Preface

This thesis was made as a final project for the study "applied economic sciences: business engineering" at Hasselt University. As several people have provided me with support both academically and personally over the years I would like to thank them in this short introduction.

First, I would like to thank my promotor Mark Vancauteren for his valuable input, useful feedback and overall help in the process of writing this thesis. Secondly, I would like to thank my family and friends for all the support during the past five years and making sure that I could focus on my studies

Abstract

Ever since its creation in 1964, the capital asset pricing model (CAPM) has been subjected to criticism of the academic community. This is mainly due to its mixed results in empirical tests and its set of unrealistic assumptions. A worrying issue as the CAPM is often taught to students in introductory courses to finance, used in government regulation and applied by companies for investment analysis.

In this thesis, the validity of the basic CAPM is researched in the setting of the Belgian stock market, both from a theoretical as an empirical perspective. In the first chapters of this thesis, the theory and assumptions behind the CAPM and testing the CAPM are handled. It is clarified that the CAPM theory still remains intact when some of its underlying assumptions do not hold in reality. More specifically, the implications of prospect theory, other restrictive assumptions and non-normality of asset returns on the CAPM are briefly summarised. The CAPM remains theoretically intact in the sense that the concepts of the security market line (SML) and capital market line (CML) still hold, but that CAPM is probably only an approximating model at best due to the non-normality of asset returns. Furthermore, it is shown that the equilibrium prices of assets will be different under prospect theory than under an expected utility framework.

In the empirical part and main focus of this thesis, the CAPM is tested on a sample of the 50 largest Belgian companies by market capitalisation using the reverse engineering approach invented by Levy and Roll (2010). It is shown that this test is theoretically more correct than the often used double-pass regression test and that the CAPM cannot be rejected empirically in the Belgian setting. This does not mean that the CAPM is correct in the Belgian setting as there still exist several issues with the reverse engineering test in this thesis, namely invariability of the asset weights, invariability of the correlation matrix, a long test horizon and no adjustments for the use of a market proxy.

Finally, it is shown that under the CAPM theory, one should use the SML relationship to estimate the expected returns of stocks instead of the historical average returns for purposes such as portfolio optimisation.

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1. Introduction

Stock and debt markets have been around for centuries. The first known instance of a common gathering place resembling an equity market dates back as far as Ancient Rome. During the second century B.C., flourishing commerce throughout the republic caused wealth to accumulate in Rome, giving rise to a demand for capital investments. The availability of commercial credit, a set of comprehensive laws on property rights and the willingness of individuals to take on risk to earn a return, caused investors to gather on the public forum. As the Roman Empire's influence fell, so did it's primitive stock markets. After the fall of the Roman Empire, many monotheistic cultures criminalised the practice of lending as usury and condemned the activity of "making money from money". It wasn't until high medieval times that financial markets emerged again in the Italian city states (Smith, 2003).

In medieval Venice, money lenders filled in important gaps left by the already established banking system (Smith, 2003). At first, moneylenders traded debts with each other, making it possible for them to exchange high-risk, high-interest yielding loans with other lenders. Later, the moneylenders started trading government issued debt and selling off loans to non-moneylending customers. In the 1300's the Venetians became the absolute leaders in this field and were the first to start trading securities issued by other governments (Puga & Trefler, 2012).

The creation of the first formal permanent "stock exchange" dates as far back as 1531. In the city of Antwerp, moneylenders and brokers would come together to trade government, business and even individual debt issues (Smith, 2003). While the idea of these so called "beurzen" spread fast to other cities like Ghent and Rotterdam, these gathering places were still fundamentally different from the stock exchanges we know today. Back then these exchanges solely dealt in promissory notes and bonds; the asset class that is nowadays known as stocks did not exist yet. There were many types of business-financier partnerships that produce cash flows like stocks do, but official shares did not exist (Beattie, 2017).

The first official issue of common stock was done by the East-Indian trading company back in 1602 on the stock exchange of Amsterdam, making it the first publicly traded company. This company would pay dividends on the proceeds of their voyages to their investors, making these shares the dividend yielding assets we know today (de la Vega, 1688).

Although stock exchanges have clearly existed for a long time, not much was known about the returns, portfolios and risks of investing in stocks (De Geeter, 2013). It wasn't until 1952 that significant progress on these topics was booked with Markowitz's portfolio theory (Markowitz, 1952). Markowitz's portfolio theory together with Tobin's separation theory (Tobin, 1958) eventually led to the creation of the CAPM by William Sharpe (1964) and John Lintner (1965).

Ever since the creation of modern portfolio theory and the emergence of the CAPM, the subject of finance has been dominated by these theories. Today the CAPM is still seen as one of the most fundamental theories in finance and is taught to students everywhere around the world in introductory courses to finance. Through the years the CAPM has been subjected to much criticism due to its set of unrealistic assumptions and contradictory performance in empirical tests. This casted doubt on its validity and questions whether it is ethically correct to teach an often empirically rejected theory to a new generation of students (Dempsey, 2013). Furthermore, The CAPM is frequently used by governments for regulatory purposes such as price regulation for public utilities. If the CAPM is inherently false, governments should use a different approach to estimate the cost of equity (Sudarsanam, 2011). Despite all the criticism and scrutiny the CAPM receives, it is still thought by many to be a valuable model (Berkman, 2013) and is often used in practice to calculate the "fair value" of a risky asset.

One of the main benefits of the CAPM is that it defines a very intuitive relationship between risk and return. The CAPM gives investors, companies and governments an idea on how the financial markets price risk and is easy to implement (Berk & DeMarzo, 2014).

The CAPM also holds benefits for decision analysis in a company. It helps managers to identify the risk factors that are important for a company's investor in order to conduct a proper analysis of a project's added value. When valuing a project, one should only take into account the so called undiversifiable market risk of the project. Project-specific risk can be diversified by investors if they hold multiple assets different from the company's common stock and should as a result not be taken into account. Portfolio diversification works because stock price changes are less than perfectly correlated with one another. Take the example of holding a stock portfolio consisting of an umbrella company and an ice cream business. Selling umbrellas is a risky business; the company might make a killing when it rains a lot, but suffers during a heat wave. Selling ice cream is no safer; the business should do well during a heat wave, but performs bad during rainy periods. By diversifying your investment across the two businesses, you can make an average level of profit come rain come shine. The effect of risk diversification will increase as more assets are added to the portfolio, ultimately resulting in a portfolio where the only remaining volatility arises due to common risk factors (Brealey, Myers & Marcus, 2015).

Using the CAPM, a company is able to calculate its weighted average cost of capital (WACC). The WACC can serve as a decision rule within a company. An expected return higher (lower) than the WACC, indicates that a project increases (decreases) value for the company's stockholders by providing a positive (negative) net present value (NPV) (Berk & DeMarzo, 2014).

Finally, the CAPM provides a useful benchmark for investors to measure the performance of portfolio managers. Popular portfolio evaluation measures based on the CAPM include Jensen's alpha and the Sharpe ratio (Sollis, 2012).

The research of Markowitz (1952) on modern portfolio theory, together with Sharpe's (1964) work on the CAPM was deemed of such importance, that in 1990 they were awarded with a Nobel prize. The next section will briefly describe the history of research on the CAPM, followed by a section that will define this thesis' central research question and elaborates on the employed research methodology.

2. Research history of the CAPM

From Sharpe's and Lintner's work on the CAPM it is clear that the CAPM is based on a series of strict, often unrealistic assumptions of the market's and the investors' behaviour. These assumptions and their implications for the CAPM's use in practice will be explained in the fifth chapter of this thesis.

The CAPM's unrealistic assumptions led to a surge of new research on advanced CAPM models, relaxing the underlying assumptions. Some noteworthy examples include Black's CAPM with restricted borrowing that permits the short sale of assets (Black, 1972), the intertemporal ICAPM allowing investments over multiple periods (Merton, 1973), the human capital CAPM (HCAPM) that tries to include non-marketable assets into the market portfolio (Mayers, 1973), the consumption-based CAPM (CCAPM) (Breeden, 1979) and the international CAPM that extends the CAPM to an international setting taking into account exchange rate risk (Adler & Dumas, 1984). This list is in no way exhaustive, the number of CAPM models available in the literature is immense. All these models have been tested repeatedly over the years with mixed results.

At first, it was still impossible to test the CAPM's validity empirically due to two simple reasons. First of all, in 1964 when Sharpe and Lintner (1965) created the CAPM, there was no such thing as a general database containing stock returns. It wasn't until the end of the 60's that initiatives were taken by the university of Chicago to construct such a database and testing could start taking place. The second reason for the inability of testing the CAPM was that no adequate statistical test was available in previous literature (De Geeter, 2013). Both the lack of necessary data and the unavailability of statistical knowledge, led to the first empirical test being created by Lintner (1965), Black, Jensen and Scholes (1972) and Fama and Macbeth (1973), the so called "double-pass" regression based test. The specifics of this statistical test as well as its limitations will be disclosed in the sixth chapter.

The contradictory results of the double-pass test on the CAPM casted doubt on the CAPM's validity and in 1977 the possibility of applications on the CAPM seemed to have reached a dead end after Roll argued that testing the CAPM is impossible due to the fact that the true market portfolio is unknown. Roll explains that the double-pass method tests in fact a joint hypothesis of the CAPM being correct and the proxy used for the market portfolio in the CAPM being equal to the true market portfolio. This makes the rejection of the hypothesis ambiguous. Rejection of the hypothesis can occur when either the CAPM does not hold, the proxy used in the model is different from the true market portfolio, or both (Roll, 1977). Roll's critique will be handled in more detail in the sixth chapter of this thesis

The rise of the less restrictive arbitrage pricing theory (APT), that uses factor models to relate expected returns to different sources of systematic risk (Ross, 1976), led to the creation of many new CAPM models with better empirical test results by adding different explanatory factors to the original CAPM model (De Geeter, 2013). One of the most famous multifactor models is the Fama and French three-factor model (1993) that controls for a company's size, determined by its market capitalisation and whether the stock is a value or growth stock, depending on its book-to-market ratio (Sollis, 2012). Even though many of these models fitted better to real-life data, they could not escape the joint hypothesis problem posed by Roll with the current two-pass regression test (De Geeter, 2013).

Besides criticism on the testability of the CAPM, the theory's fundamentals have been under attack by behavioural economists. The CAPM is indirectly based on the expected utility theory (EUT) of Von Neumann and Morgenstern (1953), making the assumption that investors are completely rational (Berk & DeMarzo, 2014). Research has however shown that investors often exhibit irrational behaviour in real life and do not maximise their utility. A good example of this is the phenomenon of home bias. Investors tend to prefer stocks that feel familiar. This will lead investors to invest too much of their capital into stocks within the same industry or country, not optimally diversifying their risk. Research conducted by French and Poterba (1991) reported that American, British and Japanese investors hold respectively 94, 92 and 98 percent of their total portfolio in domestic stocks on average. Later research confirmed this anomaly for American investors (Wolf, 2000).

Another display of irrational behaviour is the disposition effect. This effect causes investors to sell profitable stocks too early and losing stocks too late (Shefrin & Statman, 1985) (Odean, 1998). Other biases that defy the rationality assumption include under-diversification bias, overconfidence bias and herd behaviour (Berk & DeMarzo, 2014).

One of the cornerstones of behavioural economics and a possible conflicting theory with the CAPM is the Nobel prize awarded prospect theory of Kahneman and Tversky (1979) which describes the behaviour of investors dealing with uncertainty. Whether prospect theory has any effect on the validity of the CAPM will be discussed in seventh chapter of this thesis.

Recently, there has been promising development in the field of testing the CAPM. Multiple new tests have been developed, some of which can possibly escape Roll's critique as well as satisfy the behavioural economics theory. These tests include the long-term test of Ang and Chen (2007), the large effects test of Pesaran and Yamagata (2012), the crisis model of Berkman, Jacobsen & Lee (2011) that performs well in times of financial distress, tests conducted in an experimental setting (Boassaerts & Plott, 2004), the use of conditional CAPMs (De Geeter, 2013) and the reverse-engineering approach of Levy and Roll (2010).

The main focus of this thesis will be to summarise the theory behind the reverse-engineering test and apply it to the Belgian stock market. The reverse-engineering test differs from many different CAPM-tests in the sense that it makes use of ex-ante variables instead of ex-post historical data to test its hypothesis. Sharp and Lintner first intended the model to be used this way, but before Levy and Roll, no test based on ex-ante variables was available. This testing approach shows promising as it successfully escapes most critiques on testing the CAPM explained in this thesis. The test has already been successfully performed in several settings like the US market (Levy & Roll, 2010) and the Taiwanese market (Wang, Huang & Hu, 2017).

3. Research Questions and Research Methodology

3.1 Central Research Question

Central research question: Can the CAPM's use be justified both theoretically and empirically in the setting of the Belgian stock market?

In order to answer the central research question, this thesis will start by building a theoretical framework around the CAPM, its assumptions, the empirical tests and the critiques that discredit the model from both the empirical as the behavioural economics theoretical perspective. With this in mind, the central research question can be divided five sub-questions, each representing one or more different chapters in this thesis. These sub-questions, as well as the research methodology employed to answer them, will be described briefly below.

3.2 Sub-Questions

Sub-question 1: *What is the theory behind the CAPM and what are its restrictive assumptions? (chapters 4-5)*

To provide the reader with enough background on the CAPM, this thesis will start by summarising the theory that resulted in its creation. First of all, Markowitz's portfolio theory together with the concepts of risk-aversion, diversification, systematic- and idiosyncratic risk will be discussed. Secondly, Tobin's separation theory will be explained. Finally, Sharpe and Lintner's CAPM will be discussed alongside its assumptions.

Sub-question 2: *How does the "double-pass" regression based test work and what are its limitations? (chapter 6)*

To answer this question, the double-pass regression based test will be discussed alongside its technical problems as well as the critique it has received from Roll (1977). To study this topic the articles of Roll (1977), Black, Jensen and Scholes (1972), Fama and Macbeth (1973) as well as various textbooks will be used.

Sub-question 3: *What are the implications of behavioural economics for the CAPM? (chapter 7)*

After introducing Kahneman and Tversky's (1979) Prospect theory, its possible implications on the validity of the CAPM will be discussed. Just as in sub-question 2, multiple textbooks and scientific articles will be analysed to form an acceptable response to this question.

Sub-question 4: *What is the reverse-engineering approach to testing the CAPM and how does it escape the empirical and theoretical critiques of the double-pass test? (chapter 8)*

In this chapter the reverse engineering approach of Levy and Roll (2010) to testing the CAPM will be introduced. After the test is explained mathematically, a short section on its limitations and practical use will be disclosed.

Sub-question 5: *Using the reverse engineering test on Belgian stock data, does the CAPM hold empirically? (chapters 9-12)*

Before any real empirical testing can begin, assumptions will be made concerning the variables that impact the CAPM's results. First, proxies will be selected for the market portfolio and risk-free rate. The stocks used in the empirical test are stocks that are included in the market portfolio proxy. Secondly, a decision has to be made concerning the studied time period, granularity and estimation techniques.

In order to conduct the reverse-engineering test, a small program will be written in Matlab to perform the necessary statistical tests. It will also facilitate sensitivity analysis as only the test's parameters will have to be adjusted. The necessary data (Stock returns) will be collected from Yahoo finance. The test will be conducted on the set of the largest 50 Belgian stocks based on market capitalisation.

The reverse engineering test does not necessarily require the existence of a risk-free asset to test the CAPM. This thesis will cover the case wherein such an asset exists. Euribor yields with a maturity of 1 month will be used as an approximation for the risk-free rate. This data will be collected from the database of the European Central Bank.

4. Modern portfolio theory

4.1 Markowitz's portfolio selection

The first real breakthrough in what is now known as modern portfolio theory was made by Markowitz in 1952. Markowitz argued that investors seek to maximise the mean-variance relationship of their portfolios. Markowitz makes the assumption that investors are rational entities and investors are deemed rational if they abide to the following two principles:

1. For a given level of risk, a rational investor opts for the portfolio that offers the highest return.

2. Given a particular return, a rational investor will prefer the portfolio holding the lowest risk.

In the context of this thesis risk is defined as quantifiable uncertainty (Knight, 1921) and return is defined as the ratio of money gained or lost on an investment relative to the amount of money invested (Frömmel, 2011). In the next few sections the concepts of risk-aversion, diversification, expected utility, systematic- and idiosyncratic risk, the portfolio opportunity set, the efficiency frontier and the optimal risky portfolio will be handled in order to provide the reader with enough background to better understand Markowitz's portfolio selection, Tobin's separation theory and the optimal portfolio allocation.

4.2 Risk-aversion

One of the biggest assumptions underlying Markowitz's portfolio theory is the assumption that investors are risk averse. In this context risk-aversion means investors seek to maximise their utility, which is a function of wealth positively influenced by expected return and negatively influenced by risk (Levy, 2012). This concept is explained in figure 1. Consider an investor with a utility function U(W) that is displayed by the concave function in figure 1. Imagine that the investor has the opportunity to invest in a risky asset that either yields R_1 or R_2 with respective probabilities P_1, P_2 so that the expected return $(E(R) = R_1 * P_1 + R_2 * P_2)$ is located on a straight line between R_1 and R_2. For simplicity assume that these yields include initial wealth.

It can easily be deduced that a risk-averse investor attains the same level of utility by either investing in the risky asset or in an asset with a certain yield of R^* that is clearly lower than E(R). The investor is therefore willing to pay a premium $\pi = E(R) - R^*$ in order to be rid of the risk. In other words, the investor's utility is negatively influenced by risk. The magnitude of this risk premium for risk-averse investors has been approximated by Arrow (1971) and Pratt (1964) by using Taylor series approximation and is given below:

$$\pi \cong -\frac{\sigma^2}{2} * \frac{U^{''}(W+E(R))}{U^{'}(W+(E(R))} \qquad (4.1)$$

Here σ^2 is the variance of the risky asset's yield, W is the investor's initial wealth and E(R) is the expected return of a prospect in terms of wealth.

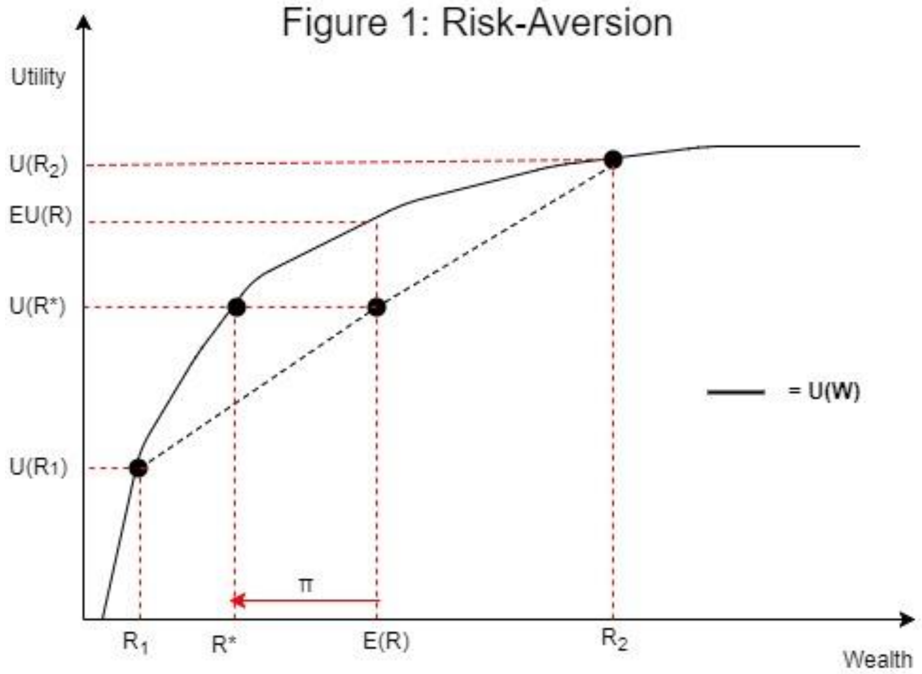

Figure 1: Risk Aversion

4.3 Risk diversification

The most important discovery that Markowitz made in his 1952 paper is probably the notion that assets are not always perfectly correlated with one another. A discovery that has led to the idea of risk diversification in asset portfolios. This means that by adding multiple assets that are not perfectly correlated together in a portfolio, the portfolio-risk measured by its variance, increases less than proportionally. As a result, investors can achieve more efficient portfolios by diversifying their investments over multiple, not perfectly correlated assets.

The idea of portfolio diversification was of vital importance for later studies on risk and has nowadays become common knowledge among people who have never studied or even heard of modern portfolio theory. This at first ground-breaking theory is today often referred to under the form of "not putting all your eggs in one basket" when investing. General formulas for the expected return and variance of a portfolio are given below (Markowitz, 1952).

$$E(R_P) = \sum_{i=1}^{N} w_i \, E(R_i) \qquad (4.2)$$

$$\sigma_P^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i \, w_j \sigma_{ij} \qquad (4.3)$$

With $E(R_P)$ the expected return of the portfolio, $E(R_i)$ the expected return of asset i in the portfolio, w the weight of the asset in the portfolio, σ_P^2 the total portfolio variance and σ_{ij} the covariance between asset i and j. Note that the expected portfolio return is the weighted average of expected asset returns while the portfolio variance is lower than the weighted average of asset variances when assets are not perfectly correlated with each other (Berk & DeMarzo, 2014).

This lower overall portfolio variance is known as the diversification effect. It can be shown theoretically that a substantial amount of the overall portfolio variance can be reduced by increasing the number of assets in a stock portfolio. Figure 2 below shows this effect in practice for the average portfolio. As the number of assets in the portfolio increase, the variance drops rapidly until it converges to a natural lower bound. This phenomenon is best explained by the two types of risk that are present in the market, namely systematic- and idiosyncratic risk. Idiosyncratic or firm-specific risk arises because many of the perils that surround an individual company are peculiar to that specific company and maybe to its direct competitors. Examples of idiosyncratic risk events are the firing of a CEO, the disapproval of a patent, a corporate scandal et cetera. Systematic or market risk arises from economy-wide perils that threaten all businesses. It is a common risk factor that explains why stocks have a tendency to move together. This stems from the fact that the returns of most stocks are positively correlated with the overall market portfolio. Here the market portfolio is defined as a market capitalisation weighted sum of all individual risky assets. A good example of this type of risk is the drop in most stock prices caused by an economic recession. During an economic recession the overall market experiences a price drop and therefore all assets that are positively correlated with the market drop in price as well. Assets that possess a negative correlation with the market portfolio can be regarded as insurance policies, providing a hedge against a market downturn (Berk & DeMarzo, 2014).

Figure 2: Diversification

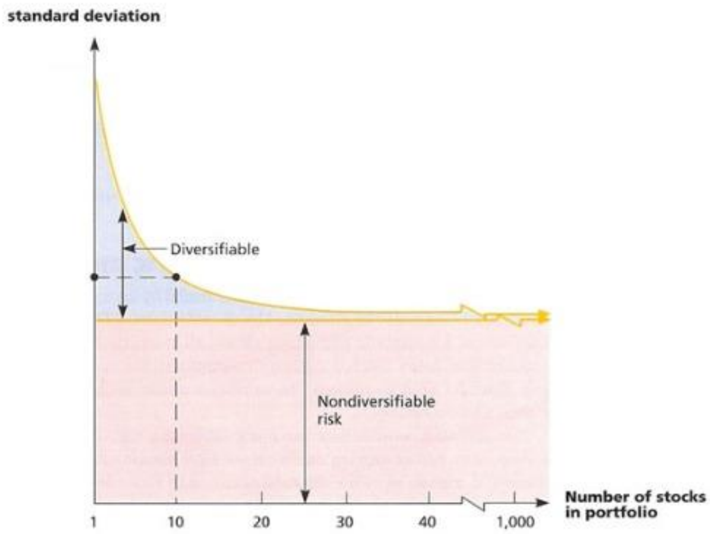

Figure 2: Diversification (Stutton, 2009)

The importance of this risk classification in two categories is that diversification only eliminates the idiosyncratic risk present in a portfolio. No matter how many securities investors hold in their portfolio, they cannot eliminate all risk. So for a well-diversified portfolio, only systematic- or market risk matters. This results in investors only being rewarded for holding this undiversifiable, market risk. In other words, assets that are subjected to a large amount of market risk require higher rates of return in order to appeal to investors. This concept lays at the foundation of the CAPM derivation in the next chapter (Brealey, Myers & Marcus, 2015).

4.4 The portfolio opportunity set & efficiency frontier

Markowitz's paper (1952) ends with the concepts of the portfolio opportunity set and the efficiency frontier. The portfolio opportunity set is given by all the possible combinations of expected portfolio return and portfolio standard deviation (σ_p). In practice, this portfolio opportunity set can be computed by varying the asset weights (w_i) under the constraint that the sum of all weights equal to unity ($\sum_{i=1}^{N} w_i = 1$). Note that this constraint does not restrict the use of short sales. The portfolio opportunity set is represented by the black line oo' in figure 3. This line is the boundary of the portfolio opportunity set, meaning that with the current selection of assets, only portfolios with a risk-return relationship underneath this line are available for investment (Sollis, 2012).

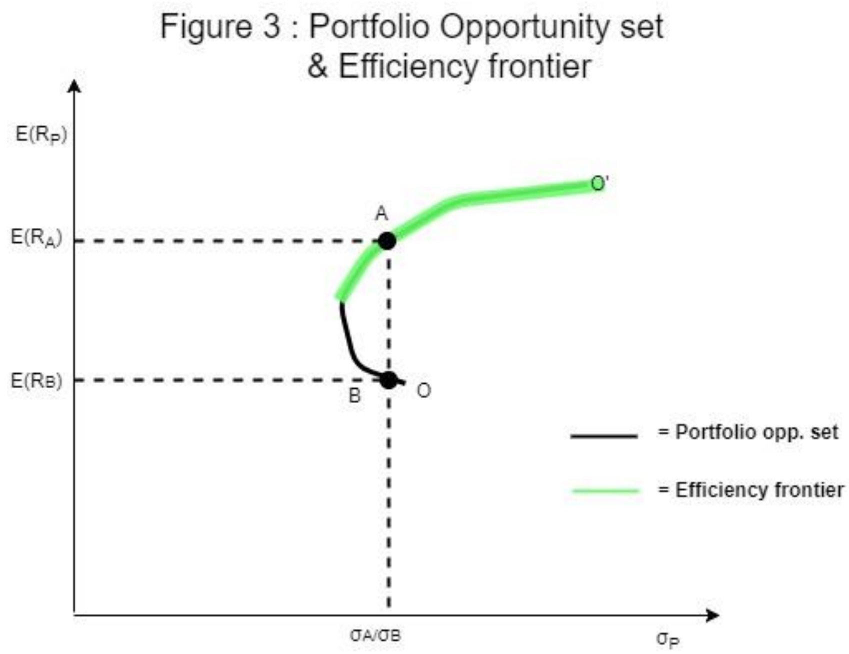

Figure 3: Portfolio Opportunity set & Efficiency frontier

Figure 3 shows another important element of portfolio selection, the efficiency frontier. The efficiency frontier can be defined as the group of preferred risky portfolios on the portfolio opportunity set. These portfolios are seen as efficient as there are no alternative portfolios the investor can hold to achieve lower/equivalent risk (σ_p) for equivalent/higher return or vice-versa (Markowitz, 1952). This concept is best explained by an example. Say an investor holds portfolio B within the portfolio opportunity set. It can easily be seen that this portfolio is inefficient as the investor could hold portfolio A that offers a higher return at the same amount of risk. As a result, rational investors should only invest in portfolios situated on the efficiency frontier to maximise the risk-return relationship (Berk & DeMarzo, 2014).

Mathematically, this frontier is formed using either of the following iterative optimisation procedures (Levy, 2012).

$$MAX_{w_i}\ E(R_P) = \sum_{i=1}^{N} w_i\, E(R_i) \text{ for any given } \sigma_p \qquad (4.4)$$

or

$$MIN_{w_i}\ \sigma_p\ = \sqrt{\left(\sum_{i=1}^{N}\sum_{j=1}^{N} w_i\, w_j \sigma_{ij}\right)} \text{ for any given } E(R_P) \qquad (4.5)$$

Here w_i represents the weight of asset i in portfolio P, σ_{ij} is the covariance between asset i and j, $E(R_i)$ is the expected return of asset i and $E(R_P)$, σ_p respectively represent the expected return and volatility of portfolio P.

4.5 Tobin's separation theorem

The next big contribution to modern finance was Tobin's so called separation theory published in 1958. Tobin argued that Markowitz's research did not include the possibility of investing in a riskless asset such as government bonds (Tobin, 1958). A risk-free asset can be defined as an asset that delivers a certain income flow, independent of the state of the world (Lengwiler, 2004). In practice, low default government securities are often used as a proxy (van Ewijk et al., 2012). However, it should be mentioned that recent developments have shown that there is no such thing as a riskless investment, even assets such as government bonds and bank deposits are subjected to risk, especially in times of economic turmoil (De Geeter, 2013). The risk carried by holding government bonds can be summarised in the following four categories (Wardenier, 2014):

1. Risk of government bankruptcy (i.e. Greece).
2. Unexpected inflation caused by an increase in money supply, lowering the real return.
3. Interest rate risk. If interest rates rise, the bond's investor experiences an opportunity cost.
4. Foreign exchange risk if the bond is denominated in a foreign currency.

Under the assumption that investors could both invest and borrow at a true risk-free rate of interest, the inclusion of a riskless asset allowed a separation of the process of searching for an optimal investment portfolio into two steps (Tobin, 1958).

1. Independent of the investor's risk preferences, define the optimal risky portfolio in the market.

2. Taking into account the investor's level of risk-aversion, compute the portion of the investor's total funds to be invested in the riskless asset.

4.5.1 Step 1: The optimal risky portfolio

The introduction of a risk-free asset in the portfolio selection decision resulted in the creation of a whole new set of efficient portfolios. To illustrate this, figure 4 shows some possible new portfolio sets that can be obtained by combining the investment of a risk-free asset with a portfolio on the efficiency frontier. Note that an infinite number of new portfolio sets could arise but one of these is clearly superior (R_fR^*). This new set of portfolios is superior as it provides the investor with the most attractive risk-return relationship and is obtained by maximising the slope of the tangency line between the risk-free rate and the efficiency frontier. This set of portfolios that the investor should hold is defined as the capital allocation line (CAL) and the tangent portfolio on the efficiency frontier is called the optimal risky portfolio (ORP). In order to maximise their risk-return relationship an investor should hold a combination of the risk-free asset and the ORP (Brealey, Myers & Marcus, 2015).

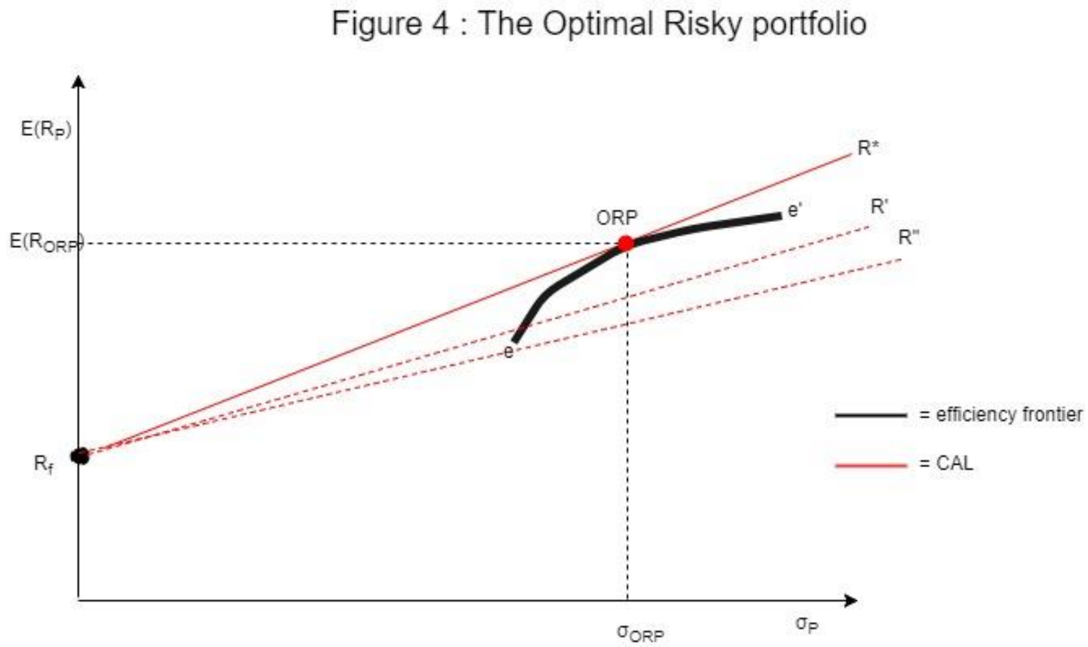

Figure 4: The Optimal Risky Portfolio

Mathematically, this problem can be solved with the following maximisation problem:

$$MAX_{w_i}\, CAL_{Slope} = \frac{E(R_P) - R_f}{\sigma_P}, \qquad (4.6)$$

$$\text{subject to } \sum_{i=1}^{N} w_i = 1$$

here $E(R_P)$ and σ_P are calculated using formulas (4.2) and (4.3) respectively, R_f represents the risk-free interest rate and w_i is the weight of asset i in portfolio p (Sollis, 2012). The ratio $\frac{E(R_P) - R_f}{\sigma_P}$ or slope of the CAL is also often referred to as the Sharpe-ratio.

In order to solve maximisation problem (4.6) one could simply take the partial derivative of the CAL slope with respect to each w_i and equate them to zero.

4.5.2 Step 2: The optimal final portfolio

After the ORP has been calculated, the only thing left to do is to determine the proportion of capital that the investor wishes to invest in the risk-free asset and the ORP. The resulting portfolio is called the optimal final portfolio (OFP). This individual preference is calculated under the assumption that investors seek to maximise their utility, which in turn is a function of total wealth, positively influenced by expected return and negatively by risk. The OFP is thus determined by the highest possible tangency point of the investor's indifference/iso-utility curves with the CAL (figure 5) (Sollis, 2012).

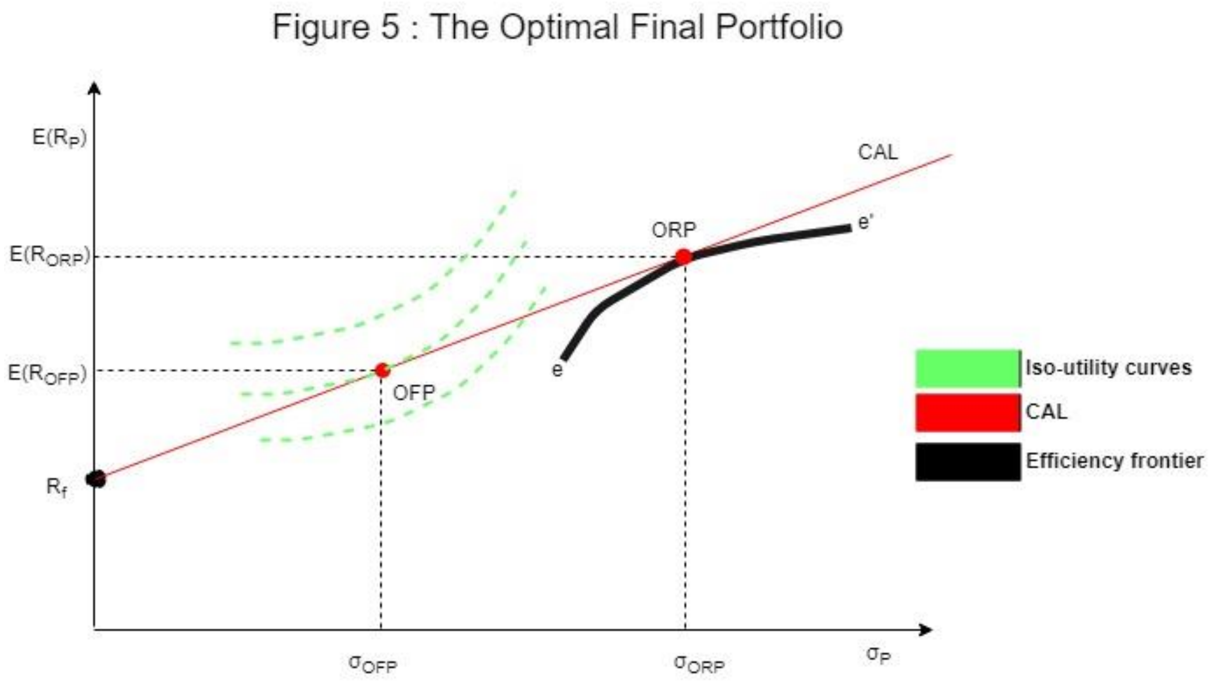

Figure 5: The Optimal Final Portfolio

In order to calculate the effect on utility of an uncertain investment, consider an utility function U(W + R). Here W is the initial level of wealth and R denotes an uncertain income coming from an investment in risky assets. The effect on utility of uncertain income R can easily be approximated by using a Taylor expansion about the value W + E(R) (Levy, 2012).

$$U(W+R) \cong U\big(W+E(R)\big) + \frac{U'(.)*(R-E(R))}{1!} + \frac{U''(.)*(R-E(R))^2}{2!} + \cdots + \frac{U^i(.)*(R-E(R))^i}{i!} \qquad (4.7)$$

Taking the expected value of both sides yields:

$$EU(W+R) \cong U\big(W+E(R)\big) + \frac{U''(.)*\sigma_R^2}{2!} + \cdots + \frac{U^i(.)*\mu_{i,R}}{i!} \quad (4.8)$$

here,

$E\big(R-E(R)\big) = E(R) - E(R) = 0$

$E\big(R-E(R)\big)^2 = \sigma_R^2$

$U(.) = U\big(W+E(R)\big)$

$E\big(R-E(R)\big)^i = \mu_{i,R}$

$\mu_{i,R}$ = *The ith central moment of the distribution of R*

In order to calculate the OFP, the existing literature often suggests the use of a quadratic approximation of the utility function (Levy, 2012). The function is approximated by using the Taylor expansion up till the second order term. The effect of other moments of the return distribution is therefore not taken into account. This yields the following expected utility function for a portfolio P:

$$E(U) \cong U(.) + \frac{U''(.)}{2} * \sigma_P^2$$

$$E(U) \cong w*(E(R_{ORP}) - R_F) + R_F - AV*w^2*\sigma_{ORP}^2 \quad (4.9)$$

Maximising this utility function with respect to portfolio weight w yields:

$$MAX_{w_i}\, E(U) = E(R_{ORP}) - R_F - AV*2w*\sigma_{ORP}^2 = 0 \quad (4.10)$$

$$\Rightarrow \quad w = \frac{E(R_{ORP}) - R_F}{2AV*\sigma_{ORP}^2} \quad (4.11)$$

With,

AV = *Risk-aversion factor*

w = *weight of investment in the ORP*

R_F = *riskfree rate*

R_P = *Portfolio rate of return*

R_{ORP} = *Optimal risky portfolio rate of return*

σ_{ORP}^2 = *Optimal risky portfolio variance*

σ_P^2 = *Portfolio variance*

The risk-aversion factor AV is specific for each investor. The higher this factor, the lower the proportion of wealth that is invested in risky assets (ORP). Mathematically, this factor depends on the second derivative of the utility function with respect to wealth, which is always negative for a risk-averse investor ($U''(W + E(R)) < 0$).

Using the found investment weight w in the ORP from equation (4.11), the expected return of the OFP is calculated by:

$$E(R_{OFP}) = (1 - w)R_F + \text{w} * E(R_{ORP}) \tag{4.12}$$

The main drawback of using the quadratic approximation is that it only takes into account the second moment of the return distribution. In reality investors may also care about higher moments of the return distribution when making investment decisions. The third moment, namely skewness has been shown to have great impact on an investor's utility (Levy, 2012). Investors usually prefer a positive skewness as this reduces the probability of facing extreme negative returns (Blau, 2017). As expected utility theory and portfolio utility maximisation are not the main topics of this thesis, these topics will not be discussed further.

4.6 Conclusion

Modern portfolio theory is developed in the expected utility framework and makes the assumption that all investors are rational and risk-averse. The process of finding the optimal final portfolio can be summarized in the following three steps:

1) Construct the efficiency frontier using either equation (4.4) or (4.5). This is an iterative procedure and can best be performed by a computer.

2) Find the CAL. This is done by solving the maximisation problem from equation (4.6). When the CAL is found, the ORP is also known.

3) Starting from the investor's utility function, determine the amount to be invested in the ORP and risk-free asset. For a quadratic approximation of the investor's utility this can be done by solving problem (4.10). The resulting weight is the investment proportion of the OFP in the ORP. The expected return of the OFP can be calculated using equation (4.12).

5. The Capital asset pricing model

5.1: The CAPM and its assumptions

Using modern portfolio theory (Markowitz, 1952) and Tobin's separation theorem (1958) explained in the previous chapter, Sharpe (1964) and Lintner (1965) were able to derive an equilibrium asset pricing model for risky assets, namely the capital asset pricing model (CAPM). This proved to be quite a challenge as the actual prices in security markets are driven by a complex mechanism that takes into account many decision variables and incorporates the opinions of millions of investors. For this reason, no theoretical model can probably fully describe the equilibrium behaviour of financial markets without imposing a set of restrictive assumptions. More simplifying assumptions will lead to a simpler model but will increase the odds that the model does not accurately reflect actual observed prices in the market as they sometimes shape an unrealistic view of how financial markets work/behave (Levy, 2012). The assumptions behind the CAPM are as follows (Sharpe, 1964, Lintner, 1965, Levy, 2012 & Sollis, 2012):

1) Investors are risk averse.
2) Investors maximise their utility which is defined as a function of total wealth influenced positively by expected return and negatively by risk (here σ^2).
3) Asset returns are normally distributed.
4) Investors are able to borrow and lend as much as they want at the risk-free interest rate, which is an exogenous variable.
5) Investors have a single period investment horizon.
6) A perfect capital market which entails the following:
    a. No transaction cost or taxes.
    b. Many buyers and sellers with none able to influence the market directly.
    c. All investors have access to the same, costless information.
    d. Divisibility: investors can buy as much securities as they want, even with a limited amount of capital.
7) Investors have homogenous expectations given the same information.

It can easily be deduced that this is a very demanding set of assumptions. Many other articles have been published that propose CAPM-related models that relax several assumptions. In general, these models are theoretically more correct but less intuitive and applicable than the CAPM (Levy, 2012). Examples are the zero-beta model (Black, 1972) that relaxes the risk-free rate assumption, the segmented CAPM (Levy, 1978 & Merton, 1987) that adjusts the divisibility assumption and incorporates incomplete information, the intertemporal CAPM (Merton, 1973) that deals with multiple investment periods, the heterogeneous belief CAPM (Levy, H., Levy, M. & Benita, 2006) and the tax-adjusted CAPM (Brennan, 1970). These different models will not be discussed further as the focus of this thesis is testing the theory behind the CAPM. It is enough to know that the model could be adjusted for these assumptions.

The concepts of risk-aversion and utility as a function of total wealth were explained in the previous chapter and find their origin in expected utility theory. These assumptions alongside the normality assumption will prove more threatening to the validity of the CAPM as they are respectively in conflict with the theory of behavioural finance and empirical findings. The implications for the CAPM of these assumptions will be handled further in chapter 7.

Finally, it is of vital importance to note that the CAPM is expressed in terms of ex-ante expected returns, which means that it determines the fair equilibrium price of risky assets conditional on all available information. This will be a cause of concern in the following chapters as most empirical tests of the CAPM use data based on ex-post, observed prices.

5.2 Derivation of the CAPM

Under the set of assumptions given in the previous paragraph, Sharpe (1964) and Lintner (1965) were able to derive the basic CAPM. Consider the situation displayed in figure 6 where there are n risky assets available in the market. Deriving the corresponding efficiency frontier using either formula (4.4) or formula (4.5) yields the line ee'. Suppose R_F is the risk-free rate in the market, maximising the Sharpe-ratio or CAL slope with equation (4.6) results in $R_F r'$ and tangency point m. Tangency point m represents a portfolio of risky assets where $\sum_{i=1}^{N} w_i$ = 1. In other words a portfolio without borrowing and lending.

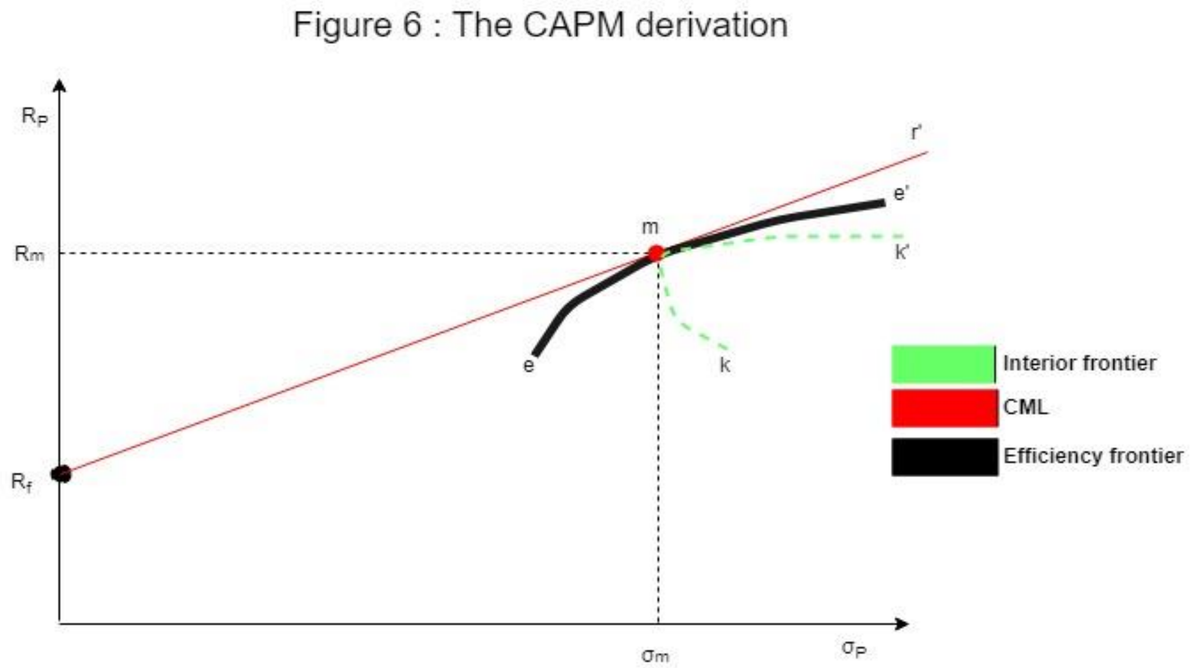

Figure 6: The CAPM derivation

As the CAPM assumes that all investors are risk averse and employ modern portfolio theory in a perfect capital market with homogeneous beliefs, it follows that assets in the market are fairly priced. The CAPM is an equilibrium model, in other words, it looks at the situation where the supply equals the demand of risky assets. If supply equals demand of risky assets and all of these assets are fairly priced, the portfolio m is none other than the market portfolio. This market portfolio consists of all risky assets weighted by their respective market capitalisation. The weight of an asset k in portfolio m is therefore $P_k * S_k / \sum_{k=1}^{N}(P_K * S_K)$, where P_k is the fair price of asset k, S_k is the total number of shares outstanding of asset k and the denominator takes the sum of all the asset's market capitalisations in the market.

Next Sharpe investigates the portfolio consisting of a single interior security k combined with portfolio m. The obtained investing frontier is given by kk' in figure 6. Here point k represents a 100 percent investment in security k, m represents an investment of 100 percent in the market portfolio m and k' is obtained by investing in portfolio m while short selling security k. Note that the frontier kk' is fully located inside the efficiency frontier with tangency point m. The expected return and variance of the portfolio consisting of portfolio m and security k is given by (Sharpe, 1964):

$$R_P = w_k * R_k + (1 - w_k) * R_m \quad (5.1)$$

$$\sigma_P^2 = w_k^2 * \sigma_k^2 + (1 - w_k)^2 * \sigma_m^2 + 2 w_k * (1 - w_k) * \sigma_{km} \quad (5.2)$$

with,

W_i = portfolio weight
σ_m^2 = market portfolio variance
R_m = return on the market portfolio
R_k = return on asset k
σ_k^2 = variance of asset k
σ_{km} = covariance between market portfolio m and asset k

As the slope of line $R_F r'$ is none other than the slope value of the CAL, the following relation holds in point m:

$$Slope_{R_F r'} = \frac{R_m - R_f}{\sigma_m} \quad (5.3)$$

Next Sharpe determines the derivatives of equations (5.1) and (5.2) in order to simultaneously define a relationship between the portfolio return and volatility of a portfolio located on kk'.

$$\frac{dR_P}{dw_k} = R_k - R_m \quad (5.4)$$

$$\frac{d\sigma_P}{dw_k} = \frac{1}{2\sigma_P}[2 w_k \sigma_k^2 - 2(1 - w_k)\sigma_m^2 + 2\sigma_{km} - 4 w_k \sigma_{km}] \quad (5.5)$$

As $w_k = 0$ and $\sigma_P = \sigma_m$ at point m on kk', equation (5.5) reduces to:

$$\frac{d\sigma_P}{dw_k} = (\sigma_{km} - \sigma_m^2) / \sigma_m \quad (5.6)$$

Equation (5.4) can be rewritten as:

$$\frac{dR_P}{dw_k} = \frac{dR_P}{d\sigma_P}\frac{d\sigma_P}{dw_k} \qquad (5.7)$$

$$\Rightarrow \quad \frac{dR_P}{d\sigma_P} = \frac{dR_P}{dw_k}\bigg/\frac{d\sigma_P}{dw_k} = \frac{(R_k - R_m)\sigma_m}{\sigma_{km} - \sigma_m^2} \qquad (5.8)$$

The right-hand side of equation (5.8) is none other than the slope of the derivative at point m. Equating formulas (5.3) and (5.8) yields:

$$\frac{R_m - R_f}{\sigma_m} = \frac{(R_k - R_m)\sigma_m}{\sigma_{km} - \sigma_m^2}$$

$$\Rightarrow \quad R_k = R_f + (R_m - R_f)\frac{\sigma_{km}}{\sigma_m^2} \qquad (5.9)$$

Note that $\frac{\sigma_{km}}{\sigma_m^2}$ is the regression coefficient of a time-series regression of R_{kt} on R_{mt}. This coefficient will be referred to as β_k.

$$\frac{\sigma_{km}}{\sigma_m^2} = \beta_k \qquad (5.10)$$

Inserting this factor into equation (5.9) yields the famous CAPM formula (5.11) which should be expressed in terms of expected returns as in (5.12). Equation (5.12) is also referred to as the security market line (SML).

$$R_k = R_f + (R_m - R_f)\beta_k \qquad (5.11)$$

$$E(R_k) = R_f + (E(R_m) - R_f)\beta_k \qquad (5.12)$$

Under the CAPM, the return R_p of the optimal final portfolio P, in other words, a combination of the efficient market portfolio m and the risk-free investment, can be expressed as:

$$R_p = R_f + \frac{(R_m - R_f)\sigma_p}{\sigma_m} \qquad (5.13)$$

or

$$R_p = R_f + (E(R_m) - R_f)\beta_P \qquad (5.14)$$

Equation (5.13) is often called the Capital market line (CML) and $\frac{\sigma_p}{\sigma_m}$ represents the weight invested in the risky market portfolio m. This equation corresponds to line $R_F r'$ in figure 6. Note that in order to derive this equation Tobin's separation theorem is needed. An investor should therefore only invest in a combination of risk-free investment and the market portfolio. Investing in any other portfolio will lead to a lower Risk-Return trade-off according to the CAPM. These inferior portfolios are represented by the area under the efficiency frontier ee'.

5.3 Beta and the Security market line

From the SML equation (5.12) it is clear that the appropriate risk measure of a portfolio or asset is determined by its β. This measure β is influenced by the variance of the market portfolio and by the covariance of the asset/portfolio with the market (Levy, 2012). It is of importance to note that this term β does not include the individual volatility/variance of a particular asset/portfolio. This means that the only risk of an asset that matters is the risk that stems from its covariance with the market. This is a direct result of the concept of diversification introduced by Markowitz (1952). In the CAPM, the market portfolio is seen as a perfectly diversified portfolio, meaning that all firm-specific risks are filtered out and only market risk remains. As all rational investors hold a stake in this market portfolio, they are only subject to shocks in the overall economy. Because firm-specific risk can be eliminated by diversification, investors should only be rewarded for holding market risk, as this is the only risk that a rational investor holds (Brealey, Myers & Marcus, 2015). The different terms of the SML equation (5.12) can therefore be interpreted as follows:

$$E(R_k) = R_f + (E(R_m) - R_f)\beta_k$$

R_f: All investors hold a combination the risk-free asset and the market portfolio. An investment in a risky asset will require an expected return higher than the risk-free rate.

$(E(R_m) - R_f)\beta_k$: The term between parenthesis is called the risk premium. This is the extra return on the market portfolio required by investors to be exposed to market risk, it can be interpreted as the market price of risk. This term is multiplied by β_k, which represents the market risk of asset k. A β_k higher (lower) than 1 implies that the expected return on the asset k tends to increase (decrease) with more (less) than 1% if the expected risk premium of the overall market increases (decreases) with 1% and the asset carries relatively more (less) market risk than the market portfolio.

In equilibrium the return on a risky asset is as a result the sum of the risk-free rate and a premium that accounts for the total of undiversifiable risk of the asset (Berk & Demarzo, 2014).

Figure 7 shows the SML equation in a β–R space. The SML is represented by the blue line. If all assets are fairly priced according the CAPM, they should all lie on this line. Point m corresponds with the market portfolio and has an expected return of $E(R_m)$ and a β of 1. If an asset a is situated above the SML, it generates a return higher than implied by its market risk and is undervalued. Investors will react by buying this asset, which increases demand and its price until it is located on position a' on the SML. If an asset is located under the SML, it is overpriced and the inverse will happen due to an increase of supply (selling) of the asset.

Figure 7: The Security market line

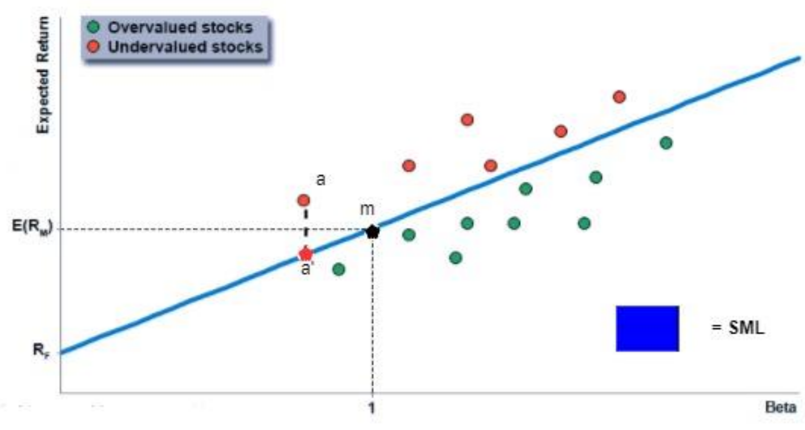

Figure 7: The Security Market Line (Smirnov, 2018)

5.4 Conclusion

This chapter introduced the theory behind the CAPM. The CAPM is an equilibrium asset pricing model for risky assets that, like any other economic model, rests on a set of restrictive assumptions. In section 5.1, it is shown that there are many extensions of the CAPM that adjust for these assumptions. While these assumptions make the CAPM more theoretically correct, they also make the model more difficult to implement. The assumptions of risk-aversion, utility maximisation as a function of total wealth and normality of asset returns will be handled in more debt in the next chapters.

Under the CAPM, the optimal portfolio choice is a combination of the risk-free asset and the market portfolio. This new set of efficient portfolios is represented by the CML in equation (5.13). Another important relationship under the CAPM is the SML, depicted in equation (5.12) and figure 7. The SML describes the relationship between the expected returns and risk of assets. Under the CAPM the investor is only rewarded for holding market risk as firm-specific risk can be eliminated by holding a diversified portfolio (the market portfolio). This market risk is measured with the factor β.

6. Testing the capital asset pricing model

6.1 Testing the CAPM: the double-pass regression test

This chapter will focus on the most employed empirical test of the CAPM, namely the "double-pass" regression based test. This test, developed by Lintner (1965), Black, Jensen and Scholes (1972) and Fama and Macbeth (1973), has resulted in mixed results in the past and has been subjected to much criticism (De Geeter, 2013). The chapter will start by explaining the double-pass regression test, followed by a short summary of the technical and theoretical limitations of the test. Finally, some insight will be provided on how empirical tests of the CAPM could be improved.

6.2 The double-pass regression test

The double-pass regression test was the first formal empirical test of the CAPM. The CAPM is supposed to be tested using the market portfolio, which should include all risky assets available. Sadly, no such measure is observable in practice and researchers use a market proxy to conduct the test. This market proxy usually consists of a stock market index. Proxies for the risk free interest rate are measures such as government bonds, interbank rates, etc. (De Geeter, 2013).

As the name of the test suggests, the test consists of two parts. In the first part of the double-pass test, a time-series regression is used to determine the β-coefficients of the different assets in the market portfolio. These β-coefficients are used directly as the regressors in a second cross-sectional regression. The next two sections will explain these regressions in more detail.

6.2.1 The time-series regression

The time-series regression of the double-pass test can be carried out by collecting real world data on a particular stock market index like the BEL20 and its components on a yearly, monthly or other basis. After selecting an appropriate proxy for the risk-free rate, the following time-series regression is run:

$$R_{k,t} - R_{f,t} = \alpha + \beta_k (R_{m,t} - R_{f,t}) + \varepsilon_{k,t} \tag{6.1}$$

here,

$R_{k,t}$ = *return on asset k at time t*

$R_{f,t}$ = *risk-free rate at time t*

$R_{m,t}$ = *return on the market proxy*

β_k = β *of asset k*

$\varepsilon_{k,t}$ = *error term*

α = *abnormal return*

Note that the excess return unaccounted for by the CAPM on asset k (α) should not be statistically insignificantly different from 0 under CAPM theory (Sollis, 2012 & Lintner, 1965). Many studies however, report the finding that this term is statistically significant (De Geeter, 2013).

6.2.2 The cross-sectional regression

The cross-sectional regression uses the found $\beta_k's$ from regression (6.1) as the regressors in a regression on the sample mean excess return of the assets. Consider the following regression:

$$\overline{R_k} - \overline{R_f} = \delta_0 + \delta_1\beta_k + \delta_2\hat{\sigma}_k^2 + \mu_k \qquad (6.2)$$

with,

$\overline{R_k}$ =*mean return on asset k*
$\overline{R_f}$=*mean return on risk-free asset*
δ_0 =*intercept parameter*
$\delta_{1,2}$ =*regression coefficients*
$\hat{\sigma}_k^2$ =*estimated variance of k*
μ_k =*error term*

If the CAPM holds, the following null hypotheses should not be rejected:

1) $H_0: \delta_0 = 0$
2) $H_0: \delta_1 = \overline{R_m} - \overline{R_f}$
3) $H_0: \delta_2 = 0$

These null hypotheses can be tested separately using sequential t-tests. The first null implies that assets do not receive an abnormal excess return, in other words a return not justified by their $\beta's$. The second null tests if the risk premium in practice is different from the theoretical one. The hypothesis $H_0: \delta_1 = 0$ is often tested to verify whether market risk is priced, this hypothesis should be rejected if the CAPM holds. The final hypothesis $\delta_2 = 0$ tests whether investors receive a premium for holding firm-specific risk. Here, firm-specific risk is measured by $\hat{\sigma}_k^2$ (Sollis, 2012) (Lintner, 1965).

Most empirical studies using this test report that risk premiums are positively related to β-values. In other words, the hypothesis $\delta_1 = 0$ is rejected, supporting the CAPM. These same studies however, tend to find that the risk premium is lower than the CAPM stated. The null $\delta_1 = \overline{R_m} - \overline{R_f}$ is rejected and more specifically studies conclude that $\delta_1 < \overline{R_m} - \overline{R_f}$ which is unsupportive of the CAPM. This finding, together with a significant α in regression (6.1) suggests that the curve formed by the SML is actually flatter than the CAPM predicts. Leading firms with a low/high β to have higher/lower returns than the CAPM predicts (Sollis, 2012 , Lintner 1965 & Levy, 2012).

Finally, studies discover that the hypothesis $\delta_2 = 0$ is rejected, meaning that investors are rewarded for holding firm-specific risk. This last finding is in direct contradiction with the theory behind the CAPM. According to the CAPM, investors should only receive a premium for exposing themselves to undiversifiable market risk (Sollis, 2012). It can be concluded that these results only partially support the CAPM at best. The next sections will discuss the technical as well as theoretical problems with the double-pass test and provide the reader a possible explanation and solution for the mixed results.

6.3 Technical Problems with the double-pass regression

The nature of the used variables and double-regression results cause several technical econometric issues that could lead to biased results. This section will focus on the three most important technical criticisms, namely cross-sectional serial correlation, measurement errors in the β–coefficients and the use of ex-post data in empirical tests.

6.3.1 Cross-sectional serial correlation

A first possible problem is the one of correlated standard-errors in the cross-sectional regression. Correlated standard-errors lead to biased estimates and unreliable t/F-tests. This problem can be a result of omitted variable bias if the omitted variables are significantly related to both the regressor (β) and regressand (excess mean return). Research has shown that the small firm effect is one of these variables (Banz, 1981). Note however, that the existence of a variable that explains systematic abnormal returns in the market is in contradiction with the CAPM. Other studies show possible other variables that systematically explain returns. Another good example is the study performed by Fama and French (1992) that determines the book-to-market value of equity as another factor that explains abnormal returns. This shows that market risk, determined by an asset's β is maybe not the only factor to be taken into account when estimating expected returns.

6.3.2 Measurement errors

A second significant issue is the measurement error in the β-coefficients. Note that the cross-sectional regression uses the $\beta's$ from the time-series regression. As β is a random variable, there will be a certain degree of uncertainty whether this β truly reflects the population β of each asset. In general a bigger sample will lead to an estimate closer to the population parameter. One could therefore obtain a better estimate by using data over a longer horizon. However, this requires the time-series model to remain stable over time which is not the case as companies change business strategies, industries, et cetera. The researcher is therefore left with the trade-off between noisy but unbiased estimates of β or tighter parameter estimates that could be biased (Levy, 2012 & Sollis, 2012). Black, Jensen & Scholes (1972) propose a possible solution to this problem by grouping assets in equally weighted, ranked β portfolios and running the time-series regression on these portfolios which are adjusted each period.

6.3.2 Ex-post data

A third drawback of the double-pass regression test is that it makes use of historical/ex-post data to infer its results. Remember the previous chapter in which it is stated that the CAPM theory holds in terms of ex-ante parameters. Therefore, one could make the argument that any test rejecting the

CAPM using ex-post parameters, especially ex-post β's is invalid. More recent studies incorporate this into their CAPM test and show that once ex-ante parameters are considered, the CAPM cannot be rejected empirically. The reverse-engineering test introduced in chapter 8 is one of these tests (Levy, 2012).

6.4 Theoretical Problems with the double-pass regression test

Over the years, the CAPM theory and double-pass regression based test have not only been subjected to criticism due to technical problems. Many articles in the extensive literature on systematic risk question both the theoretical and empirical validity of the CAPM model. These criticisms can be grouped in three broad categories:

1. Criticisms from a behavioural economics perspective.
2. Criticisms on the restrictive, often unrealistic assumptions behind the CAPM.
3. Criticisms from a methodological perspective on testing the CAPM.

As briefly shown in chapter 5, many of the restrictive assumptions behind the CAPM can be relaxed by modifying the model itself. The criticisms of behavioural economists on the CAPM will be handled in more detail in the next chapter of this thesis and the methodological shortcoming of the double-pass regression test, often referred to as Roll's Critique (De Geeter, 2013), will be explained in the next section.

6.4.1 Roll's Critique

Roll (1977) argues that the double-pass regression test will always have a positive outcome (no rejection of the CAPM) if the used proxy for the market portfolio to estimate the β's in the first-pass time-series regression is "mean-variance" efficient. This means that, as long as the proxy is located on the efficiency frontier, there will exists a risk-free rate R_f for which the second-pass cross-sectional regression will yield a linear relationship between the sample β's and the sample mean excess returns (Levy, 2012). So if the double-pass regression test is carried out on a portfolio P, located on the efficiency frontier using a sample of n risky assets, relationship (6.3) will hold for each asset i in the portfolio for a particular risk-free rate R_f .

$$\bar{R_i} = R_f + (\overline{R_p} - R_f)\,\beta \tag{6.3}$$

Consider the example in which the double-pass regression test is carried out on a sample of 10 stocks. Figure 8 shows several possible portfolio combinations in the variance-return space. Roll's finding implies that relationship (6.3) will hold for all portfolios located on the efficiency frontier. So for portfolios a, b and c there will exist a particular risk-free interest rate (intercept term) that results in a no rejection of the hypotheses of the double-pass regression test. The intercept terms for examples a, b and c are respectively given by R_{fa}, R_{fb} and R_{fc}. Using portfolios located on the inside of the frontier (m) will lead to a rejection of the hypotheses and therefore a rejection of the CAPM (Roll, 1977).

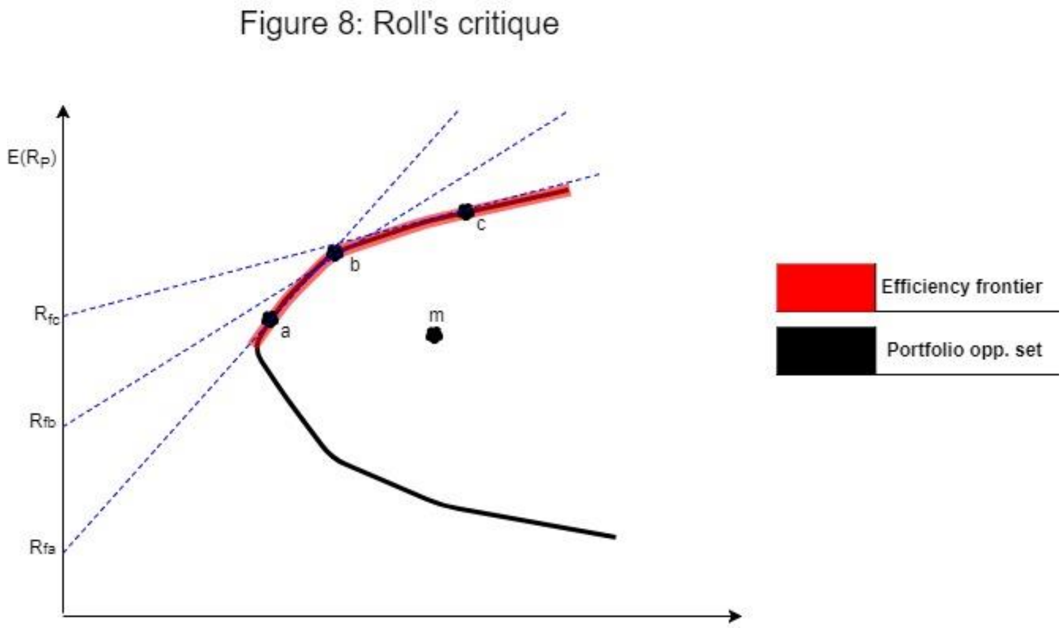

Figure 8: Roll's critique

In the preceding example a portfolio consisting only of 10 stocks is used as the proxy for the market portfolio. If the portfolio is mean-variance efficient, the double-pass test will support the theory of the CAPM. This finding, however, does not conform with the CAPM theory. According to the CAPM relationship (6.3) should only hold for the true market portfolio, which by definition cannot exist of only 10 stocks. Conducting this test on any other efficient portfolio than the market portfolio will therefore lead to a misleading result (Roll, 1977) (Levy, 2012). According to Roll, the only correct way to test the CAPM is to test whether the market portfolio is efficient. This imposes many practical implications for testing the CAPM as there is currently no way to observe the true market portfolio. Current tests have used stock market indices as proxies for the market portfolio, but the true market portfolio should also include human capital, private real estate investment, private equity, etc., on which there currently is no data (De Geeter, 2013).

In summary, the double-pass regression test is not an adequate test for the CAPM. It entails a "joint hypothesis" problem in which both the hypothesis that the CAPM theory holds and the hypothesis that the used market proxy is the true market portfolio are tested. A better test for the CAPM would be to test whether the market portfolio is efficient. This proved a difficult problem to tackle as the true market portfolio is not directly observable and the market proxies used in research are mere approximations of this portfolio. This does not necessarily mean that the CAPM cannot be tested by using market proxies like stock indices. Research has shown that big stock indices and the unobservable market portfolio are often correlated and given appropriate adjustments to the test, justify their use in empirical research. As these correlation based CAPM tests are a research subject on their own and well beyond the scope of this thesis, they will not be disclosed further (De Geeter, 2013, Kandel & Stambaugh, 1987 & Shanken, 1987).

6.5 Conclusion

First empirical tests of the CAPM were based on the double-pass regression approach. This approach is a two-step procedure. First a time-series regression is ran using formula (6.1) after which the found β-coefficients are used in a second, cross sectional regression (6.2). The double-pass test has received much criticism over the years and the empirical results are so mixed that they only represent partial support for the CAPM at best.

The main problems concerning the double-pass regression are:

1) The possibility of cross-sectional correlated standard errors.
2) Possible measurement errors in the β-coefficients.
3) The use of ex-post instead of ex-ante variables.
4) The joint hypothesis problem introduced by Roll (1977).

It can be concluded that in order to test the validity of the CAPM, another test than the double-pass regression test is needed. This new test should preferably test the hypothesis that the market portfolio is mean-variance efficient using ex-ante parameters. The use of ex-ante parameters will also correct for the measurement errors in the β-coefficients. One of these tests, the reverse engineering test is introduced in chapter 8. Note that the first technical problem, the possibility of cross-sectional correlated standard errors can also be avoided by using a non-regression based test. This will solve the econometric problem of correlated standard errors but the bias from omitting important variables in the asset pricing model will remain.

7. Behavioural Finance and Normality of Asset Returns

7.1 Behavioural Finance and the CAPM

Last chapter showed that the CAPM has received criticism over the years due to the lack of a theoretically correct test and mixed results in empirical research. Another wave of criticism however, comes from an entirely different field of study, namely behavioural economics. Prospect theory (PT), one of the cornerstones of behavioural economics and its later, modified version cumulative prospect theory (CPT) cast doubt on the expected utility theory (EUT) of Morgenstern and Von Neumann (1953). If the PT-framework is correct and the EUT is invalid, the CAPM indirectly loses ground as the CAPM is designed within the EUT-framework. Remember from chapter 5 that two of the assumptions behind the CAPM are risk-aversion and utility maximisation. This chapter will discuss the implications for the CAPM as under PT these assumptions no longer hold. Another often criticised assumption behind the CAPM, the normality assumption of asset returns, will also be discussed further in this chapter. It will be shown that even though PT and CPT contradict EUT, the CAPM is still theoretically valid.

7.2 Prospect theory

Before PT, EUT was the dominant theory to describe the behaviour of individuals in risky or probabilistic situations. When making decisions regarding wealth and uncertain prospects, this EUT was based on the following three basic principles:

1) Individuals are risk-averse, in other words, the utility function is concave (see figure 1 chapter 4).

2) The utility of an uncertain prospect is its expected utility.

3) Individuals maximise their utility which is a function of total wealth.

EUT did however not seem to explain the behaviour of individuals in all risky situations and just after its creation several paradoxes started to emerge (De Geeter, 2013). Famous examples of these include the Allais paradox (1953), which points out that investors prefer certain positive outcomes to uncertain ones, even if the expected value is lower and Roy's safety first rule (1952), which implies that individuals only seek to minimise the probability of a disastrous outcome. These anomalies, which are not explained by EUT, lead to the creation of PT(1979) and CPT (1992) by Kahneman and Tversky.

Under the CPT-framework an individual makes investment decisions (choices concerning uncertain prospects) based on the following principles:

1) Individuals maximise the expected value of a value function, not a utility function. This value function is stated in terms of the change in wealth rather than the total wealth stated by EUT.

In other words, the initial wealth of an individual does not matter when making decisions regarding uncertain prospects.

2) In order to calculate the expected value of uncertain prospects, individuals use decision weights instead of objective probabilities. These decision weights differ for each individual. Remember from chapter 4 that portfolio risk is measured by its variance. The variance of a portfolio's return can be defined as (Gujarati & Porter, 2009):

$$\sigma_P^2 = \int_{-\infty}^{+\infty} (R_P - E(R_P))^2 \, f(R_P) \, dR_P \qquad (7.1)$$

with σ_P^2 the portfolio variance, R_P the possible portfolio returns, $E(R_P)$ the expected portfolio return and $f(R_P)$ the probability density function of portfolio returns. If the objective probabilities are changed to decision weights, it can easily be deduced that the way an investor experiences risk is determined by his/her individual decision weights. Using decision weights would distort the objective probability density function $f(R_P)$ for an individual investor when evaluating the risk of a uncertain prospect.

As the individual investor's risk measurement changes, the utility the investor derives from each portfolio changes as well. The third term from equation (4.9) changes, resulting in a different investment weight in the optimal risky portfolio. The use of decision weights under CPT will therefore result in a different optimal final portfolio as calculated under EUT.

3) Individuals are risk-seeking towards losses and risk-averse towards gains. Risk-aversion does not prevail in all situations.

4) Individuals display loss aversion, meaning that losses are more impactful than gains of equal size. Therefore, the value function is steeper for negative changes in wealth than for positive changes (Levy, 2012).

Figure 9 shows the value function under CPT. Note that the x-axis represents change in wealth rather than total wealth while the y-axis shows expected value instead of expected utility. In the positive/negative domain the value function is respectively concave/convex. This implies risk-aversion for positive changes in wealth and risk-seeking behaviour towards negative changes. Note that the value function is steeper in the negative domain (Kahneman & Tversky, 1979).

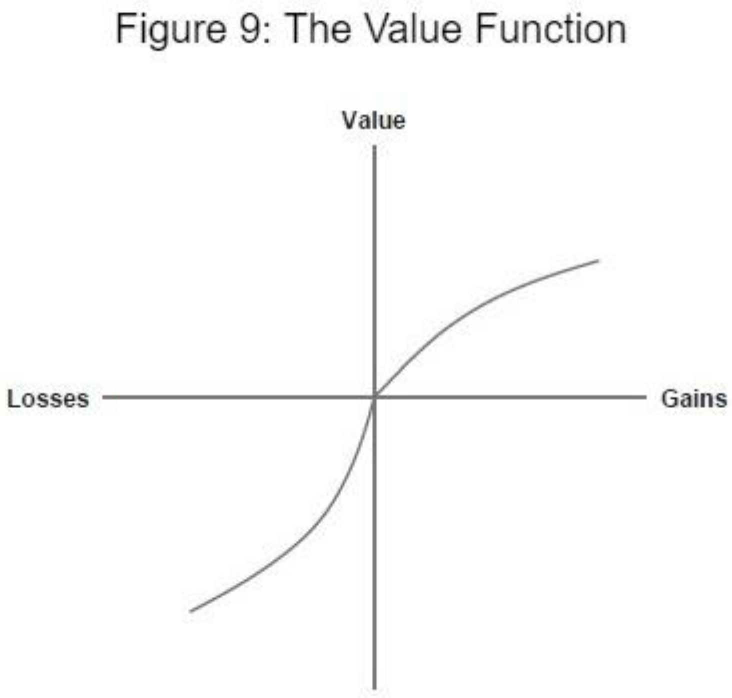

Figure 9: The Value Function (Kahneman & Tversky, 1979)

As risk-aversion no longer prevails for all prospects and utility is no longer a function of total wealth, the first two assumptions of the CAPM are discredited. The following sections will show that even in absence of these assumptions, the CAPM still holds. The third CAPM assumption, the normality assumption of asset returns, will also be impacted by CPT and discussed further. In order to facilitate the reader's understanding of the proposed arguments, the concept of stochastic dominance is handled first.

7.3 Stochastic Dominance

Stochastic dominance rules are rules that provide us the ability to order random variables according to utility when their probability distributions are known and a set of assumptions is made, making them interesting as an investment criteria (Hadar & Russell, 1969). There are many stochastic dominance rules available in the current literature but in the context of this thesis, only the concept of first order stochastic dominance (FSD) is needed (Levy, 2012).

FSD states that a prospect A dominates prospect B if and only if the following inequality holds for each value of x:

$$A(x) \leq B(X) \qquad (7.2)$$

with,

$A(x) =$ *the cumulative probability function of A*

$B(x) =$ *the cumulative probability function of B*

The underlying assumption of FSD is that it only holds for monotonic utility functions. Monotonic utility functions are utility functions that increase in function of wealth, a realistic assumption to make as it is logical to assume individuals prefer having more wealth. Figure 10 represents the intuition behind FSD graphically. The figure shows the cumulative probability function (CPF) of both prospect A and B. Remember that the CPF is nothing else than the probability that a random variable takes on a lower value than x (Gujarati & Porter, 2009). If prospect A has a lower probability to return a value lower than x in comparison to B for each value of return x, it can easily be deduced that prospect A dominates B (Levy, 2012).

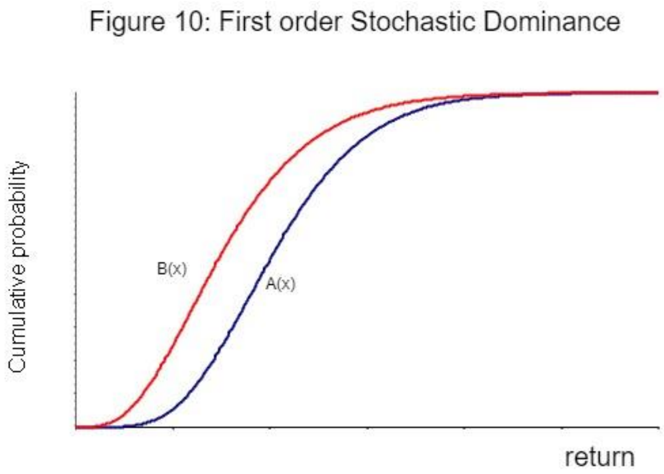

Figure 10: First order Stochastic Dominance (Epix analytics LLC, 2018)

For normally distributed variables the inequality (7.1) results in (Levy, 2010):

$$E_A(x) \geq E_B(y) \quad (7.3)$$

$$\sigma_A(x) \leq \sigma_B(y) \quad (7.4)$$

with,

at least one of the above inequalities strict
$E_i(n) =$ *the expected value of prospect i with possible outcome distribution n*
$\sigma_i(n) =$ *the standard deviation of prospect i with possible outcome distribution n*

This finding corresponds with the mean-variance optimisation to derive the efficiency frontier in chapter 4.

7.4 Change in wealth and the CAPM

The CAPM makes the assumption that an investor maximises their utility as a function of total wealth. CPT on the other hand claims that utility as a function of total wealth is simply incorrect and that investors only consider changes in wealth when making the investment decision. This section will show that even when investors only consider changes in wealth, the CAPM still holds in the sense that the optimal final portfolio still lies on the CML and that the SML and separation theory are still intact. Using change in wealth as an investment criteria will however change the location of the optimal final portfolio on the CML for each investor and therefore affect the equilibrium prices of risky assets. These changes do not affect the linear risk-return relationship stated by the CAPM in equation (5.12), only the different parameters are subject to change (Levy, De Giorgi & Hens, 2012).

To prove that CML line is still intact consider first the EUT situation in which initial wealth would be taken into account when making the investment decision. If an investor has the possibility to invest their wealth in either portfolio A, which is a combination of the market portfolio and the risk-free rate, or portfolio B, which is a portfolio located under the CML, FSD dictates that the following relationships hold under normality:

$$E_A(w+x) \geq E_B(w+y) \qquad (7.5)$$

$$\sigma_A(w+x) \leq \sigma_B(w+y) \qquad (7.6)$$

with,

at least one of the above inequalities strict
$\sigma_i(w+n)$ =*the expected value of prospect i with possible outcome distribution n and initial wealth w*
$E_i(w+n)$ = *the expected value of prospect i with possible outcome distribution n and initial wealth w*

Or more generally:

$$A(w+x) \leq B(w+x) \qquad (7.7)$$

with,

$A(w+x)$ = *the cumulative probability function of A*

$B(w+x)$ = *the cumulative probability function of B*

Note that graphically, by adding w, the two cumulative distributions depicted in figure 10 simply shift to the right by a constant w. The level of initial wealth has therefore no influence on the stochastic dominance of one prospect over the other. Mathematically:

$$A(w+x) \leq B(w+x) \Leftrightarrow A(x) \leq B(x) \qquad (7.8)$$

From inequality (7.8) it can be concluded that in a CPT setting the efficient portfolio set, represented by the CML does not change. All investors should still hold a combination of the market portfolio and the risk-free asset as these portfolios stochastically dominate all other portfolios. As the CML stays the same, the derivation of the CAPM shown in section 5.2 remains intact (Levy, 2010).

After the efficient market portfolio is located, investors are still left with the decision of how much to invest in risk-free rate. Note that while the separation theory remains intact, the resulting optimal final portfolio will change under CPT. This stems from the fact that utility functions change when initial wealth is not taken into account. Consider the example shown in figure 11. As CPT and EUT respectively lead to different utility functions and indifference curves U(x) and U(w +x), the amount invested in the risk-free asset for a certain investor will change. In the case depicted in figure 11, the investor clearly invests a higher amount in the riskless asset than under EUT.

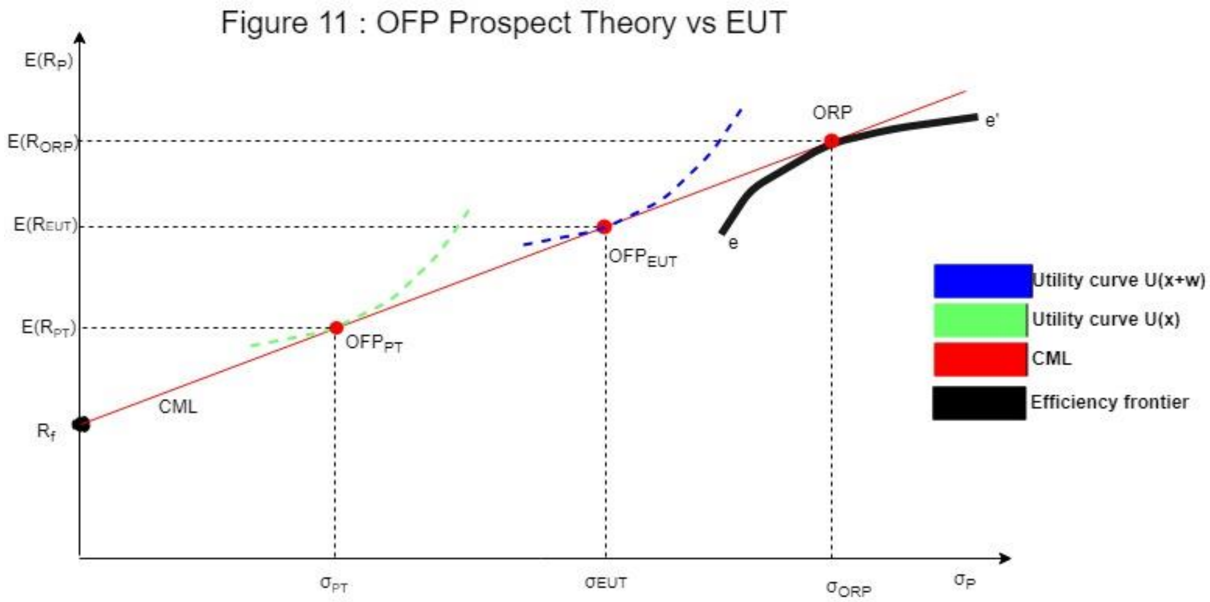

Figure 11: OFP Prospect Theory vs EUT

This change in investment behaviour will change the equilibrium price of assets. Under the CPT situation in this example, less wealth is invested in the market portfolio, lowering the demand for risky assets. Lower demand implies lower prices and a higher mean rate of return for risky assets than under EUT (Levy, De Giorgi & Hens, 2012).

7.5 Risk-aversion and the CAPM

The second assumption behind the CAPM that not holds under CPT is the assumption of risk-aversion. CPT proposes an S-shaped value function that describes risk-seeking behaviour in its negative domain (figure 10). This finding proved to be problematic as it casts doubt on the use of variance as a risk measure in the CAPM (Levy, 2012).

In order to prove that this assumption is not necessary to derive the CAPM, a similar argument to the one in section 7.4 is given. From section 7.4 it has already been made clear that, under the normality assumption, portfolios located on the CML are superior to those who are not by FSD. As a result, even in the case of risk-seeking behaviour investors will invest in a portfolio located on the CML and the CAPM can be derived. Just as in the case of change in wealth versus total wealth, the use of an S-shaped value function will impact the optimal final portfolio decision and impact risky asset prices as investor’s indifference curves change (Levy, 2010).

To derive the CAPM as an intact equilibrium pricing model, the assumption of risk-aversion is not needed. However, as regards risk-seeking behaviour, one must make the additional assumption that there is a restriction on the amount borrowed. If this restriction is not in place there might be risk-seeking individuals borrowing unlimited amounts of money, which makes equilibrium asset prices impossible. Such an assumption is realistic as borrowing limits exist in practice (Levy, 2010).

7.6 Normality of asset returns and the CAPM

The final and maybe most criticised assumption behind the CAPM is the one of normality of asset returns. Numerous studies have used goodness-of-fit tests to examine the normality of asset returns and reached the clear conclusion that asset returns are not normally distributed. More specifically, asset returns usually follow a distribution that has more skew and fatter tails than a normal distribution. Important studies on this topic include, but are not limited to Mandelbrot (1963), Fama (1965), Gray and French (1990), Zhou (1993), Focardi and Fabozzi (2003), Levy and Duchin (2004), et cetera. Even if asset returns were normally distributed, under CPT investors transform return distributions by using decisions weights instead of objective probabilities, leading the resulting distributions to be non-normal as well (Levy, De Giorgi & Hens, 2012).

The finding that asset returns are not normally distributed affects the CAPM as the CAPM is based on variance as its risk measure. If asset returns are not normally distributed, variance becomes a less informative measure of dispersion as it ignores other factors like skewness. Besides influencing the decision for the optimal final portfolio and risky asset prices, the non-normality of returns also endangers the previous statement that portfolios located on the CML are superior to those who are not by FSD. As the mean-variance optimisation used in the CAPM does not account for risk factors other than variance, the optimal risky portfolio could be sub-optimal (Levy, 2012).

Research using empirical return distributions has shown that when return distributions are non-normal, the mean-variance rule is usually an excellent approximation for expected utility. If a portfolio is selected using mean-variance optimisation, the expected utility of the resulting optimal final portfolio is extremely close to the utility obtained by maximising the investor’s utility function directly (Markowitz & Levy, 1979). The corresponding utility loss induced by this approximation has been found to be empirically negligible. In most cases the mean-variance framework can be used safely to maximise the utility of an investment (Levy, 2010).

Current research has however not yet concluded that the mean-variance framework is an excellent approximation for portfolio optimisation under CPT. It has been shown that under CPT, when an individual investor employs a decision weighting function rather than objective probabilities, the resulting return distribution carries more skew and weight in its tails. This could result in a higher importance of factors like skewness in portfolio optimisation and a less efficient approximation of the value function optimising OFP when using the mean-variance framework. Recent studies have been developing ways to include other moments of the return distribution in security pricing under CPT. These methods however, are beyond the scope of this thesis (Barberis & Huang, 2008).

As the non-normality of returns and its implications on portfolio selection is a whole research topic on its own and the main goal of this thesis is using the reverse engineering test to test the CAPM in the setting of the Belgian stock market, this thesis will continue under the assumption that the mean-variance framework is intact. This is not an unreasonable assumption because the framework provides a good approximation of the reality under non-normal return distributions and. Under this line of reasoning the CML of the CAPM remains intact as variance is still a useable, approximating measure for risk. The reader should note that the non-normality of asset returns gives support to the argument that the CAPM is not a fully correct model.

7.7 Conclusion

Behavioural economics and more specifically prospect theory show that some of the assumptions behind the CAPM do not hold in reality. In section 7.4 and 7.5 it is explained that for two of these assumptions: utility as a function of total wealth and risk-aversion, the CAPM theory is not affected. If investors are not always risk averse and utility is a function of change in wealth, the concepts of the CML and SML still hold while the equilibrium prices of assets change.

The violation of the normally distributed asset return distributions assumption proves more damaging to the CAPM theory. Non-normality of return distributions would require portfolio optimisation to be based on other risk measures than variance. One would for instance need to take into account the skewness of return distributions. Research however, has shown that under non-normally distributed returns, the mean-variance optimisation usually leads to a good approximation of the optimal final portfolio when maximising utility. Under CPT the optimality of the mean-variance framework as an approximation is less clear and forms an interesting topic for future research.

From this chapter it can be concluded that the CAPM is probably not an exact pricing model as it does not take into account the non-normality of return distributions. Note that the equilibrium prices and expected returns differ when calculation is done based on CPT- instead of the EUT-framework. The impact of these differences and which of the two methods should be preferred in practice forms an interesting topic for further research.

8. Testing the CAPM: a reverse engineering approach

8.1 The reverse engineering test

In the final theoretical chapter of this thesis, the reverse engineering test will be introduced. This empirical test, first developed by Levy and Roll (2010), will be used in this thesis' empirical part to test the CAPM in a Belgian setting.

Remember from chapter 6 that in order to test whether the CAPM empirically holds, one only needs to test if the market portfolio, measured by a market proxy, is mean-variance efficient or is located on the efficiency frontier created by all the assets included in the proxy. Two of the main technical issues of testing the CAPM, cross-sectional serial correlation and measurement errors in β, can be avoided by conducting a test using ex-ante instead of ex-post variables that is not regression based. An example of past research that corrects for the use of ex-post data is the β confidence interval (CI) approach that takes into account that β is a random variable. Under this approach, the true ex-ante $\beta's$ are assumed to be somewhere in CI around the $\beta's$ found using ex-post data. Under this method one could build a joint CI and test whether the CAPM holds (Levy, 1982). A more recent, more correct approach to correct for the use of ex-post data is the reverse engineering test explained in the next section.

8.2 The intuition behind the reverse engineering test

In their article Levy and Roll show that it takes only a small variation of sample return (μ) and standard deviation (σ) parameters, well in their respective estimation error bounds, to make a typical market proxy mean-variance efficient. Thus using a typical market proxy like the S&P index, one could not reject the CAPM when one takes into account the estimation error made by using ex-post instead of ex-ante values for μ and σ (Levy & Roll, 2010).

Many studies suggest using a trial and error approach that implies changing the value of the sample parameter μ and verifying if these changes lead to a mean-variance efficient market proxy. The methodology put forward by Levy and Roll however, takes on the reverse approach. The reverse engineering test starts with the assumption that the CAPM holds. In other words, that the linear relationship depicted in equation (5.12) is correct. Given this requirement, the method searches for the parameters μ and σ that are as close to their sample counterparts as possible and satisfy the relationship implied by the CAPM. If these parameters are within the joint statistical bounds that their distributions allow, the CAPM cannot be rejected as the true ex-ante parameters are expected to be somewhere in these bounds. One could for example look at the proportion of sample parameters that deviate from the standard estimation error bounds and the size of these deviations. An ex-ante parameter set could for instance be considered reasonably close when 95% of the these parameters are within the 95% CI of their sample counterparts. Another, more conventional approach is the use of formal statistical tests (Levy & Roll, 2010). In this thesis, joint t-tests will be conducted for all individual assets using a Bonferroni correction to deal with the problem of testing multiple hypotheses at the same time. This approach will be explained in more detail in the empirical part of this thesis.

As it turns out, only small variations in sample parameters are required to make the CAPM equation hold. Levy and Roll therefore conclude that based on a sample of the one hundred largest stocks in the U.S. market during the period 1997-2006, the CAPM cannot be rejected. This result is replicated by Wang, Huang and Hu (2017) on a sample of stocks from the Taiwanese TWSE index. Furthermore, calculated ex-ante β's are found to be close to their sample counterparts. Next section will explain the reverse engineering test mathematically.

8.3 The reverse engineering test: optimisation problem

Consider a market proxy m with vector x_m representing the vector of market portfolio weights calculated as the weight of their total market capitalisation over the total capitalisation of the market index (equation (8.4)). The vectors of sample average returns (averaged for each asset over time), standard deviations and their ex-ante counterparts are given respectively by μ^s, σ^s and μ, σ. ρ^s denotes the sample correlation matrix. For simplicity Levy and Roll's reverse engineering test allows only for variation in the mean and standard deviation of assets. The correlation matrix stays the same as in the sample over the whole testing period as this would impose additional technical difficulties (Levy & Roll, 2010). Allowing for variation in the correlation matrix would add to the validity of the reverse engineering test and might be an interesting topic for future research.

In order to obtain the parameters (μ, σ) that are the closest to their sample counterparts (μ^s, σ^s), Levy and Roll define and minimise a distance function D, given by:

$$D((\mu^s,\sigma^s),(\mu,\sigma) = \sqrt{\alpha \frac{1}{N}\sum_{i=1}^{N}\left(\frac{\mu_i-\mu_i^s}{\sigma_i^s}\right)^2 + (1-\alpha)\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\sigma_i-\sigma_i^s}{\sigma_i^s}\right)^2} \quad (8.1)$$

Here N is the total number of assets and $0 \leq \alpha \leq 1$ represents a parameter determining the relative importance assigned to deviations of the mean return in comparison with standard deviations. In previous research, researchers varied this parameter α in the 0.5-0.75 range as this proved to yield the most robust results (Giannakopoulos, 2013). In the empirical part of this thesis, the value of α will be analysed in the same range.

Both the deviation of the mean and standard deviation are divided by σ_i^s. The intuition behind this, is that as σ_i^s grows larger, the statistical error resulting from estimating the corresponding parameter increases in value as well. Dividing by σ_i^s ensures that the distance function takes into account that large deviations in parameters with a small standard deviation are weighted more heavily (Levy & Roll, 2010). The complete optimisation problem can be solved using non-linear programming methods and is given by:

$$\text{Minimize } D((\mu^s,\sigma^s),(\mu,\sigma)) = \sqrt{\alpha\ \frac{1}{N}\sum_{i=1}^{N}\left(\frac{\mu_i - \mu_i^s}{\sigma_i^s}\right)^2 + (1-\alpha)\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\sigma_i - \sigma_i^s}{\sigma_i^s}\right)^2} \qquad (8.2)$$

Subject to

$$\begin{bmatrix} \sigma_1 & 0 & \dots & 0 \\ 0 & \sigma_2 & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \dots & 0 & \sigma_N \end{bmatrix} * \rho^s * \begin{bmatrix} \sigma_1 & 0 & \dots & 0 \\ 0 & \sigma_2 & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \dots & 0 & \sigma_N \end{bmatrix} \begin{bmatrix} x_{m1} \\ x_{m2} \\ \vdots \\ x_{mN} \end{bmatrix} = q * \begin{bmatrix} \mu_1 - r_f \\ \mu_2 - r_f \\ \vdots \\ \mu_N - r_f \end{bmatrix} \qquad (8.3)$$

with,

r_f = *the risk-free rate*

$q = \frac{\sigma_m^2}{\mu_m - r_f}$

μ_m = *mean market portfolio return*

$$x_{mi} = \frac{Market\ capitalisation\ of\ firm\ i}{\sum_{j=1}^{N} market\ capitalisation\ of\ firm\ j} \qquad (8.4)$$

Note that constraint (8.3) can be rewritten as the SML equation:

$$C_{i,m} = \frac{\sigma_m^2}{\mu_m - r_f} * \mu_i - r_f$$

$$\Rightarrow \quad \mu_i = r_f + \frac{C_{i,m}}{\sigma_m^2} * (\mu_m - r_f) \ = r_f + \beta * (\mu_m - r_f)$$

Where $C_{i,m}$ is the asset covariance with the market portfolio.

While the above optimisation problem's constraint (8.3) takes into account the existence of a risk-free rate r_f, it should be mentioned that its inclusion is not needed in order to conduct the reverse engineering test. As this thesis focusses on the standard, most basic form of the CAPM, the risk-free rate will be included. The risk-free rate used in the test is an average of the risk-free returns during the period that the test is conducted. Remember from section 4.5 that in reality a risk-free investment does not exist. In order to conduct the test a suitable proxy is needed. The proxy for the risk-free rate used in this thesis is Euribor with a maturity of 1 month. The reason for using this proxy is explained in chapter 9.

It should additionally be mentioned that previous research using the reverse engineering test does not allow weights in market portfolio to vary over time. The weight of each risky asset i in the market portfolio is determined based on their market capitalisation at the end of the sample period (equation 8.4).

8.4 The ex-ante Beta

As the reverse engineering test provides a vector of both the true ex-ante asset mean returns and volatilities, one could wonder whether these parameters have any implications for asset pricing. Remember that the SML formula (5.12) from chapter 5 makes use of an asset's β calculated using sample data. If one knows the true expected return and volatility (μ, σ) an ex-ante β^* could be calculated.

β^* and β^S of asset i are then given as follows for a market portfolio consisting of N assets:

$$\beta_i^* = \frac{\sum_{j=1}^{N} x_{mj}\, \sigma_i\, \sigma_j\, \rho_{ij}}{x_m'\, C_m x_m} \tag{8.5}$$

$$\beta_i^S = \frac{\sum_{j=1}^{N} x_{mj}\, \sigma_j^S\, \sigma_i^S \rho_{ij}}{x_m'\, C_m^S x_m}\ , \tag{8.6}$$

With x_{mj} representing the weight of asset j in the market portfolio, x_m the vector of asset weights in the market portfolio, ρ_{ij} the correlation between asset i and j, σ_i, σ_j, σ_i^S and σ_j^S respectively portraying ex-ante volatilities of asset i, j and their sample ex-post counterparts. Finally C_m and C_m^S respectively depict the ex-ante and ex-post covariance matrix. These two matrices differ because of the difference in ex-ante and ex-post volatilities. Note that this formula, just like the reverse engineering test, does not take into account changes in asset correlation.

8.5 The ex-ante Beta and Asset pricing

In their paper, Roll and Levy (2010) find that these β^*'s are in fact very close to their sample counterparts (β^S's) when the number of assets used in derivation of the SML is large. The reason for this small difference being the similarity of both numerator and denominators of equations (8.5) and (8.6) and the offsetting potential of σ's when the number of assets is large. This last claim can be interpreted in the sense that in some cases $\sigma > \sigma^s$ while simultaneously in other cases $\sigma < \sigma^s$. Figure 12 shows this result for Levy and Roll's (2010) test on a sample of the one hundred largest stocks in the United States market (January 1997-December 2006, market portfolio weights based on December 2006 market capitalisations)

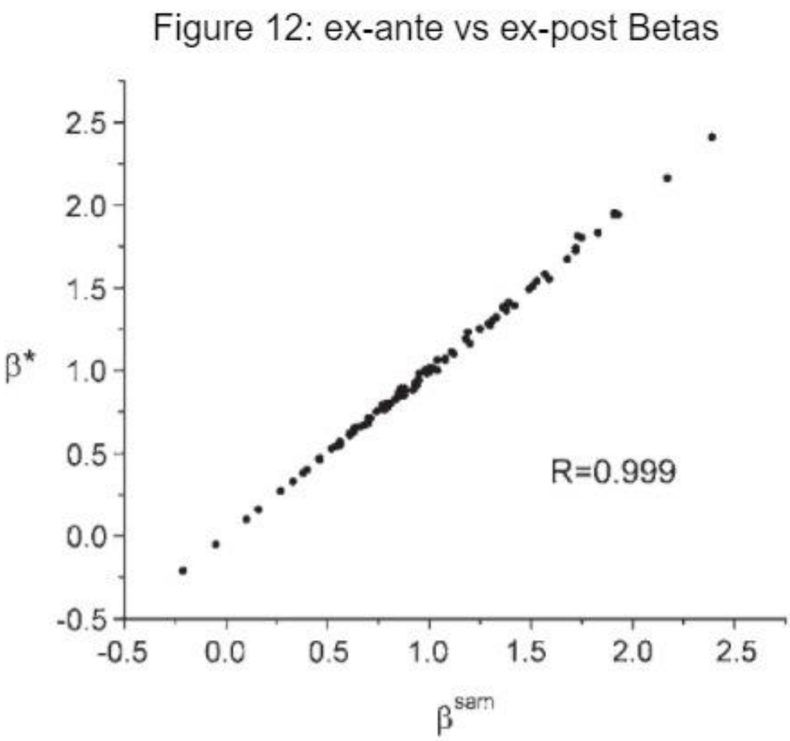

Figure 12: ex-ante vs ex-post Betas (Levy & Roll, 2010)

This similarity of β^* and β^S can provide support to the practical use of the SML equation as a tool to estimate the weighted average cost of capital for publicly listed firms. One can, assuming that the CAPM is correct, confidently employ sample $\beta^{S\prime}s$ for a good approximation of the expected return of risky assets.

Levy and Roll (2010) argue that when analysing the expected return for portfolio optimisation under the assumption that the CAPM is correct, the expected returns should be estimated from the SML equation (5.12) using sample β^S's instead of using the mean historical returns (μ^s) directly. This will yield an expected return estimate closer to the true expected return. Figure 13 shows this graphically. Panel A shows the cross-sectional relation between sample mean return and sample β^S's for all stocks in Levy and Roll's research. There is no close relationship apparent. In panel B, adjusted ex-ante mean returns μ and adjusted $\beta^{*\prime}s$ are plotted against each other. It is no surprise that a perfect relationship holds in panel B as this relationship is assumed from the constraint (8.3) in optimisation problem (8.2), small deviations from the perfect relationship in panel B are caused by rounding errors (Levy & Roll, 2010).

Figure 13: Relationship Betas and ex-ante/ex-post means

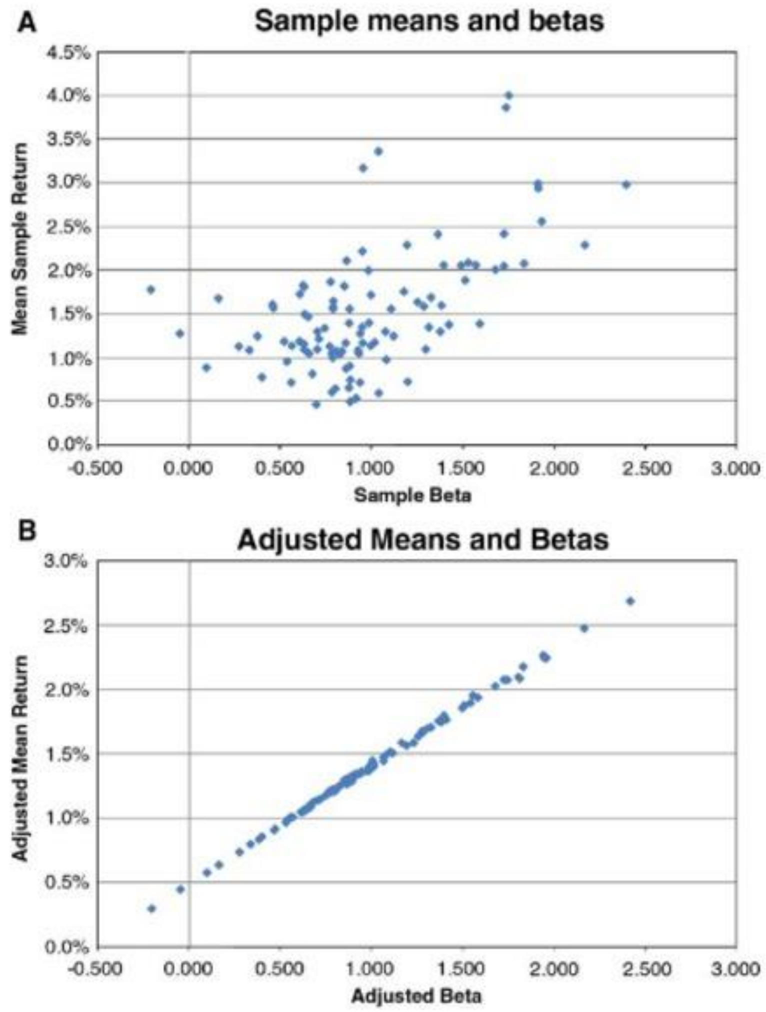

Figure 13: Relationship Betas and ex-ante/ex-post means (Levy & Roll, 2010)

To conclude, in order to estimate the expected return of a risky asset one should ignore the sample mean return μ^s as it is not closely related to the β^S (and the true β^*) at all. The expected return obtained from equation (5.12) provides a better estimate of the true ex-ante return μ as β^* and β^S appear to be closely related (figure 12). It should be stressed that this finding only holds if the CAPM is valid.

8.6 Conclusion

In this chapter the reverse engineering test was introduced. This test is both technically and theoretically more correct than the double-pass regression based test to test the CAPM. In their break-through article Levy and Roll (2010) find that when using the correct reverse engineering test, the CAPM cannot be rejected in the U.S. setting. Furthermore, they find that the use of the SML equation (5.12) is justified to estimate the expected return of a risky asset. The historical mean return on a risky asset should be ignored when estimating the expected return and cost of capital.

More practical implications and possible limitations of the reverse engineering test will be handled in more detail in the empirical part of this thesis where the reverse engineering test will be used to test whether the results from Levy's and Roll's research also hold in the setting of the Belgian stock market.

9. Sample and Data

The dataset employed to test whether the CAPM applies to the Belgian stock market consists of monthly price data of the 50 largest publicly listed Belgian companies from the period January 2008-March 2018, imported from yahoo finance. The largest companies are determined based on their total market capitalisation in March 2018 using market capitalisation data from Belfirst. Using each company's market capitalisation, their respective weight in the overall market portfolio can be calculated using formula 8.4 (appendix A). It should be noted that the largest Belgian company AB INBEV, is rather large compared to other Belgian publicly listed companies and makes up a large part of the market portfolio (45,06%). The performance of ABI is therefore a large factor in the overall performance of the Belgian market portfolio. Knowing each asset's weight in the market portfolio, the total average return and volatility of the market portfolio can be calculated using equation 4.2 and 4.3, resulting in a mean monthly market return and volatility of 1,22% and 5,40% respectively.

Appendix A not only lists the 50 largest companies by market capitalisation and their respective weights in the market portfolio, but also the defines the sector in which the company undertakes their principal activities. This categorisation is based on the ISIC standard (United Nations Statistics Division, 2018) and will allow the reverse engineering test to provide an indication whether a rejection of the CAPM is due to stocks from a particular sector.

The used proxy for the risk-free rate is the 1 month Euribor rate, imported from the online database of the European Central Bank. Euribor or the "European interbank offer rate" is the rate at which European prime rated financial institutions can lend (borrow) at (from) other prime rated European financial institutions (European Banking Federation, 2017). Euribor was chosen as a proxy due to its easy online availability (in comparison to short maturity Belgian government bonds) and close relation to the definition of a risk-free asset from section 4.5. Prime financial institutions have a low probability of default while a maturity of one month minimises both interest rate- and unexpected inflation risk. Furthermore, as Euribor is quoted in Euro, foreign exchange risk does not need to be taken into account.

As the dataset supplied the 1 month Euribor rate expressed as an annual percentage rate (APR), the rate had to be divided by 12 for usage in the reverse engineering test. As the reverse engineering test requires an average risk-free rate over the analysed period as a whole, the monthly Euribor is averaged over the sample period, resulting in a risk-free rate of approximately 0.053%.

10. The reverse engineering test: Matlab implementation

In this empirical chapter, the algorithm employed to solve the nonlinear optimisation problem of reverse engineering test (equations 8.2 and 8.3) will be discussed in more detail. In order to give the reader an idea of how the solution was obtained, a brief description of the Matlab implementation will be given. The Matlab code can be consulted in full in this thesis' appendices. The code is accompanied by explanatory comments in order to provide the reader with a step by step guide to the employed method.

10.1 Matlab implementation: fmincon

Just as in the main article on the reverse engineering test (Levy & Roll, 2010), this thesis conducts the test in a Matlab environment using Matlab's built in fmincon algorithm in the main program. The exact description of the fmincon function given by Matlab is given below (Mathworks, 2018):

Find the minimum of a non-linear programming problem specified by:

$$\min_{x} f(x) \ \ such\ that \begin{cases} c(x) \leq 0 \\ ceq(x) = 0 \\ A * x \ \leq b \\ Aeq * x = beq \\ lb \ \leq x \leq ub \end{cases} \qquad (10.1)$$

With beq en ceq vectors, A and Aeq matrices, c(x) and ceq(x) functions that return vectors, f(x) a function that returns a scalar and x, lb and ub either vectors or matrices. Aeq and Beq represent linear contraints while c(x) and ceq(x) can be nonlinear in nature. Lb and Ub are respectively the lower and upper bounds of the x values in the optimisation problem.

10.2 Matlab implementation: fmincon algorithm

In order to solve the non-linear optimisation problem, fmincon uses a non-linear programming algorithm based on Newton's interior point Algorithm (Mathworks, 2018). As operations research is not the main topic of this thesis, the exact mathematical explanation of this method is not given. Instead a more intuitive approach to explaining the algorithm will be used. If the reader is interested in the full mathematical explanation of the algorithm, the Mathworks library can be consulted.

Based on the description given by Matlab on the fmincon function, optimisation starts by calculating an initial value of the objective function f(x) using an initial solution vector X_0. For this initial solution fmincon not only calculates the objective function value (equation 8.2), but also the maximum violation of the constraints of the optimisation problem (equation 8.3). After the initial solution is found, fmincon searches the nearby area of feasible solutions taking turns in minimising both the objective function value and the maximum constraint violation. This procedure continues until the next iteration lowers the objective function by less than 10^{-6} and the maximum constraint violation is less than 10^{-6}.

Below a sample output of the Matlab program is given for an α value of 0.75. Note that for each iteration the solver computes multiple objective function values before moving on to the next iteration. Both the objective function value and maximum constraint violation decrease over each iteration resulting in a minimum objective function value of approximately 0.0941.

```
____________________________________________________________
   Diagnostic Information

Number of variables: 100

Functions
Objective:                   optimfcnchk/checkfun
Gradient:                                        finite-differencing
Hessian:                              finite-differencing (or Quasi-Newton)
Nonlinear constraints:                optimfcnchk/checkfun
Nonlinear constraints gradient:       finite-differencing

Constraints
Number of nonlinear inequality constraints: 0
Number of nonlinear equality constraints:   50

Number of linear inequality constraints:    0
Number of linear equality constraints:      0
Number of lower bound constraints:          50
Number of upper bound constraints:          0

Algorithm selected
   interior-point


____________________________________________________________
   End diagnostic information
                                                 First-order      Norm of
 Iter F-count            f(x)  Feasibility   optimality         step
    0     101    6.778514e+00    5.653e-01    1.742e-01
    1     202    5.173469e+00    2.432e-01    5.419e-01    2.558e+00
    2     303    4.278361e+00    6.573e-02    4.183e-01    9.268e-01
    3     404    3.921884e+00    1.394e-02    2.186e-01    8.216e-01
    4     505    3.688638e+00    1.366e-03    1.327e-01    2.643e-01
    5     606    2.462989e+00    5.411e-03    3.247e-01    1.344e+00
    6     707    1.811043e+00    3.085e-03    2.632e-01    8.323e-01
    7     808    9.712717e-01    1.226e-02    4.324e-01    1.608e+00
    8     909    8.312497e-01    1.508e-03    2.095e-01    6.159e-01
    9    1010    6.105781e-01    1.316e-03    1.062e-01    4.886e-01
   10    1111    5.987802e-01    4.492e-04    6.223e-02    3.467e-01
   11    1212    5.189225e-01    1.172e-04    4.375e-02    2.368e-01
   12    1313    4.906568e-01    1.143e-04    3.984e-02    9.518e-02
   13    1414    4.702131e-01    1.405e-05    2.000e-02    8.847e-02
   14    1515    1.459742e-01    3.961e-03    4.152e-01    8.529e-01
   15    1616    1.590120e-01    2.240e-04    2.103e-01    2.728e-01
   16    1718    1.394138e-01    3.849e-04    1.802e-01    2.825e-01
   17    1820    1.183390e-01    5.802e-04    2.352e-01    1.090e-01
   18    1922    1.041882e-01    3.479e-04    2.266e-01    2.384e-01
   19    2024    1.019054e-01    2.641e-04    1.472e-01    6.511e-02
```

20	2127	1.031255e-01	2.108e-04	1.046e-01	7.075e-02
21	2230	1.037921e-01	1.612e-04	8.060e-02	3.615e-02
22	2333	1.027029e-01	1.215e-04	6.564e-02	1.367e-02
23	2435	1.004426e-01	6.573e-05	5.856e-02	1.752e-02
24	2538	9.989884e-02	4.935e-05	4.508e-02	9.439e-03
25	2641	1.001181e-01	3.727e-05	3.521e-02	3.751e-03
26	2744	1.005330e-01	2.804e-05	2.229e-02	4.299e-03
27	2847	1.007043e-01	2.104e-05	1.544e-02	2.354e-03
28	2951	1.007193e-01	1.841e-05	1.335e-02	4.688e-04
29	3056	1.006718e-01	3.818e-08	4.014e-03	9.993e-04
30	3158	9.732828e-02	1.034e-05	1.253e-01	3.459e-02

Iter	F-count	f(x)	Feasibility	First-order optimality	Norm of step
31	3261	9.491493e-02	8.446e-06	1.049e-01	2.873e-02
32	3364	9.451104e-02	7.196e-06	6.906e-02	1.330e-02
33	3467	9.464862e-02	5.543e-06	8.692e-02	1.508e-02
34	3570	9.463104e-02	4.314e-06	3.200e-02	7.518e-03
35	3673	9.462940e-02	3.256e-06	3.239e-02	6.283e-03
36	3776	9.462172e-02	2.814e-06	1.306e-02	7.689e-03
37	3879	9.454728e-02	2.151e-06	1.557e-02	3.972e-03
38	3982	9.447281e-02	1.622e-06	1.433e-02	3.847e-03
39	4085	9.440227e-02	1.224e-06	1.044e-02	3.751e-03
40	4188	9.438213e-02	9.227e-07	1.287e-02	2.253e-03
41	4291	9.438334e-02	7.044e-07	8.702e-03	2.503e-03
42	4395	9.438929e-02	6.164e-07	5.432e-03	1.278e-03
43	4498	9.440504e-02	4.693e-07	4.022e-03	1.467e-03
44	4601	9.441590e-02	3.541e-07	3.347e-03	9.778e-04
45	4704	9.442156e-02	2.751e-07	2.949e-03	1.007e-03
46	4807	9.441841e-02	2.066e-07	1.679e-03	7.396e-04
47	4910	9.441041e-02	1.567e-07	1.557e-03	6.427e-04
48	5013	9.440169e-02	1.197e-07	2.262e-03	7.144e-04
49	5116	9.439920e-02	9.002e-08	1.216e-03	3.910e-04
50	5219	9.439990e-02	6.757e-08	1.233e-03	3.846e-04
51	5321	9.440486e-02	3.494e-08	9.350e-04	5.454e-04
52	5423	9.440757e-02	1.752e-08	8.842e-04	3.647e-04
53	5525	9.440693e-02	8.849e-09	8.228e-04	4.927e-04
54	5627	9.440383e-02	4.428e-09	8.004e-04	3.617e-04
55	5728	9.420850e-02	1.137e-06	1.748e-02	1.104e-02
56	5831	9.416692e-02	8.338e-07	1.182e-02	4.348e-03
57	5934	9.415009e-02	5.776e-07	9.523e-03	3.339e-03
58	6037	9.415605e-02	4.656e-07	6.393e-03	2.978e-03
59	6140	9.415971e-02	3.517e-07	4.467e-03	2.274e-03
60	6243	9.415871e-02	2.435e-07	2.149e-03	1.562e-03

Iter	F-count	f(x)	Feasibility	First-order optimality	Norm of step
61	6346	9.415625e-02	1.776e-07	2.223e-03	9.601e-04
62	6448	9.415129e-02	8.889e-08	3.004e-03	1.079e-03
63	6550	9.414688e-02	4.529e-08	2.199e-03	9.751e-04
64	6652	9.414645e-02	2.157e-08	1.338e-03	8.076e-04
65	6754	9.414910e-02	1.137e-08	8.804e-04	8.162e-04
66	6856	9.415020e-02	7.978e-09	8.685e-04	5.263e-04
67	6959	9.415018e-02	7.239e-09	8.864e-04	3.266e-04
68	7061	9.414899e-02	3.612e-09	3.728e-04	2.655e-04
69	7164	9.414839e-02	2.929e-09	1.600e-04	1.561e-04
70	7265	9.413978e-02	7.107e-08	2.254e-03	2.457e-03
71	7367	9.413942e-02	5.037e-08	3.129e-03	1.135e-03
72	7469	9.413917e-02	4.439e-08	1.483e-03	1.226e-03

```
    73     7571    9.413888e-02    3.780e-08    9.413e-04    7.224e-04
    74     7673    9.413862e-02    2.111e-08    6.415e-04    5.657e-04
    75     7775    9.413839e-02    1.092e-08    6.065e-04    3.519e-04
    76     7877    9.413818e-02    6.146e-09    2.627e-04    2.671e-04
    77     7978    9.413821e-02    2.728e-10    1.743e-04    3.187e-04
    78     8079    9.413825e-02    1.293e-10    8.759e-05    1.613e-04
    79     8181    9.413823e-02    1.435e-10    4.302e-05    7.027e-05
    80     8282    9.413821e-02    2.438e-12    3.200e-05    4.074e-05
    81     8383    9.413779e-02    1.055e-09    1.498e-04    3.943e-04
    82     8485    9.413779e-02    5.375e-10    6.961e-05    9.027e-05
    83     8586    9.413781e-02    5.741e-11    4.198e-05    9.652e-05
    84     8687    9.413780e-02    1.159e-11    8.086e-06    4.500e-05
    85     8789    9.413780e-02    5.797e-12    6.400e-06    6.772e-06
    86     8890    9.413778e-02    9.722e-11    3.652e-05    8.119e-05
    87     8991    9.413778e-02    2.118e-11    2.000e-05    4.461e-05
    88     9092    9.413778e-02    2.450e-12    1.568e-05    1.831e-05
    89     9193    9.413778e-02    2.004e-12    1.426e-05    1.670e-05
    90     9294    9.413778e-02    6.255e-13    1.280e-06    1.007e-05

                                            First-order      Norm of
 Iter F-count            f(x)  Feasibility   optimality         step
    91     9395    9.413778e-02    1.631e-12    2.815e-06    1.419e-05
    92     9496    9.413778e-02    5.122e-14    3.126e-06    2.262e-06
    93     9597    9.413778e-02    5.140e-15    1.812e-06    2.108e-06
    94     9698    9.413778e-02    5.253e-15    2.560e-07    1.076e-06
    95     9799    9.413778e-02    5.793e-14    6.919e-07    2.729e-06
    96     9900    9.413778e-02    9.563e-16    5.120e-08    2.948e-07
    97    10001    9.413778e-02    3.086e-15    7.879e-08    6.621e-07
    98    10102    9.413778e-02    1.388e-17    1.394e-08    4.028e-08
    99    10203    9.413778e-02    4.337e-18    1.524e-09    7.758e-09

Local minimum possible. Constraints satisfied.
```

10.3 Matlab implementation: Matlab code

In the previous sections of this chapter a general description of the fmincon function in Matlab was given. The following sections will deal with its implementation in the Matlab environment and describe the problems faced when conducting the reverse engineering test with fmincon. For a full walkthrough of the Matlab code, the reader can consult appendices B1, B2, B3 and B4.

10.3.1 Fmincon function

When fmincon is called as a function in Matlab the code is as follows (Mathworks, 2018):

x = fmincon(fun, X_0, A, b, Aeq, beq, lb, ub, nonlcon, options),

with x the solutions vector, fun the objective function, X_0 the initial solutions vector, A, b, Aeq and beq linear (in)equalities, lb and ub the lower and upper bounds of each variable in the optimisation problem, nonlcon the non-linear contraints and options are a set of specifications given to adapt the algorithm/output.

Before explaining the structure of the written program, it is necessary to discuss two potential problems of using fmincon for the reverse engineering test. First of all, the output x is specified as a vector, while the reverse engineering test clearly requires two different sets of outputs (μ, σ). This problem is solved by creating a 2*N vector x, where the first N terms represent the different stocks' mean returns and the terms from N+1 up till 2N represent the stocks' volatilities. For example: the first stock's ex-ante mean return and volatility will be represented respectively by element 1 and N+1 in vector x. The lower bound of the first N variables is set to $-\infty$ and 0 for the variables N+1 to 2N.

Second of all, note that using an initial solution vector X_0 can make fmincon prone to error. It could be possible that the objective function depicted in equation 8.2 has several local minima, resulting in a possible earlier termination of the fmincon algorithm, without finding the global minimum. To solve this, the Matlab script was run with several different initial solution vectors. As all different runs of the program led to the same solution for each value of α, further analysis was not needed.

10.3.2 Program structure

Figure 13 shows the structure of the written Matlab program, to provide the reader a clear, orderly comprehension of the used methods as well as maintaining an efficient program, the main program (appendix B1) was divided into several stages.

In the first stage a function FinitialSolotion is called (appendix B2) in order to calculate the initial values of the market return and volatility while an initial solution vector is created and the lower bound of the optimisation problem is specified. In appendix B2 the initial solution vector is set to zero but, in order to make sure that fmincon gives the global minimum, this initial solutions vector was varied in multiple different runs, each time leading to the same solution.

In the second stage the fmincon function is called. In order to maintain efficiency and a clear overall view of the program, two sub-functions: Fobject (appendix B3) and Fceq (appendix B4) are used, representing equation 8.2 and 8.3 respectively.

Lastly, in the final stage the output from the fmincon function is used as input in statistical t- and chi²-tests after which the ex-ante betas are calculated according to formula 8.5 (appendix B1).

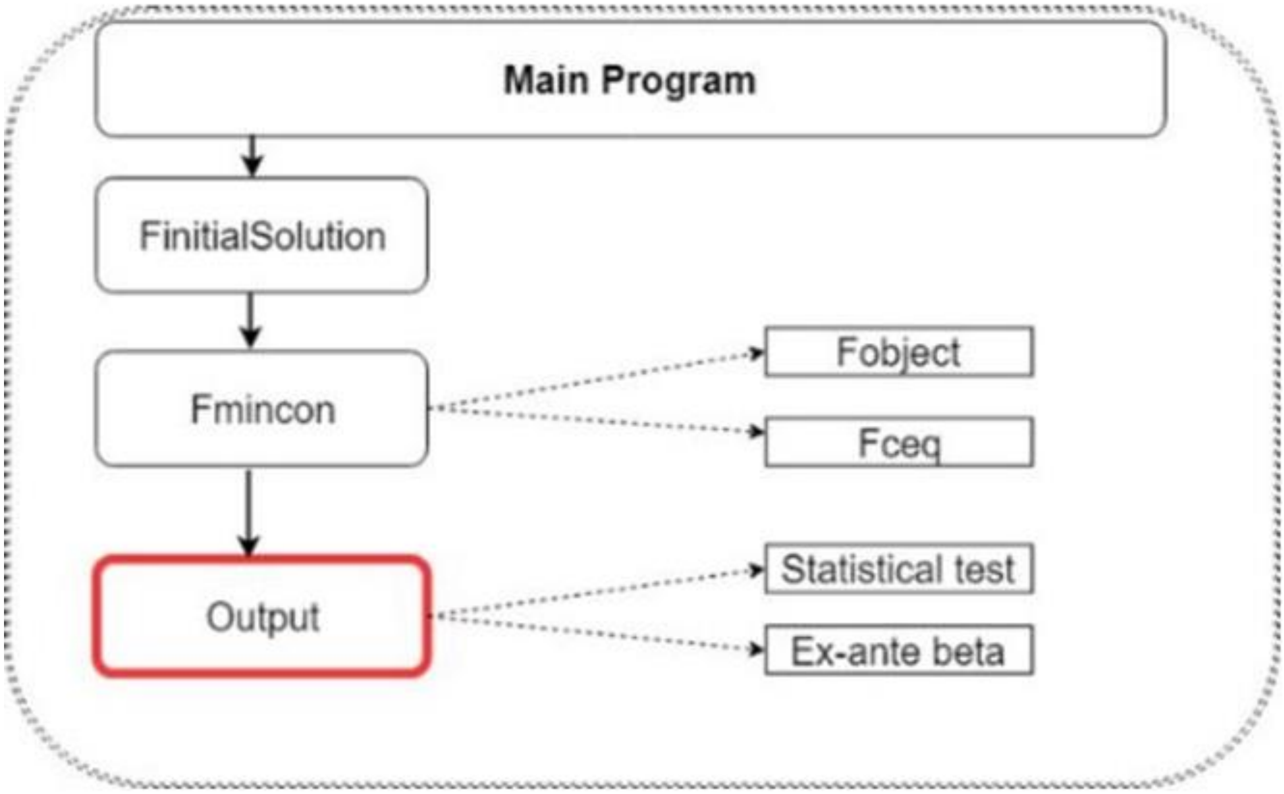

Figure 14: Main Program

11. Results & Discussion

In this thesis' chapter, the results obtained from the reverse engineering test using the methods described in chapter 10 are handled in more detail. The chapter will start by summarizing the results and drawing a conclusion concerning the validity of the CAPM model in the Belgian market. After proper statistical tests are conducted, the limitations and drawbacks of the reverse engineering test will be discussed.

11.1 Test Results

In order to conduct the reverse engineering test, a value for the parameter α needs to be determined. Remember from chapter 8 that this parameter determines the relative importance of differences in mean returns versus volatilities in objective function 8.2. α can take any value between 0 and 1, the higher (lower) the value of α, the higher the importance of deviations in mean returns (volatilities) in the reverse engineering test. In other words if α is higher, differences between sample- and ex-ante mean returns weigh more heavily than deviations in sample- and ex-ante volatilities. Previous research has shown that the reverse engineering test yields robust results for α values in the range 0.5-0.75 (Giannakopoulos, 2013). As a result, the test in this thesis is conducted for several values of α within this range (0.50, 0.55, 0.60, 0.65, 0.70 and 0.75).

Note that there are two possible ways to test the validity of the CAPM (at a significance level of 5%) using the reverse engineering test. The first method is to reject the CAPM when more than 95% of the tested parameters are statistically significantly different from their sample counterpart on the 5% significance level. In this particular case, where 100 parameters are tested simultaneously, the CAPM is rejected when more than 5 parameters are statistically significantly different from their sample counterparts. This method however, fails to take into account that the reverse engineering test actually tests a multiple-comparison hypothesis where, in this case, 100 hypotheses are tested simultaneously. As multiple hypotheses are tested at the same time, the probability of one of these hypotheses being statistically significant due simple chance increases, especially when the number of hypotheses is large. In order to mitigate this problem, one could employ the so-called Bonferonni correction. This method takes into account the extra chance for false-positive results in multiple-comparison hypotheses by rejecting the overall test when one of the individual hypotheses is rejected at the k/n significance level. Here k is the overall level of significance (0.05) and n is the number of hypotheses tested (100) (Goldman, 2008). This would mean that in this particular instance of the reverse engineering test, the CAPM would be rejected if one of the hypotheses is significant at the 0.0005 level.

For a return series of 122 observations this yields critical t-values of +- 1.979 and +-3.577 for the standard and Bonferonni approach respectively. The confidence intervals for the Chi²-test are given by [0,789;1,308] and [0,662;1,632]. These critical values are summarised in table 1.

	t		**CHI²**	
Critical Values	**Lower**	**Upper**	**Lower**	**Upper**
Standard	-1,979	+1,979	0,789	1,308
Bonferonni	-3,577	+3,577	0,662	1,632

Table 1: Critical values

Table 2 shows the results of the reverse engineering test for an α value of 0.75. The numbers shown in bold exceed critical values in the standard case. Note that in the standard case, 4 ex-ante mean returns and 1 ex-ante volatility are statistically significantly different from their sample counterparts. The stocks corresponding to these rejected parameters are Lotus Bakeries, Econocom group, VGP and Kinepolis group for the mean returns and AB Inbev for the statistically significantly different ex-ante volatility. Note that the stocks for which the ex-ante parameters are statistically significantly different from their sample counterparts are all from different sectors. As a rejection of the CAPM needs more than 5 parameters to be statistically significant, the CAPM is not rejected under the standard method. Using the more appropriate Bonferonni correction, not a single parameter is statistically significant, leading again to a no rejection of the CAPM. Table 3 summarises the results of the reverse engineering test for all different values of α. The reader can consult appendices C1, C2, C3, C4, C5 and C6 for the Matlab output of each α value.

Ticker	Ex-ante μ	Sample μ	Ex-ante σ	Sample σ	Ex-ante β	chi²	t
ABI	0,0179	0,0132	0,0860	0,0750	1,2706	**1,3129**	0,6883
KBC	0,0212	0,0129	0,1577	0,1556	1,5080	1,0278	0,5866
GBLB	0,0071	0,0058	0,0475	0,0475	0,4792	1,0012	0,2951
UCB	0,0077	0,0111	0,0804	0,0784	0,5236	1,0518	-0,4805
SOLB	0,0114	0,0093	0,0837	0,0838	0,7957	0,9984	0,2789
RTL	0,0077	0,0067	0,0763	0,0758	0,5269	1,0119	0,1590
UMI	0,0082	0,0143	0,1297	0,1275	0,5599	1,0356	-0,5250
PROX	0,0036	0,0083	0,0588	0,0577	0,2254	1,0397	-0,8967
AGS	0,0235	0,0070	0,1438	0,1489	1,6738	0,9320	1,2146
TNET	0,0072	0,0139	0,0714	0,0688	0,4881	1,0783	-1,0732
COLR	0,0016	0,0080	0,0513	0,0507	0,0764	1,0218	-1,3906
ACKB	0,0071	0,0105	0,0654	0,0640	0,4794	1,0443	-0,5862
SOF	0,0079	0,0085	0,0478	0,0473	0,5348	1,0211	-0,1585
KBCA	0,0248	0,0156	0,1816	0,1843	1,7757	0,9707	0,5533
MELE	0,0130	0,0249	0,0888	0,0833	0,9083	1,1372	-1,5803
ABLX	0,0121	0,0264	0,1653	0,1612	0,8414	1,0514	-0,9794
ELI	0,0021	0,0091	0,0444	0,0436	0,1134	1,0377	-1,7789
SOLV	0,0087	0,0087	0,0675	0,0675	0,5987	1,0025	0,0090
BEKB	0,0216	0,0285	0,2746	0,2714	1,5353	1,0239	-0,2828
COFB	0,0048	0,0014	0,0416	0,0426	0,3123	0,9537	0,8891
WDP	0,0072	0,0129	0,0523	0,0500	0,4859	1,0945	-1,2585
LOTB	0,0051	0,0223	0,0617	0,0578	0,3306	1,1403	**-3,2738**
ECONB	0,0115	0,0348	0,1273	0,1209	0,8009	1,1082	**-2,1149**
KIN	0,0070	0,0295	0,0779	0,0722	0,4701	1,1657	**-3,4308**
TESB	0,0088	0,0089	0,0855	0,0854	0,6012	1,0035	-0,0203
PIC	0,0104	0,0217	0,1229	0,1198	0,7205	1,0519	-1,0351
BEFB	0,0064	0,0061	0,0579	0,0579	0,4264	0,9998	0,0572
AED	0,0044	0,0122	0,0456	0,0436	0,2858	1,0949	-1,9470
GIMB	0,0046	0,0104	0,1141	0,1132	0,2940	1,0157	-0,5699
BAR	0,0113	0,0156	0,1249	0,1235	0,7833	1,0235	-0,3892
BNB	0,0083	0,0057	0,0636	0,0645	0,5640	0,9730	0,4426
VPG	-0,0001	0,0152	0,0694	0,0698	-0,0483	0,9893	**-2,4176**
EURN	0,0116	0,0040	0,1241	0,1261	0,8090	0,9690	0,6627
OBEL	0,0017	-0,0037	0,0806	0,0809	0,0872	0,9945	0,7438
GREEN	0,0053	0,0060	0,0750	0,0748	0,3511	1,0043	-0,0961
FAGR	0,0099	0,0142	0,1203	0,1192	0,6850	1,0191	-0,3945
IBAB	0,0129	0,0106	0,1213	0,1218	0,9046	0,9914	0,2085
RET	0,0038	0,0104	0,0449	0,0434	0,2384	1,0688	-1,6721
AGFB	0,0155	0,0075	0,1476	0,1499	1,0906	0,9696	0,5868
WEHB	0,0038	0,0114	0,0492	0,0477	0,2356	1,0655	-1,7701
COMB	0,0072	0,0084	0,0683	0,0679	0,4874	1,0112	-0,1877
VAN	0,0048	0,0073	0,0558	0,0551	0,3133	1,0221	-0,4967
SIOE	0,0102	0,0164	0,0910	0,0889	0,7068	1,0468	-0,7681
Immo	0,0138	0,0142	0,1029	0,1026	0,9672	1,0045	-0,0495
TIG	0,0064	0,0141	0,2027	0,2019	0,4255	1,0082	-0,4243
MONT	0,0034	0,0107	0,0437	0,0422	0,2069	1,0701	-1,9195
REC	0,0154	0,0137	0,1227	0,1232	1,0871	0,9919	0,1510
LEAS	0,0055	0,0090	0,0509	0,0498	0,3609	1,0431	-0,7870
INTO	0,0084	0,0066	0,0614	0,0620	0,5732	0,9783	0,3236
EVS	0,0115	0,0020	0,0917	0,0950	0,7995	0,9305	1,0986

Table 2: Test Results = α 0.75

α	Rejected # μ (95%)	Rejected # σ (95%)	Rejected # μ (bonferonni)	Rejected # σ (bonferroni)
0,50	5	0	0	0
0,55	5	0	0	0
0,60	5	0	0	0
0,65	5	0	0	0
0,70	4	0	0	0
0,75	4	1	0	0

Table 3:Test Results different α's

Note that the results are similar for all values of α except for the case of 0.75 and 0.70. In all other cases than the 0.75 or 0.70 one, 5 ex-ante mean returns are statistically significantly different from their sample counterparts when tested using separate t-tests under the standard case. In these cases, not a single ex-ante volatility differs from its sample counterpart. The corresponding stocks of the rejected parameters are Kinepolis group, Econocom group, Lotus Bakeries, VGP and Aedifica. All of these are from different sectors except VGP and Aedifica that are both active in the real estate sector. In the case of an α value of 0.70, only the ex-ante mean returns of Kinepolis group, Econocom group, Lotus Bakeries and VGP are statistically significantly different from their sample counterparts. Note that for α values lower than 0.75, the ex-ante volatility of AB Inbev is no longer statistically significantly different from its sample counterpart. This can be explained by the lower values of α that increase the importance of deviations in volatilities relative to mean returns in the objective function. As for each value of α there are still only 5 or 4 parameters statistically significantly different from their sample counterparts and none under the Bonferonni correction, the CAPM is not rejected at the 5% significance level.

11.2 The ex-ante vs sample Beta

Table 4 shows the calculated sample betas as well as the ex-ante betas resulting from the reverse engineering test together with the calculated expected mean returns from sample values using the SML equation ($E(R_k) = R_f + (E(R_m) - R_f)\beta_k$) from section 5.2 for an α value of 0.60. Remember from section 5.3 that a beta greater (less) than 1 signifies that the asset carries relatively more (less) market risk than the market portfolio. Note that the found ex-ante betas (column 2) are relatively close to their sample counterparts (column 1). The reasoning behind this was already explained in section 8.5 by the similarity in equations 8.5 and 8.6 and the offsetting potential of the volatilities. A closer look at table 4 explains the importance of using estimated sample returns from sample betas in portfolio optimisation under the CAPM theory, introduced in section 8.5.

Note the mean returns estimated from sample betas (column 3) using SML equation 5.12 are closer to the real ex-ante values obtained from the reverse engineering test (column 5) than the historical average sample values (column 4). Conducting t-tests (using real ex-ante volatility), it can be concluded that not a single estimated sample return parameter under the CAPM is statistically significantly different from the real ex-ante value on the 5% level (critical value +- 1.979 or +- 3.373 with Bonferroni correction). This provides support to the claim that if the CAPM holds, one should use the estimated return parameters from the SML equation and not historical averages in portfolio optimisation. Appendices D1, D2, D3, D4, D5 and D6 show this result for all tested values of α.

Ticker	Sample β	Ex-ante β (α=0,60)	SML μ	Sample μ	Ex ante μ (α = 0,60)	Ex-ante σ	t
ABI	1,1681	1,2318	0,0142	0,0132	0,0164	0,0817	0,2984
KBC	1,6920	1,5785	0,0203	0,0129	0,0209	0,1570	0,0382
GBLB	0,5340	0,5001	0,0068	0,0058	0,0070	0,0475	0,0457
UCB	0,5588	0,5355	0,0071	0,0111	0,0074	0,0795	0,0504
SOLB	0,8843	0,8298	0,0109	0,0093	0,0112	0,0838	0,0456
RTL	0,5822	0,5475	0,0073	0,0067	0,0076	0,0761	0,0352
UMI	0,6279	0,5841	0,0079	0,0143	0,0081	0,1286	0,0154
PROX	0,2411	0,2307	0,0034	0,0083	0,0035	0,0582	0,0289
AGS	1,8986	1,7679	0,0228	0,0070	0,0233	0,1466	0,0428
TNET	0,5113	0,4949	0,0065	0,0139	0,0069	0,0702	0,0621
COLR	0,0791	0,0773	0,0015	0,0080	0,0015	0,0510	0,0153
ACKB	0,5217	0,4942	0,0066	0,0105	0,0069	0,0647	0,0450
SOF	0,5833	0,5529	0,0074	0,0085	0,0077	0,0476	0,0696
KBCA	2,0350	1,8763	0,0244	0,0156	0,0247	0,1831	0,0222
MELE	0,9354	0,9132	0,0115	0,0249	0,0123	0,0862	0,1057
ABLX	0,9153	0,8664	0,0112	0,0264	0,0117	0,1633	0,0309
ELI	0,1193	0,1154	0,0019	0,0091	0,0020	0,0440	0,0228
SOLV	0,6667	0,6243	0,0083	0,0087	0,0086	0,0675	0,0399
BEKB	1,6688	1,5834	0,0201	0,0285	0,0209	0,2731	0,0355
COFB	0,3517	0,3280	0,0047	0,0014	0,0048	0,0421	0,0293
WDP	0,5037	0,4906	0,0064	0,0129	0,0069	0,0512	0,0924
LOTB	0,3353	0,3299	0,0045	0,0223	0,0048	0,0597	0,0607
ECONB	0,8269	0,8061	0,0102	0,0348	0,0109	0,1241	0,0634
KIN	0,4794	0,4696	0,0061	0,0295	0,0066	0,0750	0,0651
TESB	0,6642	0,6248	0,0083	0,0089	0,0086	0,0855	0,0362
PIC	0,7757	0,7392	0,0096	0,0217	0,0101	0,1214	0,0410
BEFB	0,4643	0,4408	0,0060	0,0061	0,0062	0,0580	0,0471
AED	0,2870	0,2848	0,0039	0,0122	0,0042	0,0446	0,0774
GIMB	0,3255	0,3053	0,0043	0,0104	0,0045	0,1136	0,0121
BAR	0,8688	0,8142	0,0107	0,0156	0,0110	0,1242	0,0291
BNB	0,6265	0,5886	0,0079	0,0057	0,0081	0,0641	0,0440
VPG	-0,0518	-0,0497	-0,0001	0,0152	-0,0001	0,0696	-0,0054
EURN	0,9068	0,8474	0,0111	0,0040	0,0115	0,1251	0,0273
OBEL	0,1022	0,0928	0,0017	-0,0037	0,0017	0,0808	-0,0001
GREEN	0,3875	0,3646	0,0051	0,0060	0,0052	0,0749	0,0244
FAGR	0,7391	0,7046	0,0092	0,0142	0,0096	0,1198	0,0399
IBAB	1,0043	0,9426	0,0123	0,0106	0,0127	0,1216	0,0360
RET	0,2504	0,2419	0,0035	0,0104	0,0037	0,0441	0,0472
AGFB	1,2298	1,1449	0,0149	0,0075	0,0153	0,1488	0,0270
WEHB	0,2482	0,2395	0,0034	0,0114	0,0036	0,0484	0,0416
COMB	0,5408	0,5072	0,0069	0,0084	0,0071	0,0681	0,0337
VAN	0,3405	0,3231	0,0045	0,0073	0,0047	0,0555	0,0357
SIOE	0,7702	0,7289	0,0096	0,0164	0,0099	0,0900	0,0469
Immo	1,0312	0,9922	0,0126	0,0142	0,0133	0,1028	0,0774
TIG	0,4935	0,4503	0,0063	0,0141	0,0063	0,2023	0,0015
MONT	0,2143	0,2089	0,0030	0,0107	0,0032	0,0429	0,0475
REC	1,2220	1,1382	0,0148	0,0137	0,0152	0,1230	0,0332
LEAS	0,3854	0,3694	0,0050	0,0090	0,0053	0,0503	0,0550
INTO	0,6257	0,5942	0,0079	0,0066	0,0082	0,0618	0,0600
EVS	0,8946	0,8392	0,0110	0,0020	0,0114	0,0934	0,0410

Table 4:Ex-ante Vs Ex-post Beta α = 0.60

11.3 Limitations of the reverse engineering test

While conducting the reverse engineering test, several limitations of the test became apparent that are not fully in line with the CAPM theory and economic reality. These limitations are the invariability of asset weights, the invariability of the correlation matrix, the long test horizon and the use of a market proxy. The reverse engineering test could gain more ground and test the validity of the CAPM in a more reliable way if these limitations were accounted for in future research.

11.3.1 Invariability of asset weights

When conducting the reverse engineering test, Levy and Roll (2010) used the assets' market capitalisations at the end of the sample period in order to determine their weight in the market portfolio. Note that in context of the CAPM theory, an asset's weight in the market portfolio is determined by its current market capitalisation divided by the current total value of the market or the sum of all assets' capitalisations. As asset prices vary over time, their market capitalisations do as well and the weight of each asset in the market portfolio will tend to vary over time. In order to better test the CAPM with the reverse engineering test in the future, the test should allow for variation in this parameter or should at least be divided into several periods i.e. years in which the assets' weights are recalculated.

11.3.2 Invariability of the correlation matrix

The reverse engineering test only allows the mean returns and volatilities of the assets to deviate from their sample counterparts in testing the CAPM due to mathematical difficulties. In reality, it is quite illogical that the calculated sample correlations between returns are identical to their real ex-ante counterparts as the returns themselves are the result of a stochastic process. The possible inclusion of a varying correlation matrix would be beneficial to the reliability of the reverse engineering test.

11.3.3 Long test horizon

In this thesis, the reverse engineering test is used to test the CAPM over a sample period of 122 months or roughly 10 years. This could introduce several issues as some in- and outputs of the CAPM have shown to be unstable over time. It has i.e. been shown that the betas of stocks (Groenewold & Fraser, 2000) could be unstable over longer periods of time. For the reverse engineering test to be more accurate it should include these aspects or be modified to test more advanced CAPM models with time-varying aspects like the conditional CAPM. Both are interesting topics for further research.

11.3.4 Use of a market proxy

The last experienced drawback from using the reverse engineering test is that when conducting the test, a market proxy portfolio is created consisting of all assets (here 50) in the test. Remember from section 6.4.1 that this does not necessarily pose a problem as a proxy portfolio is often strongly correlated with the market portfolio (De Geeter, 2013, Kandel & Stambaugh, 1987 & Shanken, 1987). The test should however make adjustments for this issue, which is not handled by Levy and Roll (2010). As correlation based tests of the CAPM are a whole research subject on their own and the main goal of this thesis is using and scrutinising the reverse engineering approach in the Belgian market, the possibility of the inclusion of these adjustments in the reverse engineering test will not be handled. This issue however, would form in interesting topic for future research.

12. Conclusion

In order to give the reader a clear overview of the validity of the CAPM in the Belgian setting, this chapter will provide a brief summary of both the main theoretical- and empirical findings of this thesis. This chapter is divided into three separate sections. In the first section, a summary of the findings concerning the theory behind the CAPM is given. In the second part, the issues of testing the CAPM are reviewed and finally, a general conclusion is given. Suggestions for further research are drafted over the course of this chapter.

12.1 Summary: findings CAPM theory

In the theoretical part of this thesis, the underlying theories of the CAPM were first discussed. Starting with the concepts of portfolio theory, risk-aversion, diversification, etc. in chapter 4, the CAPM framework was derived in chapter 5. Here the restrictive assumptions behind the CAPM were handled, most of which could be relaxed by adjusting the model.

Three of these assumptions could however not easily be adjusted for. These assumptions were the normality of asset returns, utility as a function of total wealth and risk-aversion. The reason why these assumptions were not in line with reality were handled in the chapter concerning cumulative prospect theory (chapter 7), after which their implications for the CAPM were discussed. It was shown that even if the assumptions of risk-aversion and utility as a function of total wealth did not hold, the CAPM theory remained intact while equilibrium asset prices change. The third assumption, normality of asset returns, proved more troubling for the CAPM theory as it criticises the use of the mean-variance framework because other aspects of the return distribution such as skewness are not taken into account. Previous research however, has shown that under the expected utility framework the mean-variance framework provides a decent approximation for maximising total utility. Under cumulative prospect theory, for reasons explained in section 7.6, the optimality of the mean-variance framework is less clear. The mean-variance framework being a good approximation of expected utility under EUT and possibly a less good approximation under CPT, gives reason to believe that the CAPM is an approximating model at best, that probably not describes the returns of risky assets exactly. The impact of CPT on the efficiency of the mean-variance framework together with research on the possibility of more exact asset pricing models are therefore interesting topics for further research.

12.2 Summary: findings CAPM tests

In chapter 6 of this thesis, the double-pass regression approach to testing the CAPM was discussed. It was shown that this test is subject to three main technical issues as well as Roll's critique. The technical issues concerning the double-pass test were: cross-sectional serial correlation, measurement errors in the $\beta's$ and the use of ex-post data. The reverse engineering test conducted in this thesis escapes these main issues as it searches for the ex-ante risky asset parameters, is not regression based and tests if the market proxy is mean-variance efficient.

Omitted variable bias, handled in section 6.3.1, can no longer pose technical issues under the reverse engineering approach but could still lead to bias if variables other than an asset's β explain the return of risky assets. As other research often points out, such variables could exist and therefore cast doubt on the validity of the CAPM. The existence of such variables would, if these variables explain expected returns correctly, provide a more correct asset pricing model. The possibility of including other variables in the reverse engineering test is therefore an interesting topic for future research.

Finally, the current limitations of the reverse engineering test were discussed. These drawbacks were: the invariability of asset weights, the invariability of the correlation matrix, the long test horizon and the uncorrected use of a market proxy. The implications of these imperfections can be consulted in section 11.3. Correcting for these could provide more support to the validity/rejection of the CAPM and is therefore a necessary subject for future research. If the reverse engineering test does not fully reflect the reality, there is always the possibility of a false (non)rejection of the CAPM.

12.3 General Conclusion

Overall, from the empirical part of this thesis it is made clear that the CAPM cannot be rejected in the context of the Belgian stock market with the current reverse engineering test in the studied time period (January 2008-March 2018). This however, does not mean that the CAPM is correct. As revealed in this thesis, several technical issues with the current form of the reverse engineering test exist. Further research is needed to determine whether the CAPM truly holds in the Belgian setting. It should be noted that there are more advanced versions of the CAPM model out there that possibly provide better predictions than the CAPM (section 5.1). As this thesis only tested the basic CAPM with the reverse engineering test, a conclusion concerning these other models in the Belgian setting could not be drawn.

Suggestions for further research were given over the course of this chapter and include correcting the limitations in the reverse engineering test, conducting the test for different asset pricing models and more. For now it can be concluded that the CAPM cannot be rejected empirically and theoretically in the Belgian setting and could still be used to approximate a company's cost of capital, taught to students to introduce financial theory, approximate expected returns of risky assets, et cetera. It should be stressed that, as reviewed in section 12.1 and 12.2, the CAPM is probably not an exact asset pricing model as the normality of asset returns assumption is violated and other, more correct asset pricing models could exist in which expected returns are explained by variables other than β.

Finally, under the CAPM theory, it is shown that when calculating the expected returns of risky assets i.e. for the purpose of portfolio optimisation, one should not use the simple average historical returns of the risky assets. One should make use of the SML-equation (equation 5.12) instead to get a better estimate of the risky asset's expected return.

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Appendices

Appendix A: Market capitalisations

Company	Ticker	Market Capitalisation	Weight in Market Portfolio	Industry (ISIC)
ANHEUSER-BUSCH INBEV	ABI	146 059 072	45,06%	Manufacturing (Beverages)
KBC GROEP	KBC	30 942 731	9,55%	Financial & Insurance activities
GROUPE BRUXELLES LAMBERT	GBLB	14 370 569	4,43%	Financial & Insurance activities
UCB	UCB	12 767 351	3,94%	Manufacturing (Pharmaceuticals)
SOLVAY	SOLB	11 762 870	3,63%	Manufacturing (Petroleum)
RTL Group SA	RTL	10 143 168	3,13%	Information & Communication (Broadcasting)
UMICORE	UMI	8 979 040	2,77%	Manufacturing (Metals)
PROXIMUS	PROX	8 643 303	2,67%	Telecommunication
AGEAS	AGS	8 606 338	2,65%	Financial Services
TELENET GROUP HOLDING	TNET	6 898 498	2,13%	Information & Communication (Telecom)
ETABLISSEMENTEN FR. COLRUYT	COLR	6 431 408	1,98%	Retail
ACKERMANS & VAN HAAREN	ACKB	4 749 861	1,47%	Financial & Insurance activities
SOFINA	SOF	4 527 850	1,40%	Financial & Insurance activities
KBC ANCORA	KBCA	3 997 282	1,23%	Financial & Insurance activities
MELEXIS	MELE	3 318 860	1,02%	Manufacturing (Electronics)
ABLYNX	ABLX	3 257 820	1,00%	Scientific activities (Pharmaceuticals)
ELIA SYSTEM OPERATOR	ELI	2 862 348	0,88%	Electricity, Gas, Steam & Air conditioning Supply
SOLVAC	SOLV	2 736 004	0,84%	Financial & Insurance activities
BEKAERT	BEKB	2 162 591	0,67%	Manufacturing (Metals)
COFINIMMO	COFB	2 128 740	0,66%	Real Estate
WAREHOUSES DE PAUW	WDP	2 126 096	0,66%	Transportation & Storage
LOTUS BAKERIES	LOTB	1 726 598	0,53%	Manufacturing (Food)
ECONOCOM GROUP	ECONB	1 644 892	0,51%	Information & Communication (Consultancy)
KINEPOLIS GROUP	KIN	1 573 499	0,49%	Arts, Entertainment & Recreation
TESSENDERLO GROUP	TESB	1 565 865	0,48%	Manufacturing (Chemicals)
PICANOL	PIC	1 557 600	0,48%	Manufacturing (Textiles)
BEFIMMO	BEFB	1 340 351	0,41%	Real Estate
AEDIFICA	AED	1 299 652	0,40%	Real Estate
GIMV	GIMB	1 245 907	0,38%	Financial & Insurance activities
BARCO	BAR	1 218 915	0,38%	Information & Communication
NATIONALE BANK VAN BELGIE	BNB	1 184 000	0,37%	Public Administration
VGP	VPG	1 092 683	0,34%	Real Estate
EURONAV	EURN	1 026 102	0,32%	Transportation & Storage
ORANGE BELGIUM	OBEL	956 630	0,30%	Information & Communication (Telecom)
GREENYARD	GREEN	871 478	0,27%	Wholesale (Food)
FAGRON	FAGR	747 895	0,23%	Wholesale (Pharmaceuticals)
ION BEAM APPLICATIONS	IBAB	711 304	0,22%	Manufacturing (Medical)
RETAIL ESTATES	RET	677 425	0,21%	Real Estate
AGFA-GEVAERT	AGFB	661 626	0,20%	Manufacturing (Electronics)
WERELDHAVE BELGIUM	WEHB	630 063	0,19%	Real Estate
COMPAGNIE DU BOIS SAUVAGE	COMB	620 919	0,19%	Financial & Insurance activities
VAN DE VELDE	VAN	556 880	0,17%	Manufacturing & Retail (Wearing Apparel)
SIOEN INDUSTRIES	SIOE	540 981	0,17%	Manufacturing (Textiles)
IMMOBEL	Immo	527 860	0,16%	Real Estate
TIGENIX	TIG	503 795	0,16%	Scientific activities (Pharmaceuticals)
MONTEA	MONT	488 803	0,15%	Real Estate
RECTICEL	REC	465 599	0,14%	Manufacturing (Chemicals)
LEASINVEST REAL ESTATE	LEAS	464 254	0,14%	Real Estate
INTERVEST OFFICES & WAREHOUSES	INTO	400 933	0,12%	Real Estate
EVS BROADCAST EQUIPMENT	EVS	391 719	0,12%	Wholesale (Electronics)

Appendix B1: Main Program

```
% Create all necessary global variables (to be used in each function)

global RiskFree
global STDmarket
global Rmarket
global LowerBound
global X0
global Weights
global alpha
global Scorrmatrix
global meanRet
global n
global t
global SVol

% Define both the risk-free rate (EURIBOR 1M) and alpha

RiskFree = 0.000532944;

% In this example alpha is set to 0.75

alpha = 0.75;

% Run FinitialSolution to calculate sample parameters and set
% initial values for X0

FinitialSolution;

% Run the Fmincon function
% Fobject is the objective function from equation 8.2
% Fceq is the set of equality constraints (eq 8.3)

options =
optimset('MaxFunEvals',100000,'MaxIter',1000,'display','iter','diagnostics','on','FunValCheck'
,'on');
[x, fval, exitflag, output] = fmincon(@Fobject,X0,[],[],[],[],LowerBound,[], @Fceq,options);


% Create a vector to store both the ex-ante mean and volatility

MeanExAnte = x(1:n);
VolExAnte = x(n+1:n+n);


% Calculate the Chi-square value

CHI = (VolExAnte.^2)./(SVol.^2);

% Calculate the t-values

t_value = (MeanExAnte - meanRet)./(SVol./sqrt(t-1));

% Calculate the ex-ante covariance matrix
```

```
EXcovmatrix = VolExAnte .* Scorrmatrix .* VolExAnte';

% Calculate the ex-ante beta for all stocks using equation 8.5 and a loop

for i = 1:n
    for j = 1:n
        Numerator(j) = Weights(j) * VolExAnte(i) * VolExAnte(j) * Scorrmatrix(i,j);
    end

  SumNumerator = sum(Numerator);
  beta_ex_ante(i) = SumNumerator / (Weights' * EXcovmatrix * Weights);
end
```

Appendix B2: FinitialSolution

```
function [] = FinitialSolution

global X0
global Weights
global LowerBound
global STDmarket
global Rmarket
global Scorrmatrix
global Scovmatrix
global meanRet
global n
global SVol
global t

% Import returns and weights from excel

Sreturns = xlsread('return');
Weights = xlsread('weig');

% Calculate the number of stocks & timeperiods

[t,n] = size(Sreturns);

% Calculate mean returns, covariance- and correlationmatrix

meanRet = mean(Sreturns);
Scorrmatrix = corrcoef(Sreturns);
Scovmatrix = cov(Sreturns);

% Calculate stock variance and volatility

SVar = diag(Scovmatrix);
SVol = sqrt(SVar);
SVol = SVol';

% Calculate the mean market return and volatility

Rmarket = meanRet * Weights;
```

```
STDmarket = sqrt(Weights' * Scovmatrix * Weights);

% The initial solution vector is set to zero in this example. In order to
% verify that this initial solution does not yield a local minimum, the
% initial solution vector was varied over multiple runs.

X0 = zeros(2*n);
X0 = diag(X0);
X0 = X0';

% Define a lower bound for the fmincon algorithm, this is minus
% infinity for stock returns and zero for stock volatility

LowerBound = [-inf*ones(1,n),zeros(1,n)];

end
```

Appendix B3: Fobject

```
function [Solution] = Fobject(mean_vol)

global alpha
global meanRet
global n
global SVol

% The fmincon function requires an input objective function specified using
% a vector, mean_vol is a 2*n vector with the first, last n values
% respectively representing the unkown ex-ante mean return & volatilities
% of the selected stocks

i = 1:n;

% Define the ex-ante parameters seperately for use in the objective
% function

mean = mean_vol(1:n);
vol = mean_vol(n+1:n+n);

% Calculate the sum for the first term in eq 8.2

FirstTerm(i) = ((mean(i) - meanRet(i))./SVol(i)).^2;
SumFirstTerm = sum(FirstTerm);

% Calculate the sum for the second term in eq 8.2

SecondTerm(i) = ((vol(i)- SVol(i))./SVol(i)).^2;
SumSecondTerm = sum(SecondTerm);

% Define the solution to the objective distance function

Solution = sqrt(alpha*(1/n)* SumFirstTerm + (1-alpha)*(1/n)*SumSecondTerm);
```

```
end
```

Appendix B4: Fceq

```
function [c,ceq] = Fceq(mean_vol)

global RiskFree
global STDmarket
global Rmarket
global Weights
global Scorrmatrix
global n

% Define ceq as the constraint equated to zero, therefore c equals zero

c = [];

mean = mean_vol(1:n);
vol = mean_vol(n+1:n+n);

% Create a diagonal matrix for unknown ex-ante Volalitity called SigmaDiag

SigmaDiag = diag(vol);

% For the system of constraints the inverse of the weights vector is needed


% Calculate q as in 8.3

q = (STDmarket.^2)/(Rmarket - RiskFree);

% Create a vector RHS (and inverse) for the right-hand side from eq 8.3


RHS = mean - RiskFree;
RHSInv = RHS';

ceq = SigmaDiag * Scorrmatrix * SigmaDiag * Weights - q * RHSInv;


end
```

Appendix C1: Matlab output α = 0.50

Ticker	Ex-ante μ	Sample μ	Ex-ante σ	Sample σ	Ex-ante β	chi²	t
ABI	0,0158	0,0132	0,0798	0,0750	1,2143	1,1309	0,3724
KBC	0,0207	0,0129	0,1567	0,1556	1,6101	1,0144	0,5557
GBLB	0,0069	0,0058	0,0475	0,0475	0,5094	1,0024	0,2564
UCB	0,0073	0,0111	0,0791	0,0784	0,5415	1,0184	-0,5326
SOLB	0,0111	0,0093	0,0838	0,0838	0,8449	1,0016	0,2410
RTL	0,0075	0,0067	0,0760	0,0758	0,5569	1,0051	0,1266
UMI	0,0080	0,0143	0,1282	0,1275	0,5957	1,0124	-0,5413
PROX	0,0035	0,0083	0,0581	0,0577	0,2334	1,0133	-0,9266
AGS	0,0232	0,0070	0,1474	0,1489	1,8062	0,9805	1,1965
TNET	0,0068	0,0139	0,0697	0,0688	0,4989	1,0265	-1,1404
COLR	0,0015	0,0080	0,0509	0,0507	0,0777	1,0071	-1,4058
ACKB	0,0068	0,0105	0,0645	0,0640	0,5014	1,0155	-0,6325
SOF	0,0076	0,0085	0,0475	0,0473	0,5612	1,0089	-0,2233
KBCA	0,0246	0,0156	0,1835	0,1843	1,9206	0,9914	0,5410
MELE	0,0121	0,0249	0,0852	0,0833	0,9180	1,0467	-1,7011
ABLX	0,0116	0,0264	0,1626	0,1612	0,8791	1,0173	-1,0125
ELI	0,0020	0,0091	0,0438	0,0436	0,1164	1,0122	-1,8020
SOLV	0,0085	0,0087	0,0675	0,0675	0,6359	1,0018	-0,0260
BEKB	0,0207	0,0285	0,2726	0,2714	1,6062	1,0087	-0,3176
COFB	0,0047	0,0014	0,0423	0,0426	0,3347	0,9854	0,8699
WDP	0,0067	0,0129	0,0508	0,0500	0,4937	1,0322	-1,3591
LOTB	0,0047	0,0223	0,0591	0,0578	0,3308	1,0449	**-3,3450**
ECONB	0,0107	0,0348	0,1231	0,1209	0,8107	1,0354	**-2,1867**
KIN	0,0064	0,0295	0,0741	0,0722	0,4713	1,0533	**-3,5101**
TESB	0,0085	0,0089	0,0854	0,0854	0,6356	1,0018	-0,0530
PIC	0,0099	0,0217	0,1209	0,1198	0,7487	1,0174	-1,0781
BEFB	0,0061	0,0061	0,0580	0,0579	0,4473	1,0008	0,0148
AED	0,0041	0,0122	0,0443	0,0436	0,2851	1,0307	**-2,0313**
GIMB	0,0044	0,0104	0,1135	0,1132	0,3106	1,0053	-0,5822
BAR	0,0109	0,0156	0,1240	0,1235	0,8288	1,0082	-0,4179
BNB	0,0081	0,0057	0,0642	0,0645	0,5993	0,9921	0,4084
VPG	-0,0001	0,0152	0,0697	0,0698	-0,0502	0,9967	**-2,4127**
EURN	0,0114	0,0040	0,1255	0,1261	0,8640	0,9904	0,6426
OBEL	0,0017	-0,0037	0,0808	0,0809	0,0954	0,9982	0,7441
GREEN	0,0052	0,0060	0,0749	0,0748	0,3708	1,0017	-0,1184
FAGR	0,0095	0,0142	0,1196	0,1192	0,7139	1,0067	-0,4331
IBAB	0,0126	0,0106	0,1217	0,1218	0,9596	0,9979	0,1777
RET	0,0036	0,0104	0,0439	0,0434	0,2440	1,0225	-1,7230
AGFB	0,0152	0,0075	0,1492	0,1499	1,1687	0,9905	0,5675
WEHB	0,0036	0,0114	0,0482	0,0477	0,2417	1,0213	-1,8146
COMB	0,0070	0,0084	0,0681	0,0679	0,5163	1,0042	-0,2189
VAN	0,0046	0,0073	0,0554	0,0551	0,3278	1,0076	-0,5317
SIOE	0,0098	0,0164	0,0896	0,0889	0,7398	1,0161	-0,8166
Immo	0,0131	0,0142	0,1028	0,1026	1,0031	1,0031	-0,1193
TIG	0,0063	0,0141	0,2022	0,2019	0,4619	1,0028	-0,4259
MONT	0,0032	0,0107	0,0427	0,0422	0,2102	1,0226	-1,9704
REC	0,0151	0,0137	0,1231	0,1232	1,1612	0,9980	0,1235
LEAS	0,0052	0,0090	0,0502	0,0498	0,3736	1,0147	-0,8428
INTO	0,0081	0,0066	0,0619	0,0620	0,6031	0,9942	0,2743
EVS	0,0113	0,0020	0,0940	0,0950	0,8553	0,9781	1,0742

Appendix C2: Matlab output α = 0.55

Ticker	Ex-ante μ	Sample μ	Ex-ante σ	Sample σ	Ex-ante β	chi²	t
ABI	0,0161	0,0132	0,0807	0,0750	1,2224	1,1557	0,4157
KBC	0,0208	0,0129	0,1568	0,1556	1,5954	1,0166	0,5605
GBLB	0,0070	0,0058	0,0475	0,0475	0,5051	1,0025	0,2622
UCB	0,0074	0,0111	0,0793	0,0784	0,5387	1,0223	-0,5257
SOLB	0,0112	0,0093	0,0838	0,0838	0,8379	1,0016	0,2468
RTL	0,0076	0,0067	0,0761	0,0758	0,5525	1,0061	0,1312
UMI	0,0080	0,0143	0,1284	0,1275	0,5902	1,0150	-0,5392
PROX	0,0035	0,0083	0,0581	0,0577	0,2322	1,0163	-0,9226
AGS	0,0233	0,0070	0,1471	0,1489	1,7887	0,9755	1,2010
TNET	0,0068	0,0139	0,0699	0,0688	0,4970	1,0324	-1,1318
COLR	0,0015	0,0080	0,0509	0,0507	0,0775	1,0087	-1,4037
ACKB	0,0069	0,0105	0,0646	0,0640	0,4980	1,0188	-0,6264
SOF	0,0076	0,0085	0,0475	0,0473	0,5573	1,0106	-0,2142
KBCA	0,0247	0,0156	0,1833	0,1843	1,9001	0,9892	0,5435
MELE	0,0122	0,0249	0,0856	0,0833	0,9156	1,0570	-1,6859
ABLX	0,0116	0,0264	0,1629	0,1612	0,8731	1,0212	-1,0083
ELI	0,0020	0,0091	0,0439	0,0436	0,1159	1,0149	-1,7989
SOLV	0,0085	0,0087	0,0675	0,0675	0,6305	1,0020	-0,0209
BEKB	0,0208	0,0285	0,2728	0,2714	1,5955	1,0105	-0,3128
COFB	0,0047	0,0014	0,0422	0,0426	0,3316	0,9820	0,8733
WDP	0,0068	0,0129	0,0509	0,0500	0,4921	1,0393	-1,3461
LOTB	0,0047	0,0223	0,0593	0,0578	0,3303	1,0551	**-3,3363**
ECONB	0,0108	0,0348	0,1235	0,1209	0,8084	1,0433	**-2,1777**
KIN	0,0065	0,0295	0,0745	0,0722	0,4704	1,0655	**-3,5006**
TESB	0,0085	0,0089	0,0855	0,0854	0,6305	1,0021	-0,0483
PIC	0,0100	0,0217	0,1211	0,1198	0,7442	1,0213	-1,0725
BEFB	0,0062	0,0061	0,0580	0,0579	0,4443	1,0008	0,0209
AED	0,0042	0,0122	0,0444	0,0436	0,2849	1,0377	**-2,0205**
GIMB	0,0044	0,0104	0,1136	0,1132	0,3081	1,0064	-0,5806
BAR	0,0110	0,0156	0,1241	0,1235	0,8220	1,0100	-0,4140
BNB	0,0081	0,0057	0,0642	0,0645	0,5944	0,9901	0,4138
VPG	-0,0001	0,0152	0,0696	0,0698	-0,0500	0,9959	**-2,4134**
EURN	0,0114	0,0040	0,1253	0,1261	0,8563	0,9881	0,6459
OBEL	0,0017	-0,0037	0,0808	0,0809	0,0942	0,9978	0,7441
GREEN	0,0052	0,0060	0,0749	0,0748	0,3679	1,0020	-0,1153
FAGR	0,0096	0,0142	0,1197	0,1192	0,7095	1,0082	-0,4278
IBAB	0,0126	0,0106	0,1216	0,1218	0,9517	0,9973	0,1822
RET	0,0036	0,0104	0,0440	0,0434	0,2430	1,0275	-1,7164
AGFB	0,0152	0,0075	0,1490	0,1499	1,1577	0,9883	0,5708
WEHB	0,0036	0,0114	0,0483	0,0477	0,2406	1,0261	-1,8088
COMB	0,0070	0,0084	0,0681	0,0679	0,5121	1,0051	-0,2145
VAN	0,0047	0,0073	0,0554	0,0551	0,3256	1,0093	-0,5269
SIOE	0,0099	0,0164	0,0898	0,0889	0,7347	1,0196	-0,8102
Immo	0,0132	0,0142	0,1028	0,1026	0,9980	1,0035	-0,1092
TIG	0,0063	0,0141	0,2022	0,2019	0,4565	1,0034	-0,4257
MONT	0,0032	0,0107	0,0428	0,0422	0,2095	1,0277	-1,9639
REC	0,0152	0,0137	0,1231	0,1232	1,1505	0,9975	0,1277
LEAS	0,0053	0,0090	0,0502	0,0498	0,3716	1,0179	-0,8353
INTO	0,0081	0,0066	0,0618	0,0620	0,5990	0,9926	0,2819
EVS	0,0113	0,0020	0,0938	0,0950	0,8479	0,9730	1,0789

Appendix C3: Matlab output α = 0.60

Ticker	Ex-ante μ	Sample μ	Ex-ante σ	Sample σ	Ex-ante β	chi²	t
ABI	0,0164	0,0132	0,0817	0,0750	1,2318	1,1848	0,4664
KBC	0,0209	0,0129	0,1570	0,1556	1,5785	1,0191	0,5658
GBLB	0,0070	0,0058	0,0475	0,0475	0,5001	1,0026	0,2688
UCB	0,0074	0,0111	0,0795	0,0784	0,5355	1,0271	-0,5176
SOLB	0,0112	0,0093	0,0838	0,0838	0,8298	1,0014	0,2533
RTL	0,0076	0,0067	0,0761	0,0758	0,5475	1,0072	0,1365
UMI	0,0081	0,0143	0,1286	0,1275	0,5841	1,0183	-0,5367
PROX	0,0035	0,0083	0,0582	0,0577	0,2307	1,0200	-0,9180
AGS	0,0233	0,0070	0,1466	0,1489	1,7679	0,9690	1,2056
TNET	0,0069	0,0139	0,0702	0,0688	0,4949	1,0396	-1,1215
COLR	0,0015	0,0080	0,0510	0,0507	0,0773	1,0107	-1,4013
ACKB	0,0069	0,0105	0,0647	0,0640	0,4942	1,0229	-0,6192
SOF	0,0077	0,0085	0,0476	0,0473	0,5529	1,0125	-0,2036
KBCA	0,0247	0,0156	0,1831	0,1843	1,8763	0,9865	0,5461
MELE	0,0123	0,0249	0,0862	0,0833	0,9132	1,0698	-1,6677
ABLX	0,0117	0,0264	0,1633	0,1612	0,8664	1,0259	-1,0032
ELI	0,0020	0,0091	0,0440	0,0436	0,1154	1,0184	-1,7953
SOLV	0,0086	0,0087	0,0675	0,0675	0,6243	1,0023	-0,0151
BEKB	0,0209	0,0285	0,2731	0,2714	1,5834	1,0127	-0,3073
COFB	0,0048	0,0014	0,0421	0,0426	0,3280	0,9777	0,8770
WDP	0,0069	0,0129	0,0512	0,0500	0,4906	1,0481	-1,3307
LOTB	0,0048	0,0223	0,0597	0,0578	0,3299	1,0681	**-3,3257**
ECONB	0,0109	0,0348	0,1241	0,1209	0,8061	1,0534	**-2,1669**
KIN	0,0066	0,0295	0,0750	0,0722	0,4696	1,0808	**-3,4889**
TESB	0,0086	0,0089	0,0855	0,0854	0,6248	1,0025	-0,0429
PIC	0,0101	0,0217	0,1214	0,1198	0,7392	1,0261	-1,0658
BEFB	0,0062	0,0061	0,0580	0,0579	0,4408	1,0008	0,0279
AED	0,0042	0,0122	0,0446	0,0436	0,2848	1,0465	**-2,0076**
GIMB	0,0045	0,0104	0,1136	0,1132	0,3053	1,0079	-0,5787
BAR	0,0110	0,0156	0,1242	0,1235	0,8142	1,0122	-0,4095
BNB	0,0081	0,0057	0,0641	0,0645	0,5886	0,9876	0,4199
VPG	-0,0001	0,0152	0,0696	0,0698	-0,0497	0,9949	**-2,4143**
EURN	0,0115	0,0040	0,1251	0,1261	0,8474	0,9852	0,6496
OBEL	0,0017	-0,0037	0,0808	0,0809	0,0928	0,9973	0,7440
GREEN	0,0052	0,0060	0,0749	0,0748	0,3646	1,0024	-0,1116
FAGR	0,0096	0,0142	0,1198	0,1192	0,7046	1,0100	-0,4216
IBAB	0,0127	0,0106	0,1216	0,1218	0,9426	0,9965	0,1875
RET	0,0037	0,0104	0,0441	0,0434	0,2419	1,0339	-1,7087
AGFB	0,0153	0,0075	0,1488	0,1499	1,1449	0,9855	0,5744
WEHB	0,0036	0,0114	0,0484	0,0477	0,2395	1,0322	-1,8020
COMB	0,0071	0,0084	0,0681	0,0679	0,5072	1,0061	-0,2094
VAN	0,0047	0,0073	0,0555	0,0551	0,3231	1,0113	-0,5214
SIOE	0,0099	0,0164	0,0900	0,0889	0,7289	1,0239	-0,8026
Immo	0,0133	0,0142	0,1028	0,1026	0,9922	1,0039	-0,0977
TIG	0,0063	0,0141	0,2023	0,2019	0,4503	1,0041	-0,4254
MONT	0,0032	0,0107	0,0429	0,0422	0,2089	1,0342	-1,9561
REC	0,0152	0,0137	0,1230	0,1232	1,1382	0,9967	0,1324
LEAS	0,0053	0,0090	0,0503	0,0498	0,3694	1,0219	-0,8266
INTO	0,0082	0,0066	0,0618	0,0620	0,5942	0,9906	0,2904
EVS	0,0114	0,0020	0,0934	0,0950	0,8392	0,9665	1,0839

Appendix C4: Matlab output α = 0.65

Ticker	Ex-ante μ	Sample μ	Ex-ante σ	Sample σ	Ex-ante β	chi²	t
ABI	0,0168	0,0132	0,0829	0,0750	1,2427	1,2194	0,5266
KBC	0,0210	0,0129	0,1572	0,1556	1,5588	1,0217	0,5718
GBLB	0,0070	0,0058	0,0475	0,0475	0,4943	1,0025	0,2763
UCB	0,0075	0,0111	0,0797	0,0784	0,5319	1,0332	-0,5078
SOLB	0,0113	0,0093	0,0838	0,0838	0,8203	1,0010	0,2607
RTL	0,0076	0,0067	0,0762	0,0758	0,5416	1,0084	0,1427
UMI	0,0081	0,0143	0,1289	0,1275	0,5770	1,0225	-0,5336
PROX	0,0035	0,0083	0,0584	0,0577	0,2292	1,0247	-0,9124
AGS	0,0234	0,0070	0,1459	0,1489	1,7430	0,9605	1,2099
TNET	0,0070	0,0139	0,0705	0,0688	0,4926	1,0489	-1,1089
COLR	0,0015	0,0080	0,0511	0,0507	0,0770	1,0133	-1,3985
ACKB	0,0070	0,0105	0,0649	0,0640	0,4898	1,0281	-0,6105
SOF	0,0077	0,0085	0,0476	0,0473	0,5478	1,0148	-0,1912
KBCA	0,0248	0,0156	0,1827	0,1843	1,8486	0,9828	0,5487
MELE	0,0125	0,0249	0,0868	0,0833	0,9110	1,0860	-1,6454
ABLX	0,0118	0,0264	0,1638	0,1612	0,8589	1,0320	-0,9970
ELI	0,0020	0,0091	0,0441	0,0436	0,1148	1,0230	-1,7909
SOLV	0,0086	0,0087	0,0675	0,0675	0,6171	1,0024	-0,0083
BEKB	0,0211	0,0285	0,2735	0,2714	1,5695	1,0155	-0,3007
COFB	0,0048	0,0014	0,0420	0,0426	0,3237	0,9721	0,8810
WDP	0,0069	0,0129	0,0514	0,0500	0,4889	1,0593	-1,3120
LOTB	0,0049	0,0223	0,0602	0,0578	0,3297	1,0850	**-3,3126**
ECONB	0,0111	0,0348	0,1249	0,1209	0,8038	1,0663	**-2,1537**
KIN	0,0067	0,0295	0,0757	0,0722	0,4690	1,1007	**-3,4745**
TESB	0,0086	0,0089	0,0855	0,0854	0,6181	1,0028	-0,0366
PIC	0,0102	0,0217	0,1218	0,1198	0,7336	1,0323	-1,0578
BEFB	0,0063	0,0061	0,0580	0,0579	0,4368	1,0007	0,0361
AED	0,0043	0,0122	0,0449	0,0436	0,2849	1,0579	**-1,9919**
GIMB	0,0045	0,0104	0,1137	0,1132	0,3020	1,0098	-0,5764
BAR	0,0111	0,0156	0,1244	0,1235	0,8053	1,0149	-0,4040
BNB	0,0082	0,0057	0,0640	0,0645	0,5819	0,9842	0,4267
VPG	-0,0001	0,0152	0,0696	0,0698	-0,0493	0,9936	**-2,4152**
EURN	0,0115	0,0040	0,1249	0,1261	0,8368	0,9814	0,6537
OBEL	0,0017	-0,0037	0,0807	0,0809	0,0912	0,9966	0,7440
GREEN	0,0053	0,0060	0,0750	0,0748	0,3608	1,0029	-0,1073
FAGR	0,0097	0,0142	0,1199	0,1192	0,6990	1,0122	-0,4143
IBAB	0,0128	0,0106	0,1215	0,1218	0,9320	0,9953	0,1935
RET	0,0037	0,0104	0,0443	0,0434	0,2408	1,0422	-1,6992
AGFB	0,0153	0,0075	0,1485	0,1499	1,1300	0,9818	0,5783
WEHB	0,0037	0,0114	0,0486	0,0477	0,2383	1,0400	-1,7937
COMB	0,0071	0,0084	0,0682	0,0679	0,5016	1,0074	-0,2034
VAN	0,0047	0,0073	0,0555	0,0551	0,3203	1,0139	-0,5147
SIOE	0,0100	0,0164	0,0902	0,0889	0,7224	1,0294	-0,7935
Immo	0,0135	0,0142	0,1029	0,1026	0,9853	1,0043	-0,0842
TIG	0,0063	0,0141	0,2024	0,2019	0,4432	1,0051	-0,4251
MONT	0,0033	0,0107	0,0431	0,0422	0,2082	1,0427	-1,9466
REC	0,0153	0,0137	0,1230	0,1232	1,1239	0,9957	0,1378
LEAS	0,0053	0,0090	0,0505	0,0498	0,3669	1,0270	-0,8161
INTO	0,0082	0,0066	0,0617	0,0620	0,5885	0,9878	0,3001
EVS	0,0114	0,0020	0,0930	0,0950	0,8287	0,9581	1,0891

Appendix C5: Matlab output α = 0.70

Ticker	Ex-ante μ	Sample μ	Ex-ante σ	Sample σ	Ex-ante β	chi²	t
ABI	0,0173	0,0132	0,0843	0,0750	1,2554	1,2613	0,5991
KBC	0,0211	0,0129	0,1575	0,1556	1,5356	1,0246	0,5787
GBLB	0,0071	0,0058	0,0475	0,0475	0,4874	1,0021	0,2850
UCB	0,0076	0,0111	0,0800	0,0784	0,5280	1,0411	-0,4958
SOLB	0,0114	0,0093	0,0838	0,0838	0,8091	1,0001	0,2692
RTL	0,0077	0,0067	0,0762	0,0758	0,5349	1,0100	0,1501
UMI	0,0081	0,0143	0,1292	0,1275	0,5690	1,0280	-0,5299
PROX	0,0036	0,0083	0,0586	0,0577	0,2274	1,0309	-0,9055
AGS	0,0234	0,0070	0,1450	0,1489	1,7124	0,9488	1,2133
TNET	0,0071	0,0139	0,0709	0,0688	0,4903	1,0612	-1,0933
COLR	0,0016	0,0080	0,0511	0,0507	0,0767	1,0168	-1,3949
ACKB	0,0070	0,0105	0,0651	0,0640	0,4849	1,0350	-0,5998
SOF	0,0078	0,0085	0,0477	0,0473	0,5419	1,0176	-0,1764
KBCA	0,0248	0,0156	0,1823	0,1843	1,8156	0,9778	0,5513
MELE	0,0127	0,0249	0,0877	0,0833	0,9091	1,1075	-1,6172
ABLX	0,0119	0,0264	0,1644	0,1612	0,8506	1,0401	-0,9894
ELI	0,0021	0,0091	0,0442	0,0436	0,1141	1,0291	-1,7856
SOLV	0,0087	0,0087	0,0675	0,0675	0,6087	1,0026	-0,0003
BEKB	0,0213	0,0285	0,2740	0,2714	1,5536	1,0190	-0,2927
COFB	0,0048	0,0014	0,0418	0,0426	0,3186	0,9645	0,8852
WDP	0,0070	0,0129	0,0518	0,0500	0,4873	1,0741	-1,2886
LOTB	0,0049	0,0223	0,0608	0,0578	0,3298	1,1078	**-3,2959**
ECONB	0,0113	0,0348	0,1259	0,1209	0,8019	1,0837	**-2,1369**
KIN	0,0068	0,0295	0,0766	0,0722	0,4690	1,1276	**-3,4558**
TESB	0,0087	0,0089	0,0855	0,0854	0,6104	1,0032	-0,0292
PIC	0,0103	0,0217	0,1222	0,1198	0,7274	1,0405	-1,0479
BEFB	0,0063	0,0061	0,0580	0,0579	0,4320	1,0004	0,0457
AED	0,0043	0,0122	0,0452	0,0436	0,2851	1,0732	-1,9723
GIMB	0,0045	0,0104	0,1139	0,1132	0,2983	1,0123	-0,5735
BAR	0,0112	0,0156	0,1246	0,1235	0,7951	1,0185	-0,3974
BNB	0,0082	0,0057	0,0638	0,0645	0,5739	0,9796	0,4343
VPG	-0,0001	0,0152	0,0695	0,0698	-0,0488	0,9918	**-2,4163**
EURN	0,0116	0,0040	0,1246	0,1261	0,8243	0,9763	0,6581
OBEL	0,0017	-0,0037	0,0807	0,0809	0,0894	0,9957	0,7439
GREEN	0,0053	0,0060	0,0750	0,0748	0,3563	1,0035	-0,1022
FAGR	0,0098	0,0142	0,1201	0,1192	0,6925	1,0151	-0,4054
IBAB	0,0128	0,0106	0,1214	0,1218	0,9195	0,9937	0,2004
RET	0,0037	0,0104	0,0445	0,0434	0,2396	1,0532	-1,6874
AGFB	0,0154	0,0075	0,1481	0,1499	1,1122	0,9767	0,5825
WEHB	0,0037	0,0114	0,0489	0,0477	0,2370	1,0506	-1,7834
COMB	0,0072	0,0084	0,0682	0,0679	0,4951	1,0091	-0,1963
VAN	0,0048	0,0073	0,0556	0,0551	0,3170	1,0174	-0,5067
SIOE	0,0101	0,0164	0,0905	0,0889	0,7151	1,0367	-0,7824
Immo	0,0136	0,0142	0,1029	0,1026	0,9771	1,0045	-0,0683
TIG	0,0064	0,0141	0,2025	0,2019	0,4350	1,0064	-0,4248
MONT	0,0033	0,0107	0,0433	0,0422	0,2075	1,0540	-1,9347
REC	0,0153	0,0137	0,1229	0,1232	1,1071	0,9942	0,1439
LEAS	0,0054	0,0090	0,0506	0,0498	0,3641	1,0338	-0,8032
INTO	0,0083	0,0066	0,0615	0,0620	0,5816	0,9840	0,3111
EVS	0,0114	0,0020	0,0925	0,0950	0,8158	0,9466	1,0943

Appendix C6: Matlab output α = 0.75

Ticker	Ex-ante μ	Sample μ	Ex-ante σ	Sample σ	Ex-ante β	chi²	t
ABI	0,0179	0,0132	0,0860	0,0750	1,2706	**1,3129**	0,6883
KBC	0,0212	0,0129	0,1577	0,1556	1,5080	1,0278	0,5866
GBLB	0,0071	0,0058	0,0475	0,0475	0,4792	1,0012	0,2951
UCB	0,0077	0,0111	0,0804	0,0784	0,5236	1,0518	-0,4805
SOLB	0,0114	0,0093	0,0837	0,0838	0,7957	0,9984	0,2789
RTL	0,0077	0,0067	0,0763	0,0758	0,5269	1,0119	0,1590
UMI	0,0082	0,0143	0,1297	0,1275	0,5599	1,0356	-0,5250
PROX	0,0036	0,0083	0,0588	0,0577	0,2254	1,0397	-0,8967
AGS	0,0235	0,0070	0,1438	0,1489	1,6738	0,9320	1,2146
TNET	0,0072	0,0139	0,0714	0,0688	0,4881	1,0783	-1,0732
COLR	0,0016	0,0080	0,0513	0,0507	0,0764	1,0218	-1,3906
ACKB	0,0071	0,0105	0,0654	0,0640	0,4794	1,0443	-0,5862
SOF	0,0079	0,0085	0,0478	0,0473	0,5348	1,0211	-0,1585
KBCA	0,0248	0,0156	0,1816	0,1843	1,7757	0,9707	0,5533
MELE	0,0130	0,0249	0,0888	0,0833	0,9083	1,1372	-1,5803
ABLX	0,0121	0,0264	0,1653	0,1612	0,8414	1,0514	-0,9794
ELI	0,0021	0,0091	0,0444	0,0436	0,1134	1,0377	-1,7789
SOLV	0,0087	0,0087	0,0675	0,0675	0,5987	1,0025	0,0090
BEKB	0,0216	0,0285	0,2746	0,2714	1,5353	1,0239	-0,2828
COFB	0,0048	0,0014	0,0416	0,0426	0,3123	0,9537	0,8891
WDP	0,0072	0,0129	0,0523	0,0500	0,4859	1,0945	-1,2585
LOTB	0,0051	0,0223	0,0617	0,0578	0,3306	1,1403	**-3,2738**
ECONB	0,0115	0,0348	0,1273	0,1209	0,8009	1,1082	**-2,1149**
KIN	0,0070	0,0295	0,0779	0,0722	0,4701	1,1657	**-3,4308**
TESB	0,0088	0,0089	0,0855	0,0854	0,6012	1,0035	-0,0203
PIC	0,0104	0,0217	0,1229	0,1198	0,7205	1,0519	-1,0351
BEFB	0,0064	0,0061	0,0579	0,0579	0,4264	0,9998	0,0572
AED	0,0044	0,0122	0,0456	0,0436	0,2858	1,0949	-1,9470
GIMB	0,0046	0,0104	0,1141	0,1132	0,2940	1,0157	-0,5699
BAR	0,0113	0,0156	0,1249	0,1235	0,7833	1,0235	-0,3892
BNB	0,0083	0,0057	0,0636	0,0645	0,5640	0,9730	0,4426
VPG	-0,0001	0,0152	0,0694	0,0698	-0,0483	0,9893	**-2,4176**
EURN	0,0116	0,0040	0,1241	0,1261	0,8090	0,9690	0,6627
OBEL	0,0017	-0,0037	0,0806	0,0809	0,0872	0,9945	0,7438
GREEN	0,0053	0,0060	0,0750	0,0748	0,3511	1,0043	-0,0961
FAGR	0,0099	0,0142	0,1203	0,1192	0,6850	1,0191	-0,3945
IBAB	0,0129	0,0106	0,1213	0,1218	0,9046	0,9914	0,2085
RET	0,0038	0,0104	0,0449	0,0434	0,2384	1,0688	-1,6721
AGFB	0,0155	0,0075	0,1476	0,1499	1,0906	0,9696	0,5868
WEHB	0,0038	0,0114	0,0492	0,0477	0,2356	1,0655	-1,7701
COMB	0,0072	0,0084	0,0683	0,0679	0,4874	1,0112	-0,1877
VAN	0,0048	0,0073	0,0558	0,0551	0,3133	1,0221	-0,4967
SIOE	0,0102	0,0164	0,0910	0,0889	0,7068	1,0468	-0,7681
Immo	0,0138	0,0142	0,1029	0,1026	0,9672	1,0045	-0,0495
TIG	0,0064	0,0141	0,2027	0,2019	0,4255	1,0082	-0,4243
MONT	0,0034	0,0107	0,0437	0,0422	0,2069	1,0701	-1,9195
REC	0,0154	0,0137	0,1227	0,1232	1,0871	0,9919	0,1510
LEAS	0,0055	0,0090	0,0509	0,0498	0,3609	1,0431	-0,7870
INTO	0,0084	0,0066	0,0614	0,0620	0,5732	0,9783	0,3236
EVS	0,0115	0,0020	0,0917	0,0950	0,7995	0,9305	1,0986

Appendix D1: Ex-ante vs Sample Beta with α = 0.50

Ticker	Sample β	Ex-ante β (α=0,50)	SML μ	Sample μ	Ex ante μ (α = 0,50)	Ex-ante σ	t
ABI	1,1681	1,2143	0,0142	0,0132	0,0158	0,0798	0,2168
KBC	1,6920	1,6101	0,0203	0,0129	0,0207	0,1567	0,0283
GBLB	0,5340	0,5094	0,0068	0,0058	0,0069	0,0475	0,0333
UCB	0,5588	0,5415	0,0071	0,0111	0,0073	0,0791	0,0357
SOLB	0,8843	0,8449	0,0109	0,0093	0,0111	0,0838	0,0333
RTL	0,5822	0,5569	0,0073	0,0067	0,0075	0,0760	0,0253
UMI	0,6279	0,5957	0,0079	0,0143	0,0080	0,1282	0,0109
PROX	0,2411	0,2334	0,0034	0,0083	0,0035	0,0581	0,0204
AGS	1,8986	1,8062	0,0228	0,0070	0,0232	0,1474	0,0333
TNET	0,5113	0,4989	0,0065	0,0139	0,0068	0,0697	0,0438
COLR	0,0791	0,0777	0,0015	0,0080	0,0015	0,0509	0,0109
ACKB	0,5217	0,5014	0,0066	0,0105	0,0068	0,0645	0,0319
SOF	0,5833	0,5612	0,0074	0,0085	0,0076	0,0475	0,0501
KBCA	2,0350	1,9206	0,0244	0,0156	0,0246	0,1835	0,0170
MELE	0,9354	0,9180	0,0115	0,0249	0,0121	0,0852	0,0741
ABLX	0,9153	0,8791	0,0112	0,0264	0,0116	0,1626	0,0217
ELI	0,1193	0,1164	0,0019	0,0091	0,0020	0,0438	0,0161
SOLV	0,6667	0,6359	0,0083	0,0087	0,0085	0,0675	0,0289
BEKB	1,6688	1,6062	0,0201	0,0285	0,0207	0,2726	0,0253
COFB	0,3517	0,3347	0,0047	0,0014	0,0047	0,0423	0,0219
WDP	0,5037	0,4937	0,0064	0,0129	0,0067	0,0508	0,0651
LOTB	0,3353	0,3308	0,0045	0,0223	0,0047	0,0591	0,0424
ECONB	0,8269	0,8107	0,0102	0,0348	0,0107	0,1231	0,0444
KIN	0,4794	0,4713	0,0061	0,0295	0,0064	0,0741	0,0452
TESB	0,6642	0,6356	0,0083	0,0089	0,0085	0,0854	0,0261
PIC	0,7757	0,7487	0,0096	0,0217	0,0099	0,1209	0,0290
BEFB	0,4643	0,4473	0,0060	0,0061	0,0061	0,0580	0,0340
AED	0,2870	0,2851	0,0039	0,0122	0,0041	0,0443	0,0546
GIMB	0,3255	0,3106	0,0043	0,0104	0,0044	0,1135	0,0086
BAR	0,8688	0,8288	0,0107	0,0156	0,0109	0,1240	0,0207
BNB	0,6265	0,5993	0,0079	0,0057	0,0081	0,0642	0,0323
VPG	-0,0518	-0,0502	-0,0001	0,0152	-0,0001	0,0697	-0,0039
EURN	0,9068	0,8640	0,0111	0,0040	0,0114	0,1255	0,0202
OBEL	0,1022	0,0954	0,0017	-0,0037	0,0017	0,0808	0,0000
GREEN	0,3875	0,3708	0,0051	0,0060	0,0052	0,0749	0,0176
FAGR	0,7391	0,7139	0,0092	0,0142	0,0095	0,1196	0,0285
IBAB	1,0043	0,9596	0,0123	0,0106	0,0126	0,1217	0,0261
RET	0,2504	0,2440	0,0035	0,0104	0,0036	0,0439	0,0332
AGFB	1,2298	1,1687	0,0149	0,0075	0,0152	0,1492	0,0201
WEHB	0,2482	0,2417	0,0034	0,0114	0,0036	0,0482	0,0293
COMB	0,5408	0,5163	0,0069	0,0084	0,0070	0,0681	0,0243
VAN	0,3405	0,3278	0,0045	0,0073	0,0046	0,0554	0,0255
SIOE	0,7702	0,7398	0,0096	0,0164	0,0098	0,0896	0,0332
Immo	1,0312	1,0031	0,0126	0,0142	0,0131	0,1028	0,0558
TIG	0,4935	0,4619	0,0063	0,0141	0,0063	0,2022	0,0011
MONT	0,2143	0,2102	0,0030	0,0107	0,0032	0,0427	0,0335
REC	1,2220	1,1612	0,0148	0,0137	0,0151	0,1231	0,0242
LEAS	0,3854	0,3736	0,0050	0,0090	0,0052	0,0502	0,0391
INTO	0,6257	0,6031	0,0079	0,0066	0,0081	0,0619	0,0438
EVS	0,8946	0,8553	0,0110	0,0020	0,0113	0,0940	0,0309

Appendix D2: Ex-ante vs Sample Beta with α = 0.55

Ticker	Sample β	Ex-ante β (α=0,55)	SML μ	Sample μ	Ex ante μ (α = 0,55)	Ex-ante σ	t
ABI	1,1681	1,2224	0,0142	0,0132	0,0161	0,0807	0,2548
KBC	1,6920	1,5954	0,0203	0,0129	0,0208	0,1568	0,0330
GBLB	0,5340	0,5051	0,0068	0,0058	0,0070	0,0475	0,0391
UCB	0,5588	0,5387	0,0071	0,0111	0,0074	0,0793	0,0425
SOLB	0,8843	0,8379	0,0109	0,0093	0,0112	0,0838	0,0391
RTL	0,5822	0,5525	0,0073	0,0067	0,0076	0,0761	0,0299
UMI	0,6279	0,5902	0,0079	0,0143	0,0080	0,1284	0,0130
PROX	0,2411	0,2322	0,0034	0,0083	0,0035	0,0581	0,0243
AGS	1,8986	1,7887	0,0228	0,0070	0,0233	0,1471	0,0380
TNET	0,5113	0,4970	0,0065	0,0139	0,0068	0,0699	0,0522
COLR	0,0791	0,0775	0,0015	0,0080	0,0015	0,0509	0,0129
ACKB	0,5217	0,4980	0,0066	0,0105	0,0069	0,0646	0,0379
SOF	0,5833	0,5573	0,0074	0,0085	0,0076	0,0475	0,0591
KBCA	2,0350	1,9001	0,0244	0,0156	0,0247	0,1833	0,0195
MELE	0,9354	0,9156	0,0115	0,0249	0,0122	0,0856	0,0885
ABLX	0,9153	0,8731	0,0112	0,0264	0,0116	0,1629	0,0259
ELI	0,1193	0,1159	0,0019	0,0091	0,0020	0,0439	0,0192
SOLV	0,6667	0,6305	0,0083	0,0087	0,0085	0,0675	0,0340
BEKB	1,6688	1,5955	0,0201	0,0285	0,0208	0,2728	0,0300
COFB	0,3517	0,3316	0,0047	0,0014	0,0047	0,0422	0,0254
WDP	0,5037	0,4921	0,0064	0,0129	0,0068	0,0509	0,0776
LOTB	0,3353	0,3303	0,0045	0,0223	0,0047	0,0593	0,0507
ECONB	0,8269	0,8084	0,0102	0,0348	0,0108	0,1235	0,0530
KIN	0,4794	0,4704	0,0061	0,0295	0,0065	0,0745	0,0542
TESB	0,6642	0,6305	0,0083	0,0089	0,0085	0,0855	0,0307
PIC	0,7757	0,7442	0,0096	0,0217	0,0100	0,1211	0,0345
BEFB	0,4643	0,4443	0,0060	0,0061	0,0062	0,0580	0,0401
AED	0,2870	0,2849	0,0039	0,0122	0,0042	0,0444	0,0650
GIMB	0,3255	0,3081	0,0043	0,0104	0,0044	0,1136	0,0102
BAR	0,8688	0,8220	0,0107	0,0156	0,0110	0,1241	0,0246
BNB	0,6265	0,5944	0,0079	0,0057	0,0081	0,0642	0,0378
VPG	-0,0518	-0,0500	-0,0001	0,0152	-0,0001	0,0696	-0,0046
EURN	0,9068	0,8563	0,0111	0,0040	0,0114	0,1253	0,0236
OBEL	0,1022	0,0942	0,0017	-0,0037	0,0017	0,0808	0,0000
GREEN	0,3875	0,3679	0,0051	0,0060	0,0052	0,0749	0,0207
FAGR	0,7391	0,7095	0,0092	0,0142	0,0096	0,1197	0,0337
IBAB	1,0043	0,9517	0,0123	0,0106	0,0126	0,1216	0,0307
RET	0,2504	0,2430	0,0035	0,0104	0,0036	0,0440	0,0396
AGFB	1,2298	1,1577	0,0149	0,0075	0,0152	0,1490	0,0234
WEHB	0,2482	0,2406	0,0034	0,0114	0,0036	0,0483	0,0349
COMB	0,5408	0,5121	0,0069	0,0084	0,0070	0,0681	0,0286
VAN	0,3405	0,3256	0,0045	0,0073	0,0047	0,0554	0,0302
SIOE	0,7702	0,7347	0,0096	0,0164	0,0099	0,0898	0,0395
Immo	1,0312	0,9980	0,0126	0,0142	0,0132	0,1028	0,0658
TIG	0,4935	0,4565	0,0063	0,0141	0,0063	0,2022	0,0013
MONT	0,2143	0,2095	0,0030	0,0107	0,0032	0,0428	0,0399
REC	1,2220	1,1505	0,0148	0,0137	0,0152	0,1231	0,0284
LEAS	0,3854	0,3716	0,0050	0,0090	0,0053	0,0502	0,0464
INTO	0,6257	0,5990	0,0079	0,0066	0,0081	0,0618	0,0514
EVS	0,8946	0,8479	0,0110	0,0020	0,0113	0,0938	0,0358

Appendix D3: Ex-ante vs Sample Beta with α = 0.60

Ticker	Sample β	Ex-ante β (α=0,60)	SML μ	Sample μ	Ex ante μ (α = 0,60)	Ex-ante σ	t
ABI	1,1681	1,2318	0,0142	0,0132	0,0164	0,0817	0,2984
KBC	1,6920	1,5785	0,0203	0,0129	0,0209	0,1570	0,0382
GBLB	0,5340	0,5001	0,0068	0,0058	0,0070	0,0475	0,0457
UCB	0,5588	0,5355	0,0071	0,0111	0,0074	0,0795	0,0504
SOLB	0,8843	0,8298	0,0109	0,0093	0,0112	0,0838	0,0456
RTL	0,5822	0,5475	0,0073	0,0067	0,0076	0,0761	0,0352
UMI	0,6279	0,5841	0,0079	0,0143	0,0081	0,1286	0,0154
PROX	0,2411	0,2307	0,0034	0,0083	0,0035	0,0582	0,0289
AGS	1,8986	1,7679	0,0228	0,0070	0,0233	0,1466	0,0428
TNET	0,5113	0,4949	0,0065	0,0139	0,0069	0,0702	0,0621
COLR	0,0791	0,0773	0,0015	0,0080	0,0015	0,0510	0,0153
ACKB	0,5217	0,4942	0,0066	0,0105	0,0069	0,0647	0,0450
SOF	0,5833	0,5529	0,0074	0,0085	0,0077	0,0476	0,0696
KBCA	2,0350	1,8763	0,0244	0,0156	0,0247	0,1831	0,0222
MELE	0,9354	0,9132	0,0115	0,0249	0,0123	0,0862	0,1057
ABLX	0,9153	0,8664	0,0112	0,0264	0,0117	0,1633	0,0309
ELI	0,1193	0,1154	0,0019	0,0091	0,0020	0,0440	0,0228
SOLV	0,6667	0,6243	0,0083	0,0087	0,0086	0,0675	0,0399
BEKB	1,6688	1,5834	0,0201	0,0285	0,0209	0,2731	0,0355
COFB	0,3517	0,3280	0,0047	0,0014	0,0048	0,0421	0,0293
WDP	0,5037	0,4906	0,0064	0,0129	0,0069	0,0512	0,0924
LOTB	0,3353	0,3299	0,0045	0,0223	0,0048	0,0597	0,0607
ECONB	0,8269	0,8061	0,0102	0,0348	0,0109	0,1241	0,0634
KIN	0,4794	0,4696	0,0061	0,0295	0,0066	0,0750	0,0651
TESB	0,6642	0,6248	0,0083	0,0089	0,0086	0,0855	0,0362
PIC	0,7757	0,7392	0,0096	0,0217	0,0101	0,1214	0,0410
BEFB	0,4643	0,4408	0,0060	0,0061	0,0062	0,0580	0,0471
AED	0,2870	0,2848	0,0039	0,0122	0,0042	0,0446	0,0774
GIMB	0,3255	0,3053	0,0043	0,0104	0,0045	0,1136	0,0121
BAR	0,8688	0,8142	0,0107	0,0156	0,0110	0,1242	0,0291
BNB	0,6265	0,5886	0,0079	0,0057	0,0081	0,0641	0,0440
VPG	-0,0518	-0,0497	-0,0001	0,0152	-0,0001	0,0696	-0,0054
EURN	0,9068	0,8474	0,0111	0,0040	0,0115	0,1251	0,0273
OBEL	0,1022	0,0928	0,0017	-0,0037	0,0017	0,0808	-0,0001
GREEN	0,3875	0,3646	0,0051	0,0060	0,0052	0,0749	0,0244
FAGR	0,7391	0,7046	0,0092	0,0142	0,0096	0,1198	0,0399
IBAB	1,0043	0,9426	0,0123	0,0106	0,0127	0,1216	0,0360
RET	0,2504	0,2419	0,0035	0,0104	0,0037	0,0441	0,0472
AGFB	1,2298	1,1449	0,0149	0,0075	0,0153	0,1488	0,0270
WEHB	0,2482	0,2395	0,0034	0,0114	0,0036	0,0484	0,0416
COMB	0,5408	0,5072	0,0069	0,0084	0,0071	0,0681	0,0337
VAN	0,3405	0,3231	0,0045	0,0073	0,0047	0,0555	0,0357
SIOE	0,7702	0,7289	0,0096	0,0164	0,0099	0,0900	0,0469
Immo	1,0312	0,9922	0,0126	0,0142	0,0133	0,1028	0,0774
TIG	0,4935	0,4503	0,0063	0,0141	0,0063	0,2023	0,0015
MONT	0,2143	0,2089	0,0030	0,0107	0,0032	0,0429	0,0475
REC	1,2220	1,1382	0,0148	0,0137	0,0152	0,1230	0,0332
LEAS	0,3854	0,3694	0,0050	0,0090	0,0053	0,0503	0,0550
INTO	0,6257	0,5942	0,0079	0,0066	0,0082	0,0618	0,0600
EVS	0,8946	0,8392	0,0110	0,0020	0,0114	0,0934	0,0410

Appendix D4: Ex-ante vs Sample Beta with α = 0.65

Ticker	Sample β	Ex-ante β (α=0,65)	SML μ	Sample μ	Ex ante μ (α = 0,65)	Ex-ante σ	t
ABI	1,1681	1,2427	0,0142	0,0132	0,0168	0,0829	0,3489
KBC	1,6920	1,5588	0,0203	0,0129	0,0210	0,1572	0,0442
GBLB	0,5340	0,4943	0,0068	0,0058	0,0070	0,0475	0,0533
UCB	0,5588	0,5319	0,0071	0,0111	0,0075	0,0797	0,0600
SOLB	0,8843	0,8203	0,0109	0,0093	0,0113	0,0838	0,0531
RTL	0,5822	0,5416	0,0073	0,0067	0,0076	0,0762	0,0414
UMI	0,6279	0,5770	0,0079	0,0143	0,0081	0,1289	0,0184
PROX	0,2411	0,2292	0,0034	0,0083	0,0035	0,0584	0,0344
AGS	1,8986	1,7430	0,0228	0,0070	0,0234	0,1459	0,0474
TNET	0,5113	0,4926	0,0065	0,0139	0,0070	0,0705	0,0741
COLR	0,0791	0,0770	0,0015	0,0080	0,0015	0,0511	0,0182
ACKB	0,5217	0,4898	0,0066	0,0105	0,0070	0,0649	0,0535
SOF	0,5833	0,5478	0,0074	0,0085	0,0077	0,0476	0,0819
KBCA	2,0350	1,8486	0,0244	0,0156	0,0248	0,1827	0,0249
MELE	0,9354	0,9110	0,0115	0,0249	0,0125	0,0868	0,1264
ABLX	0,9153	0,8589	0,0112	0,0264	0,0118	0,1638	0,0369
ELI	0,1193	0,1148	0,0019	0,0091	0,0020	0,0441	0,0270
SOLV	0,6667	0,6171	0,0083	0,0087	0,0086	0,0675	0,0467
BEKB	1,6688	1,5695	0,0201	0,0285	0,0211	0,2735	0,0421
COFB	0,3517	0,3237	0,0047	0,0014	0,0048	0,0420	0,0334
WDP	0,5037	0,4889	0,0064	0,0129	0,0069	0,0514	0,1102
LOTB	0,3353	0,3297	0,0045	0,0223	0,0049	0,0602	0,0728
ECONB	0,8269	0,8038	0,0102	0,0348	0,0111	0,1249	0,0759
KIN	0,4794	0,4690	0,0061	0,0295	0,0067	0,0757	0,0784
TESB	0,6642	0,6181	0,0083	0,0089	0,0086	0,0855	0,0424
PIC	0,7757	0,7336	0,0096	0,0217	0,0102	0,1218	0,0488
BEFB	0,4643	0,4368	0,0060	0,0061	0,0063	0,0580	0,0554
AED	0,2870	0,2849	0,0039	0,0122	0,0043	0,0449	0,0923
GIMB	0,3255	0,3020	0,0043	0,0104	0,0045	0,1137	0,0144
BAR	0,8688	0,8053	0,0107	0,0156	0,0111	0,1244	0,0345
BNB	0,6265	0,5819	0,0079	0,0057	0,0082	0,0640	0,0510
VPG	-0,0518	-0,0493	-0,0001	0,0152	-0,0001	0,0696	-0,0064
EURN	0,9068	0,8368	0,0111	0,0040	0,0115	0,1249	0,0315
OBEL	0,1022	0,0912	0,0017	-0,0037	0,0017	0,0807	-0,0001
GREEN	0,3875	0,3608	0,0051	0,0060	0,0053	0,0750	0,0287
FAGR	0,7391	0,6990	0,0092	0,0142	0,0097	0,1199	0,0472
IBAB	1,0043	0,9320	0,0123	0,0106	0,0128	0,1215	0,0420
RET	0,2504	0,2408	0,0035	0,0104	0,0037	0,0443	0,0563
AGFB	1,2298	1,1300	0,0149	0,0075	0,0153	0,1485	0,0311
WEHB	0,2482	0,2383	0,0034	0,0114	0,0037	0,0486	0,0496
COMB	0,5408	0,5016	0,0069	0,0084	0,0071	0,0682	0,0397
VAN	0,3405	0,3203	0,0045	0,0073	0,0047	0,0555	0,0423
SIOE	0,7702	0,7224	0,0096	0,0164	0,0100	0,0902	0,0557
Immo	1,0312	0,9853	0,0126	0,0142	0,0135	0,1029	0,0909
TIG	0,4935	0,4432	0,0063	0,0141	0,0063	0,2024	0,0018
MONT	0,2143	0,2082	0,0030	0,0107	0,0033	0,0431	0,0567
REC	1,2220	1,1239	0,0148	0,0137	0,0153	0,1230	0,0386
LEAS	0,3854	0,3669	0,0050	0,0090	0,0053	0,0505	0,0653
INTO	0,6257	0,5885	0,0079	0,0066	0,0082	0,0617	0,0699
EVS	0,8946	0,8287	0,0110	0,0020	0,0114	0,0930	0,0465

Appendix D5: Ex-ante vs Sample Beta with α = 0.70

Ticker	Sample β	Ex-ante β (α=0,70)	SML μ	Sample μ	Ex ante μ (α = 0,70)	Ex-ante σ	t
ABI	1,1681	1,2554	0,0142	0,0132	0,0173	0,0843	0,4080
KBC	1,6920	1,5356	0,0203	0,0129	0,0211	0,1575	0,0510
GBLB	0,5340	0,4874	0,0068	0,0058	0,0071	0,0475	0,0621
UCB	0,5588	0,5280	0,0071	0,0111	0,0076	0,0800	0,0716
SOLB	0,8843	0,8091	0,0109	0,0093	0,0114	0,0838	0,0617
RTL	0,5822	0,5349	0,0073	0,0067	0,0077	0,0762	0,0487
UMI	0,6279	0,5690	0,0079	0,0143	0,0081	0,1292	0,0221
PROX	0,2411	0,2274	0,0034	0,0083	0,0036	0,0586	0,0411
AGS	1,8986	1,7124	0,0228	0,0070	0,0234	0,1450	0,0512
TNET	0,5113	0,4903	0,0065	0,0139	0,0071	0,0709	0,0889
COLR	0,0791	0,0767	0,0015	0,0080	0,0016	0,0511	0,0216
ACKB	0,5217	0,4849	0,0066	0,0105	0,0070	0,0651	0,0639
SOF	0,5833	0,5419	0,0074	0,0085	0,0078	0,0477	0,0965
KBCA	2,0350	1,8156	0,0244	0,0156	0,0248	0,1823	0,0275
MELE	0,9354	0,9091	0,0115	0,0249	0,0127	0,0877	0,1521
ABLX	0,9153	0,8506	0,0112	0,0264	0,0119	0,1644	0,0443
ELI	0,1193	0,1141	0,0019	0,0091	0,0021	0,0442	0,0322
SOLV	0,6667	0,6087	0,0083	0,0087	0,0087	0,0675	0,0546
BEKB	1,6688	1,5536	0,0201	0,0285	0,0213	0,2740	0,0499
COFB	0,3517	0,3186	0,0047	0,0014	0,0048	0,0418	0,0378
WDP	0,5037	0,4873	0,0064	0,0129	0,0070	0,0518	0,1321
LOTB	0,3353	0,3298	0,0045	0,0223	0,0049	0,0608	0,0880
ECONB	0,8269	0,8019	0,0102	0,0348	0,0113	0,1259	0,0914
KIN	0,4794	0,4690	0,0061	0,0295	0,0068	0,0766	0,0950
TESB	0,6642	0,6104	0,0083	0,0089	0,0087	0,0855	0,0499
PIC	0,7757	0,7274	0,0096	0,0217	0,0103	0,1222	0,0584
BEFB	0,4643	0,4320	0,0060	0,0061	0,0063	0,0580	0,0650
AED	0,2870	0,2851	0,0039	0,0122	0,0043	0,0452	0,1107
GIMB	0,3255	0,2983	0,0043	0,0104	0,0045	0,1139	0,0173
BAR	0,8688	0,7951	0,0107	0,0156	0,0112	0,1246	0,0410
BNB	0,6265	0,5739	0,0079	0,0057	0,0082	0,0638	0,0588
VPG	-0,0518	-0,0488	-0,0001	0,0152	-0,0001	0,0695	-0,0075
EURN	0,9068	0,8243	0,0111	0,0040	0,0116	0,1246	0,0361
OBEL	0,1022	0,0894	0,0017	-0,0037	0,0017	0,0807	-0,0002
GREEN	0,3875	0,3563	0,0051	0,0060	0,0053	0,0750	0,0338
FAGR	0,7391	0,6925	0,0092	0,0142	0,0098	0,1201	0,0559
IBAB	1,0043	0,9195	0,0123	0,0106	0,0128	0,1214	0,0491
RET	0,2504	0,2396	0,0035	0,0104	0,0037	0,0445	0,0676
AGFB	1,2298	1,1122	0,0149	0,0075	0,0154	0,1481	0,0354
WEHB	0,2482	0,2370	0,0034	0,0114	0,0037	0,0489	0,0595
COMB	0,5408	0,4951	0,0069	0,0084	0,0072	0,0682	0,0468
VAN	0,3405	0,3170	0,0045	0,0073	0,0048	0,0556	0,0502
SIOE	0,7702	0,7151	0,0096	0,0164	0,0101	0,0905	0,0666
Immo	1,0312	0,9771	0,0126	0,0142	0,0136	0,1029	0,1067
TIG	0,4935	0,4350	0,0063	0,0141	0,0064	0,2025	0,0022
MONT	0,2143	0,2075	0,0030	0,0107	0,0033	0,0433	0,0679
REC	1,2220	1,1071	0,0148	0,0137	0,0153	0,1229	0,0448
LEAS	0,3854	0,3641	0,0050	0,0090	0,0054	0,0506	0,0778
INTO	0,6257	0,5816	0,0079	0,0066	0,0083	0,0615	0,0812
EVS	0,8946	0,8158	0,0110	0,0020	0,0114	0,0925	0,0521

Appendix D6: Ex-ante vs Sample Beta with α = 0.75

Ticker	Sample β	Ex-ante β (α=0,75)	SML μ	Sample μ	Ex ante μ (α = 0,75)	Ex-ante σ	t
ABI	1,1681	1,2706	0,0142	0,0132	0,0179	0,0860	0,4780
KBC	1,6920	1,5080	0,0203	0,0129	0,0212	0,1577	0,0587
GBLB	0,5340	0,4792	0,0068	0,0058	0,0071	0,0475	0,0722
UCB	0,5588	0,5236	0,0071	0,0111	0,0077	0,0804	0,0862
SOLB	0,8843	0,7957	0,0109	0,0093	0,0114	0,0837	0,0715
RTL	0,5822	0,5269	0,0073	0,0067	0,0077	0,0763	0,0576
UMI	0,6279	0,5599	0,0079	0,0143	0,0082	0,1297	0,0268
PROX	0,2411	0,2254	0,0034	0,0083	0,0036	0,0588	0,0496
AGS	1,8986	1,6738	0,0228	0,0070	0,0235	0,1438	0,0530
TNET	0,5113	0,4881	0,0065	0,0139	0,0072	0,0714	0,1077
COLR	0,0791	0,0764	0,0015	0,0080	0,0016	0,0513	0,0259
ACKB	0,5217	0,4794	0,0066	0,0105	0,0071	0,0654	0,0769
SOF	0,5833	0,5348	0,0074	0,0085	0,0079	0,0478	0,1141
KBCA	2,0350	1,7757	0,0244	0,0156	0,0248	0,1816	0,0297
MELE	0,9354	0,9083	0,0115	0,0249	0,0130	0,0888	0,1848
ABLX	0,9153	0,8414	0,0112	0,0264	0,0121	0,1653	0,0538
ELI	0,1193	0,1134	0,0019	0,0091	0,0021	0,0444	0,0387
SOLV	0,6667	0,5987	0,0083	0,0087	0,0087	0,0675	0,0640
BEKB	1,6688	1,5353	0,0201	0,0285	0,0216	0,2746	0,0597
COFB	0,3517	0,3123	0,0047	0,0014	0,0048	0,0416	0,0421
WDP	0,5037	0,4859	0,0064	0,0129	0,0072	0,0523	0,1597
LOTB	0,3353	0,3306	0,0045	0,0223	0,0051	0,0617	0,1075
ECONB	0,8269	0,8009	0,0102	0,0348	0,0115	0,1273	0,1114
KIN	0,4794	0,4701	0,0061	0,0295	0,0070	0,0779	0,1167
TESB	0,6642	0,6012	0,0083	0,0089	0,0088	0,0855	0,0588
PIC	0,7757	0,7205	0,0096	0,0217	0,0104	0,1229	0,0705
BEFB	0,4643	0,4264	0,0060	0,0061	0,0064	0,0579	0,0765
AED	0,2870	0,2858	0,0039	0,0122	0,0044	0,0456	0,1339
GIMB	0,3255	0,2940	0,0043	0,0104	0,0046	0,1141	0,0208
BAR	0,8688	0,7833	0,0107	0,0156	0,0113	0,1249	0,0490
BNB	0,6265	0,5640	0,0079	0,0057	0,0083	0,0636	0,0674
VPG	-0,0518	-0,0483	-0,0001	0,0152	-0,0001	0,0694	-0,0089
EURN	0,9068	0,8090	0,0111	0,0040	0,0116	0,1241	0,0410
OBEL	0,1022	0,0872	0,0017	-0,0037	0,0017	0,0806	-0,0003
GREEN	0,3875	0,3511	0,0051	0,0060	0,0053	0,0750	0,0399
FAGR	0,7391	0,6850	0,0092	0,0142	0,0099	0,1203	0,0667
IBAB	1,0043	0,9046	0,0123	0,0106	0,0129	0,1213	0,0573
RET	0,2504	0,2384	0,0035	0,0104	0,0038	0,0449	0,0819
AGFB	1,2298	1,0906	0,0149	0,0075	0,0155	0,1476	0,0400
WEHB	0,2482	0,2356	0,0034	0,0114	0,0038	0,0492	0,0720
COMB	0,5408	0,4874	0,0069	0,0084	0,0072	0,0683	0,0554
VAN	0,3405	0,3133	0,0045	0,0073	0,0048	0,0558	0,0600
SIOE	0,7702	0,7068	0,0096	0,0164	0,0102	0,0910	0,0802
Immo	1,0312	0,9672	0,0126	0,0142	0,0138	0,1029	0,1257
TIG	0,4935	0,4255	0,0063	0,0141	0,0064	0,2027	0,0026
MONT	0,2143	0,2069	0,0030	0,0107	0,0034	0,0437	0,0822
REC	1,2220	1,0871	0,0148	0,0137	0,0154	0,1227	0,0520
LEAS	0,3854	0,3609	0,0050	0,0090	0,0055	0,0509	0,0933
INTO	0,6257	0,5732	0,0079	0,0066	0,0084	0,0614	0,0941
EVS	0,8946	0,7995	0,0110	0,0020	0,0115	0,0917	0,0571

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