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Peer-reviewed author version

NAPOLES RUIZ, Gonzalo; Mosquera, Carlos; Falcon, Rafael; Grau, Isel; Bello, Rafael & VANHOOF, Koen (2018) Fuzzy-rough cognitive networks. In: NEURAL NETWORKS, 97, p. 19-27.

DOI: 10.1016/j.neunet.2017.08.007

Handle: <http://hdl.handle.net/1942/25527>

# Fuzzy-Rough Cognitive Networks

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## Abstract

*Rough Cognitive Networks* (RCNs) are a kind of granular neural network that augment the reasoning rule present in Fuzzy Cognitive Maps with crisp information granules coming from Rough Set Theory. While RCNs have shown promise in solving different classification problems, this model is still very sensitive to the similarity threshold upon which the rough information granules are built. In this paper, we cast the RCN model within the framework of *fuzzy rough sets* in an attempt to eliminate the need for a user-specified similarity threshold while retaining the model's discriminatory power. As far as we know, this is the first study that brings fuzzy sets into the domain of rough cognitive mapping. Numerical results in presence of 140 well-known pattern classification problems reveal that our approach, referred to as *Fuzzy-Rough Cognitive Networks*, is capable of outperforming most traditional classifiers used for benchmarking purposes. Furthermore, we explore the impact of using different heterogeneous distance functions and fuzzy operators over the performance of our granular neural network.

*Key words:* fuzzy cognitive maps, fuzzy rough sets, rough cognitive mapping, pattern classification, granular classifiers

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## 1. Introduction

*Pattern classification* [1] is one of the most popular field within Artificial Intelligence as a result of its link with real-world problems. In short, it may be defined as the process of identifying the right category (among those in a predefined set) to which an observation belongs. The ease with which we recognize our beloved black cat from hundreds similar to it or read handwritten characters belies the astoundingly complex processes that underlie these scenarios. That is why researchers have been focused on developing a wide spectrum of classification algorithms called *classifiers* with the goal of solving these problems with the best possible accuracy.

The literature on classification models [2] is vast and offers a myriad of techniques that approach the classification problem from multiple angles. Regrettably, some of the most accurate classifiers do not provide any mechanism to explain how they arrived at each conclusion and behave like *black boxes*. This means that their reasoning mechanism is not transparent, therefore negatively affecting their practical usability in scenarios where understanding the decision process is required. According to the terminology discussed in [3], *transparency* can be understood as the classifier’s ability to explain its reasoning mechanism, whereas *interpretability* refers to the classifier’s ability to explain the problem domain at the attribute level.

Recently, Nápoles and his collaborators [4] introduced the *Rough Cognitive Networks* (RCNs) in an attempt to develop an accurate, transparent classifier. Such granular neural networks augment the reasoning scheme present in Fuzzy Cognitive Maps (FCMs) [5] with information granules coming from Rough Set Theory (RST) [6]. Although RCNs can be considered as recurrent neural systems that fit the McCulloch-Pitts’ scheme [7], there are important differences with regards to other neural models.

Classical neural networks regularly perform like black boxes, where neither neurons nor connections have any clear specific meaning for the problem itself [8]. However, all the neurons and connections in an RCN have a precise meaning at a granular level, therefore making it possible to understand the underlying decision process at a granular (symbolic) level. The absence of hidden neurons and the lazy learning approach are other distinctive features attached to these granular, recurrent neural systems.

While RCNs have shown promise in solving different pattern classification problems [4] [9], their performance is still very sensitive to an input parameter denoting the similarity threshold upon which the rough information granules

are built. The proper estimation of this parameter is essential in presence of numerical attributes since it defines whether two objects are deemed similar or not. Aiming at overcoming this drawback, Nápoles et. al. [4] proposed an optimization-based hyperparameter learning scheme to estimate the value of this parameter from a hold-out test set. However, this strategy may become impractical for large datasets since it requires rebuilding the information granules for each parameter value to be evaluated.

In [10] the authors proposed a granular ensemble named *Rough Cognitive Ensembles* (RCEs) to deal with the parametric requirements of RCN-based classifiers. This classification model employs a collection of RCNs, each operating at a different granularity degree. While this approach involves a more elaborated solution, the ensemble architecture and the bagging strategy used to improved the diversity among the base classifiers irremediably harm the transparency of RCNs, thus becoming another black-box.

In this paper, we cast the RCN approach within the framework of Fuzzy Rough Set Theory (FRST) [11] [12] [13] [14] in an attempt to eliminate the need for a user-specified similarity threshold while retaining the model’s discriminatory power. Fuzzy rough sets are an extension of classical rough sets in which fuzzy sets are used to characterize the degree to which an object belongs to each information granule. The inclusion of the fuzzy approach into the RCN model allows coping with both the vagueness (fuzzy sets) and inconsistency (rough sets) of the information typically found in pattern classification environments. Besides, it allows designing a more elegant solution for the parametric issues of RCN-based classifiers.

Numerical simulations using 140 datasets reveal that the proposed model, referred to as *Fuzzy-Rough Cognitive Networks* (FRCNs), is capable of outperforming the standard RCNs using a fixed, reasonable similarity threshold value. The results also suggest that FRCNs remain competitive with regards to RCEs and other black-box classifiers adopted for comparison purposes. More importantly, the challenging process of estimating a precise value for the similarity threshold parameter is no longer a concern.

The rest of this paper is organized as follows. Section 2 briefly describes the RCN algorithm and the motivation behind our proposal. The fuzzy RCN classifier is unveiled in Section 3, whereas Section 4 introduces the numerical simulations and their ensuing discussion. Towards the end, Section 5 outlines some concluding remarks and future work directions.

## 74 2. Rough Cognitive Mapping

75 This section discusses the technical background relevant to this study and  
76 explains the motivation behind the fuzzy approach.

### 77 2.1. Theoretical Background

78 *Rough cognitive mapping* is a recently introduced concept[4] that brings  
79 together RST and FCMs. RCNs are granular FCMs whose topology is defined  
80 by the abstract semantics of the three-way decision rules [15] [16]. The set  
81 of input neurons in an RCN represent the positive, boundary and negative  
82 regions of the decision classes in the problem under consideration. The output  
83 neurons describe the set of decision classes. The topology (both concepts and  
84 weights) is entirely computed from historical data, thus removing the need  
85 for expert intervention during the classifier’s construction.

86 The first step in the RCN learning process is related to the *input data*  
87 *granulation* using RST. The positive, boundary and negative regions of each  
88 decision class according to a subset of attributes are computed using the  
89 training data set and a predefined similarity relation.

90 The second step is concerned with *topology design* where a sigmoid FCM  
91 is automatically created from the discovered information granules by using  
92 a set of predefined rules; see [4] for more details. In principle, an RCN will  
93 be composed of at most  $4|\mathcal{D}|$  neurons and  $3|\mathcal{D}|(1 + |D|)$  causal relationships,  
94 with  $\mathcal{D} = \{D_1, \dots, D_K\}$  being the set of decision classes.

95 The last step refers to the *network exploitation*, which simply means com-  
96 puting the response vector  $\mathcal{A}_x(\mathcal{D}) = \{A_x(D_1), \dots, A_x(D_k), \dots, A_x(D_K)\}$  for  
97 some unlabeled object. The new object  $x$  is presented to the RCN as an input  
98 vector  $A^{(0)}$  that activates input neurons. Each element in  $A^{(0)}$  is computed  
99 on the basis of the inclusion degree of  $x$  to each rough granular region. After  
100 this, the input vector is propagated through the RCN using the McCulloch-  
101 Pitts reasoning model [7] and next the decision class with the highest value  
102 in the response vector is then assigned to the test object.

### 103 2.2. Motivation for the FRCN Approach

104 The notion of *rough cognitive mapping* opened up a new research avenue  
105 in the field of granular-neural classifiers. However, their performance is highly  
106 sensitive to the similarity threshold used to determine whether two instances  
107 can be gathered together into the same similarity class.

108 Nápoles et. al. [4] used a parameter tuning method based on the Harmony  
 109 Search (HS) optimizer to estimate the similarity threshold. Nevertheless, the  
 110 evaluation of every candidate solution requires recalculating the lower and  
 111 upper approximations of each RST-based region for each decision class, which  
 112 could be computationally prohibitive for large datasets.

113 Let us assume that  $\mathcal{U}_1 \subset \mathcal{U}$  is the training set and  $\mathcal{U}_2 \subset \mathcal{U}$  is the hold-out  
 114 test (validation) set such that  $\mathcal{U}_1 \cap \mathcal{U}_2 = \emptyset$ . The computational complexity  
 115 of building the lower and upper approximations is  $O(|\Phi||\mathcal{U}_1|^2)$ , with  $\Phi$  being  
 116 the attribute set, whereas the complexity of building the network topology  
 117 is  $O(|\mathcal{D}|^2)$ , with  $\mathcal{D}$  being the set of decision classes. Besides, the complexity  
 118 of exploiting the granular network for  $|\mathcal{U}_2|$  instances is  $O(|\mathcal{U}_2||\Phi||\mathcal{U}_1|^2)$ . This  
 119 implies that the temporal complexity of evaluating a single parameter value is  
 120  $O(\max\{|\Phi||\mathcal{U}_1|^2, |\mathcal{D}|^2, |\mathcal{U}_2||\Phi||\mathcal{U}_1|^2\})$ . Due to the fact that  $|\mathcal{U}_1| \geq |\mathcal{U}_2|$  in most  
 121 machine learning scenarios, we can conclude that the overall complexity of  
 122 this parameter learning method is  $O(T|\Phi||\mathcal{U}_1|^3)$ , where  $T$  is the number of  
 123 learning cycles. Regrettably, this may negatively affect the practical usability  
 124 of RCNs in solving real-world pattern classification problems.

125 The key goal behind this research is to remove the estimation of the sim-  
 126 ilarity threshold without affecting the overall RCN's discriminatory power.  
 127 Being more explicit, we aim to arrive at a parameterless classifier (and hence  
 128 suppressing the need for a parameter tuning strategy) without degrading the  
 129 RCN's performance in classification problems.

### 130 3. Fuzzy-Rough Cognitive Mapping

131 This section presents the notion of *fuzzy-rough cognitive mapping* in order  
 132 to remove the requirement of estimating the similarity threshold in an RCN.  
 133 With this goal in mind, we first describe the mathematical foundations be-  
 134 hind this approach. Afterwards, we explain how to construct an FRCN for  
 135 solving pattern classification problems.

#### 136 3.1. Fuzzy-Rough Set Theory

137 The hybridization between rough sets and fuzzy sets was originally in-  
 138 vestigated by Dubois and Prade [11], and later extended and/or modified by  
 139 several authors. In this paper, we adopt the approach proposed by Inuiguchi  
 140 et al. [14] since it includes some mathematical properties that may be con-  
 141 venient when designing our fuzzy-rough classifier.

Let us assume the universe  $\mathcal{U}$ , a fuzzy set  $X \in \mathcal{U}$  and a fuzzy binary relation  $P \in \mathcal{F}(\mathcal{U} \times \mathcal{U})$ , where  $\mu_X(x)$  and  $\mu_P(y, x)$  denote their respective membership functions. The function  $\mu_X : \mathcal{U} \rightarrow [0, 1]$  computes the membership degree to which  $x \in \mathcal{U}$  is a member of  $X$ , whereas  $\mu_P : \mathcal{U} \times \mathcal{U} \rightarrow [0, 1]$  denotes the degree to which  $y$  is presumed to be a member of  $X$  from the fact that  $x$  is a member of the fuzzy set  $X$ . For the sake of simplicity,  $P(x)$  is defined by its membership function  $\mu_{P(x)}(y) = \mu_P(y, x)$ .

In order to define the lower and upper approximations of a set in fuzzy environments, we should consider the consistency degree of  $x$  being a member of  $X$  under the knowledge  $P$ . This degree can be measured by the truth value of the statement “ $y \in P(x)$  implies  $y \in X$ ” under fuzzy sets  $P(x)$  and  $X$ . To do that, we use a necessity measure  $\inf_{y \in \mathcal{U}} \mathcal{I}(\mu_P(y, x), \mu_{X_k}(y))$  with an implication function  $\mathcal{I} : [0, 1] \times [0, 1] \rightarrow [0, 1]$  such that  $\mathcal{I}(0, 0) = \mathcal{I}(0, 1) = \mathcal{I}(1, 0) = \mathcal{I}(1, 1) = 0$ , where  $\mathcal{I}(\cdot, a)$  decreases and  $\mathcal{I}(a, \cdot)$  increases,  $\forall a \in [0, 1]$ . In this formulation,  $X_k$  is the set comprising all objects labeled with the  $k$ -th decision class. Equation (1) displays the membership function for the lower approximation  $P_*(X)$  associated with the fuzzy set  $X$ .

$$\mu_{P_*(X_k)}(x) = \min \left\{ \mu_{X_k}(x), \inf_{y \in \mathcal{U}} \mathcal{I}(\mu_P(y, x), \mu_{X_k}(y)) \right\} \quad (1)$$

Analogously to the lower approximation, we can derive the membership function for the upper approximation assuming that  $X$  is a fuzzy set and  $P$  is a fuzzy binary relation. By doing so, we should measure the truth value of the statement “ $\exists y \in \mathcal{U}$  such that  $x \in P(y)$ ” under fuzzy sets  $P(x)$  and  $X$ . The true value of this statement can be obtained by a possibility measure  $\sup_{y \in \mathcal{U}} \mathcal{T}(\mu_P(x, y), \mu_{X_k}(y))$  with a conjunction function  $\mathcal{T} : [0, 1] \times [0, 1] \rightarrow [0, 1]$  such that  $\mathcal{T}(0, 0) = \mathcal{T}(0, 1) = \mathcal{T}(1, 0) = \mathcal{T}(1, 1) = 0$ , where both  $\mathcal{T}(\cdot, a)$  and  $\mathcal{T}(a, \cdot)$  increase,  $\forall a \in [0, 1]$ . Equation (2) shows the membership function for the upper approximation  $P^*(X)$  associated with  $X$ .

$$\mu_{P^*(X_k)}(x) = \max \left\{ \mu_{X_k}(x), \sup_{y \in \mathcal{U}} \mathcal{T}(\mu_P(x, y), \mu_{X_k}(y)) \right\} \quad (2)$$

It should be remarked that the intersection of two fuzzy sets  $X$  and  $Y$  is regularly defined as  $\mu_{X \cap Y} = \min\{\mu_X(x), \mu_Y(x)\}, \forall x \in \mathcal{U}$ , whereas their union takes the form  $\mu_{X \cup Y} = \max\{\mu_X(x), \mu_Y(x)\}, \forall x \in \mathcal{U}$ . However, some researchers replace the *min* operator with a t-norm and the *max* operator with a t-conorm [14]. On the other hand, note that Inuiguchi’s model does

not assume that  $\mu_P(x, x) = 1, \forall x \in \mathcal{U}$ . Instead, we compute the minimum between  $\mu_X(x)$  and  $\inf_{y \in \mathcal{U}} \mathcal{I}(\mu_P(y, x), \mu_{X_k}(y))$  when computing  $\mu_{P_*(X_k)}(x)$ , and the maximum between  $\mu_X(x)$  and  $\sup_{y \in \mathcal{U}} \mathcal{T}(\mu_P(x, y), \mu_{X_k}(y))$  when computing  $\mu_{P^*(X_k)}(x)$ . This feature allows preserving the inclusiveness of  $P_*(X)$  in the fuzzy set  $X$  and the inclusiveness of  $X$  in  $P^*(X)$ .

Based on the above elements, one can define the three fuzzy-rough regions that comprise the core of the granulation stage. Equation (3), (4) and (5) display the membership functions associated with the fuzzy-rough positive, negative and boundary regions, respectively.

$$\mu_{POS(X_k)}(x) = \mu_{P_*(X_k)}(x) \quad (3)$$

$$\mu_{NEG(X_k)}(x) = 1 - \mu_{P_*(X_k)}(x) \quad (4)$$

$$\mu_{BND(X_k)}(x) = \mu_{P^*(X_k)}(x) - \mu_{P_*(X_k)}(x) \quad (5)$$

These memberships functions allow computing more flexible information granules. As such, abrupt transitions between classes are replaced with gradual ones, therefore allowing an element to belong to more than one class with varying degrees. Next, we explain how to exploit these fuzzy-rough information granules by using a cognitive neural network.

### 3.2. Fuzzy-Rough Cognitive Networks

The proposed FRCN model transforms the attribute space into a fuzzy-rough one, which is exploited by a recurrent neural network. Under these fuzzy conditions, objects are categorized into information granules with soft boundaries, and therefore, a strict similarity threshold is no longer required. This suggests that the first step when constructing an FRCN is related with the fuzzy granulation of the available information.

Let  $X = \{X_1, \dots, X_k, \dots, X_M\}$  be a partition of  $\mathcal{U}$  according to the values of the decision attribute such that the subset  $X_k$  comprises those objects labeled as  $D_k$ . Based on this partition, we can define the membership degree of  $x \in \mathcal{U}$  to a subset  $X_k$  (see Equation 6). We assume that all objects labeled as  $D_k$  have maximum membership degree to the  $k$ -th subset; however, more sophisticated variants can be formalized as well.

$$\mu_{X_k}(x) = \begin{cases} 1 & , y \in X_k \\ 0 & , y \notin X_k \end{cases} \quad (6)$$



Another pivotal element to be defined is the membership function  $\mu_P(y, x)$  associated with the fuzzy binary relation. Equation (7) shows the function adopted in this paper, which depends on the membership degree of object  $x$  to  $X$ , and the similarity degree between  $x$  and  $y$ . The similarity degree  $\varphi(x, y)$  denotes the complement of the normalized distance  $\delta(x, y)$  between two instances  $x$  and  $y$ . Section 4.2 describes some heterogeneous distance functions explored in this study that allow comparing instances comprising both numerical and nominal attributes.

$$\mu_P(y, x) = \mu_{X_k}(x)\varphi(x, y) = \mu_{X_k}(x)(1 - \delta(x, y)) \quad (7)$$

Let us assume that the universe of discourse  $\mathcal{U}$  is composed of those objects comprised into the training dataset and  $\Theta : \mathcal{U} \rightarrow \mathcal{D}$  is a function that returns the decision class attached to each training instance, such that  $\mathcal{D} = \{D_1, \dots, D_K\}$ . Algorithm 1 summarizes the steps for granulating the information space under the fuzzy settings described above.

**Algorithm 1. Fuzzy-rough information granulation.**

```

214  FOREACH  $x \in \mathcal{U}$  DO
215    IF  $\Theta(x) = D_k$  THEN
216       $X_k \leftarrow X_k \cup \{x\}$ 
217    END IF
218    Compute  $\mu_{X_k}(x)$  according to Equation 6
219  END
220  FOREACH  $x \in \mathcal{U}$  DO
221    FOREACH subset  $X_k$  DO
222      Compute  $\mu_{POS(X_k)}(x)$  according to Equation 3
223      Compute  $\mu_{NEG(X_k)}(x)$  according to Equation 4
224      Compute  $\mu_{BND(X_k)}(x)$  according to Equation 5
225    END
226  END

```

After granulating the information space, the resultant fuzzy-rough constructs are used to build a neural network. Similarly to RCN models, input neurons denote positive or negative fuzzy-rough regions, whereas output neurons comprise the decision classes for the problem at hand. During preliminary simulations we noticed that including the fuzzy-rough boundary regions into the modeling did not significantly increase the classifier's discriminatory

ability. This behavior is not surprising because in crisp-rough environments the hesitant evidence is more conclusive when compared to the evidence coming from fuzzy-rough granules. Therefore, the neural network topology can be constructed by using the following rules:

- $(R_1^*)$  IF  $C_i = P_k^*$  AND  $C_j = D_k$  THEN  $w_{ij} = 1.0$
- $(R_2^*)$  IF  $C_i = N_k^*$  AND  $C_j = D_k$  THEN  $w_{ij} = -1.0$
- $(R_3^*)$  IF  $C_i = P_k^*$  AND  $C_j = D_{v \neq k}$  THEN  $w_{ij} = -1.0$
- $(R_4^*)$  IF  $C_i = P_k^*$  AND  $C_j = P_{v \neq k}$  THEN  $w_{ij} = -1.0$

where  $C_i$  is the  $i$ -th neural processing entity,  $D_k$  represents  $k$ -th decision class, while  $P_k^*$  and  $N_k^*$  are neurons denoting the positive and negative fuzzy-rough region associated to the  $k$ -th decision class.

Figure 1 shows the network topology of FRCNs for binary classification problems. Without loss of generality, any FRCN comprises  $2|\mathcal{D}|$  input neurons,  $|\mathcal{D}|$  output neurons and  $|\mathcal{D}|(4 + |\mathcal{D}|)$  causal weights. Observe that the number of neurons in the causal network is not determined by the number of features but by the number of decision classes.

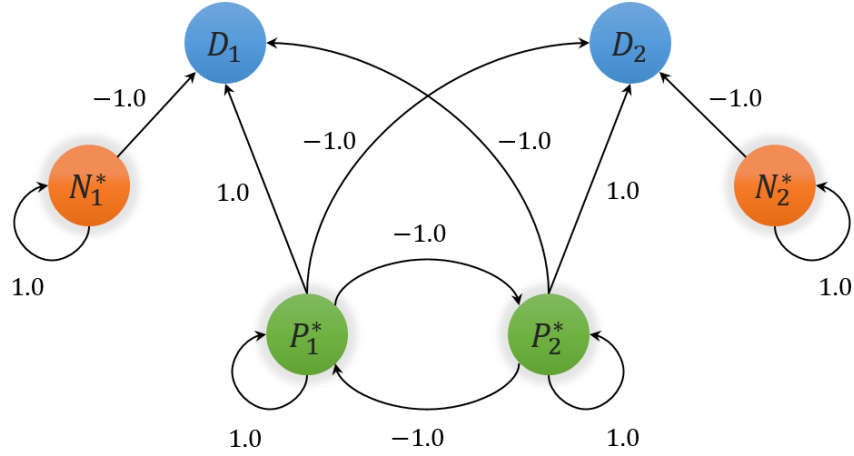


Figure 1: Fuzzy-Rough Cognitive Network for binary classification problems.

Algorithm 2 shows the steps required to build the topology of the granular neural network from discovered information granules.

**Algorithm 2. Network construction procedure.**

```

253   FOREACH subset  $X_k$  DO
254       Add a neuron  $P_k$  as the  $k$ th positive region
255       Add a neuron  $N_k$  as the  $k$ th positive region
256       Add a neuron  $B_k$  as the  $k$ th positive region
257   END
258   FOREACH decision  $D_k$  DO
259       Add a neuron  $D_k$  as the  $k$ th decision
260   END
261   FOREACH neuron  $C_i$  DO
262       FOREACH neuron  $C_j$  DO
263           Configure  $w_{ij}$  according to rules  $R_1^* - R_4^*$ 
264       END
265   END
266 
```

Once the network has been constructed, we can perform the classification for new (unlabeled) instances by activating the input-type neurons and performing the reasoning process. In order to activate these neurons, we use the similarity degree between the object  $y$  and  $x \in \mathcal{U}$  as well as the membership degree of  $x$  to each fuzzy-rough granular region.

Figure 2 and 3 illustrate the semantics behind this activation mechanism for the  $k$ -th positive and negative region, respectively. More explicitly, such figures show the degree to which  $y$  belongs to the fuzzy intersection defined from the membership functions  $\mu_{POS(X_k)}(x)$  (or  $\mu_{NEG(X_k)}(x)$ ), and the fuzzy similarity relation between the new instance  $y$  and  $x \in X$ . As a further step, we calculate the inclusion degree of the fuzzy intersection set into the  $k$ -th fuzzy-rough region. This procedure produces a normalized value that will be used to activate the input neurons in the causal network.

Equation (8) formalizes a generalized measure to compute the activation value of the  $k$ -th positive neuron, where  $\mathcal{T}_2$  denotes a t-norm,  $\varphi(x, y)$  is the similarity degree between  $x$  and  $y$  whereas  $\mu_{POS(X_k)}(x)$  represents the membership grade of  $x$  to the  $k$ -th fuzzy-rough positive region. A t-norm is a conjunction function  $\mathcal{T}_2 : [0, 1] \times [0, 1] \rightarrow [0, 1]$  that fulfills three conditions: (i)  $\forall a \in [0, 1], \mathcal{T}_2(a, 1) = \mathcal{T}_2(1, a) = a$ , (ii)  $\forall a, b \in [0, 1], \mathcal{T}_2(a, b) = \mathcal{T}_2(b, a)$ , and (iii)  $\forall a, b, c \in [0, 1], \mathcal{T}_2(a, \mathcal{T}_2(b, c)) = \mathcal{T}_2(\mathcal{T}_2(a, b), c)$ . Similarly, we can activate neurons denoting fuzzy-rough negative regions. Only output neurons remain inactive at the outset of the neural reasoning process.

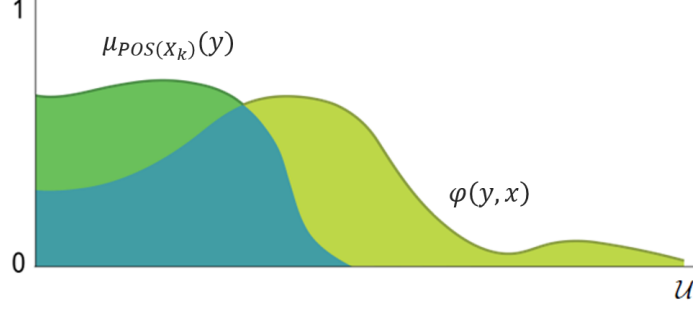


Figure 2: Inclusion degree of  $y$  into the  $k$ -th positive region.

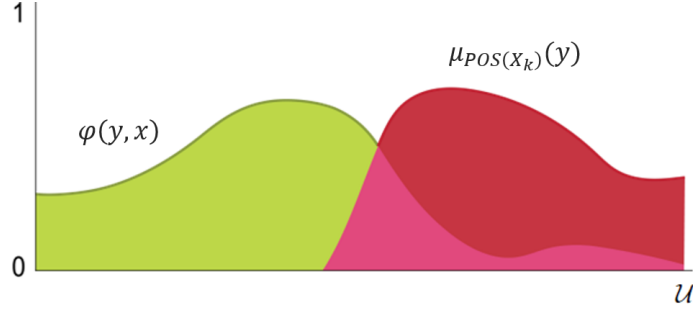


Figure 3: Inclusion degree of  $y$  into the  $k$ -th negative region.

$$\mathcal{A}(P_k^*) = \frac{\int \mathcal{T}_2(\varphi(x, y), \mu_{POS(X_k)}(x)) dx}{\int \mu_{POS(X_k)}(x) dx} \quad (8)$$

289 However, due to the fact that the universe of discourse  $\mathcal{U}$  is rather finite,  
 290 the use of integrals may not be convenient. Rules  $(R_5^*)$  and  $(R_6^*)$  show a more  
 291 practical mechanism to activate the granular classifier.

- 292 •  $(R_5^*)$  IF  $C_i = P_k^*$  THEN  $A_i^{(0)} = \frac{\sum_{x \in \mathcal{U}} \mathcal{T}_2(\varphi(x, y), \mu_{POS(X_k)}(x))}{\sum_{x \in \mathcal{U}} \mu_{POS(X_k)}(x)}$
- 293 •  $(R_6^*)$  IF  $C_i = N_k^*$  THEN  $A_i^{(0)} = \frac{\sum_{x \in \mathcal{U}} \mathcal{T}_2(\varphi(x, y), \mu_{NEG(X_k)}(x))}{\sum_{x \in \mathcal{U}} \mu_{NEG(X_k)}(x)}$

294 Once the initial activation vector  $A^{(0)}$  associated with the object  $y$  has  
 295 been computed, we perform the neural reasoning process until (i) a fixed-  
 296 point attractor is discovered, or alternatively (ii) a maximal number of iter-  
 297 ations is reached. At that point, the label of the output neuron having the  
 298 highest activation value is assigned to the target object.

299 Algorithm 3a shows the first step towards exploiting the neural network,  
 300 that is, the activation of input neurons for a new test instance  $x$ . Similarly,  
 301 Algorithm 3b illustrates how to determine the decision class from outputs  
 302 neurons once the input neurons have been activated.

303 **Algorithm 3a. Network activation procedure.**

```

304   FOREACH decision  $D_k$  DO
305       Calculate  $A_x^{(0)}(P_k)$  according to rule  $R_5^*$ 
306       Calculate  $A_x^{(0)}(N_k)$  according to rule  $R_6^*$ 
307   END
  
```

309 **Algorithm 3b. Network reasoning procedure.**

```

310   FOR  $t = 0$  TO  $T$  DO
311        $converged \leftarrow TRUE$ 
312       FOREACH neuron  $C_i$  DO
313           Compute  $A_i^{(t+1)} = f\left(\sum_{j=1}^M w_{ji} A_j^{(t)}\right)$ 
314           IF  $A_i^{(t)} \neq A_i^{(t+1)}$  THEN
315                $converged \leftarrow FALSE$ 
316           END
317       END
318       IF  $converged$  THEN
319           RETURN  $argmax_k\{\mathcal{A}_x^{(t+1)}(D_k)\}$ 
320       END
321   END
322   IF not  $converged$  THEN
323       RETURN  $argmax_k\{\mathcal{A}_x^{(T)}(D_k)\}$ 
324   END
  
```

326 It is worth mentioning that the FRCN algorithm can operate in either a  
 327 lazy or inductive fashion. In a lazy setting, both the fuzzy-rough granules  
 328 and the network topology can be constructed when the new instance arrives.  
 329 This is however not efficient since the granules and the topology can be  
 330 reused to classify new instances. In the inductive approach, the knowledge  
 331 is stored into the discovered granules and the causal weight matrix, which is  
 332 prescriptively determined by construction rules  $(R_1^*) - (R_4^*)$ . Adjusting such  
 333 causal weights using a supervised learning algorithm is a promising research  
 334 direction to be explored as a future work.

## 335 4. Numerical Simulations

336 In this section, we conduct several simulations to evaluate the predictive  
 337 capability of the proposed fuzzy-rough neural network. As a first experiment,  
 338 we investigate the impact of using different fuzzy operators and distance  
 339 functions across 140 pattern classification data sets. Afterward, we compare  
 340 the prediction capability of the best-performing fuzzy-rough model against  
 341 17 well-established state-of-the-art classifiers.

### 342 4.1. Dataset Characterization

343 We leaned upon 140 classification datasets taken from the KEEL [17] and  
 344 UCI ML [18] repositories. These problems comprise different characteristics  
 345 and allow evaluating the predictive power of both state-of-the-art and the  
 346 granular classifiers under consideration.

347 In the adopted datasets <sup>1</sup>, the number of attributes ranges from 2 to 262,  
 348 the number of decision classes from 2 to 100, and the number of instances  
 349 from 14 to 12,906. They involve 13 noisy and 47 imbalanced datasets, with  
 350 the imbalance ratio fluctuating between 5:1 and 2160:1. To avoid the out-of-  
 351 range issues, the numerical attributes have been normalized. Furthermore,  
 352 we replaced missing values with the mean or the mode depending on whether  
 353 the attribute was numerical or nominal, respectively.

354 As a final element, each dataset has been partitioned using a standard  
 355 10-fold cross-validation procedure, i.e., each problem has been split into 10  
 356 folds, each containing 10% of the instances.

### 357 4.2. Heterogeneous Distance Functions

358 The distance function plays a pivotal role when designing instance-based  
 359 classifiers. Next, we briefly describe three distance functions [19] [10] used in  
 360 our experiments that allow comparing heterogeneous instances, i.e., objects  
 361 comprising both numerical and nominal attributes.

- 362 • *Heterogeneous Euclidean-Overlap Metric (HEOM)*. This distance func-  
 363 tion computes the normalized Euclidean distance between numerical  
 364 attributes and an overlap metric for nominal attributes.

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<sup>1</sup>The reader can find a complete characterization of such datasets in [10]

- *Heterogeneous Manhattan-Overlap Metric (HMOM)*. This heterogeneous variant is similar to the HEOM function but it replaces the Euclidean distance with the Manhattan distance when computing the dissimilarity between two numerical values.
- *Heterogeneous Value Difference Metric (HVDM)*. This function involves a stronger strategy for quantifying the dissimilarity between nominal attributes. Instead of using a matching approach, it measures the correlation between attributes and the decision classes.

#### 4.3. Determining the Best-Performing Fuzzy Model

The first experiment is oriented to determining the combination of fuzzy operators leading to the best prediction rates. The FRCN algorithm requires the specification of a fuzzy implicator and two t-norms. The  $\mathcal{I}$  implicator is used to compute the membership degree of an object to the lower approximations, the  $\mathcal{T}_1$  t-norm is used to compute the membership degree of an object to the upper approximations whereas the  $\mathcal{T}_2$  t-norm is used to activate the neural processing entities. For the sake of simplicity, we use the same t-norm to compute the membership degree to the upper approximations as well as to exploit the neural network. Tables 1 and 2 display the t-norms and fuzzy implicators included in this first simulation.

Table 1: T-norms explored in this paper.

<b>T-norm</b>	<b>Formulation</b>
Standard intersection	$\mathcal{T}(x, y) = \min\{x, y\}$
Algebraic product	$\mathcal{T}(x, y) = xy$
Lukasiewicz	$\mathcal{T}(x, y) = \max\{0, x + y - 1\}$
Drastic product	$\mathcal{T}(x, y) = \begin{cases} x & , y = 1 \\ y & , x = 1 \\ 0 & , otherwise \end{cases}$

To measure the classifiers' prediction capability, we computed the Kappa coefficient. Cohen's Kappa coefficient [20] measures the inter-rater agreement for categorical items. It is usually deemed a more robust measure than the standard accuracy since this coefficient takes into account the agreement occurring by chance. Figure 4 shows the average Kappa coefficient achieved by each model for different combinations of fuzzy operators using the HMOM distance as the standard dissimilarity functional.

Table 2: Fuzzy implicators explored in this paper.

Implicator	Formulation
Standard	$\mathcal{I}(x, y) = \begin{cases} 1 & , x \leq y \\ 0 & , x > y \end{cases}$
Kleene-Dienes	$\mathcal{I}(x, y) = \max\{1 - x, y\}$
Lukasiewicz	$\mathcal{I}(x, y) = \min\{1 - x + y, 1\}$
Zadeh	$\mathcal{I}(x, y) = \max\{1 - x, \min\{x, y\}\}$
Godel	$\mathcal{I}(x, y) = \begin{cases} 1 & , x \leq y \\ y & , x > y \end{cases}$
Larsen	$\mathcal{I}(x, y) = xy$
Mamdani	$\mathcal{I}(x, y) = \min\{x, y\}$
Reichenbach	$\mathcal{I}(x, y) = 1 - x + xy$
Yager	$\mathcal{I}(x, y) = \begin{cases} 1 & , x = y = 0 \\ y^x & , otherwise \end{cases}$
Goguen	$\mathcal{I}(x, y) = \begin{cases} 1 & , x \leq y \\ y/x & , otherwise \end{cases}$

391 From the above results we can notice that the FRCN method computes  
392 the best prediction rates when the Lukasiewicz t-norm is used to activate the  
393 input neurons regardless of the fuzzy operator attached to the membership  
394 functions  $\mu_{P_*(X_k)}(x)$  and  $\mu_{P^*(X_k)}(x)$ . Consequently, we adopt the Lukasiewicz  
395 implicator and the Lukasiewicz t-norm as standard fuzzy operators in the rest  
396 of the simulations conducted in this paper.

397 The following experiment is devoted to comparing the prediction capabil-  
398 ity of the proposed classifier with respect to the crisp variant (RCN) using a  
399 reasonably, fixed similarity threshold equal to 0.98. Figure 5 summarizes the  
400 average Kappa measure attained by each classifier for different distance func-  
401 tions. The simulations confirm that the FRCN models always report better  
402 prediction rates regardless of the underlying distance function, although the  
403 HMOM function seems to stand as the best choice.

404 Aiming at conducting a more rigorous analysis, we compute the Friedman  
405 two-way analysis of variances by ranks [21]. The test advocates for the rejec-  
406 tion of the null hypothesis ( $p\text{-value} = 8.1268E - 10 < 0.05$ ) for a confidence  
407 interval of 95%, hence we can conclude that there are significant differences  
408 between at least two models across datasets.



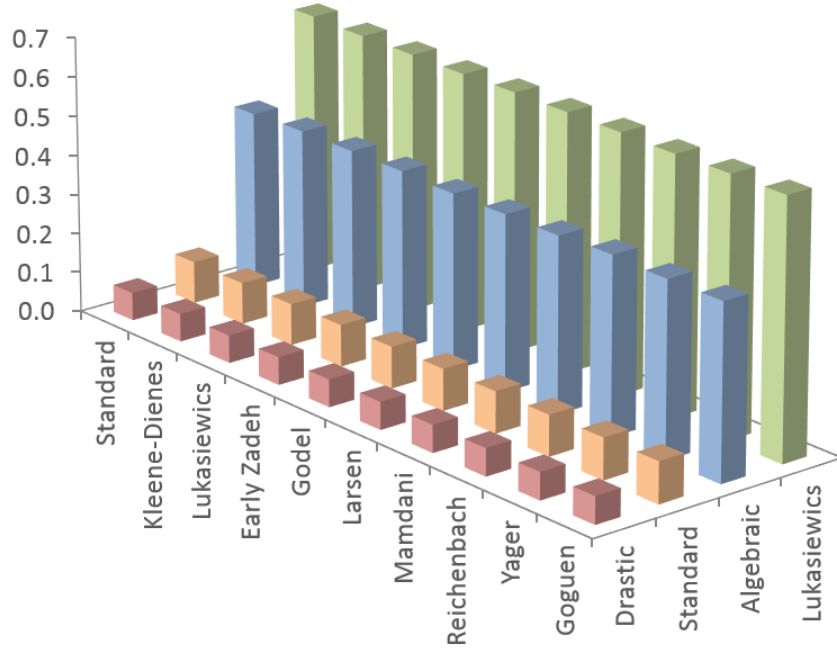


Figure 4: Average Kappa measure computed for the proposed fuzzy-rough classifier using the HMOM distance function with different t-norm and fuzzy implicators.

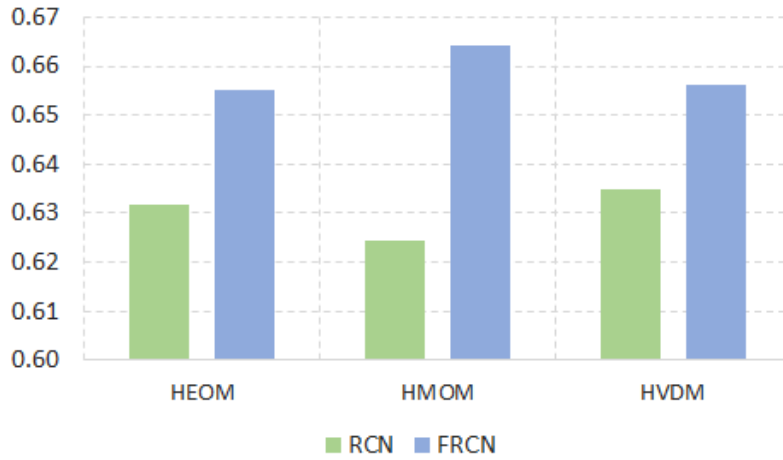


Figure 5: Average Kappa measure for different distance functions.

409 The next experiment is focused on determining whether the superiority  
410 of the FRCN-HMOM classifier is statistically significant or not. To that end,  
411 we resorted to the Wilcoxon signed rank test [22] and post-hoc procedures  
412 to adjust the  $p$ -values instead of using mean-ranks approaches, as recently  
413 suggested by Benavoli and collaborators [23].

414 Table 3 reports the unadjusted  $p$ -value computed by the Wilcoxon signed  
415 rank test and the corrected  $p$ -values associated with each pairwise comparison  
416 using FRCN-HMOM as the control method. In this paper, we assume that  
417 a null hypothesis can be rejected if at least one of the post-hoc procedures  
418 advocates for the rejection. The statistical analysis confirms FRCN-HMOM’s  
419 superiority as all the null hypotheses were rejected.

Table 3: Adjusted  $p$ -values according to different post-hoc procedures using the best-performing rough classifier (FRCN-HMOM) as the control method.

Algorithm	$p$ -value	Bonferroni	Holm	Holland	Null Hypothesis
RCN-HEOM	2.15E-07	0.000001	0.000001	0.000001	Rejected
RCN-HMOM	2.50E-07	0.000001	0.000001	0.000001	Rejected
RCN-HVDM	0.000003	0.000015	0.000009	0.000009	Rejected
FRCN-HEOM	0.000076	0.000380	0.000152	0.000152	Rejected
FRCN-HVDM	0.007897	0.039485	0.007897	0.007897	Rejected

420 The above simulations suggest that the proposed FRCN algorithm, like  
421 the *Rough Cognitive Ensembles* [10], is capable of suppressing the parametric  
422 requirements of RCNs without harming their performance. But are they sim-  
423 ilar in performance? In order to answer this question we can use the Wilcoxon  
424 signed rank test for pairwise comparisons. The test suggests accepting the  
425 conservative hypothesis ( $p$ -value=0.7387 > 0.05) using a confidence interval  
426 of 95%. Therefore, we can conclude that both approaches perform similarly  
427 for the datasets adopted in the empirical comparison.

428 However, the fuzzy approach proposed in this paper is preferred since it  
429 fits best the parsimony principle: *the simpler the better*. The bagging scheme  
430 and the ensemble model itself make the RCE algorithm less transparent  
431 than the fuzzy variant, thus notably reducing one of the main contributions  
432 attached to rough cognitive classifiers. In other words, we can achieve the  
433 same prediction rates by using a single fuzzy-rough classifier rather than an  
434 ensemble composed of several crisp models!

#### 4.4. Comparison Against State-of-the-Art Classifiers

In this section, we compare the prediction ability of the best-performing fuzzy model (FRCN-HMOM, hereinafter simply called FRCN) against the following well-known state-of-the-art classifiers:

- **Rule-based models**

- *Decision Table (DT)* [24]. The algorithm searches for matches in the body using a subset of attributes. If no instances are found, the majority class in the table is returned; otherwise, the majority class of all matching instances is returned.

- **Bayesian models**

- *Naïve Bayes (NB)* [25]. A probabilistic classification algorithm using estimator classes, where numeric estimator precision values are chosen based on the analysis of the training data.
- *Naïve Bayes Updateable (NBU)* [25]. Implements an incremental NB classifier that learns one instance at a time. Instead of using normal density measures for numerical attributes, this algorithm employs a kernel estimator without discretization.

- **Function-based models**

- *Simple Logistic (SL)* [26]. A classifier building linear logistic regression models. LogitBoost with simple regression functions as base learners is used for fitting the logistic models.
- *Multilayer Perceptron (MLP)* [27]. Neural network that uses the backpropagation algorithm to train the model.
- *Support Vector Machines (SMO)*. [28] Implements John Platt’s sequential minimal optimization algorithm for training a support vector classifier. In our research, we adopted a quadratic polynomial kernel to perform the numerical simulations.

- **Tree-based models**

- *Decision Tree (J48)* [29]. Induces classification rules in the form of a pruned/unpruned decision tree.

- 465 – *Random Tree (RT)* [30]. Decision tree without pruning that con-  
466 sideres  $k$  randomly chosen attributes at each node.
- 467 – *Random Forest (RF)* [31]. Bagging of random trees.
- 468 – *Fast Decision Tree (FDT)* [32]. Builds a tree using information  
469 gain and prunes it using reduced-error pruning.
- 470 – *Best-first Decision Tree (BFT)* [33]. Classification trees that use  
471 binary split for both nominal and numeric attributes.
- 472 – *Logistic Model Tree (LMT)* [34]. Decision trees for classification  
473 that use logistic regression functions at the leaves.

#### 474 • Instance-based models

- 475 – *Nearest Neighbor (NN)* [35]. Instance-based (lazy) classifier that  
476 simply chooses the closest instance to the test instance and returns  
477 its class.
- 478 – *k-Nearest Neighbors (kNN)* [35]. Lazy learner that computes the  
479 predicted class based upon the classes of the  $k$  training instances  
480 that are most similar to the test instance, as determined by a  
481 similarity function.
- 482 –  *$K^*$  classifier ( $K^*$ )* [36]. Instance-based classifier similar to  $k$ NN  
483 that uses an entropy-based distance function.

#### 484 • Fuzzy-rough models

- 485 – *Fuzzy-Rough k-Nearest Neighbors (FRNN)* [37]. Nearest neighbor  
486 model that utilizes the lower and upper approximations from fuzzy  
487 rough set theory to classify test instances.
- 488 – *Vaguely-quantified k-Nearest Neighbors (VQNN)* [38]. Fuzzy-rough  
489 model that emulates the linguistic quantifiers *some* and *most* when  
490 performing the classification process.

491 In our simulations, we retain the default parameter settings implemented  
492 in Weka v3.6.11 [39], therefore no classification algorithm explicitly performs  
493 parameter tuning. Despite the fact that a proper parametric setting often in-  
494 creases the algorithm’s performance over multiple data sources [40], a robust  
495 classifier should be able to produce good results even when its parameters  
496 might not have been optimized for a specific problem.

497 Analogously to the previous simulations, we utilized Cohen’s Kappa coef-  
 498 ficient to quantify the algorithms’ performance. Figure 6 displays the average  
 499 Kappa measure attained by each classification algorithm across the selected  
 500 datasets. The results show that FRCN is the second-best ranked algorithm  
 501 whereas LMT arises as the best-performing classifier.

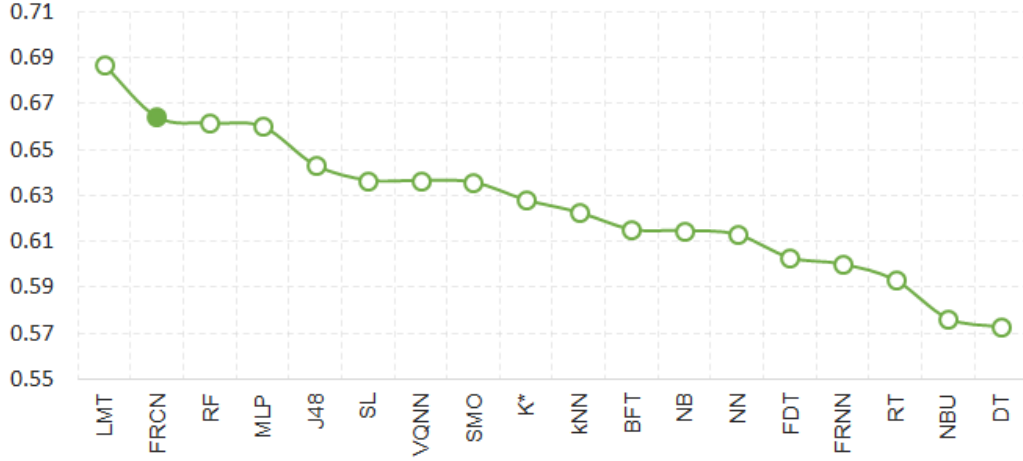


Figure 6: Average Kappa values reported by each classifier.

502 For this experiment, the Friedman test suggests rejecting the null hypoth-  
 503 esis ( $p\text{-value} = 1.4396928E - 10 < 0.1$ ) for a confidence interval of 90%. This  
 504 suggests that there are significant differences between at least two algorithms  
 505 across the 140 datasets adopted for simulation.

506 Table 4 summarizes the  $p$ -values reported by the Wilcoxon signed rank  
 507 test and the corrected  $p$ -values according to several post-hoc procedures us-  
 508 ing FRCNs as the control method. The results indicate that LMT is the  
 509 best-performing classifier in our study, with no significant differences spot-  
 510 ted between our proposal and MLP, RF, SMO and SL, as the null hypothesis  
 511 was accepted in each of these pairwise comparisons.

512 The superiority of LMT is quite interesting. This method allows inducing  
 513 trees with linear-logistic regression models at the leaves. During the training  
 514 process, it determines the appropriate number of boosting iterations by in-  
 515 ternally cross-validating the model until the performance ceases to increase.  
 516 This is somehow similar to the RCNs’ parameter tuning step that our fuzzy  
 517 approach attempts suppressing, so one may question whether including the  
 518 LMT algorithm in our simulations is fair at all.

Table 4: Adjusted  $p$ -values according to different post-hoc procedures using the proposed fuzzy-rough classifier (FRCNs) as the control method.

Algorithm	$p$ -value	Bonferroni	Holm	Holland	Null Hypothesis
RT	1.29E-11	2.34E-10	2.21E-10	2.21E-10	Rejected
DT	4.94E-11	8.91E-10	7.92E-10	7.91E-10	Rejected
NBU	3.79E-08	6.82E-07	5.68E-07	5.68E-07	Rejected
FDT	6.31E-07	1.13E-05	0.000008	8.83E-06	Rejected
FRNN	8.94E-07	0.000016	0.000011	1.16E-05	Rejected
NN	2.15E-06	0.000038	0.000025	2.57E-05	Rejected
NB	5.51E-06	0.000099	0.000060	6.06E-05	Rejected
BFT	3.20E-05	0.000576	0.000320	0.000319	Rejected
$k$ NN	7.13E-05	0.001285	0.000642	0.000642	Rejected
$K^*$	0.005752	0.103543	0.046019	0.045103	Rejected
LMT	0.006376	0.114778	0.046019	0.045103	Rejected
J48	0.010528	0.189511	0.063170	0.061531	Rejected
VQNN	0.010947	0.197052	0.063170	0.061531	Rejected
SL	0.109578	1.000000	0.438314	0.371388	Failed to reject
SMO	0.273587	1.000000	0.820761	0.616689	Failed to reject
RF	0.940694	1.000000	1.000000	0.996482	Failed to reject
MLP	1.000000	1.000000	1.000000	1.000000	Failed to reject

On the other hand, FRCN’s superiority upon other instance-based classifiers such as  $k$ NN or  $K^*$  is remarkable. We conjecture that this could be a direct result of using all the available evidence to infer the most likely decision for a new instance, instead of only using the information contributed by the positive region (e.g., the  $k$  closest neighbors). Combining such evidence in a nonlinear manner as the FRCN neurons do is likely another key piece towards the attainment of high prediction rates.

Equally important is the fact that our classification algorithm provides an introspection mechanism into its decision process, which stands as its chief advantage over comparably accurate black-box classifiers. It is fair to mention that the literature includes several neural models that provide such explanatory features. For example, the *Evolving Fuzzy Neural Networks* [41], the *Dynamic Evolving Neural-Fuzzy Inference System* [42] and the *Evolving Spiking Neural Networks* [43] all rely on low-level fuzzy rules to extract knowledge from the problem domain. This cannot be naturally achieved with our high-level approach. However, in presence of high-dimensional problems, these algorithms induce a large number of fuzzy rules with many antecedents,

536 which are difficult to interpret in practice. The number of causal rules cod-  
 537 ified into an FRCN does not depend on the number of attributes but on  
 538 the number of decision classes in the problem at hand. This guarantees that  
 539 the introspection mechanism attached to FRCNs remains fairly interpretable  
 540 and unaffected by the problem dimensionality.

## 541 5. Conclusions

542 In this paper, we introduced the notion of *fuzzy-rough cognitive mapping*  
 543 in an attempt to get rid of the parameter learning requirements of RCN-based  
 544 models. In the FRCN algorithm, information granules have soft boundaries,  
 545 thus leading to gradual transitions between the classes as opposed to abrupt  
 546 transitions that regularly occur in crisp environments.

547 The results have shown that the proposed fuzzy classifier is capable of out-  
 548 performing the crisp RCN variant regardless of the adopted distance function.  
 549 In spite of that, the Lukasiewicz operators and the HMOM distance function  
 550 stand as the best choices. From the comparison between the best-performing  
 551 fuzzy model and 17 state-of-the-art classifiers, we concluded that FRCNs are  
 552 as accurate as the most successful black boxes. The main advantage of our  
 553 granular neural network relies on its ability to elucidate its decision process  
 554 using inclusion degrees and causal relations. It is worth mentioning that  
 555 our classifier performs better than other instance-based learners across the  
 556 datasets adopted for simulation purposes.

557 More importantly, the results support the hypothesis behind our research:  
 558 that the fuzzy-rough approach allows completely suppressing the parametric  
 559 requirements behind rough cognitive mapping without either harming its  
 560 performance or significantly increasing its computational complexity.

561 Of course, the classifier presented in this paper is *no panacea*. While the  
 562 foundations underpinning FRCNs seem quite intuitive for mathematicians,  
 563 it may not be intuitive enough for experts with no background in Computer  
 564 Science or related areas. Besides, computing a transparent decision model  
 565 does not necessarily imply that we can understand the problem domain at  
 566 a low level. As a future work, we will investigate other strategies to au-  
 567 tomatically construct FCM-based classifiers from historical data. Deriving  
 568 FCM-based models with lower abstraction levels leads to truly interpretable  
 569 classifiers although their accuracy may be compromised.

## 570 Acknowledgments

571 This work was supported by the Research Council of Hasselt University.  
572 The authors would like to thank the anonymous reviewers for their construc-  
573 tive remarks throughout the revision process.

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