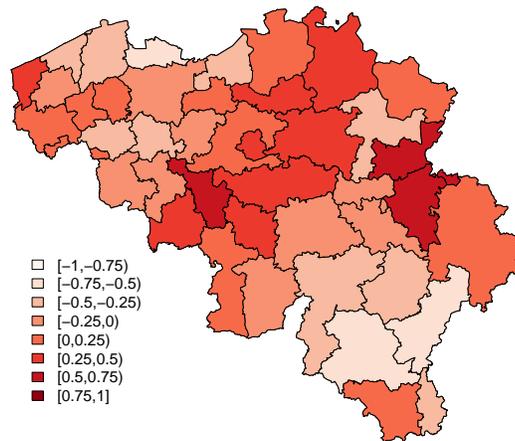
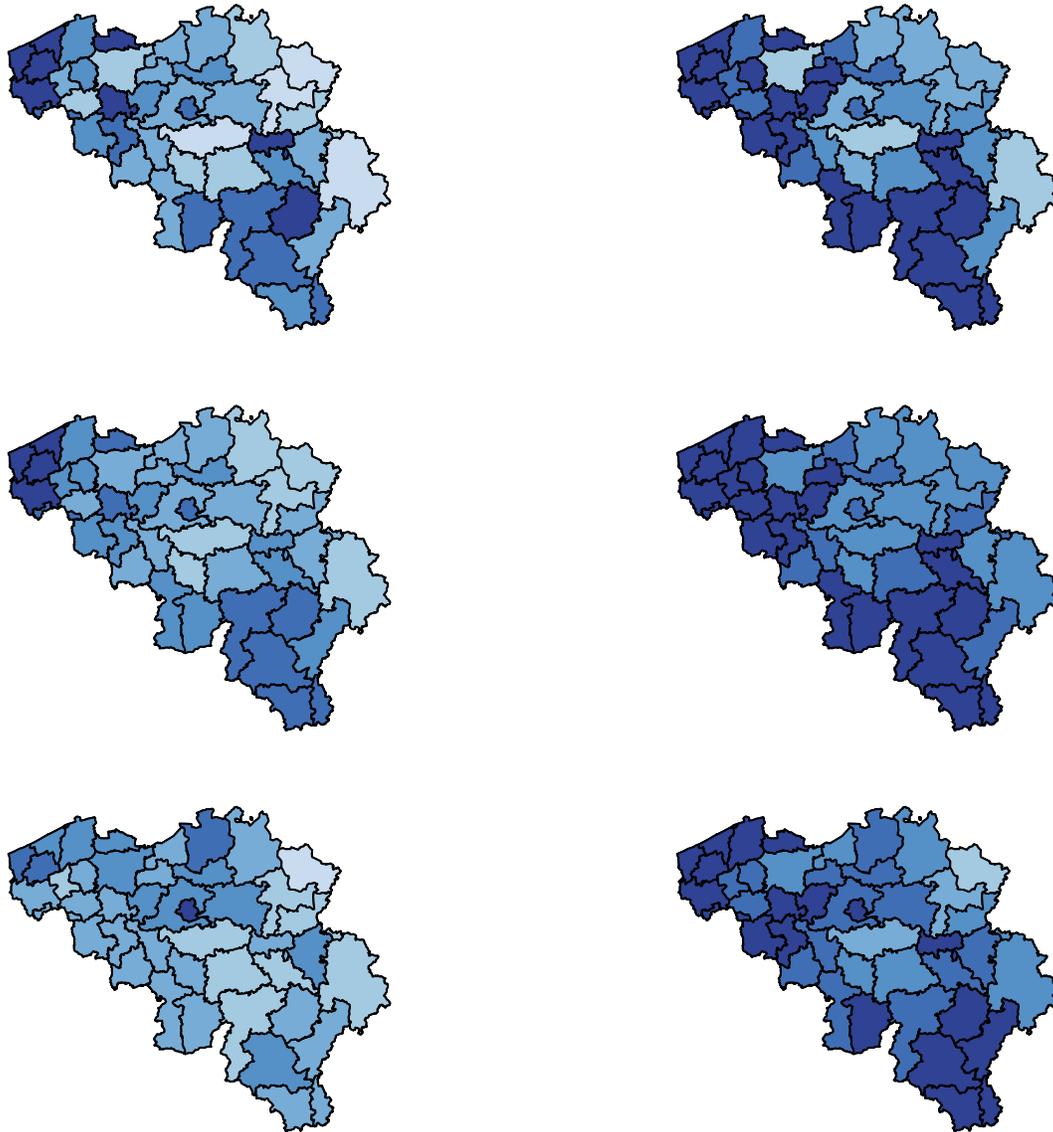
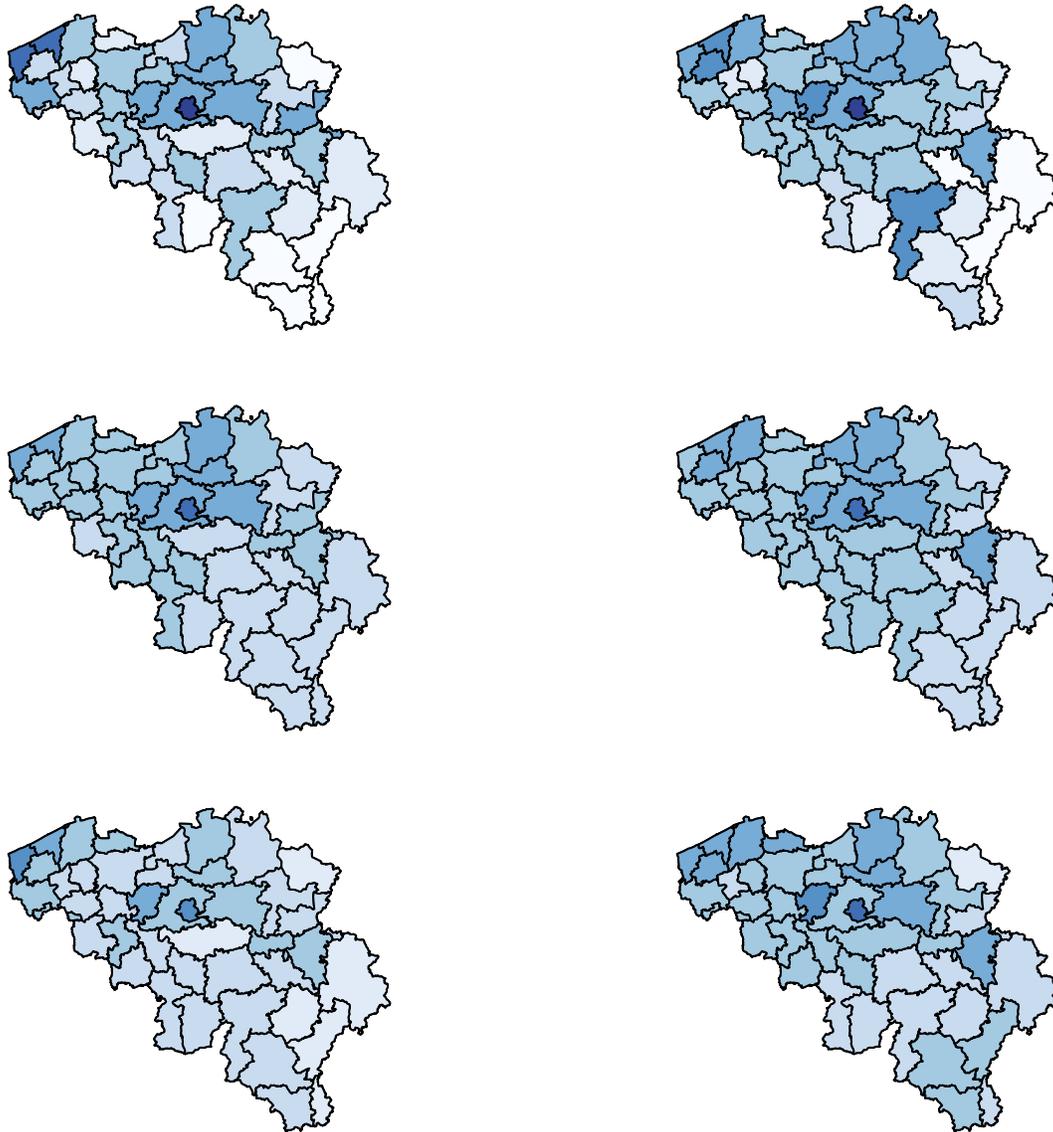


Figure 2: *Geographic distribution of the simulated spatial random effect used in scenario S4, S6 and S8.*



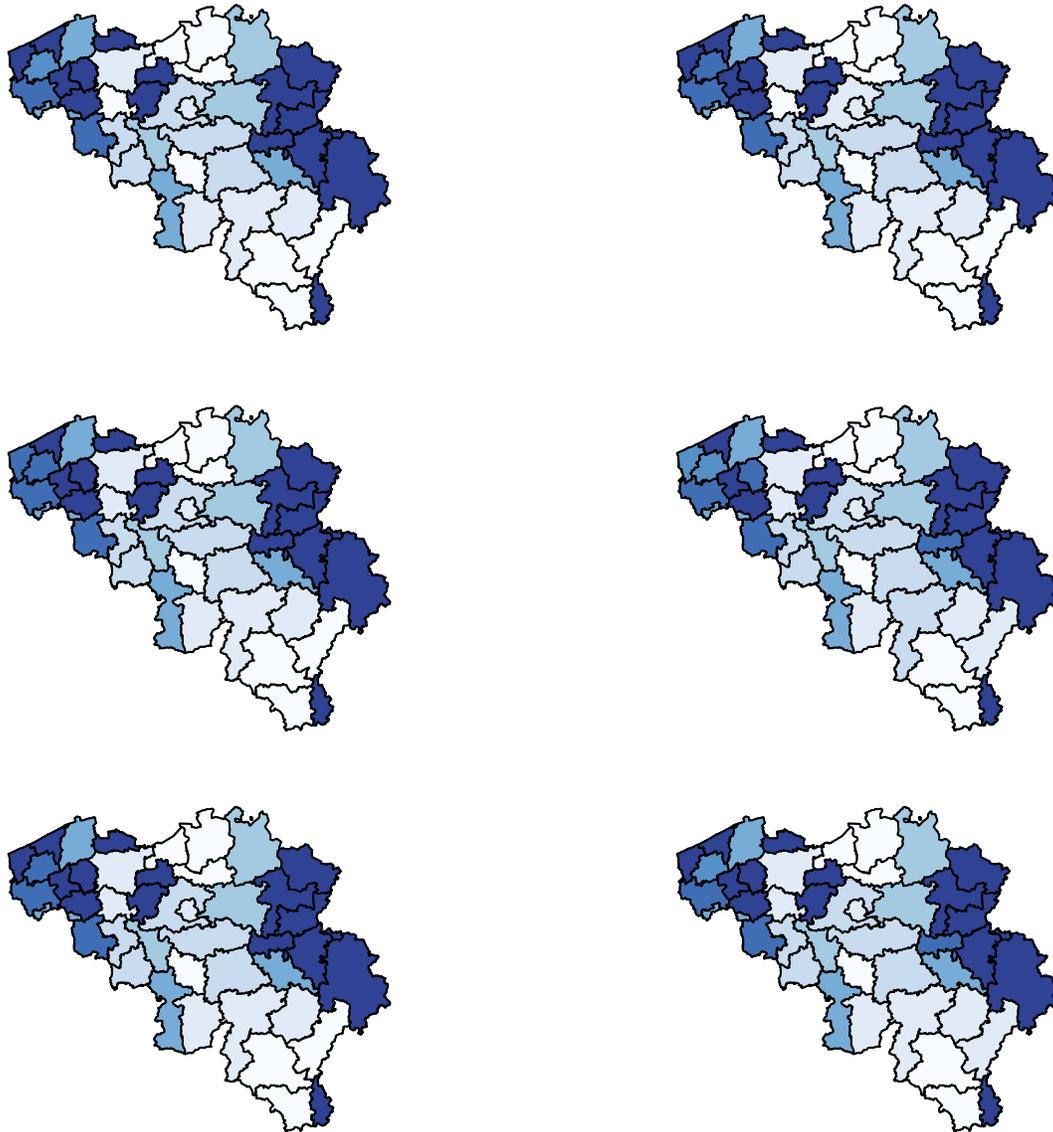
[0.5,0.53)
 [0.53,0.54)
 [0.54,0.55)
 [0.55,0.56)
 [0.56,0.57)
 [0.57,0.58)
 [0.58,0.59)
 [0.59,0.65]

Figure 3: *Spatial maps displaying the estimated prevalence for the HT estimator (top row), AN estimator (middle row), MB1 (RW1) estimator (bottom left) and MB3 (SP + OD) estimator (bottom right), analysed under the M1 simulation mechanism and S1 (left column) and S2 (right column) missingness mechanism with 60% missingness.*



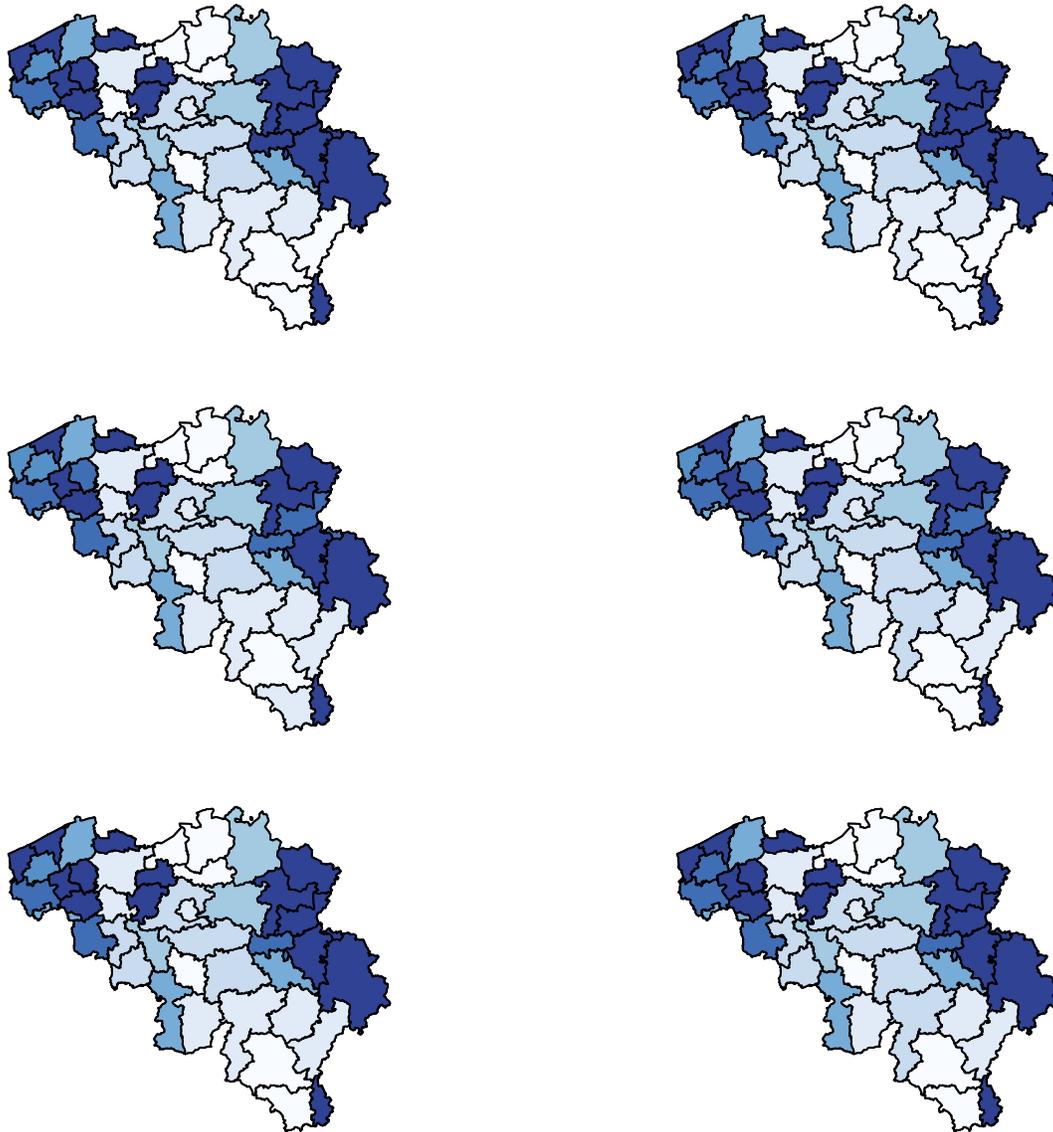
[0.5,0.53)
 [0.53,0.54)
 [0.54,0.55)
 [0.55,0.56)
 [0.56,0.57)
 [0.57,0.58)
 [0.58,0.59)
 [0.59,0.65]

Figure 4: *Spatial maps displaying the estimated prevalence for the HT estimator (top row), AN estimator (middle row) and MB3 (SP + OD) estimator (bottom row), analysed under the M1 simulation mechanism and S3 (left column) and S4 (right column) missingness mechanism with 60% missingness.*



[0.05,0.2)
 [0.2,0.3)
 [0.3,0.4)
 [0.4,0.5)
 [0.5,0.6)
 [0.6,0.7)
 [0.7,0.8)
 [0.8,0.975]

Figure 5: Spatial maps displaying the estimated prevalence for the HT estimator (top row), AN estimator (middle row) and MB3 (SP + OD) estimator (bottom row), analysed under the M2 simulation mechanism and S1 (left column) and S2 (right column) missingness mechanism with 60% missingness.



[0.05,0.2)
 [0.2,0.3)
 [0.3,0.4)
 [0.4,0.5)
 [0.5,0.6)
 [0.6,0.7)
 [0.7,0.8)
 [0.8,0.975]

Figure 6: Spatial maps displaying the estimated prevalence for the HT estimator (top row), AN estimator (middle row) and MB3 (SP + OD) estimator (bottom row), analysed under the M2 simulation mechanism and S3 (left column) and S4 (right column) missingness mechanism with 60% missingness.

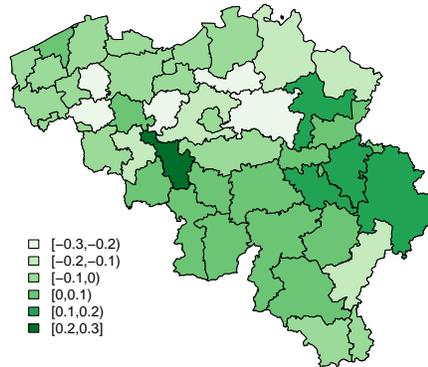


Figure 7: *Geographic distribution of the estimated spatial random effect $u_k + v_k$ of the HIS application.*