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Maastricht University

Faculty of Sciences
School for Information Technology
Master of Statistics

Masterthesis

Gifted Children Underachieving in Mathematics

Nahid Sultana
Thesis presented in fulfillment of the requirements for the degree of Master of Statistics, specialization Biostatistics

SUPERVISOR :
dr. Liesbeth BRUCKERS
SUPERVISOR :
Ms. Tessa KIEBOOM

Transnational University Limburg is a unique collaboration of two universities in two countries: the University of Hasselt and Maastricht University.

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Dedicated To

My

Beloved Parents and Husband

Acknowledgement

Firstly, I express my deep gratitude to Almighty Allah who has given me strength,patience and capability to complete this thesis.

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Nahid Sultana
June 15, 2018.

Abstract

The purpose of the current study is to investigate the extent to which mathematical gaps ascertained by highly gifted young people in primary school give rise to underachieving in high school and in which particular students these gaps for mathematics are found. The study used a dataset containing information on 1013 children who were at last year of their primary school. The nature of the data was hierarchical, i.e students were nested within classrooms, classrooms were nested within schools. To asses any underlying structure in data, clustering of variables were used. Fifteen clusters of variables was identified with highest number of variables in cluster one (twelve variables). Correlation matrix for the four different learning domains were calculated where the pattern of the association between learning domains seem to be similar between gifted and non-gifted students except for the group of students who follows no written strategy. Correlations between the learning domains found to be significantly different for this group. To answer two of the main research questions whether gifted children have less or more problems with mathematics in primary school than non-gifted children and to find if gifted children use their own method to solve a mathematical problem more frequently than non-gifted children, Linear mixed models with random intercept were fitted which show that keeping other covariate fixed (language and sex), score for gifted children are higher most of the time than non gifted group. But the scores for number of incorrect answer was higher for non-gifted children. Also results explain that gifted children use their own method to solve a mathematical problem more frequently than non-gifted children. A generalized linear mixed effect model(GLMM) with random intercept was fitted for a binary response (whether the gifted student agrees for a three year follow up or not) which shows that all covariate (gender,age,language,intelligence score) is insignificant at 5% significance level which means students not participating in the second phase of the research seem not to be different from students that do participate.

Key Words: Gifted, GLMM, LMM, Non-gifted

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List of Acronyms

EDA	Exploratory Data Analysis
LMM	Linear mixed model
GLMM	Generalized Linear Mixed Models
SAS	Statistical Analysis Software
MD	Mathematical difficulty
MATH	Mathematical concepts+equality sign
NUM	Numberstructure+number axis+estimations
FRAC	Fraction/rational numbers+calculation with rational numbers
PER	Percent calculation
PERP	Percent calculation+proportion
ADD	Adding+adding numericals
SUB	Subtraction+subtraction numericals
MULT	Multiply+multiply numerical
DIVI	Dividing+dividing numerical
MENT	Mental calculations
NUMR	Numerical calculations
LMVA	Length+mass+volumn+area
TIME	Time calculation+clock reading
POL	Polygons
EX	Exercise in a context
PCA	Principle component analysis
LR	Likelihood ratio
SE	Standard error

1 Introduction

Mathematics is defined as a language that is used to express relations between and among objects, events, and times. The language of mathematics employs a set of symbols and rules to express these relations (Clarke& Shinn, 2004). There is some evidence that numerical concepts children acquire in early childhood lay the foundation for later acquisition of advanced mathematical concepts (Ginsburg & Allardice, 1984; Griffin et al., 1994), and that success or failure in acquiring early numerical concepts influences the interest and confidence students bring to new mathematical tasks and may fundamentally alter a students success in mathematics throughout the elementary grades (Jordan, 1995). Research shows that some talented young students, who, despite possible differentiation at school, have gaps in mathematics that could lead to underachieving. Thus, over the past years, researchers have tried to assess the most salient aspects of a childs understanding of basic numerical relationships and operations and develop potential screening measures.

In theory, mathematical difficulty(MD) can result from deficits in the ability to represent or process information used in one or all of the many areas of mathematics (e.g., arithmetic, geometry), or in one or a set of individual domains (e.g., theorems vs. graphing) within each of these areas. To make the study of MD tractable, scientists have focused primarily on mathematical domains for which competency development in academically-normal children is well understood. These domains include number, counting, and arithmetic (Geary& Hoard, 2002). Children who exhibit mathematics difficulties include those performing in the low average range (e.g., at or below the 35th percentile) as well as those performing well below average (Gersten et al.,2005).

Intelligence testing is the estimation of a student's current intellectual functioning through a performance of various tasks designed to assess different types of reasoning [https://www. verywellfamily.com/understanding-intelligence-testing-for-children-2162161]. Intelligence involves the ability to think, solve problems, analyze situations, and understand social values, customs, and norms. Today, there are more than 20 individual-based and group-based tests for measuring intelligence available in Dutch[https://www.tandfonline.com/doi/full/10.1080 /21683603.2016.1166754]. The majority of these tests measure primarily the cognitive capacities of individuals, even though the authors of these tests often acknowledge the fact that non cognitive factors, such as motivation and perseverance, are necessary for being able to cope with the above-mentioned challenges in daily life as well. Some of the Dutch intelligence tests were originally developed in another language (primarily in English) and were translated, adapted, and normed for use in this country.

The current study aims at exploring the extent to which mathematical difficulties in primary school ascertained by highly gifted young people causes underachieving in high school along with identifying the particular group of students in whom these mathematical difficulties are found. The research was designed in two phases. During the first phase of the research, a set of exercises which focuses on the four learning domains (Numbers, Calculations, Measuring counting and geometry) as well as an intelligence screening was designed for children in the last year of primary school. A thousand children had to fill out this mathematical questionnaire. Not only the correctness of the answers was looked at, but also the strategy used to solve the exercises was compared to the teachers' strategy. This double correction was used to know if the child uses the learned strategy or his own. Depending on score in intelligence screening, some of the students are characterized as "Gifted" and others as "Non-gifted". Second phase of the study was to follow up the gifted children for a period of three years for which some gifted children agrees and others not. The current study is basically based on the first phase of the research.

1.1 Objectives

Following objectives were set to analyze in this study:

- The exercises included in the mathematical questionnaire were grouped in four 'learning domains'. Is there an underlying structure in the data that supports this grouping of the exercises/variables in the learning domains?
- Is the association between the different exercises or learning domains similar for gifted and non-gifted children ?
- Have gifted children less or more problems with mathematics in primary school than non-gifted children?
- Do gifted children use their own method to solve a mathematical problem more frequently than non-gifted children?
- Do gifted students who are not participating in the second phase (a 3 year follow-up study) of the research different from students that do participate (e.g. in terms of gender, age, intelligence score, etc.) ?

1.2 Organization of the study

This study is organized as follows: Section 1 for introduction and objectives; Section 2 deals with variables description and methodology. The results of the study are presented in Section 3 and finally Section 4 includes the discussion and conclusion of findings and ends with some concluding remarks on the basis of findings.

2 Methods and Materials

2.1 Data Description

This study used a dataset containing information on 1013 children who were at last year of their primary school. All children have to answer a mathematical questionnaire which tries to asses the students on a variety of mathematical learning domains. The dataset were made up with four sections depending on the strategies that students applied. First section comprise of number of correct answers by students who are following their teachers strategy, second section belongs also to students answer correctly but they used their own strategy to solve the exercise, third group of students answer the exercise correctly but without any written strategy and lastly fourth group were those students who answer incorrectly. The test was done in two parts: in October/November 2016 the math-questionnaire was filled out by students under the supervision of the class teacher. In the period February-May 2017 an intelligence screening was conducted developed by Thomas More - Educational Psychology in Antwerp. Students were divided into two groups, namely "Gifted" and "Non-Gifted" depending on their scores in the intelligence screening. The intelligence screening was made of two parts. One part is mathematical, the other one was more focused on verbal intelligence. In the mathematical screening the children had row of figures with blank spaces which they have to fill in. This was called intelligence screening (figure series). The other screening was about contradictions: for each word that is given, the child gets 4 words and one of them is the opposite of the given word. The children have to point out which word is different. Each part of the intelligence screening was scored from 1 to 9. Children who had 15 or more in total, were added to the group of "Gifted". Children with 8 or 9 on one part of the two screenings were also considered to be "Gifted". The exercises were divided into four learning domains, namely Numbers,Calculations, Measuring counting and Geometry. These learning domains are made by summing the scores on the exercises. Variables that are used in this study along with the formation of learning domains are listed in (Table 2). Demographic characteristics of students are listed in (Table 3). The dataset used in this study is a three-level clustered data set. Clustered data arise when observations are made on subjects within the same randomly selected group. The nature of the data was hierarchical, i.e students were nested within classrooms, classrooms were nested within schools. When trying to meet the objectives of the study, the hierarchical nature of the data (children, classes, school) has to be taken into account. In this study, the hierarchy of the data can be describe as follows (Table 1):

Table 1: Description of the hierarchy of the data

Subject/unit of analysis (i)	**Cluster of units (j)**	**Cluster of clusters (k)**
Student	Classroom	School

To answer the research questions, i.e to compare gifted and non-gifted students in terms of their difficulties with mathematics and to evaluate whether gifted children use their own method in answering questions more frequently than their counterpart (non-gifted), the four learning domains which asses a student in his/her score in mathematical screening (Numbers,Calculations, Measuring counting and Geometry), were used as dependent variable. For the final research question, a binary outcome (gifted children whether they are participating in the second phase of research or not) was considered.

Table 2: Description of variables with formation of learning domains

Variable	
Mathematical concepts&equality sign(MATH)	Intelligence screening (figure series)
Intelligence screening(contradiction)	exercise in a context (EX)
Variable	**Learning Domain**
Numberstructur+number axis+estimations(NUM)	Numbers
Fractions/rational numbers+calculation with rational numbers(FRAC)	
Percent calculations(PER)	
Percent calculations+proportions(PERP)	
Adding+adding numericals(ADD)	Calculations
Subtraction+subtraction numerical(SUB)	
Multiply+multiply numerical(MULT)	
Dividing+dividing numerical(DIVI)	
Mental calculations(MENT)	
Numerical calculations(NUMR)	
Length+mass+volumn+area(LMVA)	Measuring and counting counting
Time calculation+clock reading(TIME)	
Polygons(POL)	Geometry

Table 3: Demographic characteristics of students

Variable	**Description**
Sex	Whether the child is male or female
Age	Age of the students
Language	Whether the students speaks Dutch at home or other language
Gifted	Whether the student is characterized as gifted or not

2.2 Software

Analysis of data was performed using statistical software SAS 9.4 and R software version 3.3.1. A significance level of 5% was used for statistical decision making.

2.3 Exploratory data analysis

An exploratory data analysis was done, descriptive statistics and graphical techniques were employed in order to gain insight and understanding into the data set.

2.4 Clustering variables

To find out any underlying structure of data cluster analysis of variables was conducted. Clustering refers to a very broad set of techniques for finding subgroups, or clusters, in a data set. The aim of the clustering of variables is to detect subset of correlated variables. This is another way to structure the data. A kind of dual analysis of clustering individuals.Thus, the variables which provide the same kind of information belong the same group. The groups of variables reveal the main dimensionality of the data. In a certain sense, it is more powerful than a factor analysis (e.g. principal component analysis) because it overcomes the orthogonality constraint between the factors. The homogeneity criterion of a cluster is defined as the sum of correlation ratios (for qualitative variables) and squared correlations (for quantitative variables) to a synthetic quantitative variable, summarizing "as good as possible" the variables in the cluster[Chavent et al.,2011]. Two commonly used methods for clustering of variables are: a hierarchical clustering algorithm and a k-means type partitioning algorithm. Both clustering algorithms aim at maximizing the same homogeneity criterion: a cluster of variables is defined as homogeneous when the variables in the cluster are strongly linked to a central quantitative synthetic variable. This link is measured by the squared Pearson correlation for the quantitative variables and by the correlation ratio for the qualitative variables.

2.4.1 The Hierarchical Clustering Algorithm

In hierarchical clustering, in advance it is not known how many clusters are present in data; in fact, the method end up with a tree-like visual representation of the observations, called a dendrogram, that allows to view at once the clusterings obtained for each possible number of clusters. Bottom-up or agglomerative clustering is used in this study. This is the most common type of hierarchical clustering, and refers to the fact that a dendrogram is built starting from the leaves and combining clusters up to the trunk. The hierarchical clustering dendrogram is obtained via an extremely simple algorithm. It begin by defining some sort of dissimilarity measure between each pair of observations. Most often, Euclidean distance is used. The algorithm proceeds iteratively. Starting out at the bottom of the dendrogram, each of the variables is treated as its own cluster. The two clusters that are most similar to each other are then fused. Next the two clusters that are most similar to each other are fused again. The algorithm proceeds in this fashion until all of the observations belong to one single cluster, and the dendrogram is complete.

2.5 Linear Mixed models (LMM)

To answer two of the main research questions whether gifted children have less or more problems with mathematics in primary school than non-gifted children and to find if gifted children use their own method to solve a mathematical problem more frequently than non-gifted children, Linear mixed models were applied which takes into account the hierarchical nature of the data. A linear mixed model (LMM) is a parametric linear model for clustered, longitudinal, or repeated-measures data that quantifies the relationships between a continuous dependent variable and various predictor variables. LMM may include both fixed-effect parameters associated with one or more continuous or categorical covariates and random effects associated with one or more random factors[West et al.,2014]. In LMM, the residuals are normally distributed but may not be independent or have constant variance. The name linear mixed models comes from the fact that these models are linear in the parameters, and that the covariates, or independent variables, may involve a mix of fixed and random effects. Estimation of fixed effect parameters in LMMs is generally of intrinsic interest, because they indicate the relationships of the covariates with the continuous outcome variable. When the levels of a factor can be thought of as having been sampled from a sample space, such that each particular level is not of intrinsic interest (e.g., classrooms that are randomly sampled from a larger population of classrooms), the effects associated with the levels of those factors can be modeled as random effects in an LMM. In contrast to fixed effects, which are represented by constant parameters in an LMM, random effects are represented by (unobserved) random variables, which are usually assumed to follow a normal distribution. In this study, following random intercept models were used for the four group of students who differ according to the strategy they apply in solving the exercises:

$$Numbers_{ijk} = \beta_o + \beta_1 * Gifted_{ijk} + \beta_2 * Language_{ijk} + \beta_3 * Sex_{ijk} + \mu_k + \mu_{j|k} + \epsilon_{ijk}$$

$$Calculations_{ijk} = \beta_o + \beta_1 * Gifted_{ijk} + \beta_2 * Language_{ijk} + \beta_3 * Sex_{ijk} + \mu_k + \mu_{j|k} + \epsilon_{ijk}$$

$$Measuringcounting_{ijk} = \beta_o + \beta_1 * Gifted_{ijk} + \beta_2 * Language_{ijk} + \beta_3 * Sex_{ijk} + \mu_k + \mu_{j|k} + \epsilon_{ijk}$$

$$Geometry_{ijk} = \beta_o + \beta_1 * Gifted_{ijk} + \beta_2 * Language_{ijk} + \beta_3 * Sex_{ijk} + \mu_k + \mu_{j|k} + \epsilon_{ijk}$$

Numbers, Calculations, Measuring counting and Geometry are the score on the learning domain for the student i in classroom j nested within school k;

β_0 through β_3 represent the fixed intercept and the fixed effects of the covariates

$Gifted_{ijk}$ is the status of the i^{th} student whether he/she is gifted or not

$Language_{ijk}$ indicates whether the students speaks Dutch or not

Sex_{ijk} is the sex of the student

μ_k is the random effect associated with the intercept for school k; $\mu_{j|k}$is the random effect associated with the intercept for classroom j within school k; ϵ_{ijk} represents the residual.

The distribution of the random effects associated with the schools is written as $\mu_k \sim N(0, \sigma^2_{school})$, where σ^2_{school} represents the variance of the school-specific random intercepts; The distribution of the random effects associated with classrooms nested within a given school is $\mu_{j|k} \sim N(0, \sigma^2_{classroom})$, where $\sigma^2_{classroom}$ represents the variance of the random classroom-specific intercepts at any given school; The distribution of the residuals associated with the student-level observations is $\epsilon_{ijk} \sim N(0, \sigma^2)$, where σ^2 represents the residual variance.

Model estimation
For the analysis of the dataset first three-level model with a mean structure and random classroom-specific and school-specific intercept model was fitted for all the four learning domains within four section of the dataset, a total of sixteen models. These models includes the fixed effects of language, sex and status of the student (gifted or not). These models also includes two random effects associated with the intercept for each classroom and school and residual associated with each observation. The residuals are assumed to be independent and identically distributed, with constant variance. In the next step it was tested whether the random effects associated with the intercept should be omitted from analysis or not.

2.6 Generalized Linear Mixed models (GLMM)

The generalized linear model (GLM) (McCullagh and Nelder, 1989; Nelder and Wedderburn,1972) is a generalization of ordinary least squares (OLS) regression. It allows the outcome probability distribution to be any member of an exponential family of distributions. By selecting an appropriate link function and outcome probability distribution, many commonly used statistical models can be subsumed under the name of generalized linear models. The generalized linear model is a collection of fixed-effect models that assumes all observations on outcome measures are independent of each other. This assumption is inappropriate for multilevel or hierarchically structured data. As such, the generalized linear model is further extended to two modeling frameworks: 1) marginal models called generalized estimating equations (GEE), and 2) generalized linear mixed effects models (GLMM). GLMM can also be viewed as extensions of the linear multilevel model or linear mixed model. The link functions for GLMM are the same as for GLM; however, the outcome distributions are now conditional distributions or distributions given random effects, U[Wang et al.,2011]. GLMM can be written as

$$g[E(Y|U)] = \eta = X\beta + ZU$$

where g(.) is a link function, β and U are vectors of fixed and random effects parameters, and X and Z are design matrix for fixed and random effects, respectively. As in multilevel linear models, the random effects, U, are usually assumed to have a normal distribution with zero mean and covariance matrix G. In this study the following logistic generalized linear mixed model was used

$$Logit(\pi_{ijk}) = \beta_o + \beta_1 * Gender_{ijk} + \beta_2 * Age_{ijk} + \beta_3 * Intelligencescore(figureseries)_{ijk} + \beta_4 * Intelligencescore(contradiction)_{ijk} + \beta_5 * Language + \mu_k + \mu_{j|k}$$

Where Logit$\pi_{ijk} = log[\frac{\pi_{ijk}}{1-\pi_{ijk}}]$.

Y_{ijk} is the binary indicator for students whether he/she agrees for the follow up period or not

$Gender_{ijk}$ is the indicator variable describing whether the student is male or female

Intelligence score(figure series)$_{ijk}$ is the students score on this variable

Intelligence score(contradiction)$_{ijk}$ is the students score on this variable

Age$_{ijk}$ is the age of the student

Language$_{ijk}$ indicates whether the student speaks Dutch or not

β_0 through β_5 represent the fixed intercept and the fixed effects of the covariates

μ_k is the random effect associated with the intercept for school k

$\mu_{j|k}$is the random effect associated with the intercept for classroom j within school k.

Model Estimation

The inclusion of random effects in linear form of logit scale created addition complexity in estimation of model parameters because the likelihood does not have the closed functional form, thus approximation is necessary. In context of binary data, the numerical approximations such as Gaussian and adaptive Gaussian quadrature were shown to be more accurate compared to an approximation based on marginal functional form (either by conditioning or ignoring the random effects) where their accuracy depend on either response approximately linear and /or there is large number of observations per subject (Molenberghs and Verbeke, 2005). The estimation of the model was done using adaptive Gaussian quadrature method implemented in proc GLIMMIX of sas 9.4 (SAS, 2017).

3 Results

3.1 Exploratory data analysis(EDA)

The dataset used in this study contains information on 1013 children who were at last year of their primary school. The study was conducted in thirty cities in Belgium including 39 schools and 58 classes. Two hundred and nineteen children were identified as gifted and they were asked for a three year follow up. Among these 1013 students , 9 students had complete missing information so in the final analysis they were skipped. (Table 4),(Table 5),(Table 6)and (Table 7) show descriptive statistics corresponding to four groups of students, i.e students who answer correctly with teachers strategy, students who answer correctly with their own strategy, students who answer correctly without written strategy and the last group are those students who answer incorrectly. In those tables, mean of the learning domains are the average score of students according to their sex and gift status. For example, in the learning domain "Numbers"; gifted male students on average correctly solve 21.3 exercise using the teachers strategy (Table 4), 3.61 exercise correctly using their own strategy (Table 5); on average 4.51 exercises correctly without reporting the strategy (Table 6), and 7.9 exercises were incorrect (Table 7). It appear from those tables that average score of students are higher for gifted children than non-gifted for all the learning domains. This pattern is seen both in female and male students. But in (Table 7) the opposite picture was seen. This table summarize average incorrect answer where the mean is higher for non-gifted children than gifted. Highest score for each of the learning domain was also calculated which was 37, 97, 16 and 16 for "Numbers", "Calculations", "Measuring counting" and "Geometry" respectively. First score was attained by non-gifted female students, second score by gifted male students, third score by both gifted male and female students and lastly for the learning domain "Geometry", highest score 16 was achieved by non-gifted male students.

Table 4: Average score of students who follows teacher strategy

Gifted	**Sex**	**No of Observation**	**Mean (numbers)**	**Mean (calculation)**	**Mean (measuring &measuring counting)**	**Mean (geometry)**
No	Male	376	14.91	50.49	6.40	5.10
	female	410	15.06	49.94	6.51	4.93
Yes	Male	124	21.32	63.94	10.19	7.60
	Female	94	20.86	62.66	9.23	6.96

Table 5: Average score of students who use their own strategy

Gifted	Sex	No of Observation	Mean (numbers)	Mean (calculation)	Mean (measuring &measuring counting)	Mean (geometry)
No	Male	376	2.12	34.68	0.45	0.10
	female	410	1.78	34.05	0.45	0.10
Yes	Male	124	3.61	39.48	0.54	0.15
	Female	94	2.60	37.96	0.53	0.18

Table 6: Average score of students who answer without written strategy

Gifted	Sex	No of Observation	Mean (numbers)	Mean (calculation)	Mean (measuring &measuring counting)	Mean (geometry)
No	Male	376	4.60	40.13	2.64	0.72
	female	410	2.86	36.70	1.48	0.48
Yes	Male	124	4.51	44.87	2.13	0.61
	Female	94	2.65	40.47	1.80	0.51

Table 7: Average score of students who answer wrong

Gifted	Sex	No of Observation	Mean (numbers)	Mean (calculation)	Mean (measuring &measuring counting)	Mean (geometry)
No	Male	376	9.62	41.15	3.52	6.14
	female	410	10.40	42.06	4.07	6.50
Yes	Male	124	7.90	41.89	2.16	4.29
	Female	94	7.98	42.01	2.69	4.70

The boxplot of scores of students for the gifted and non-gifted part is given in Appendix Figure(2-17). Both male and female gifted students seem to have higher correct no of responses than non-gifted children. The distribution of responses appears to be roughly symmetric at each level of gifted and sex.

3.2 Cluster analysis

The dataset was comprised of four groups of students according to what strategy they follow in solving the exercises. For each group, fifteen exercises/variables were used which measure students performance in the screening, so a total of sixty variables were used for this cluster analysis. In order to have an idea of the links between these sixty quantitative variables,a hierarchy of the variables are constructed. Figure 1 is the dendogram of variables which shows formation of the clusters. The clustering procedure divides the numeric variables into disjoint or hierarchical clusters. Associated with each cluster is a linear combination of the variables in the cluster which is first principal component (PCA). A total of fifteen clusters was identified. For each cluster, (Table 8) displays the number of variables in the cluster, the total explained variation, and the proportion of the total variance explained by the variables in the cluster. The variance explained by the variables in a cluster is similar to the variance explained by a factor in common factor analysis, but it includes contributions only from the variables in the cluster rather than from all variables. Total variation explained was found to be 41.17 which gives the sum of the explained variation over all clusters. The Proportion of variation was 0.68, indicates that about 68% of the total variation in the data can be accounted for by the fifteen cluster components. (Table 9) shows how the variables are clustered, it also displays the value of R-square for each variable with its own cluster (R own) and the R-square value with its nearest cluster (R next) in the parenthesis. The R-square value for a variable with the nearest cluster should be low if the clusters are well separated. R next values of variables seem to be quite low indicating clusters are well separated. The purpose of the clustering of variables was to identify any underlying structure in the data that supports the grouping of exercises in the learning domains, but the cluster membership of variables (Table 9) doesn't seem to support this grouping of variables.

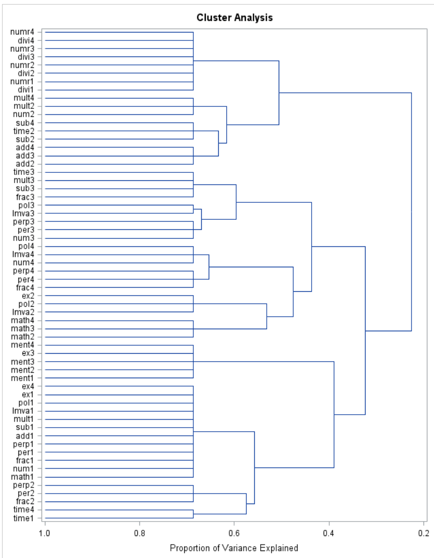

Figure 1: Cluster dendogram of variables

Table 8: Cluster Summary for 15 Clusters

Cluster	Members	Variation Explained	Proportion Explained	Second Eigenvalue
1	12	7.687989	0.6407	0.8147
2	8	6.516528	0.8146	0.6184
3	3	2.233475	0.7445	0.5247
4	5	4.009177	0.8018	0.9908
5	3	2.075178	0.6917	0.7531
6	3	2.350431	0.7835	0.5196
7	3	2.159717	0.7199	0.6322
8	3	1.588034	0.5293	0.9827
9	2	1.704811	0.8524	0.2952
10	3	1.770964	0.5903	0.6582
11	4	2.189947	0.5475	0.8266
12	3	1.846915	0.6156	0.9113
13	3	1.629971	0.5433	0.9234
14	3	1.798558	0.5995	0.6748
15	2	1.616356	0.8082	0.3836

Table 9: Cluster membership with R-Square Values

Cluster	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Variable (R own,R next)	MATH$_1$ (0.65,0.27) NUM$_1$ (0.61,0.16) FRAC$_1$ (0.72,0.17) PER$_1$ (0.63,0.25) PERP$_1$ (0.72,0.24) ADD$_1$ (0.55,0.19) SUB$_1$ (0.60,0.17) MULT$_1$ (0.75,0.24) LMVA$_1$ (0.67,0.18) POL$_1$ (0.54,0.29) EX$_1$ (0.73,0.32) EX$_4$ (0.46,0.26)	DIVI$_1$ (0.59,0.41) NUMR$_1$ (0.93,0.43) DIVI$_2$ (0.81,0.15) NUMR$_2$ (0.93,0.43) DIVI$_3$ (0.74,0.17) NUMR$_3$ (0.93,0.43) DIVI$_4$ (0.60,0.21) NUMR$_4$ (0.93,0.4)	FRAC$_4$ (0.63,0.22) PER$_4$ (0.83,0.17) PERP$_4$ (0.77,0.21)	MENT$_1$ (0.99,0.21) MENT$_2$ (0.99,0.21) MENT$_3$ (0.99,0.21) EX$_3$ (0.01,0.003) MENT$_4$ (0.99,0.21)	NUM$_3$ (0.41,0.1) PER$_3$ (0.77,0.15) PERP$_3$ (0.88,0.23)	MATH$_2$ (0.91,0.08) MATH$_3$ (0.75,0.05) MATH$_4$ (0.67,0.23)	ADD$_2$ (0.87,0.19) ADD$_3$ (0.57,0.29) ADD$_4$ (0.71,0.17)	LMVA$_2$ (0.74,0.05) POL$_2$ (0.77,0.03) EX$_2$ (0.06,0.005)	TIME1 (0.85,0.39) TIME$_4$ (0.85,0.15)	FRAC$_2$ (0.54,0.28) PER$_2$ (0.59,0.04) PERP$_2$ (0.62,0.13)	FRAC$_2$ (0.50,0.1) SUB$_3$ (0.64,0.10) MULT$_3$ (0.59,0.27) TIME$_3$ (0.43,0.08)	NUM$_2$ (0.17,0.03) MULT$_2$ (0.83,0.42) MULT$_4$ (0.83,0.25)	SUB$_2$ (0.73,0.07) TIME$_2$ (0.18,0.03) SUB$_4$ (0.70,0.10)	NUM$_4$ (0.52,0.18) LMVA$_4$ (0.65,0.14) POL$_4$ (0.62,0.15)	LMVA$_3$ (0.80,0.23) POL$_3$(0.03,0.09)

R own= R-square value of each variable with its own cluster, R next= R-square value with its nearest cluster
In the subscript, 1=variable from the group who follows teachers strategy,2=variable from the group who follows own strategy, 3=variable from the group who follows no written strategy, 4=variable from the group who answer incorrectly

3.3 Correlation Matrix

One of the objectives of the study was to assess whether the association between the different exercises or learning domains are similar for gifted and non-gifted children. To answer this question Pearson's correlation matrix for the four different learning domains were calculated both for gifted and non-gifted children. A formal testing of correlation was done which seeks to reject the null hypothesis that true correlation between the variables is zero. P-value of each tests is indicated in the tables within parenthesis and all the P-values are highly significant referring to the rejection of null hypothesis. For the first group who answer correctly and follows teachers strategy (Table 10) and (Table 11) correlation among the learning domains seem to be quite positive for both gifted and non-gifted children. For the second group who answer correctly but follows their own strategy (Table 12) and (Table 13) correlation among the learning domains seem to be positive but not so strongly for both gifted and non-gifted children. For the third group who answer correctly but without any written strategy (Table 14) and (Table 15) correlation among the learning domains again seem to be positive both for gifted and non-gifted children. At last for the last group who answer incorrectly (Table 16) and (Table 17) correlation among the learning domains again seem to be quite similar both for gifted and non-gifted children. Appendix Figure(18-25) are graphical presentation of theses correlations. Intensity of the colors in the graphs show the strengthens of the correlation. Blue color means variables are highly correlated, white means no correlation and red means inverse correlation. Graphs show that the association between the different exercises or learning domains was similar for gifted and non-gifted children. After that formal correlation tests was conducted between two independent group (gifted and non-gifted students) of different sample sizes. The aim was to test the null hypothesis, "the two correlations calculated by gifted and non-gifted group for any two learning domains are not significantly different". This test is recommended when the correlations are conducted on the same variables by two different groups, and if both correlations are found to be statistically significant. To do this, correlation coefficient values, or r values of the groups, were transformed into z scores. This transformation, also known as Fishers r to z transformation, was done so that the z scores can be compared and analyzed for statistical significance by determining the observed Z test statistic. The test statistic that was used is

$$Z_{observed} = \frac{(Z_1 - Z_2)}{(\sqrt{\frac{1}{N_1 - 3} + \frac{1}{N_2 - 3}})}$$

where Z_1 and Z_2 correspond to the Fishers transformation of the correlation coefficients (r) for the two groups and N_1 and N_2 are the sample sizes of the groups.

Once the observed Z value has been determined, statistical significance can be assessed by checking to see if the observed value is greater than the critical value(± 1.96). If $Z_{observed}$ falls into the rejection region and is greater than critical value; then the null hypothesis that the two correlations are not significantly different is rejected. (Table 18) and (Table 19) show the observed Z value for all the leaning domains where N_1 and N_2 was 219 and 786 for gifted and non-gifted group respectively. It was observed that the hypothesis was rejected for the group of students who follows no written strategy. For example correlation between domains Numbers and Calculation for gifted group were 0.36 (Table 14) and 0.47 for non-gifted (Table 15). The observed Z value was -2 which falls into critical region with 5% significance level (Table 19), it means the correlation between these domains was different for gifted and non-gifted students.

Table 10: Gifted Children who answers correctly and follows teachers strategy

	Numbers	**Calculations**	**Measuring counting**	**Geometry**
Numbers	1.00	0.78 (0.00)	0.67(0.00)	0.54 (0.00)
Calculations		1.000	0.68(0.00)	0.50(0.007)
Measuring counting			1.00	0.65(0.00)
Geometry				1.00

Table 11: Non-gifted Children who answers correctly and follows teachers strategy

	numbers	**Calculations**	**Measuring counting**	**Geometry**
Numbers	1.00	0.76(0.00)	0.74(0.00)	0.55(0.00)
Calculations		1.00	0.71(0.00)	0.53(0.00)
Measuring counting			1.00	0.59(0.00)
Geometry				1.00

Table 12: Gifted Children who answers correctly but follows own strategy

	numbers	**Calculations**	**Measuring counting**	**Geometry**
Numbers	1.00	0.17(0.009)	0.16(0.01)	0.20(0.002)
Calculations		1.00	0.09(0.01)	0.04(0.053)
Measuring counting			1.00	0.54(0.00)
Geometry				1.00

Table 13: Non-gifted Children who answers correctly but follows own strategy

	numbers	**Calculations**	**Measuring counting**	**Geometry**
Numbers	1.00	0.26(0.00)	0.27(0.00)	0.19(0.00)
Calculations		1.00	0.11(0.001)	0.08(0.02)
Measuring counting			1.00	0.44(0.00)
Geometry				1.00

Table 14: Gifted Children who answers correctly but no written strategy

	numbers	**Calculations**	**Measuring counting**	**Geometry**
Numbers	1.00	0.36(0.00)	0.44(0.00)	0.16(0.01)
Calculations		1.00	0.19(0.002)	0.13()0.03
Measuring counting			1.00	0.51(0.023)
Geometry				1.00

Table 15: Non-gifted Children who answers correctly but no written strategy

	numbers	**Calculations**	**Measuring counting**	**Geometry**
Numbers	1.00	0.47(0.00)	0.56(0.00)	0.37(0.00)
Calculations		1.00	0.33(0.00)	0.27(0.00)
Measuring counting			1.00	0.58(0.00)
Geometry				1.00

Table 16: Gifted Children who answers incorrectly

	numbers	**Calculations**	**Measuring counting**	**Geometry**
Numbers	1.00	0.11(0.08)	0.36(0.00)	0.44(0.00)
Calculations		1.00	0.14(0.03)	0.16(0.01)
Measuring counting			1.00	0.49(0.00)
Geometry				1.00

Table 17: Non-gifted Children who answers incorrectly

	numbers	**Calculations**	**Measuring counting**	**Geometry**
Numbers	1.00	0.18(0.00)	0.50(0.00)	0.44(0.00)
Calculations		1.00	0.18(0.003)	0.12(0.005)
Measuring counting			1.00	0.37(0.00)
Geometry				1.00

Table 18: Comparing correlation coefficient between gifted and non-gifted group

	Students who follow Teachers strategy			Students who follows own strategy		
	Calculation	Measuring counting	Geometry	Calculation	Measuring counting	Geometry
Numbers	0.62	-0.13	0.42	-1.28	-1.57	0.14
Calculation		-0.82	-0.85		-0.28	-0.57
Measuring counting			1.42			1.85

Table 19: Comparing correlation coefficient between gifted and non-gifted group

	Students who follow no written strategy			Students who answer incorrectly		
	Calculation	Measuring counting	Geometry	Calculation	Measuring counting	Geometry
Numbers	-2	-2.28	-3.14	-1	-1.42	0
Calculation		2.14	-2		-0.57	0.57
Measuring counting			-1.42			1.14

3.4 Linear Mixed Models

The data is this study is a three-level clustered data set: students (units of analysis) were nested within classes and classes are nested within schools. In this study, number of correctly solving math problem by students within the same class and classes within the same school are likely to be correlated because they share same environment. When LMM was fitted first likelihood ratio test(LR) was done to see if random intercept for classroom are needed or not. Likelihood ratio test statistic was calculated by subtracting the value of the 2 REML log-likelihood for the reference model(model include random classroom effect) from the value for the nested model (excluding the random classroom effects). Both the nested and reference models was fitted using REML estimation. Appendix (Table 31) and (Table 32) show the results of LR tests, as the tests are highly significant, the random effects associated with classrooms was retained in models. The random school effects was also retained, without testing them, to reflect the hierarchical structure of the data in the model specification. (Table 20),(Table 21),(Table 22) and (Table 23) show the parameter estimates of models which indicate that gifted children group significantly explain the variability of the four learning domains. It also explains that keeping language and sex constant, score for gifted children are higher most of the time for the four groups. It means that gifted children are experiencing less problems with mathematics in primary school than non-gifted children. An opposite picture was seen in (Table 23), where the coefficient for gifted children is negative meaning that scores for incorrect answer is less for gifted children than non-gifted. (Table 21) shows the parameter estimates for the group of students who use their own strategy. As the coefficient of the variable "Gifted" is positive, it means gifted students are using their own method more frequently than non-gifted.

Table 20: Parameter estimates for students who answer correctly and follows teachers strategy

	Numbers	Calculations	Measuring counting	Geometry
Covariate	Estimate(S.E,P.V)	Estimate(S.E,P.V)	Estimate(S.E,P.V)	Estimate(S.E,P.V)
(Intercept)	13.27(0.77,0.00)	47.79(1.69,0.00)	5.59(0.37,0.00)	4.31(0.27,0.00)
Gifted	5.45 (0.48,0.00)	12.46(1.07,0.00)	3.07(0.24,0.00)	2.23(0.17,0.00)
Language	1.94(0.61,0.001)	3.01(1.36,0.02)	1.02(0.30,0.00)	0.87(0.22,0.00)
Sex	0.37(0.38,0.32)	-0.36(0.85,0.66)	0.002(0.19,0.98)	-0.16(0.14,0.25)

S.E=Standard error,P.V=P value

Table 21: Parameter estimates for students who answer correctly and follows their own strategy

	Numbers	Calculations	Measuring counting	Geometry
Covariate	Estimate(S.E,P.V)	Estimate(S.E,P.V)	Estimate(S.E,P.V)	Estimate(S.E,P.V)
(Intercept)	2.15(0.23,0.00)	34.23(0.88,0.00)	0.43(0.08,0.00)	0.10(0.04,0.00)
Gifted	1.01 (0.13,0.00)	4.73(0.63,0.00)	0.05(0.05,0.35)	0.03(0.02,0.12)
Language	0.08(0.17,0.60)	0.51(0.79,0.52)	0.03(0.06,0.65)	-0.007(0.03,0.80)
Sex	-1.40(0.10,0.00)	-1.04(0.50,0.03)	0.01(0.04,0.71)	0.01(0.02,0.60)

S.E=Standard error,P.V=P value

Table 22: Parameter estimates for students who answer correctly but without written strategy

	Numbers	Calculations	Measuring counting	Geometry
Covariate	Estimate(S.E,P.V)	Estimate(S.E,P.V)	Estimate(S.E,P.V)	Estimate(S.E,P.V)
(Intercept)	4.67(0.42,0.00)	40.14(1.02,0.00)	2.79(0.19,0.00)	0.38(0.06,0.00)
Gifted	0.36(0.21,0.09)	4.60(0.73,0.00)	-0.03(0.13,0.79)	-0.02(0.04,0.63)
Language	-0.03(0.27,0.90)	-0.05(0.92,0.94)	-0.31(0.17,0.06)	0.03(0.05,0.55)
Sex	-1.74(0.17,0.00)	-3.66(0.58,0.00)	-0.92(0.10,0.00)	-0.16(0.03,0.00)

S.E=Standard error,P.V=P value

Table 23: Parameter estimates for students who answer incorrectly

	Numbers	Calculations	Measuring counting	Geometry
Covariate	Estimate(S.E,P.V)	Estimate(S.E,P.V)	Estimate(S.E,P.V)	Estimate(S.E,P.V)
(Intercept)	10.45(0.55,0.00)	40.90(0.92,0.00)	3.67(0.22,0.00)	6.98(0.29,0.00)
Gifted	-2.53(0.35,0.00)	0.65(0.64,0.30)	-1.48(0.16,0.00)	-1.95(0.20,0.00)
Language	-0.84(0.45,0.06)	0.32(0.81,0.68)	-0.12(0.20,0.55)	-0.93(0.25,0.00)
Sex	0.75(0.28,0.00)	0.46(0.51,0.36)	0.51(0.13,0.00)	0.31(0.16,0.04)

S.E=Standard error,P.V=P value

(Table 24),(Table 25),(Table 26) and (Table 27) show random intercept variance estimates along with their standard errors for the four group of students. Roughly speaking, the variability between intercept is higher for the group who follows teachers strategy.

Table 24: Covariance Parameter Estimates for students who follows teachers strategy

		Numbers	Calculation	Measuring counting	Geometry
Cov Parm	Subject	Estimate(S.E)	Estimate(S.E)	Estimate(S.E)	Estimate(S.E)
Intercept	Schoolnumber	7.1968(4.7003)	25.0404(13.592)	2.0156(0.9022)	1.144(0.43)
Intercept	Classgroup (Schoolnumber)	7.5113(3.4420)	18.3110(9.3904)	1.0134(0.5202)	0.387(0.216)
Residual		34.3736(1.6008)	173.06 (8.0634)	8.6070(0.4007)	4.6172(0.2148)

S.E=Standard error

Table 25: Covariance Parameter Estimates for students who follows own strategy

		Numbers	Calculation	Measuring counting	Geometry
Cov Parm	Subject	Estimate(S.E)	Estimate(S.E)	Estimate(S.E)	Estimate(S.E)
Intercept	Schoolnumber	0.1700(0.1241)	6.1784(2.8932)	0.007(0.020)	0.005(0.003)
Intercept	Classgroup (Schoolnumber)	0.2731(0.1287)	3.4848(2.0741)	0.03927(0.021)	0.002(0.003)
Residual		2.7638(0.1288)	60.4812 (2.8159)	0.4776(0.0222)	0.1023(0.0047)

S.E=Standard error

Table 26: Covariance Parameter Estimates for students who follows no written strategy

		Numbers	Calculation	Measuring counting	Geometry
Cov Parm	Subject	Estimate(S.E)	Estimate(S.E)	Estimate(S.E)	Estimate(S.E)
Intercept	Schoolnumber	1.0827(0.6161)	8.1924(3.4450)	0.1464(0.1699)	0.006734(0.010)
Intercept	Classgroup (Schoolnumber)	1.6636(0.58)	4.1115(2.5397)	0.4461(0.1805)	0.0238(0.012)
Residual		6.7363(0.3136)	80.5883 (3.7538)	2.7810(0.1296)	0.3316(0.0154)

S.E=Standard error

Table 27: Covariance Parameter Estimates for students who answer incorrectly

		Numbers	Calculation	Measuring counting	Geometry
Cov Parm	Subject	Estimate(S.E)	Estimate(S.E)	Estimate(S.E)	Estimate(S.E)
Intercept	Schoolnumber	2.5181(1.2833)	7.6106(3.4512)	0.04927(0.1079)	0.9191(0.3838)
Intercept	Classgroup (Schoolnumber)	3.3552(1.2158)	5.1553(2.4821)	0.4318(0.1652)	0.4241(0.2500)
Residual		19.1459(0.8911)	62.0989 (2.8900)	4.1789(0.1944)	6.1416(0.2860)

S.E=Standard error

3.5 Generalized Linear Mixed Models

A generalized linear mixed effect model was fitted to answer the research question whether or not students not participating in the second phase (a 3 year follow-up study) of the research differ from students that do participate (e.g. in terms of gender, age, intelligence score, etc.). (Table 28) shows the type 3 test for fixed effect of logistic generalized linear mixed model with only random intercept. All covariates are insignificant at 5% significant level. (Table 29) gives the estimated change of response in logit scale associated by unit change in value of covariate. By exponentiating these estimates correspond to change of odds. As no covariate was significant, so students not participating in the second phase of the research seem not to be different from students that do participate. Random intercept variances are given in(Table 30). Roughly speaking, the variability between intercept for schoolnumber and classgroup nested in school are quite similar (1.55 and 1.14 respectively).

Table 28: Type 3 Test for Fixed Effect in Random Effect Model

Effect	**Num DF**	**Den DF**	**F Value**	**Pr >F**
Age	1	160	0.03	0.8535
Sex	1	160	3.57	0.0607
Intelligence figure series	1	160	0.12	0.7248
Intelligence contradiction	1	160	0.76	0.3849
Language	1	160	0.64	0.5207

Table 29: Fixed Effects Parameter Estimate for Random Mixture Model

Effect	**Estimate**	**Standard Error**	**DF**	**t value**	**P value**
Intercept	-2.8764	3.1507	15	-0.91	0.3757
Age	0.03871	0.2092	160	0.18	0.8535
Sex(male)	0.7901	0.4183	160	1.89	0.0607
Intelligence figure series	-0.00716	0.02029	160	-0.35	0.7248
Intelligence contradiction	0.04539	0.05209	160	0.87	0.3849
Language	0.5198	0.8076	160	0.64	0.5207

Table 30: Covariance Parameter Estimates

Cov Parm	**Subject**	**Estimate**	**StandardError**
Intercept	Schoolnumber	2.1876	1.55
Intercept	Classgroup(Schoolnumber)	1.1458	1.1426

4 Discussions and Conclusions

To answer different kind of questions included in this study, several methodologies were applied. To see if there is an underlying structure in the data, Clustering of variable was performed. Fifteen clusters of variables were discovered with highest number of variables included in cluster one (twelve variables). The purpose of the clustering of variables was to identify any underlying structure in the data that supports the grouping of exercises in the learning domains, but the cluster membership of variables doesn't support this grouping of variables. Second objective of the study was to see if the association between the different learning domains similar for gifted and non-gifted children. Correlation matrix for the four different learning domains were calculated both for gifted and non-gifted children to answer this question. The pattern of the association between learning domains seem to be similar between gifted and non-gifted students except for the group of students who follows no written strategy. For this group of students, the null hypothesis that correlations which were calculated between learning domains (for gifted and non-gifted students) are not significantly different was rejected. To answer the questions do gifted children have less or more problems with mathematics in primary school than non-gifted children and do gifted children use their own method to solve a mathematical problem more frequently than non-gifted children, Linear mixed models with random intercept were applied for the four learning domains which also takes into account the hierarchical nature of the data. Results show that keeping language and sex constant, score for gifted children were higher most of the time than the other group, but the scores for number of incorrect answer was higher for non-gifted group. Also results explains that gifted children use their own method to correctly solve a mathematical problem more frequently than non-gifted children. Lastly to answer if gifted students not participating in the second phase (a 3 year follow-up study) of the research differ from students that do participate terms of gender, age, intelligence score, etc., a generalized linear mixed effect model(GLMM) was fitted as the response was a binary variable whether the gifted student agrees to the follow up or not. Type 3 test for fixed effect of logistic generalized linear mixed model with only random intercept showed that all covariate were insignificant at 5%significant level. As no covariates were significant students not participating in the second phase of the research seem not to be different from students that do participate.

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Appendix -Tables and Figures

Table 31: Likelihood ratio test for classroom random intercept

	Students who follow teacher strategy				Students who follow own strategy			
	Numbers	Calculation	Measuring counting	Geometry	Numbers	Calculation	Measuring counting	Geometry
LR statistic	37.8	14.6	16.8	10.6	13.9	6.7	10.3	0.9
P value	0.00000	0.00006	16.8	0.00056	0.00009	0.00482	0.00066	0.017139

Table 32: Likelihood ratio test for classroom random intercept

	Students who follow no written strategy				Students who answer incorrectly			
	Numbers	Calculation	Measuring counting	Geometry	Numbers	Calculation	Measuring counting	Geometry
LR statistic	51.5	5.6	27.8	9.3	37.3	1092.6	21.2	7.7
P value	0.00000	0.00898	0.00000	0.00114	0.00000	0.00000	0.00000	0.00276

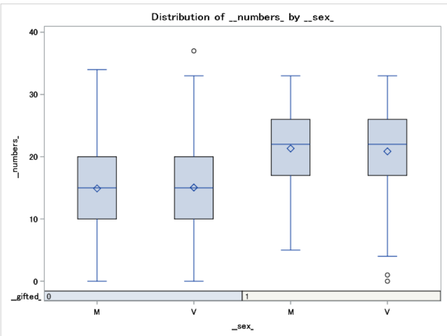


Figure 2: Boxplots of Numbers score of the group who follows teachers strategy for levels of gifted by sex

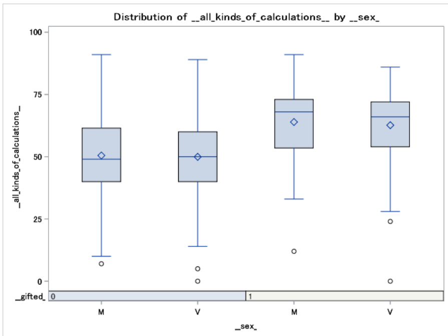


Figure 3: Boxplots of Calculation score of the group who follows teachers strategy for levels of gifted by sex

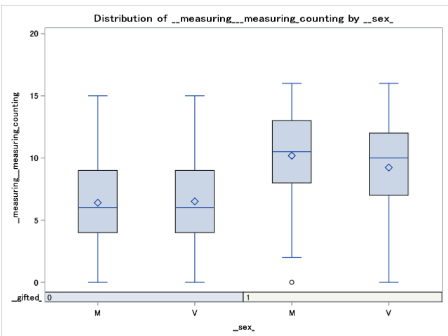


Figure 4: Boxplots of Counting score of the group who follows teachers strategy for levels of gifted by sex

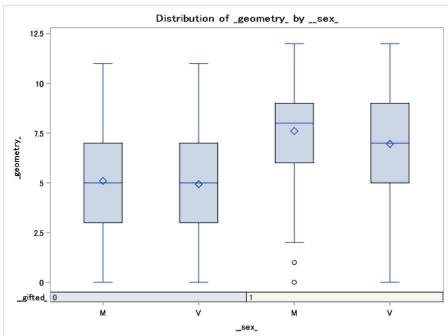


Figure 5: Boxplots of Geomery score of the group who follows teachers strategy for levels of gifted by sex

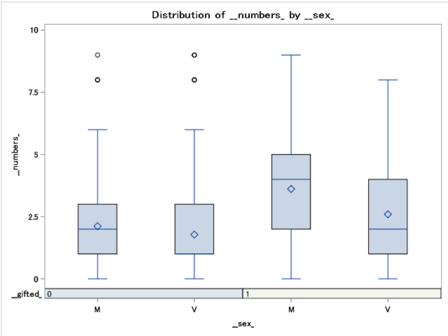


Figure 6: Boxplots of Numbers score of the group who follows own strategy for levels of gifted by sex

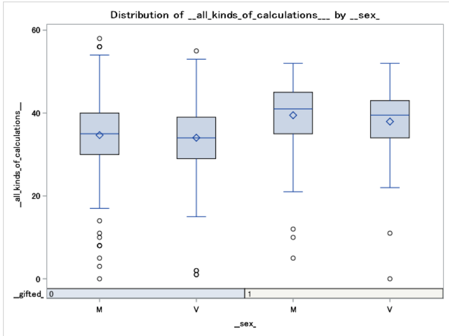


Figure 7: Boxplots of Calculation score of the group who follows own strategy for levels of gifted by sex

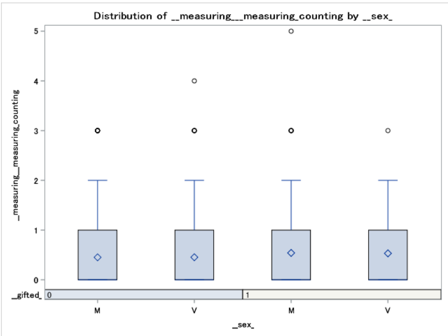


Figure 8: Boxplots of Counting score of the group who follows own strategy for levels of gifted by sex

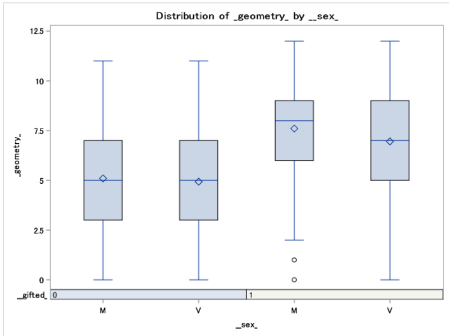


Figure 9: Boxplots of Geometry score of the group who follows own strategy for levels of gifted by sex

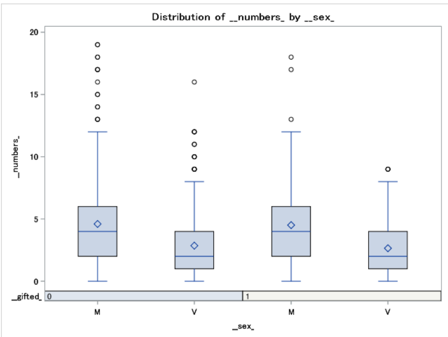


Figure 10: Boxplots of Numbers score of the group who avoid written strategy for levels of gifted by sex

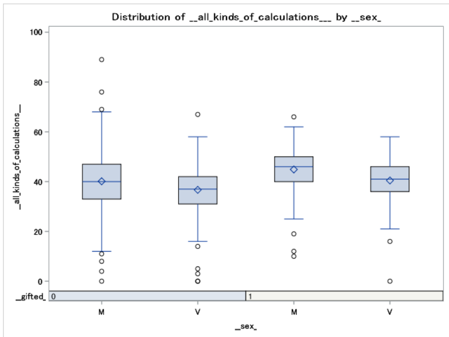
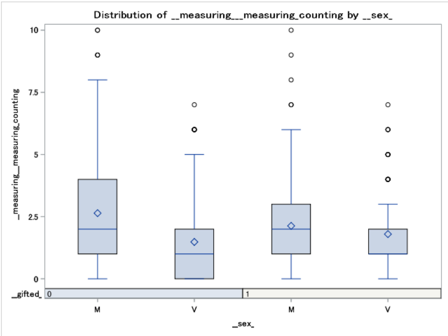


Figure 11: Boxplots of Calculation score of the group who avoid written strategy for levels of gifted by sex



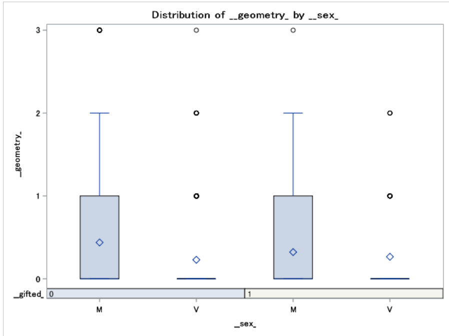
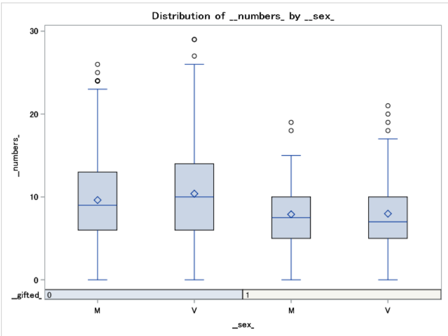


Figure 12: Boxplots of Counting score of the group who avoid written strategy for levels of gifted by sex

Figure 13: Boxplots of Geometry score of the group who avoid written strategy for levels of gifted by sex



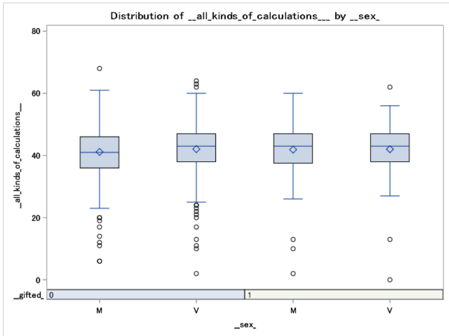
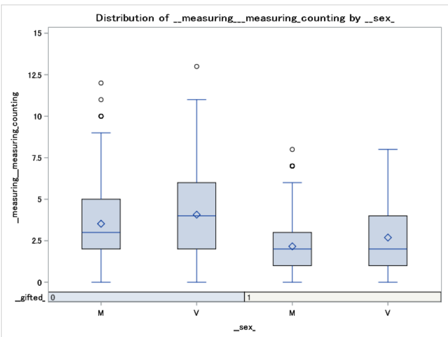


Figure 14: Boxplots of Numbers score of the group who answer incorrectly for levels of gifted by sex

Figure 15: Boxplots of Calculation score of the group who answer incorrectly for levels of gifted by sex



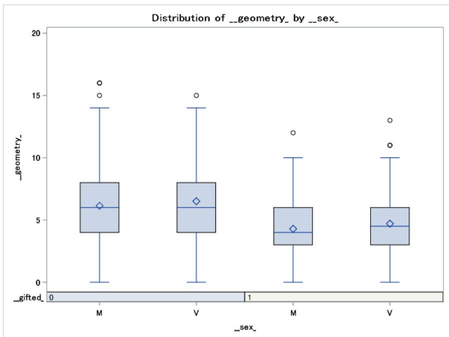
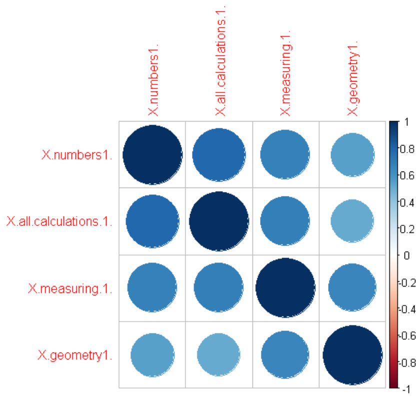


Figure 16: Boxplots of Counting score of the group who answer incorrectly for levels of gifted by sex

Figure 17: Boxplots of Geometry score of the group who answer incorrectly for levels of gifted by sex



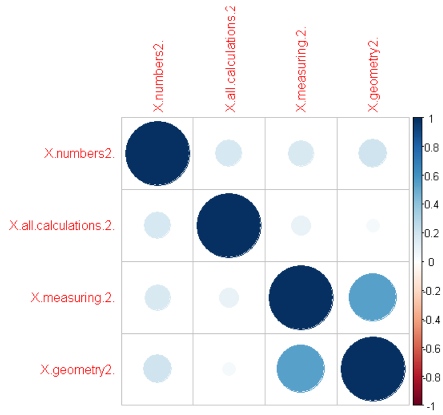
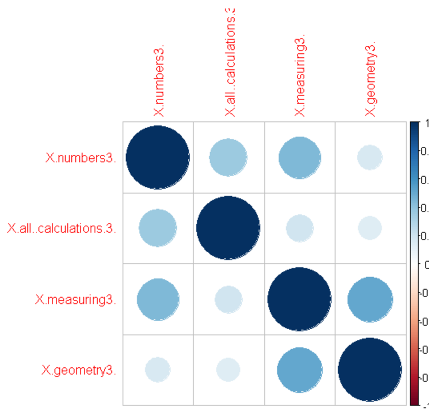


Figure 18: Correlation between variables of gifted children for the group who follows teachers strategy

Figure 19: Correlation between variables of gifted children for the group who follows their own strategy



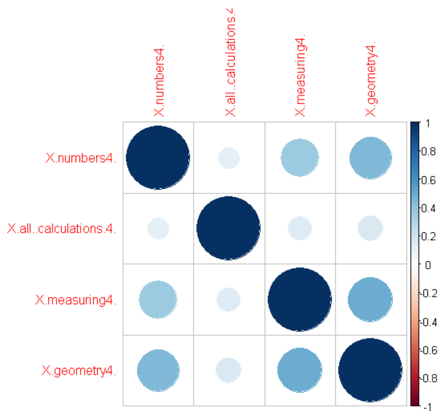


Figure 20: Correlation between variables of gifted children for the group with no written strategy

Figure 21: Correlation between variables of gifted children for the group who answer incorrectly

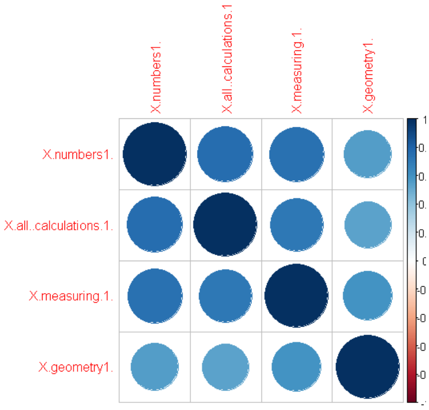


Figure 22: Correlation between variables of non-gifted children for the group who follows teachers strategy

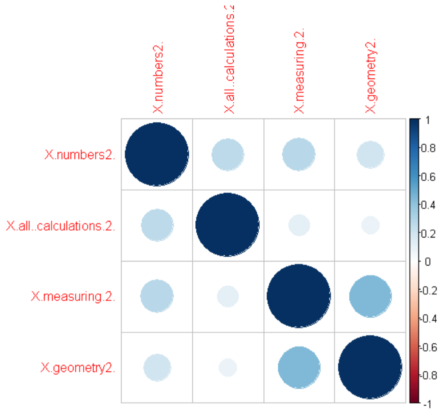


Figure 23: Correlation between variables of non-gifted children for the group who follows their own strategy

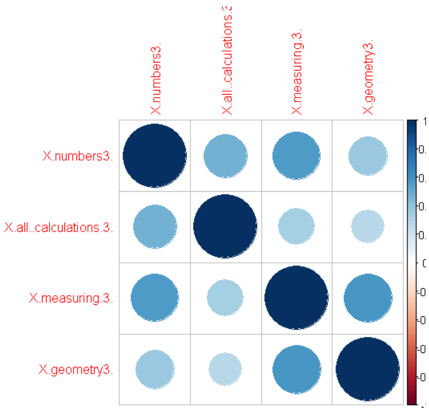


Figure 24: Correlation between variables of non-gifted children for the group with no written strategy

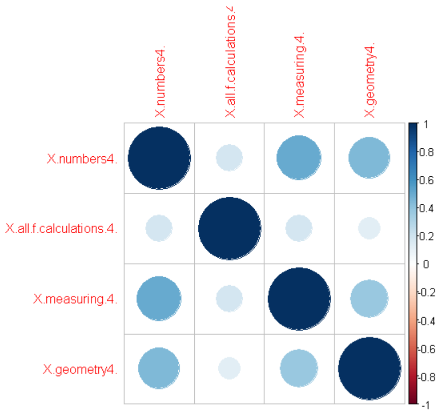


Figure 25: Correlation between variables of non-gifted children for the group who answer incorrectly

Appendix - R/SAS code

```
*******************************
clustering of variables
*******************************

PROC IMPORT OUT= WORK.classroom5
DATAFILE= "G:\SEMESTER IV\THESIS\cluster2.csv"
DBMS=CSV REPLACE;
GETNAMES=YES;
DATAROW=2;
RUN;
proc varclus data=WORK.classroom5 maxclusters=15  outtree=tree;
var math1--ex4;
run;
*******************************
association of variables
**********************************
a<-cor(relation[,c(20,26,30,32)])
cor.test(relation$X.numbers1.,relation$X.all.calculations.1.)
cor.test(relation$X.numbers1.,relation$X.measuring.1. )
cor.test(relation$X.numbers1.,relation$X.geometry1.)
cor.test(relation$X.all.calculations.1.,relation$X.measuring.1.)
cor.test(relation$X.all.calculations.1.,relation$X.geometry1.)
cor.test(relation$X.geometry1.,relation$X.measuring.1.)
b<-cor(relation[,c(39,45,49,51)])
c<-cor(relation[,c(58,64,68,70)])
d<-cor(relation[,c(77,83,87,89)])

library(corrplot)
corrplot(a)
***********************************************
LMM for students who follows teachers strategy;
***********************************************
PROC IMPORT OUT= WORK.classroom1
DATAFILE= "G:\SEMESTER IV\THESIS\long1.csv"
DBMS=CSV REPLACE;
GETNAMES=YES;
DATAROW=2;
RUN;
```

```
title "Model 4.2";
proc mixed data = work.classroom1 noclprint covtest;
class Classgroup Schoolnumber sex;
model numbers= gifted sex language / solution;
random intercept / subject = Schoolnumber v vcorr;
random intercept / subject = Classgroup(Schoolnumber);
run;

proc mixed data = work.classroom1 noclprint covtest;
class Classgroup Schoolnumber sex;
model all_kinds_of_calculations= gifted sex language / solution;
random intercept / subject = Schoolnumber v vcorr;
random intercept / subject = Classgroup(Schoolnumber);
run;

proc mixed data = work.classroom1 noclprint covtest;
class Classgroup Schoolnumber sex;
model __measuring___measuring_counting= gifted sex language / solution;
random intercept / subject = Schoolnumber v vcorr;
random intercept / subject = Classgroup(Schoolnumber);
run;

proc mixed data = work.classroom1 noclprint covtest;
class Classgroup Schoolnumber sex;
model _geometry_= gifted sex language / solution;
random intercept / subject = Schoolnumber v vcorr;
random intercept / subject = Classgroup(Schoolnumber);
run;

title summary statistics for numbers by gift and sex;
proc means data=work.classroom1 maxdec=2;
class gifted sex;
var numbers;
run;


******************************************
LMM for students who follow own strategy;
******************************************
PROC IMPORT OUT= WORK.classroom2
DATAFILE= "G:\SEMESTER IV\THESIS\long2.csv"
```

```
DBMS=CSV REPLACE;
GETNAMES=YES;
DATAROW=2;
RUN;

title "Model 4.2";
proc mixed data = work.classroom2 noclprint covtest;
class Classgroup Schoolnumber sex;
model __numbers_= gifted sex language / solution;
random intercept / subject = Schoolnumber v vcorr;
random intercept / subject = Classgroup(Schoolnumber);
run;

proc mixed data = work.classroom2 noclprint covtest;
class Classgroup Schoolnumber sex;
model __all_kinds_of_calculations___= gifted sex language / solution;
random intercept / subject = Schoolnumber v vcorr;
random intercept / subject = Classgroup(Schoolnumber);
run;

proc mixed data = work.classroom2 noclprint covtest;
class Classgroup Schoolnumber sex;
model __measuring___measuring_counting= gifted sex language / solution;
random intercept / subject = Schoolnumber v vcorr;
random intercept / subject = Classgroup(Schoolnumber);
run;

proc mixed data = work.classroom2 noclprint covtest;
class Classgroup Schoolnumber sex;
model __geometry_= gifted sex language / solution;
random intercept / subject = Schoolnumber v vcorr;
random intercept / subject = Classgroup(Schoolnumber);
run;

title summary statistics for numbers by gift and sex;
proc means data=work.classroom2 maxdec=2;
class gifted sex;
var numbers;
run;

***********************************************
```

```
LMM for students who follows no written strategy;
************************************************

PROC IMPORT OUT= WORK.classroom3
DATAFILE= "G:\SEMESTER IV\THESIS\long3.csv"
DBMS=CSV REPLACE;
GETNAMES=YES;
DATAROW=2;
RUN;

title "Model 4.2";
proc mixed data = work.classroom3 noclprint covtest;
class Classgroup Schoolnumber sex;
model __numbers_= gifted sex language / solution;
random intercept / subject = Schoolnumber;
random intercept / subject = Classgroup_(__Schoolnumber);
run;

proc mixed data = work.classroom3 noclprint covtest;
class Classgroup Schoolnumber sex;
model __all_kinds_of_calculations_= _gifted sex language / solution;
random intercept / subject = Schoolnumber v vcorr;
random intercept / subject = Classgroup(Schoolnumber);
run;

proc mixed data = work.classroom3 noclprint covtest;
class Classgroup Schoolnumber sex;
model __measuring___measuring_counting= gifted sex language / solution;
random intercept / subject = Schoolnumber v vcorr;
random intercept / subject = Classgroup(Schoolnumber);
run;

proc mixed data = work.classroom3 noclprint covtest;
class Classgroup Schoolnumber sex;
model __geometry_=gifted sex language / solution;
random intercept / subject = Schoolnumber v vcorr;
random intercept / subject = Classgroup(Schoolnumber);
run;

title summary statistics for numbers by gift and sex3;
proc means data=work.classroom3 maxdec=2;
```

```
class gifted sex;
var numbers;
run;

*****************************************
LMM for students who answer incorrectly;
*****************************************
PROC IMPORT OUT= WORK.classroom4
DATAFILE= "G:\SEMESTER IV\THESIS\long4.csv"
DBMS=CSV REPLACE;
GETNAMES=YES;
DATAROW=2;
RUN;

title "Model 4.2";
proc mixed data = work.classroom4 noclprint covtest;
class Classgroup Schoolnumber sex;
model __numbers_= gifted sex language / solution;
random intercept / subject = Schoolnumber v vcorr;
random intercept / subject = Classgroup(Schoolnumber);
run;

proc mixed data = work.classroom4 noclprint covtest;
class Classgroup Schoolnumber sex;
model __all_kinds_of_calculations___= gifted sex language / solution;
random intercept / subject = Schoolnumber v vcorr;
random intercept / subject = Classgroup(Schoolnumber);
run;

proc mixed data = work.classroom4 noclprint covtest;
class Classgroup Schoolnumber sex;
model __measuring___measuring_counting= gifted sex language / solution;
random intercept / subject = Schoolnumber v vcorr;
random intercept / subject = Classgroup(Schoolnumber);
run;

proc mixed data = work.classroom4 noclprint covtest;
class Classgroup Schoolnumber sex;
model __geometry_= gifted sex language / solution;
random intercept / subject = Schoolnumber v vcorr;
random intercept / subject = Classgroup(Schoolnumber);
```

```
run;

title summary statistics for numbers by gift and sex4;
proc means data=work.classroom4 maxdec=2;
class gifted sex;
var numbers;
run;

******************************
GLMM
*****************************;

PROC IMPORT OUT= WORK.last
DATAFILE= "G:\SEMESTER IV\THESIS\last question sas.csv"
DBMS=CSV REPLACE;
GETNAMES=YES;
DATAROW=2;
RUN;
proc glimmix data=WORK.last method=quad(qpoints=5);
class Classgroup Schoolnumber sex;
model gifted_twice = Age sex intelligencescreening_figure
intelligence_screening___contr language/ dist=binomial link=logit solution;
random intercept  / subject = Schoolnumber;
random intercept / subject = Classgroup(Schoolnumber);
run;
```

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Gifted Children Underachieving in Mathematics

Richting: **Master of Statistics-Biostatistics**
Jaar: **2018**

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Sultana, Nahid

Datum: **15/06/2018**