Identification of the minimum effective dose for normally distributed data using a Bayesian variable selection approach

Supplementary material

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Supplementary appendix

Martin Otava et al.

Table S1: The set of 16 possible monotonic dose-response models for an experiment with five dose levels (including placebo). Denote μ_i the mean response of the dose level. The model g_0 represents the null model of no dose effect.

Model	Up: Mean Structure	Down: Mean Structure
g_0	$\mu_0 = \mu_1 = \mu_2 = \mu_3 = \mu_4$	$\mu_0 = \mu_1 = \mu_2 = \mu_3 = \mu_4$
g_1	$\mu_0 < \mu_1 = \mu_2 = \mu_3 = \mu_4$	$\mu_0 > \mu_1 = \mu_2 = \mu_3 = \mu_4$
g_2	$\mu_0 = \mu_1 < \mu_2 = \mu_3 = \mu_4$	$\mu_0 = \mu_1 > \mu_2 = \mu_3 = \mu_4$
g_3	$\mu_0 < \mu_1 < \mu_2 = \mu_3 = \mu_4$	$\mu_0 > \mu_1 > \mu_2 = \mu_3 = \mu_4$
g_4	$\mu_0 = \mu_1 = \mu_2 < \mu_3 = \mu_4$	$\mu_0 = \mu_1 = \mu_2 > \mu_3 = \mu_4$
g_5	$\mu_0 < \mu_1 = \mu_2 < \mu_3 = \mu_4$	$\mu_0 > \mu_1 = \mu_2 > \mu_3 = \mu_4$
g_6	$\mu_0 = \mu_1 < \mu_2 < \mu_3 = \mu_4$	$\mu_0 = \mu_1 > \mu_2 > \mu_3 = \mu_4$
g_7	$\mu_0 < \mu_1 < \mu_2 < \mu_3 = \mu_4$	$\mu_0 > \mu_1 > \mu_2 > \mu_3 = \mu_4$
g_8	$\mu_0 = \mu_1 = \mu_2 = \mu_3 < \mu_4$	$\mu_0 = \mu_1 = \mu_2 = \mu_3 > \mu_4$
g_9	$\mu_0 < \mu_1 = \mu_2 = \mu_3 < \mu_4$	$\mu_0 > \mu_1 = \mu_2 = \mu_3 > \mu_4$
g_{10}	$\mu_0 = \mu_1 < \mu_2 = \mu_3 < \mu_4$	$\mu_0 = \mu_1 > \mu_2 = \mu_3 > \mu_4$
g_{11}	$\mu_0 < \mu_1 < \mu_2 = \mu_3 < \mu_4$	$\mu_0 > \mu_1 > \mu_2 = \mu_3 > \mu_4$
g_{12}	$\mu_0 = \mu_1 = \mu_2 < \mu_3 < \mu_4$	$\mu_0 = \mu_1 = \mu_2 > \mu_3 > \mu_4$
g_{13}	$\mu_0 < \mu_1 = \mu_2 < \mu_3 < \mu_4$	$\mu_0 > \mu_1 = \mu_2 > \mu_3 > \mu_4$
g_{14}	$\mu_0 = \mu_1 < \mu_2 < \mu_3 < \mu_4$	$\mu_0 = \mu_1 > \mu_2 > \mu_3 > \mu_4$
g_{15}	$\mu_0 < \mu_1 < \mu_2 < \mu_3 < \mu_4$	$\mu_0 > \mu_1 > \mu_2 > \mu_3 > \mu_4$

S1 Introduction

This document contains additional materials that were not presented in the main manuscript and provides detailed information about various topics mentioned there. In Section S2, the possible profiles with $K = 4$ and $K = 5$ are visualized, while in Section S3 the penalties for various information criteria (IC) are shown. Section S4 provides details about settings of simulation study and results of several configurations that were not presented in manuscript.

S2 Possible monotone profiles

All the possible monotonically increasing profiles with $K = 4$ are shown in the Figure S1. The respective profiles with $K = 5$ are visualized in the Figure S2. Table S1 shows a decomposition of an increasing or decreasing alternative hypothesis, together with a null model g_0 that represents a null hypothesis.

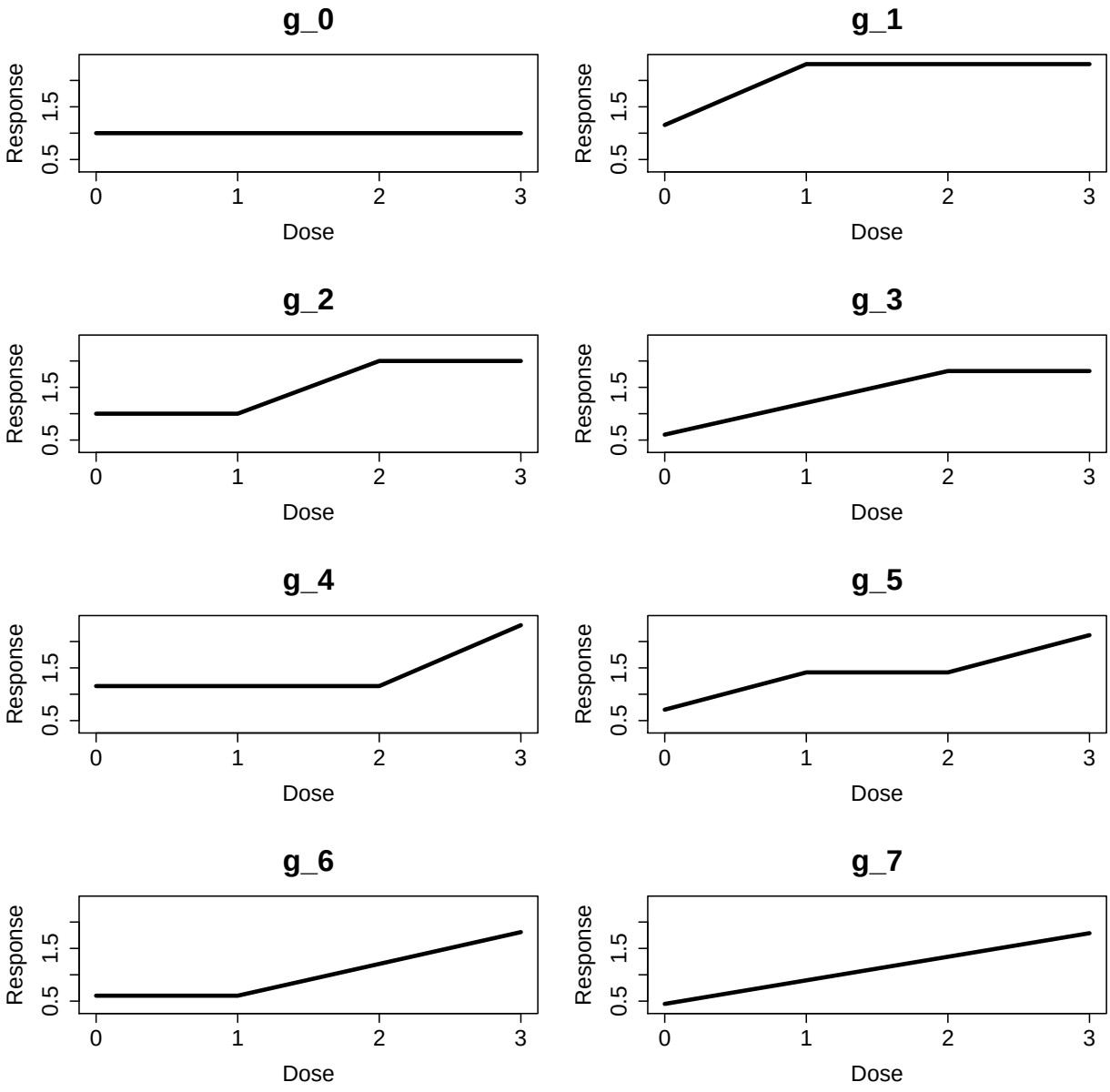

Figure S1: *The mean structure for simulation study with* $K = 4$ *and* $\lambda = 1$.

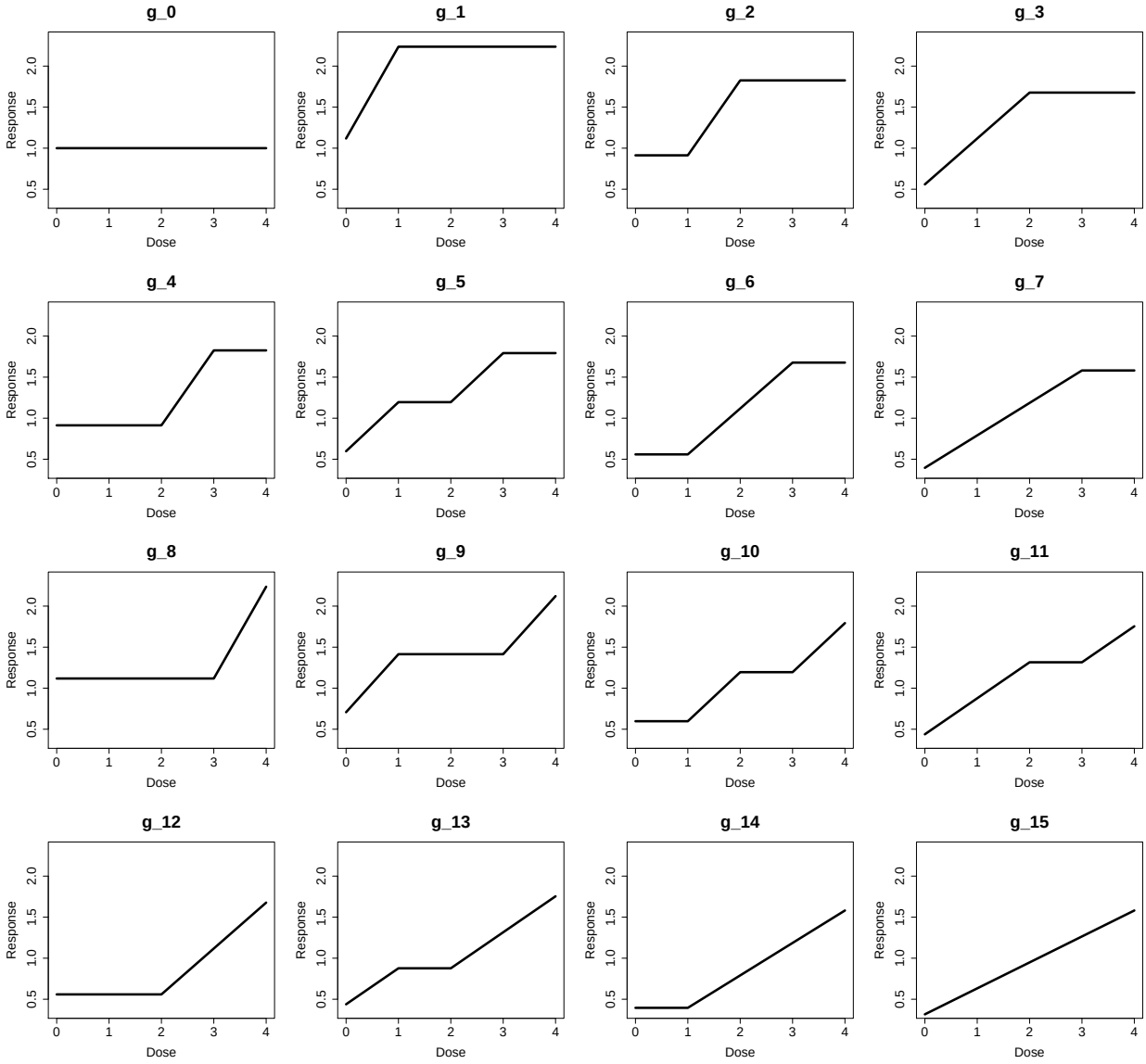

Figure S2: *The mean structure for simulation study with* $K = 5$ *and* $\lambda = 1$.

S3 Penalties

As explained in Section 3.1 in the manuscript, all IC are constructed as combination of the likelihood and the penalty term. The difference between specific IC is the choice of the penalty term that determines the properties of the IC. The values of penalties for Angina data set are shown in Table S2 and for Toxicity data set in Table S3 (note that we list only the models that were possible to fit for this particular data set). We can see that the penalty is smallest for GORIC which causes its preference for more complex models compared to other IC criteria.

Table S2: Penalties for different models fitted for the Angina data. First column: Order restricted log likelihood.

Profile	ORLL	GORIC	AIC	BIC
g_0	-149.77	2.00	4	7.82
g_1	-144.55	2.50	6	11.74
g_2	-141.46	2.50	6	11.74
g_3	-140.80	2.79	8	15.65
g_4	-138.65	2.50	6	11.74
g_5	-136.92	2.86	8	15.65
g_6	-137.39	2.77	8	15.65
g_7	-136.61	3.03	10	19.56
g_8	-135.97	2.50	6	11.74
g_9	-132.31	2.92	8	15.65
g_{10}	-131.99	2.86	8	15.65
g_{11}	-131.01	3.14	10	19.56
g_{12}	-133.01	2.79	8	15.65
g_{13}	-130.82	3.14	10	19.56
g_{14}	-131.42	3.03	10	19.56
g_{15}	-130.43	3.28	12	23.47

Table S3: Penalties for different models fitted for the Toxicity data. First column: Order restricted log likelihood.

Profile	ORLL	GORIC	AIC	BIC
g_0	-82.98	2.00	4	6.36
g_1	-80.32	2.50	6	9.53
g_4	-81.28	2.50	6	9.53
g_5	-79.51	2.89	8	12.71

Table S4: The mean structure for simulation study with $K = 4$ and $\lambda = 1$ (rounded to two decimal places).

Profile	Dose 0	Dose 1	Dose 2	Dose 3
g_1	1.15	2.31	2.31	2.31
g_2	1.00	1.00	2.00	2.00
g_3	0.60	1.21	1.81	1.81
g_4	1.15	1.15	1.15	2.31
g_5	0.71	1.41	1.41	2.12
g_6	0.60	0.60	1.21	1.81
g_7	0.45	0.89	1.34	1.79

S4 Simulation study

S4.1 Simulation configurations

The configuration with $K = 4$ follows the simulation study of Marcus (1976). Specifically, we start with vector $\boldsymbol{v}_r$ of non-decreasing integers according to particular model g_r (e.g. for model g_5 it is vector $\boldsymbol{v}_r = (1, 2, 2, 3)$). Then, the final configuration is obtained through the equation

$$\boldsymbol{s}_r = \boldsymbol{v}_r \cdot \frac{\sqrt{K}}{\sqrt{\sum_{j>i}(v_{rj} - v_{ri})^2}}. \tag{1}$$

For model g_5, we get $\boldsymbol{s}_r = (1, 2, 2, 3) \cdot \frac{\sqrt{4}}{\sqrt{1+1+4+1+1}} = (1, 2, 2, 3) \cdot \frac{1}{\sqrt{2}}$. The configurations for all models for $K = 4$ and $\lambda = 1$ are shown in Table S4 and visualized in Figure S1. The configuration for g_0 is simply $(1, 1, 1, 1)$. The configurations for the $K = 5$ were computed using the same formulas and they are shown in Table S5 and visualized in Figure S2.

S4.2 Results

Table S6 shows the rate at which the true underlying model is selected as the best or the second best model. In Table S7 and Table S8, we can see the probabilities that models would be selected as the best model (or among top two models, respectively), given the true underlying model. These tables show what is the most usual misclassification of the models. The following tables show the results of additional settings of simulation study, analogous to the one presented in the main manuscript, but with varying value of dose levels K and

Table S5: The mean structure for simulation study with $K = 5$ and $\lambda = 1$ (rounded to two decimal places).

Profile	Dose 0	Dose 1	Dose 2	Dose 3	Dose 4
g_1	1.12	2.24	2.24	2.24	2.24
g_2	0.91	0.91	1.83	1.83	1.83
g_3	0.56	1.12	1.68	1.68	1.68
g_4	0.91	0.91	0.91	1.83	1.83
g_5	0.60	1.20	1.20	1.79	1.79
g_6	0.56	0.56	1.12	1.68	1.68
g_7	0.40	0.79	1.19	1.58	1.58
g_8	1.12	1.12	1.12	1.12	2.24
g_9	0.71	1.41	1.41	1.41	2.12
g_{10}	0.60	0.60	1.20	1.20	1.79
g_{11}	0.44	0.88	1.32	1.32	1.75
g_{12}	0.56	0.56	0.56	1.12	1.68
g_{13}	0.44	0.88	0.88	1.32	1.75
g_{14}	0.40	0.40	0.79	1.19	1.58
g_{15}	0.32	0.63	0.95	1.26	1.58

number of observations per dose n. Table S9, Table S10, Table S11, Table S12, Table S13 and Table S14 show results for correct model selection and correct MED selection for $K = 4$ and Table S15, Table S16, Table S17, Table S18, Table S19, Table S20, Table S21 and Table S22 show analogous results for $K = 5$. All results are consistent with the results presented in the manuscript.

References

Marcus, R. (1976), "The Powers of Some Tests of the Equality of Normal Means against an Ordered Alternative," *Biometrika*, 63, 177–183.

Table S6: Comparison of the proportion of time the true model is selected as the best or the second best model based on 1000 simulated data sets for BVS, GORIC, AIC and BIC criterion for $K = 4$, $n = 3$.

λ	Profile	BVS	GORIC	AIC	BIC
	g_0	0.90	0.82	0.91	0.93
1	g_1	0.77	0.70	0.73	0.74
	g_2	0.64	0.58	0.65	0.66
	g_3	0.23	0.37	0.25	0.20
	g_4	0.76	0.69	0.72	0.72
	g_5	0.32	0.39	0.30	0.25
	g_6	0.21	0.34	0.23	0.19
	g_7	0.00	0.13	0.02	0.01
2	g_1	0.93	0.85	0.91	0.92
	g_2	0.89	0.78	0.87	0.89
	g_3	0.64	0.69	0.64	0.62
	g_4	0.93	0.85	0.89	0.91
	g_5	0.74	0.73	0.72	0.70
	g_6	0.63	0.68	0.63	0.60
	g_7	0.08	0.53	0.21	0.15
3	g_1	0.97	0.88	0.95	0.96
	g_2	0.96	0.82	0.92	0.94
	g_3	0.86	0.86	0.86	0.85
	g_4	0.97	0.88	0.93	0.94
	g_5	0.91	0.89	0.91	0.91
	g_6	0.87	0.85	0.86	0.85
	g_7	0.41	0.84	0.63	0.54

Table S7: Selection by the BVS for $K = 4$ and $n = 3$. The probability that a specified model has the highest posterior probability among all candidate models. Rows: The true models. Columns: Selected as the model with the highest posterior probability by BVS. Correct model is shown in bold. Note that the probabilities on the diagonal correspond to the probabilities for the BVS model presented in Table 4 in the manuscript.

λ	Profile	g_0	g_1	g_2	g_3	g_4	g_5	g_6	g_7
	g_0	**0.73**	0.10	0.08	0.00	0.09	0.00	0.00	0.00
1	g_1	0.21	**0.57**	0.10	0.02	0.07	0.03	0.00	0.00
	g_2	0.22	0.13	**0.46**	0.02	0.14	0.02	0.01	0.00
	g_3	0.19	0.32	0.30	**0.03**	0.10	0.04	0.01	0.00
	g_4	0.22	0.07	0.10	0.00	**0.55**	0.04	0.02	0.00
	g_5	0.18	0.30	0.14	0.02	0.27	**0.08**	0.02	0.00
	g_6	0.20	0.10	0.30	0.01	0.32	0.04	**0.02**	0.00
	g_7	0.17	0.24	0.27	0.03	0.22	0.06	0.02	**0.00**
2	g_1	0.01	**0.83**	0.03	0.06	0.00	0.06	0.00	0.00
	g_2	0.02	0.03	**0.78**	0.06	0.03	0.02	0.06	0.00
	g_3	0.01	0.30	0.34	**0.22**	0.02	0.09	0.02	0.00
	g_4	0.01	0.01	0.02	0.00	**0.82**	0.08	0.06	0.00
	g_5	0.01	0.23	0.05	0.04	0.18	**0.43**	0.05	0.01
	g_6	0.01	0.02	0.34	0.03	0.29	0.08	**0.23**	0.00
	g_7	0.01	0.14	0.28	0.11	0.12	0.22	0.11	**0.01**
3	g_1	0.00	**0.88**	0.00	0.07	0.00	0.05	0.00	0.00
	g_2	0.00	0.00	**0.84**	0.07	0.00	0.00	0.07	0.00
	g_3	0.00	0.15	0.17	**0.59**	0.00	0.06	0.01	0.02
	g_4	0.00	0.00	0.00	0.00	**0.86**	0.08	0.06	0.00
	g_5	0.00	0.06	0.01	0.03	0.05	**0.79**	0.03	0.03
	g_6	0.00	0.00	0.18	0.02	0.14	0.06	**0.57**	0.02
	g_7	0.00	0.03	0.16	0.20	0.02	0.32	0.18	**0.09**

Table S8: Selection by the BVS for $K = 4$ and $n = 3$. The probability that a specified model has one of the two highest posterior probabilities among all candidate models. Rows: The true models. Columns: Selected as the model with the highest or the second highest posterior probability by BVS. Correct model is shown in bold.

λ	Profile	g_0	g_1	g_2	g_3	g_4	g_5	g_6	g_7
	g_0	**0.90**	0.42	0.24	0.02	0.38	0.03	0.02	0.00
1	g_1	0.48	**0.77**	0.19	0.17	0.13	0.23	0.03	0.00
	g_2	0.42	0.26	**0.64**	0.17	0.24	0.07	0.18	0.00
	g_3	0.38	0.52	0.44	**0.23**	0.19	0.15	0.10	0.00
	g_4	0.48	0.15	0.17	0.02	**0.76**	0.24	0.18	0.00
	g_5	0.37	0.46	0.25	0.09	0.42	**0.32**	0.09	0.01
	g_6	0.39	0.20	0.45	0.10	0.49	0.15	**0.21**	0.00
	g_7	0.36	0.40	0.39	0.14	0.35	0.21	0.15	**0.00**
2	g_1	0.09	**0.93**	0.05	0.44	0.02	0.46	0.00	0.01
	g_2	0.08	0.07	**0.89**	0.43	0.08	0.04	0.41	0.01
	g_3	0.04	0.46	0.48	**0.64**	0.05	0.20	0.10	0.04
	g_4	0.10	0.02	0.04	0.00	**0.93**	0.46	0.42	0.01
	g_5	0.04	0.43	0.11	0.10	0.37	**0.74**	0.12	0.09
	g_6	0.04	0.05	0.52	0.09	0.43	0.19	**0.63**	0.04
	g_7	0.03	0.26	0.40	0.31	0.24	0.39	0.28	**0.08**
3	g_1	0.01	**0.97**	0.00	0.50	0.00	0.51	0.00	0.02
	g_2	0.00	0.01	**0.96**	0.52	0.01	0.01	0.47	0.02
	g_3	0.00	0.34	0.42	**0.86**	0.00	0.14	0.05	0.19
	g_4	0.01	0.00	0.00	0.00	**0.97**	0.52	0.48	0.01
	g_5	0.00	0.29	0.02	0.07	0.25	**0.91**	0.08	0.39
	g_6	0.00	0.00	0.45	0.05	0.32	0.13	**0.87**	0.18
	g_7	0.00	0.10	0.30	0.35	0.09	0.44	0.32	**0.41**

Table S9: Comparison of proportion of time the true model is selected based on 1000 simulated data sets for BVS, GORIC, AIC and BIC criterion for $K = 4$, $n = 4$.

λ	Profile	BVS	GORIC	AIC	BIC
	g_0	0.81	0.64	0.80	0.85
1	g_1	0.63	0.56	0.61	0.58
	g_2	0.51	0.47	0.54	0.50
	g_3	0.03	0.22	0.07	0.03
	g_4	0.56	0.51	0.55	0.51
	g_5	0.10	0.25	0.13	0.08
	g_6	0.03	0.17	0.05	0.02
	g_7	0.00	0.05	0.00	0.00
2	g_1	0.90	0.66	0.84	0.88
	g_2	0.84	0.57	0.79	0.83
	g_3	0.30	0.56	0.40	0.30
	g_4	0.88	0.62	0.80	0.86
	g_5	0.52	0.61	0.59	0.50
	g_6	0.26	0.51	0.35	0.26
	g_7	0.02	0.34	0.07	0.04
3	g_1	0.92	0.66	0.84	0.89
	g_2	0.87	0.57	0.79	0.85
	g_3	0.67	0.73	0.74	0.69
	g_4	0.91	0.62	0.80	0.87
	g_5	0.88	0.70	0.85	0.86
	g_6	0.66	0.68	0.72	0.67
	g_7	0.15	0.72	0.35	0.23

Table S10: Comparison of proportion of time the true MED is selected on 1000 simulated data sets for BVS, GORIC, AIC and BIC criterion for $K = 4$, $n = 4$.

λ	Profile	BVS	GORIC	AIC	BIC
	g_0	0.81	0.64	0.80	0.85
1	g_1	0.66	0.79	0.68	0.62
	g_2	0.52	0.56	0.56	0.51
	g_3	0.42	0.59	0.44	0.38
	g_4	0.56	0.51	0.55	0.51
	g_5	0.41	0.60	0.44	0.37
	g_6	0.37	0.44	0.40	0.36
	g_7	0.32	0.48	0.35	0.29
2	g_1	0.98	0.99	0.98	0.97
	g_2	0.88	0.72	0.86	0.88
	g_3	0.66	0.86	0.71	0.64
	g_4	0.88	0.62	0.80	0.86
	g_5	0.75	0.90	0.80	0.74
	g_6	0.60	0.63	0.64	0.63
	g_7	0.50	0.76	0.57	0.49
3	g_1	1.00	1.00	1.00	1.00
	g_2	0.93	0.72	0.87	0.91
	g_3	0.85	0.97	0.90	0.86
	g_4	0.91	0.62	0.80	0.87
	g_5	0.95	0.99	0.96	0.95
	g_6	0.82	0.70	0.81	0.81
	g_7	0.68	0.90	0.78	0.70

Table S11: Comparison of proportion of time the true model is selected based on 1000 simulated data sets for BVS, GORIC, AIC and BIC criterion for $K = 4$, $n = 5$.

λ	Profile	BVS	GORIC	AIC	BIC
	g_0	0.85	0.67	0.81	0.89
1	g_1	0.67	0.62	0.68	0.62
	g_2	0.56	0.51	0.60	0.56
	g_3	0.03	0.25	0.06	0.03
	g_4	0.63	0.58	0.63	0.57
	g_5	0.10	0.29	0.17	0.08
	g_6	0.03	0.23	0.06	0.02
	g_7	0.00	0.07	0.00	0.00
2	g_1	0.92	0.68	0.84	0.90
	g_2	0.88	0.57	0.81	0.88
	g_3	0.35	0.61	0.48	0.34
	g_4	0.91	0.65	0.82	0.89
	g_5	0.59	0.67	0.70	0.58
	g_6	0.34	0.59	0.46	0.33
	g_7	0.02	0.42	0.09	0.04
3	g_1	0.93	0.68	0.84	0.90
	g_2	0.91	0.57	0.81	0.89
	g_3	0.78	0.73	0.83	0.79
	g_4	0.92	0.64	0.82	0.89
	g_5	0.90	0.73	0.88	0.89
	g_6	0.76	0.73	0.79	0.76
	g_7	0.21	0.83	0.45	0.29

Table S12: Comparison of proportion of time the true model is selected based on 1000 simulated data sets for BVS, GORIC, AIC and BIC criterion for $K = 4$, $n = 5$.

λ	Profile	BVS	GORIC	AIC	BIC
	g_0	0.85	0.67	0.81	0.89
1	g_1	0.70	0.85	0.75	0.66
	g_2	0.57	0.62	0.62	0.57
	g_3	0.42	0.62	0.44	0.38
	g_4	0.63	0.58	0.63	0.57
	g_5	0.41	0.63	0.47	0.38
	g_6	0.36	0.48	0.42	0.37
	g_7	0.31	0.49	0.36	0.29
2	g_1	0.99	1.00	0.99	0.98
	g_2	0.91	0.75	0.88	0.91
	g_3	0.66	0.90	0.74	0.64
	g_4	0.91	0.65	0.82	0.89
	g_5	0.79	0.94	0.86	0.78
	g_6	0.66	0.69	0.71	0.68
	g_7	0.51	0.77	0.60	0.50
3	g_1	1.00	1.00	1.00	1.00
	g_2	0.94	0.75	0.88	0.93
	g_3	0.89	0.98	0.94	0.90
	g_4	0.92	0.64	0.82	0.89
	g_5	0.96	1.00	0.98	0.97
	g_6	0.88	0.74	0.84	0.87
	g_7	0.69	0.94	0.81	0.72

Table S13: Comparison of proportion of time the true model is selected based on 1000 simulated data sets for BVS, GORIC, AIC and BIC criterion for $K = 4$, $n = 10$.

λ	Profile	BVS	GORIC	AIC	BIC
	g_0	0.90	0.67	0.79	0.91
1	g_1	0.82	0.65	0.80	0.80
	g_2	0.78	0.59	0.78	0.78
	g_3	0.05	0.38	0.18	0.05
	g_4	0.82	0.64	0.80	0.81
	g_5	0.17	0.49	0.35	0.17
	g_6	0.05	0.38	0.16	0.04
	g_7	0.00	0.16	0.01	0.00
2	g_1	0.96	0.66	0.85	0.95
	g_2	0.94	0.61	0.84	0.94
	g_3	0.65	0.74	0.79	0.66
	g_4	0.94	0.65	0.84	0.93
	g_5	0.86	0.70	0.87	0.86
	g_6	0.64	0.75	0.78	0.65
	g_7	0.08	0.76	0.36	0.10
3	g_1	0.96	0.66	0.85	0.95
	g_2	0.95	0.61	0.84	0.94
	g_3	0.96	0.76	0.92	0.96
	g_4	0.95	0.65	0.83	0.93
	g_5	0.97	0.71	0.90	0.96
	g_6	0.95	0.77	0.90	0.94
	g_7	0.57	0.97	0.86	0.64

Table S14: Comparison of proportion of time the true MED is selected based on 1000 simulated data sets for BVS, GORIC, AIC and BIC criterion for $K = 4$, $n = 10$.

λ	Profile	BVS	GORIC	AIC	BIC
	g_0	0.90	0.67	0.79	0.91
1	g_1	0.85	0.96	0.90	0.83
	g_2	0.79	0.75	0.82	0.79
	g_3	0.46	0.72	0.54	0.42
	g_4	0.82	0.64	0.80	0.81
	g_5	0.48	0.76	0.61	0.46
	g_6	0.46	0.59	0.54	0.49
	g_7	0.30	0.59	0.41	0.29
2	g_1	1.00	1.00	1.00	1.00
	g_2	0.97	0.78	0.92	0.96
	g_3	0.79	0.97	0.90	0.79
	g_4	0.94	0.65	0.84	0.93
	g_5	0.93	1.00	0.97	0.93
	g_6	0.83	0.77	0.86	0.84
	g_7	0.60	0.90	0.74	0.61
3	g_1	1.00	1.00	1.00	1.00
	g_2	0.97	0.78	0.91	0.96
	g_3	0.99	1.00	1.00	0.98
	g_4	0.95	0.65	0.83	0.93
	g_5	1.00	1.00	1.00	1.00
	g_6	0.97	0.77	0.91	0.96
	g_7	0.84	0.99	0.95	0.86

Table S15: Comparison of proportion of time the true model is selected based on 1000 simulated data sets for BVS, GORIC, AIC and BIC criterion with $K = 5$, $n = 3$.

λ	Profile	BVS	GORIC	AIC	BIC
	g_0	0.73	0.58	0.74	0.81
1	g_1	0.53	0.46	0.49	0.44
	g_2	0.39	0.36	0.42	0.39
	g_3	0.02	0.11	0.03	0.02
	g_4	0.39	0.36	0.42	0.38
	g_5	0.03	0.13	0.05	0.03
	g_6	0.01	0.11	0.02	0.01
	g_7	0.00	0.01	0.00	0.00
	g_8	0.49	0.40	0.44	0.41
	g_9	0.08	0.18	0.10	0.06
	g_{10}	0.04	0.13	0.06	0.04
	g_{11}	0.00	0.04	0.00	0.00
	g_{12}	0.01	0.09	0.02	0.01
	g_{13}	0.00	0.03	0.00	0.00
	g_{14}	0.00	0.02	0.00	0.00
	g_{15}	0.00	0.00	0.00	0.00
2	g_1	0.81	0.57	0.75	0.78
	g_2	0.73	0.46	0.70	0.75
	g_3	0.15	0.37	0.21	0.14
	g_4	0.73	0.47	0.70	0.73
	g_5	0.26	0.37	0.35	0.27
	g_6	0.14	0.35	0.23	0.16
	g_7	0.00	0.16	0.01	0.00
	g_8	0.81	0.53	0.72	0.78
	g_9	0.44	0.46	0.46	0.40
	g_{10}	0.26	0.36	0.31	0.26
	g_{11}	0.01	0.20	0.05	0.02
	g_{12}	0.15	0.34	0.19	0.13
	g_{13}	0.01	0.18	0.04	0.02
	g_{14}	0.00	0.15	0.01	0.00
	g_{15}	0.00	0.05	0.00	0.00
3	g_1	0.86	0.58	0.78	0.82
	g_2	0.82	0.46	0.73	0.81
	g_3	0.48	0.54	0.54	0.46
	g_4	0.81	0.48	0.73	0.79
	g_5	0.64	0.50	0.65	0.63
	g_6	0.49	0.50	0.55	0.50
	g_7	0.03	0.39	0.13	0.07
	g_8	0.86	0.53	0.74	0.82
	g_9	0.77	0.56	0.73	0.72
	g_{10}	0.61	0.47	0.61	0.60
	g_{11}	0.12	0.43	0.24	0.15
	g_{12}	0.48	0.51	0.51	0.45
	g_{13}	0.11	0.44	0.24	0.15
	g_{14}	0.04	0.37	0.13	0.07
	g_{15}	0.00	0.23	0.01	0.00

Table S16: Comparison of proportion of time the true MED is selected based on 1000 simulated data sets for BVS, GORIC, AIC and BIC criterion with $K = 5$, $n = 3$.

λ	Profile	BVS	GORIC	AIC	BIC
	g_0	0.73	0.58	0.74	0.81
1	g_1	0.58	0.70	0.57	0.48
	g_2	0.41	0.48	0.45	0.40
	g_3	0.38	0.50	0.36	0.31
	g_4	0.40	0.42	0.42	0.39
	g_5	0.34	0.47	0.32	0.28
	g_6	0.32	0.40	0.34	0.32
	g_7	0.29	0.41	0.28	0.24
	g_8	0.49	0.40	0.44	0.41
	g_9	0.36	0.52	0.35	0.29
	g_{10}	0.27	0.37	0.32	0.28
	g_{11}	0.31	0.44	0.31	0.25
	g_{12}	0.27	0.31	0.30	0.28
	g_{13}	0.26	0.40	0.26	0.21
	g_{14}	0.23	0.32	0.27	0.24
	g_{15}	0.25	0.37	0.24	0.20
2	g_1	0.95	0.98	0.95	0.93
	g_2	0.80	0.70	0.80	0.82
	g_3	0.58	0.78	0.60	0.54
	g_4	0.77	0.59	0.74	0.76
	g_5	0.59	0.77	0.63	0.55
	g_6	0.54	0.58	0.57	0.55
	g_7	0.43	0.65	0.47	0.40
	g_8	0.81	0.53	0.72	0.78
	g_9	0.70	0.84	0.73	0.66
	g_{10}	0.55	0.59	0.58	0.57
	g_{11}	0.48	0.70	0.52	0.46
	g_{12}	0.51	0.49	0.54	0.53
	g_{13}	0.43	0.64	0.47	0.40
	g_{14}	0.39	0.49	0.44	0.40
	g_{15}	0.37	0.57	0.39	0.34
3	g_1	1.00	1.00	1.00	1.00
	g_2	0.91	0.72	0.87	0.91
	g_3	0.76	0.92	0.81	0.74
	g_4	0.86	0.61	0.80	0.84
	g_5	0.82	0.94	0.86	0.82
	g_6	0.71	0.68	0.73	0.72
	g_7	0.58	0.81	0.64	0.58
	g_8	0.86	0.53	0.74	0.82
	g_9	0.91	0.98	0.93	0.90
	g_{10}	0.79	0.69	0.79	0.80
	g_{11}	0.63	0.86	0.70	0.63
	g_{12}	0.71	0.57	0.70	0.70
	g_{13}	0.62	0.83	0.70	0.62
	g_{14}	0.54	0.60	0.59	0.56
	g_{15}	0.48	0.73	0.56	0.49

Table S17: Comparison of proportion of time the true model is selected based on 1000 simulated data sets for BVS, GORIC, AIC and BIC criterion with $K = 5$, $n = 4$.

λ	Profile	BVS	GORIC	AIC	BIC
	g_0	0.81	0.61	0.76	0.86
1	g_1	0.60	0.51	0.58	0.54
	g_2	0.45	0.38	0.48	0.46
	g_3	0.02	0.14	0.04	0.01
	g_4	0.40	0.36	0.44	0.41
	g_5	0.04	0.15	0.07	0.04
	g_6	0.01	0.12	0.02	0.01
	g_7	0.00	0.03	0.00	0.00
	g_8	0.55	0.49	0.54	0.49
	g_9	0.09	0.22	0.14	0.07
	g_{10}	0.04	0.16	0.07	0.04
	g_{11}	0.00	0.04	0.00	0.00
	g_{12}	0.02	0.13	0.03	0.01
	g_{13}	0.00	0.04	0.00	0.00
	g_{14}	0.00	0.02	0.00	0.00
	g_{15}	0.00	0.00	0.00	0.00
2	g_1	0.88	0.58	0.79	0.86
	g_2	0.80	0.47	0.72	0.81
	g_3	0.20	0.44	0.29	0.18
	g_4	0.80	0.44	0.73	0.81
	g_5	0.34	0.41	0.43	0.33
	g_6	0.18	0.37	0.27	0.18
	g_7	0.01	0.21	0.03	0.01
	g_8	0.87	0.58	0.79	0.85
	g_9	0.53	0.52	0.59	0.50
	g_{10}	0.33	0.42	0.41	0.33
	g_{11}	0.02	0.25	0.08	0.03
	g_{12}	0.18	0.40	0.26	0.17
	g_{13}	0.02	0.26	0.07	0.03
	g_{14}	0.00	0.17	0.02	0.00
	g_{15}	0.00	0.09	0.00	0.00
3	g_1	0.90	0.58	0.79	0.88
	g_2	0.85	0.48	0.73	0.84
	g_3	0.59	0.58	0.65	0.59
	g_4	0.85	0.44	0.74	0.84
	g_5	0.74	0.48	0.72	0.74
	g_6	0.58	0.48	0.62	0.59
	g_7	0.07	0.48	0.20	0.11
	g_8	0.90	0.58	0.80	0.87
	g_9	0.86	0.59	0.81	0.84
	g_{10}	0.72	0.51	0.72	0.72
	g_{11}	0.18	0.52	0.36	0.23
	g_{12}	0.58	0.56	0.62	0.54
	g_{13}	0.16	0.50	0.35	0.20
	g_{14}	0.06	0.45	0.18	0.09
	g_{15}	0.00	0.32	0.04	0.01

Table S18: Comparison of proportion of time the true MED is selected based on 1000 simulated data sets for BVS, GORIC, AIC and BIC criterion with $K = 5$, $n = 4$.

λ	Profile	BVS	GORIC	AIC	BIC
	g_0	0.81	0.61	0.76	0.86
1	g_1	0.65	0.78	0.67	0.57
	g_2	0.47	0.52	0.53	0.48
	g_3	0.39	0.55	0.41	0.34
	g_4	0.41	0.45	0.46	0.41
	g_5	0.36	0.52	0.38	0.31
	g_6	0.35	0.43	0.38	0.35
	g_7	0.30	0.45	0.32	0.26
	g_8	0.55	0.49	0.54	0.49
	g_9	0.39	0.58	0.42	0.34
	g_{10}	0.30	0.40	0.35	0.30
	g_{11}	0.32	0.46	0.34	0.28
	g_{12}	0.27	0.33	0.30	0.28
	g_{13}	0.28	0.43	0.30	0.22
	g_{14}	0.25	0.35	0.30	0.25
	g_{15}	0.26	0.39	0.28	0.20
2	g_1	0.97	0.99	0.98	0.96
	g_2	0.86	0.72	0.84	0.87
	g_3	0.61	0.84	0.66	0.57
	g_4	0.83	0.60	0.79	0.83
	g_5	0.64	0.83	0.70	0.60
	g_6	0.58	0.62	0.61	0.59
	g_7	0.45	0.71	0.51	0.41
	g_8	0.87	0.58	0.79	0.85
	g_9	0.76	0.89	0.81	0.74
	g_{10}	0.61	0.63	0.64	0.63
	g_{11}	0.53	0.75	0.56	0.49
	g_{12}	0.54	0.51	0.57	0.55
	g_{13}	0.45	0.72	0.52	0.43
	g_{14}	0.43	0.51	0.48	0.44
	g_{15}	0.37	0.64	0.43	0.34
3	g_1	1.00	1.00	1.00	1.00
	g_2	0.93	0.73	0.88	0.93
	g_3	0.81	0.96	0.87	0.80
	g_4	0.90	0.60	0.82	0.89
	g_5	0.86	0.97	0.90	0.87
	g_6	0.77	0.70	0.79	0.78
	g_7	0.61	0.86	0.70	0.61
	g_8	0.90	0.58	0.80	0.87
	g_9	0.94	0.99	0.97	0.95
	g_{10}	0.85	0.71	0.83	0.86
	g_{11}	0.67	0.89	0.76	0.66
	g_{12}	0.77	0.58	0.74	0.76
	g_{13}	0.67	0.87	0.77	0.68
	g_{14}	0.59	0.62	0.63	0.62
	g_{15}	0.49	0.79	0.60	0.50

Table S19: Comparison of proportion of time the true model is selected based on 1000 simulated data sets for BVS, GORIC, AIC and BIC criterion with $K = 5$, $n = 5$.

λ	Profile	BVS	GORIC	AIC	BIC
	g_0	0.84	0.62	0.78	0.87
1	g_1	0.64	0.55	0.63	0.57
	g_2	0.50	0.43	0.55	0.50
	g_3	0.02	0.17	0.05	0.02
	g_4	0.47	0.42	0.52	0.48
	g_5	0.06	0.20	0.10	0.05
	g_6	0.01	0.16	0.04	0.01
	g_7	0.00	0.04	0.00	0.00
	g_8	0.64	0.53	0.62	0.58
	g_9	0.12	0.28	0.20	0.10
	g_{10}	0.05	0.19	0.10	0.05
	g_{11}	0.00	0.07	0.00	0.00
	g_{12}	0.02	0.16	0.04	0.01
	g_{13}	0.00	0.06	0.00	0.00
	g_{14}	0.00	0.04	0.00	0.00
	g_{15}	0.00	0.01	0.00	0.00
2	g_1	0.90	0.60	0.80	0.88
	g_2	0.84	0.49	0.76	0.84
	g_3	0.27	0.48	0.39	0.25
	g_4	0.84	0.48	0.75	0.85
	g_5	0.42	0.48	0.54	0.42
	g_6	0.25	0.46	0.38	0.25
	g_7	0.00	0.27	0.05	0.01
	g_8	0.90	0.59	0.80	0.88
	g_9	0.64	0.56	0.69	0.62
	g_{10}	0.41	0.45	0.52	0.42
	g_{11}	0.03	0.33	0.13	0.04
	g_{12}	0.23	0.48	0.37	0.21
	g_{13}	0.03	0.33	0.11	0.04
	g_{14}	0.00	0.26	0.05	0.01
	g_{15}	0.00	0.13	0.00	0.00
3	g_1	0.91	0.60	0.79	0.88
	g_2	0.88	0.48	0.77	0.86
	g_3	0.68	0.60	0.74	0.66
	g_4	0.88	0.48	0.75	0.87
	g_5	0.80	0.53	0.77	0.80
	g_6	0.69	0.55	0.72	0.70
	g_7	0.10	0.55	0.30	0.14
	g_8	0.92	0.59	0.80	0.89
	g_9	0.89	0.60	0.82	0.88
	g_{10}	0.82	0.50	0.77	0.81
	g_{11}	0.26	0.59	0.48	0.32
	g_{12}	0.69	0.60	0.71	0.66
	g_{13}	0.22	0.59	0.49	0.28
	g_{14}	0.10	0.55	0.29	0.15
	g_{15}	0.00	0.42	0.06	0.02

Table S20: Comparison of proportion of time the true MED is selected on 1000 simulated data sets for BVS, GORIC, AIC and BIC criterion with $K = 5$, $n = 5$.

λ	Profile	BVS	GORIC	AIC	BIC
	g_0	0.84	0.62	0.78	0.87
1	g_1	0.67	0.83	0.72	0.61
	g_2	0.51	0.60	0.59	0.51
	g_3	0.40	0.57	0.43	0.35
	g_4	0.48	0.51	0.53	0.48
	g_5	0.36	0.56	0.39	0.30
	g_6	0.35	0.46	0.40	0.35
	g_7	0.30	0.46	0.32	0.25
	g_8	0.64	0.53	0.62	0.58
	g_9	0.40	0.60	0.45	0.34
	g_{10}	0.30	0.45	0.39	0.32
	g_{11}	0.31	0.49	0.35	0.27
	g_{12}	0.31	0.39	0.37	0.33
	g_{13}	0.26	0.45	0.30	0.22
	g_{14}	0.22	0.36	0.30	0.24
	g_{15}	0.25	0.41	0.28	0.21
2	g_1	0.99	1.00	0.99	0.98
	g_2	0.90	0.75	0.88	0.90
	g_3	0.62	0.87	0.70	0.59
	g_4	0.88	0.63	0.82	0.88
	g_5	0.67	0.88	0.76	0.64
	g_6	0.60	0.68	0.66	0.61
	g_7	0.44	0.72	0.53	0.42
	g_8	0.90	0.59	0.80	0.88
	g_9	0.81	0.94	0.86	0.79
	g_{10}	0.68	0.68	0.72	0.69
	g_{11}	0.51	0.77	0.58	0.48
	g_{12}	0.58	0.58	0.63	0.59
	g_{13}	0.48	0.74	0.58	0.46
	g_{14}	0.44	0.57	0.51	0.45
	g_{15}	0.38	0.65	0.44	0.35
3	g_1	1.00	1.00	1.00	1.00
	g_2	0.94	0.75	0.89	0.94
	g_3	0.84	0.97	0.92	0.82
	g_4	0.92	0.63	0.82	0.90
	g_5	0.90	0.98	0.95	0.90
	g_6	0.83	0.74	0.84	0.84
	g_7	0.62	0.90	0.73	0.62
	g_8	0.92	0.59	0.80	0.89
	g_9	0.97	1.00	0.98	0.97
	g_{10}	0.89	0.74	0.86	0.89
	g_{11}	0.68	0.94	0.80	0.68
	g_{12}	0.84	0.62	0.79	0.83
	g_{13}	0.69	0.92	0.82	0.71
	g_{14}	0.63	0.68	0.70	0.66
	g_{15}	0.50	0.81	0.62	0.51

Table S21: Comparison of proportion of time the true model is selected based on 1000 simulated data sets for BVS, GORIC, AIC and BIC criterion with $K = 5$, $n = 10$.

λ	Profile	BVS	GORIC	AIC	BIC
	g_0	0.90	0.60	0.77	0.90
1	g_1	0.80	0.57	0.75	0.77
	g_2	0.72	0.50	0.71	0.73
	g_3	0.02	0.32	0.14	0.02
	g_4	0.71	0.47	0.70	0.73
	g_5	0.09	0.32	0.23	0.08
	g_6	0.02	0.29	0.11	0.02
	g_7	0.00	0.11	0.00	0.00
	g_8	0.81	0.59	0.75	0.78
	g_9	0.19	0.42	0.36	0.18
	g_{10}	0.09	0.32	0.22	0.08
	g_{11}	0.00	0.14	0.01	0.00
	g_{12}	0.02	0.32	0.12	0.02
	g_{13}	0.00	0.12	0.01	0.00
	g_{14}	0.00	0.09	0.00	0.00
	g_{15}	0.00	0.02	0.00	0.00
2	g_1	0.94	0.58	0.80	0.93
	g_2	0.92	0.52	0.78	0.92
	g_3	0.56	0.62	0.69	0.54
	g_4	0.91	0.47	0.77	0.91
	g_5	0.73	0.49	0.76	0.74
	g_6	0.59	0.55	0.70	0.60
	g_7	0.02	0.51	0.22	0.03
	g_8	0.93	0.60	0.79	0.92
	g_9	0.87	0.60	0.83	0.86
	g_{10}	0.75	0.55	0.78	0.75
	g_{11}	0.11	0.55	0.38	0.13
	g_{12}	0.55	0.60	0.67	0.52
	g_{13}	0.09	0.55	0.37	0.11
	g_{14}	0.02	0.53	0.20	0.02
	g_{15}	0.00	0.33	0.02	0.00
3	g_1	0.95	0.58	0.80	0.93
	g_2	0.93	0.52	0.77	0.92
	g_3	0.91	0.65	0.85	0.90
	g_4	0.92	0.47	0.76	0.91
	g_5	0.94	0.50	0.82	0.93
	g_6	0.92	0.57	0.83	0.92
	g_7	0.42	0.70	0.71	0.45
	g_8	0.94	0.60	0.78	0.92
	g_9	0.96	0.60	0.85	0.95
	g_{10}	0.95	0.55	0.83	0.94
	g_{11}	0.65	0.69	0.82	0.68
	g_{12}	0.91	0.63	0.82	0.89
	g_{13}	0.62	0.68	0.80	0.65
	g_{14}	0.39	0.73	0.68	0.44
	g_{15}	0.03	0.77	0.32	0.05

Table S22: Comparison of proportion of time the true MED is selected based on 1000 simulated data sets for BVS, GORIC, AIC and BIC criterion with $K = 5$, $n = 10$.

λ	Profile	BVS	GORIC	AIC	BIC
	g_0	0.90	0.60	0.77	0.90
1	g_1	0.83	0.95	0.89	0.81
	g_2	0.73	0.74	0.78	0.75
	g_3	0.43	0.70	0.50	0.38
	g_4	0.72	0.61	0.73	0.73
	g_5	0.38	0.68	0.50	0.36
	g_6	0.45	0.57	0.50	0.46
	g_7	0.26	0.56	0.36	0.24
	g_8	0.81	0.59	0.75	0.78
	g_9	0.47	0.75	0.60	0.44
	g_{10}	0.44	0.57	0.53	0.46
	g_{11}	0.31	0.59	0.42	0.28
	g_{12}	0.41	0.50	0.48	0.44
	g_{13}	0.26	0.52	0.38	0.23
	g_{14}	0.31	0.46	0.39	0.32
	g_{15}	0.21	0.48	0.30	0.19
2	g_1	1.00	1.00	1.00	1.00
	g_2	0.96	0.79	0.91	0.97
	g_3	0.74	0.96	0.86	0.72
	g_4	0.94	0.63	0.84	0.93
	g_5	0.84	0.97	0.92	0.82
	g_6	0.78	0.77	0.83	0.79
	g_7	0.52	0.86	0.68	0.50
	g_8	0.93	0.60	0.79	0.92
	g_9	0.94	1.00	0.97	0.93
	g_{10}	0.88	0.77	0.87	0.89
	g_{11}	0.58	0.90	0.73	0.56
	g_{12}	0.78	0.62	0.78	0.78
	g_{13}	0.59	0.88	0.74	0.58
	g_{14}	0.56	0.70	0.66	0.57
	g_{15}	0.42	0.76	0.57	0.40
3	g_1	1.00	1.00	1.00	1.00
	g_2	0.97	0.78	0.91	0.97
	g_3	0.96	1.00	1.00	0.96
	g_4	0.95	0.63	0.83	0.93
	g_5	0.99	1.00	1.00	0.99
	g_6	0.96	0.77	0.90	0.96
	g_7	0.76	0.97	0.90	0.77
	g_8	0.94	0.60	0.78	0.92
	g_9	1.00	1.00	1.00	1.00
	g_{10}	0.97	0.77	0.91	0.96
	g_{11}	0.83	0.99	0.94	0.84
	g_{12}	0.94	0.63	0.83	0.92
	g_{13}	0.88	0.99	0.96	0.88
	g_{14}	0.81	0.76	0.85	0.83
	g_{15}	0.62	0.93	0.78	0.63