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LITIGATION AND THE PRODUCT RULE: A RENT SEEKING APPROACH

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Abstract: This paper examines the suppression of the product rule in litigation from a rent seeking perspective. We show that there are some important arguments in favor of not applying it. First, the expected judgment is always lower when the product rule is used, especially for relatively strong cases. Second, litigation expenditures are often larger when the product rule is used, again especially for relatively strong cases. Both of these factors decrease the plaintiff's expected value for such cases. Third, when the product rule is suppressed, the plaintiff files all cases that he or she should win. This is not so when the product rule is applied. Fourth, for many of the weakest cases (the ones in which the quality of all issues is rather weak), the expected value of the plaintiff's case is larger when the product rule is used. The main argument in favor of the application of the product rule is that when the rule is suppressed, plaintiffs file more cases in which the quality of one issue is weak and the quality of the other issue is strong. However, the influence of this factor on the ex ante incentives of the injurer is relatively small.

Keywords: product rule, litigation costs, rent seeking
JEL codes: K13, K41

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1. Introduction

Deciding a legal case often requires combining estimates about two or more elements of interest to law. For example, if the law holds an injurer liable when he or she is negligent and when his or her negligence can be said to have (proximately) caused the injury, a fact finder must evaluate and somehow combine the likelihood of the injurer's negligence and the likelihood of the causal link between his negligent behavior and the injury. The mathematics of the matter tells us that, following what is known as the "product rule" for combining independent probabilistic assessments,[1] the two probabilities need to be multiplied (see Cohen, 1989). The injurer should only be held liable if this product of the two probabilities is larger than the hurdle established by the standard of proof (e.g. preponderance of the evidence, or 0.5). However, the law does not seem to abide by this rule. In the example above, most lawyers think that the law calls for liability as soon as *each* element of the plaintiff's case is established by the relevant standard of proof.[2] Imagine for example that the fact finding in a case generates a conclusion that there is an 80 percent chance of negligence and a 60 percent chance of causation. If the product rule is used, the defendant will not be held liable under the preponderance of the evidence standard (i.e., $0.8 \times 0.6 = 0.48 < 0.5$); whereas if each element is assessed separately, the defendant will be held liable (i.e., $0.8>0.5$ and $0.6>0.5$).

The suppression of the product rule is a puzzling legal phenomenon which has no easy explanation (Cohen, 1977). For that reason, it has provoked an extensive scholarly debate during the last four decades (see e.g. Kaye, 1979; Levmore, 2001; Stein, 2001; Allen and Stein, 2013; Cheng, 2013; Clermont, 2013). When fact finders are not instructed to multiply the probabilities attached

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[1] In reality, the issues will not always be independent.

[2] See Stein (2001) at 1204 n.6 (citing case law and pattern jury instructions that suppress the product rule). For example, ELEVENTH CIRCUIT CIVIL PATTERN JURY INSTRUCTIONS, Basic Jury Instruction 3.7.1 (2013) ("In this case it is the responsibility of the [Plaintiff] [party bringing any claim] to prove every essential part of [his, her, its] claim[s] by a preponderance of the evidence.")

to discrete elements of a lawsuit, the law allows plaintiffs to win cases based upon aggregate probabilities that fall well below fifty percent. Consequently –so the argument goes—courts deliver, over the run of cases, more incorrect decisions than correct ones (Kaye, 1979; Schoeman, 1987; Robertson and Vignaux, 1993). Some authors have tried to rationalize the suppression of the product rule. For example, Levmore (2001) relies on Condercet's theorem and Clermont (2013) on fuzzy logic and belief functions as justifications. Others, however, have criticized these justifications (see e.g. Allen and Jehl, 2003).

In this article, we explore the differences in rent seeking behavior of litigants when the product rule is and is not used and show that there are some important arguments in favor of suppressing the product rule.[3] First, the expected judgment is always smaller when the product rule is applied, a result that is especially true for relatively strong cases. Second, also for relatively strong cases, the litigation expenditures are often larger when the product rule is used. Both of these factors work in the direction of decreasing the plaintiff's expected value for strong cases. Third, when the product rule is suppressed, the plaintiff files all cases he or she should win. This is not so when the product rule is applied. Fourth, for many of the weakest cases (the ones in which the quality of all issues is rather weak), the expected value of the plaintiff's case is larger when the product rule is used. The main argument in favor of the application of the product rule is that if it is suppressed, plaintiffs file more cases in which the quality of one issue is weak and the quality of the other issue is strong. However, we show that the influence of this factor on the ex ante incentives of the injurer is relatively small.

[3] Congleton, Hillman, and Konrad (2008, p. 41) point out that "civil law proceedings are rent-seeking contests in which the 'prize' is dissipated through conflict." Recently, Parisi and Luppi (2013) wrote: "Similar to a rent-seeking contest, in most litigated disputes, the parties invest efforts to win (or to avoid losing) a case. In civil cases, the stakes of the case add up to a fixed amount – one party's winnings are proportional to the other party's losses. Thus, the decision of a plaintiff to file a case, and the defendant's choice to defend it, may be framed as an endogenous decision to participate to a rent-seeking game. Additionally, the outcome of litigation depends on the objective merits of the case. Litigation in civil cases is affected by institutional features that have no counterparts in other litigation contexts, making civil litigation an interesting and unique field of application for rent-seeking models."

This article unfolds as follows. In the next section, we provide some theoretical background regarding the existence of multiple tests to determine liability. Section 3 then examines the two rules (suppression and non-suppression of the product rule) in the context of a legal contest and determines the equilibrium litigation expenditures of both parties. Given these results, Section 4 uses simulations to compare the two rules in terms of the expected judgment, litigation expenditures, the expected value of the parties' cases, and the ex ante incentives of the injurer. Finally, section 5 concludes.

2. Theoretical Background

This section provides the theoretical basis for the analysis in subsequent sections. In the context of accident law, the general question before the court is whether, in the event of an accident, the injurer should be held responsible for the victim's losses. The law has evolved various ways of answering this question. Under a negligence standard, the victim generally has to prove that the injurer was both negligent, and that his or her negligence was proximate cause of the accident. In the economic analysis of law, proximate cause has been modeled in several ways. However, in the approach proposed by Miceli (1996), one can show that either test—negligence or proximate cause—alone should be sufficient for assigning liability in an efficient manner.

To see why, consider a simple unilateral care model in which x=the cost of injurer care; $p(x)$= the probability of an accident, with $p'<0$ and $p''>0$; and L=the victim's loss from an accident.[4] The cost-minimizing level of injurer care, x^*, therefore solves the equation $1=-p'(x^*)L$. According to the marginal Hand test for negligence, the injurer would be found negligent if, for some actual choice of care, x, the court determines that

$$1 < -p'(x)L, \qquad (1)$$

[4] Making the model bilateral care would not qualitatively alter the present argument.

which will be true if and only if $x<x^*$.[5] As has been shown by standard economic models of accident law, no further inquiry is needed.

But suppose that, for whatever reason, the court adds the additional requirement that the injurer's negligence must also be proximate cause of the victim's harm. One test for proximate cause is to ask whether the injurer could have reasonably foreseen that his or her negligence would result in an accident. In formal terms, this question concerns the functional relationship between injurer care and accident risk as embodied in the probability function $p(x)$. Thus, we might formulate a test along these lines which says that, for any given choice of care by the injurer, he or she is proximate cause of the accident if and only if

$$-p'(x) > T \qquad (2)$$

for some threshold T.[6] This amounts to asking whether, from the injurer's perspective prior to the accident, additional care could have reduced the probability of an accident by "enough" for a reasonable person to have foreseen it.

In applying (2) as the test for proximate cause, the question is how the threshold T should be determined. Comparison of (1) and (2) immediately reveals that the tests for negligence and proximate cause are equivalent if we set $T=1/L$. Thus, they are, in principle, redundant in the sense that they should arrive at the same result regarding injurer liability. However, if the court treats them as independent tests, then errors are possible.[7] Suppose, for example, that the court sets $T>1/L$. Then some injurers who are negligent according to the Hand test will be absolved of liability by the proximate cause test. And in the opposite case where $T<1/L$, all negligent injurers will also be judged proximate cause, but so will some injurers who are *not* negligent. However, as long as

[5] The Hand test is usually stated as $B<PL$. Here, $B=1$ and $PL=-p'(x)L$.
[6] Shavell (1985) adds the complicating issue that the accident might have been caused by a "natural" source other than the injurer's actions. In that case, the left-hand side of (2) would be the conditional probability that the injurer was the cause. This does not change the logic of our argument, however, because the conditional probability would also be downward sloping in the injurer's care.
[7] This will necessarily happen if the same T is set for all cases, given that L will vary.

priority is given to the Hand test (i.e., as long as an injurer must be negligent to be liable), then the outcome will be efficient in this case in the sense that no injurers with $x>x^*$ will be held liable.

The preceding analysis has presumed that the two tests, although redundant, can at least be applied deterministically. In reality, this will not be possible because of legal and evidentiary uncertainty. The fact that the tests will be probabilistic in practice raises the question of how the court should combine them—that is the problem posed in the introduction. The remainder of this paper therefore takes as given the existence of multiple tests and examines the consequences for litigant behavior and judicial outcomes of the particular rule the court adopts for combining the tests.

3. **Model of Litigant Behavior**[8]

Consider a tort case with evidentiary uncertainty and with two independent issues at stake (e.g., negligence and causation). For each issue, we can use Bayes' rule to compute the conditional

[8] Several other papers have introduced similar but not identical contest success functions in the litigation context as the ones we develop in this section. Some examples include Katz, 1988 (inter alia analyzing the determinants of the litigants' expenditures such as the inherent merits of the case and the amount at stake), Farmer and Pecorino, 1999 (comparing the American and English fee shifting rules), Hirshleifer and Osborne, 2001 (examining litigation effort under two different protocols, Nash-Cournot and Stackelberg), and Guerra, Luppi and Parisi, 2018 (analyzing how different standards of proof affect litigation choices). One difference with these papers is that our contest success functions need to incorporate two issues rather than one. For that reason, we explicitly model the conditional probability that the court will consider the plaintiff to be right for *each* issue (with symbol $P_{1,i}$ with i the issue at stake). Then, with a preponderance of the evidence standard, we use these probabilities for both issues to calculate the probability of a plaintif victory (a) when the product rule is not applied ($P_{NPR}(X_1,X_2,Y_1,Y_2)$ = prob ($P_{1,1} > 1/2$) × prob ($P_{1,2} > 1/2$)) and (b) when the product rule is applied ($P_{PR}(X_1,X_2,Y_1,Y_2) = prob\ (P_{1,1} \times P_{1,2} > 1/2)$). If we were to consider only one issue, the probability of a plaintiff victory would obviously be the same with or without the product rule, and we would find a CSF much like the one used in e.g. Hirshleifer and Osborne (2001). With respect to Guerra, Luppi and Parisi (2018), another difference concerns the standard of proof. While that article examines rent seeking under different standards of proof, we hold the standard of proof fixed (preponderance of the evidence).

probability that the court will consider the plaintiff to be right about that issue, denoted $P_{1,i}$, with i the issue at stake and thus i=1 or i=2, as follows[9]:

$$P_{1,i} = \frac{P_{0,i}X_iF_i}{P_{0,i}X_iF_i + (1 - P_{0,i})Y_i(1 - F_i)} \tag{3}$$

where

$P_{0,i}$ = court's prior probability that the plaintiff's assertions about issue i are correct, $P_{0,i} \in [0,1]$;

F_i = index of the inherent quality of issue i, normalized so that $F_i \in [0,1]$;

X_i = plaintiff's litigation effort for issue i;

Y_i = defendant's litigation effort for issue i.

In this formulation, $P_{0,i}$ can be interpreted as a measure of the court's "bias" regarding issue i. Specifically, if $P_{0,i}$=½, the court is unbiased, whereas if $P_{0,i}$>(<)½, it can be said to have a pro-plaintiff (pro-defendant) bias. Alternatively, one could interpret $P_{0,i}$ as reflecting noise in the court's assessment of the evidence presented at trial. Under such an interpretation, if $P_{0,i}$ = 1/2, the court attaches equal weight to each party's evidence, whereas if $P_{0,i}$ > (<) 1/2, the court gives more weight to the evidence produced by the plaintiff (defendant). This could, for example, be the case when judges think that on average, evidence produced by the plaintiff (defendant) is more reliable. Finally, $E_p=X_iF_i$ can be interpreted as the "evidence for" the plaintiff, which depends positively on both the factual evidence in favor of him and the plaintiff's litigation effort. Likewise, $Ed=Y_i(1–F_i)$ is the "evidence for" the defendant.

[9] Note that there are various positive justifications of the randomness implicit in an imperfect discriminating contest function (see e.g. Jia, Skaperdas and Vaidya, 2013). Skaperdas and Vaidya (2012) demonstrate how the contest functions representing win probabilities can be justified as natural outcomes of an audience's Bayesian inference process in a trial setting involving a plaintiff, a defendant and a judge. To illustrate, in the first stage, both parties expend efforts to gather evidence favorable to their cause. Using evidence production functions, each side obtains a piece of evidence and presents it to the court. In our model, the evidence production function for the plaintiff (Ep) is the product of litigation effort of that party and the inherent quality of the issue, and for the defendant (Ed) it is the product of litigation effort of that party and 1 minus the inherent quality of the issue. Thus, when the inherent strength becomes stronger from a party's point of view, that party finds more evidence in her or his favor for a given level of effort. In the second stage, based on the pieces of evidence provided by the parties, the court assesses the likelihood ratio of guilt (Ep/Ed) and uses it, together with its prior belief about guilt, to determine its posterior probability of guilt using Bayes' rule. The court does not observe the efforts chosen by the parties, nor the evidence production functions. It only receives the evidence pair presented at trial. The litigants are aware of the court's inference process. The randomness of the trial outcome therefore results from the fact that the litigants do not know the court's prior with certainty, but only the distribution of the prior.

The plaintiff should win an issue if and only if the inherent quality of that issue exceeds 1/2. For example, if the defendant took just enough care to satisfy the due care standard, F=1/2.[10] Thus, F decreases below 1/2 as the defendant took more than due care, and in the limit where F reaches zero, the plaintiff can never win the due-care issue, no matter how much he or she spends relative to the defendant. Conversely, F increases above 1/2 as the defendant took less care than due care (or when the defendant intentionally caused the harm), and at the point where F reaches one, the plaintiff wins the due-care issue with certainty, no matter how much the defendant spends. The plaintiff should win the whole case if and only if the inherent quality of each issue is at least 1/2. The court thus reaches an efficient judgment if the plaintiff wins when F_1 >1/2 and F_2 >1/2 and the defendant wins otherwise.

As noted in the introduction, the specific question of interest here is how the two rules affect the litigation efforts of the parties—that is, the plaintiff's choice of X_i and the defendant's choice of Y_i. To answer this question, we first need to account for the uncertain outcome of a trial. Specifically, the parties have to form an expectation about their chances of winning under either rule. To do this, we suppose that the source of uncertainty at trial is the court's bias. Thus, we assume that F is common knowledge,[11] but that P_0 is a random variable whose distribution is known by both parties. For simplicity, we will assume that P_0 is uniformly distributed on [0,1].[12]

3.1. Product Rule Not Applied

[10] F=1/2 means that intrinsically, there is an equal number of arguments in favor of the plaintiff as in favor of the defendant. When F > (<) 1/2, there are more arguments favoring the position of the plaintiff (defendant). Note that F=1/2 implies that if the parties do not spend anything on evidence production for a certain issue, or if they spend an equal amount on evidence production, the parties have an equal chance of winning that issue. However, if the defendant took adequate care, his unit costs of evidence production will usually be lower than the unit costs of evidence production of the plaintiff, and this will lead to a probability of winning the issue that is larger than 0.5 for the defendant.

[11] This is a common assumption in the rent-seeking literature on litigation. This can be justified by supposing that the facts of the case are made public during pre-trial discovery. Of course, in some cases discovery may be too costly, and asymmetric information may still be present.

[12] We adopt the assumption of a uniform distribution for simplicity. Assuming a unimodal distribution (normal or triangular) would not qualitatively affect our conclusions, nor would restricting the support of the distribution as long as it is symmetric around 1/2.

In the situation in which the product rule is not applied, the plaintiff's objective is to choose X_1 and X_2 to maximize

$$P_{NPR}(X_1,X_2,Y_1,Y_2)J - X_1 - X_2, \qquad (4)$$

while the defendant chooses Y_1 *and* Y_2 to minimize

$$P_{NPR}(X_1,X_2,Y_1,Y_2)J + Y_1 + Y_2 \qquad (5)$$

where $P_{NPR}(X_1,X_2,Y_1,Y_2)$ is the plaintiff's probability of winning, and J is the amount at stake. The first-order conditions defining the reaction functions for the plaintiff and defendant, respectively, are

$$(P_{NPR})_{Xi} J = 1 \qquad (6)$$

$$(P_{NPR})_{Yi} J = -1. \qquad (7)$$

Simultaneous solution of these four equations determines the Nash equilibrium levels of effort[13]: $(X_1^*,X_2^*,Y_1^*,Y_2^*)$.

The probability of a plaintiff victory in the case where the product rule is not applied, given a preponderance-of-the-evidence standard, is given by

$$P_{NPR}(X_1,X_2,Y_1,Y_2) = \text{prob}\ (P_{1,1} > 1/2) \times \text{prob}\ (P_{1,2} > 1/2)$$

$$= \text{prob}\ \left(P_{0,1} > \frac{Y_1(1-F_1)}{Y_1(1-F_1)+X_1F_1}\right) \times \text{prob}\ \left(P_{0,2} > \frac{Y_2(1-F_2)}{Y_2(1-F_2)+X_2F_2}\right) \qquad (8)$$

For a uniform distribution of the court's prior, this becomes

$$P_{NPR}(X_1,X_2Y,Y_2) = \left(1-\frac{Y_1(1-F_1)}{Y_1(1-F_1)+X_1F_1}\right)\left(1-\frac{Y_2(1-F_2)}{Y_2(1-F_2)+X_2F_2}\right) = \frac{X_1F_1}{X_1F_1+Y_1(1-F_1)}\frac{X_2F_2}{X_2F_2+Y_2(1-F_2)} \qquad (9)$$

Note that this function is increasing in X_i and decreasing in Y_i, and that the investments in X_i and Y_i have diminishing returns.[14]

Using this expression in (6) and (7), we obtain the Nash equilibrium effort levels[15]

$$X_1^* = Y_1^* = F_1(1-F_1)F_2J \quad \text{and} \quad X_2^* = Y_2^* = F_2(1-F_2)F_1J \qquad (10)$$

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[13] The vast majority of articles focusing on rent seeking in litigation adopt the Nash equilibrium solution concept. Katz (1988, p. 128) explains why the Nash equilibrium approach is appropriate.

[14] Thus $dP_{NPR}/dX_i > 0$, $dP_{NPR}/dY_i < 0$, $d^2P_{NPR}/dX_i^2 < 0$ and $d^2P_{NPR}/dY_i^2 > 0$. Proof on file with the authors.

[15] The second-order sufficiency conditions for X_i^* and Y_i^* are satisfied given the characteristics of P_{NPR} described in the previous footnote. The Nash equilibrium is interior and unique. On the existence and uniqueness of a pure Nash equilibrium in rent-seeking games, see e.g. Szidarovsky and Okuguchi (1997).

These values can then be used to calculate the expected judgment at trial, $P_{NPR}(X_1,X_2,Y_1,Y_2)J = F_1F_2J$, the overall expected value of trial to the plaintiff, and the expected cost of trial to the defendant (i.e., the optimized values of the expressions in (4) and (5)). These are $F_1F_2J - F_1(1-F_1)F_2J - F_2(1-F_2)F_1J$ and $F_1F_2J + F_1(1-F_1)F_2J + F_2(1-F_2)F_1J$, respectively. Interestingly, note that the expected judgment turns out to be proportional to the product of the inherent quality of the several issues.

3.2. Product Rule Applied

In a situation in which the product rule is used, the plaintiff's objective is to choose X_1 *and* X_2 to maximize

$$P_{PR}(X_1,X_2,Y_1,Y_2)J - X_1 - X_2, \tag{11}$$

while the defendant chooses Y_1 *and* Y_2 to minimize

$$P_{PR}(X_1,X_2,Y_1,Y_2)J + Y_1 + Y_2 \tag{12}$$

where $P_{PR}(X_1,X_2,Y_1,Y_2) = prob\ (P_{1,1} \times P_{1,2} > 1/2)$

$$= prob(P_{0,2} > \frac{1+P_{0,1}(a-1)}{(P_{0,1}ab+a+b-1)-(b-1)}) \text{ with } a = \frac{X_1F_1}{Y_1(1-F_1)} \text{ and } b = \frac{X_2F_2}{Y_2(1-F_2)}$$

For a uniform distribution, the probability equals:

$$(1-\frac{1}{a+1}) - \int_{\frac{1}{a+1}}^{1} \frac{1+P_{0,1}(a-1)}{((ab+a+b-1)P_{0,1}-(b-1))} dP_{0,1}$$

$$= \frac{a}{a+1} - \frac{2ab(\ln(a+1)+\ln(b+1)-\ln 2)+(a-1)ab+\frac{a}{a+1}(a-1)^2}{(ab+a+b-1)^2} \tag{13}$$

This function is increasing in X_i and decreasing in Y_i, and the investments in X_i and Y_i have diminishing returns.[16] Note that the limiting cases (a=0 or b=0 and a=∞ or b=∞) lead to the simple

[16] Thus $dP_{PR}/dX_i > 0$, $dP_{PR}/dY_i < 0$, $d^2P_{PR}/dX_i^2 < 0$ and $d^2P_{PR}/dY_i^2 > 0$. This can for example be easily seen when graphing this function. Graphs on file with the authors. We can also see this by looking at the function from

formulas we would expect to find. Specifically, if a=0 or b=0, the plaintiff can't win one of the issues, and so his or her probability of winning the trial should be zero. This is indeed what we find if we set a=0 or b=0 in (13). At the other extreme, when a=∞, we would expect only the second issue to be relevant, and indeed, when we set a=∞ in (13), we find a probability of winning of $\frac{b}{b+1}$, which is exactly the probability of winning if issue 2 were the only issue. A similar argument applies for the case of b=∞.

The first-order conditions for the plaintiff's and defendant's problems can again be solved simultaneously to obtain the Nash equilibrium effort levels, $(X_1^{**}, X_2^{**}, Y_1^{**}, Y_2^{**})$, which in this case turn out to be rather cumbersome, and so they are reported in the Appendix.[17] These equilibrium effort levels can then be used, as above, to compute the expected judgment, the plaintiff's expected value, and the defendant's expected cost, of a trial. The complexity of the expressions precludes drawing conclusions regarding the outcomes under the two rules analytically. However, the next section uses numerical simulations to reveal the key differences.

4. Implications

4.1. Expected judgment

Result 1: *The expected judgment is always lower when the product rule is applied.*

The reason is that, compared to non-use of the rule, the plaintiff has to surpass a higher hurdle. Interestingly, the difference in expected judgment is lower for cases in which the inherent quality of both issues is very weak and for cases in which the inherent quality of at least one issue is very high, as compared to cases in which both issues are of intermediate strength (e.g. F_1=0.7 and F_2=0.75). This makes sense since, if the court views *either one* of the two issues as slightly weaker

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which the CSF in (13) is derived, $P_{PR}(X_1,X_2,Y_1,Y_2)$ = *prob* $(P_{1,1} \times P_{1,2} > 1/2)$, given that $dP_{1,i}/dX_i > 0$, $dP_{1,i}/dY_i < 0$, $d^2P_{1,i}/dX_i^2 < 0$ and $d^2P_{1,i}/dY_i^2 > 0$.

[17] The conditions for a unique interior Nash equilibrium are also satisfied in this case.

than it actually is, surpassing the evidentiary hurdle becomes very problematic when the product rule is applied, but not when the product rule is eschewed.

The figures below illustrate the preceding conclusions. In the graphs, the X-axis represents the inherent quality of issue 1, and the Y-axis the inherent quality of issue 2. The dark area in Figure 1 shows the cases for which the difference between the expected judgments is smaller than 5 % of the amount at stake. The dark area in Figure 2 shows the cases for which the difference between the expected judgments is smaller than 10 % of the amount at stake. The graphs verify that the product rule lowers the expected judgment most for cases of intermediate strength (i.e., cases in the non-darkened areas).

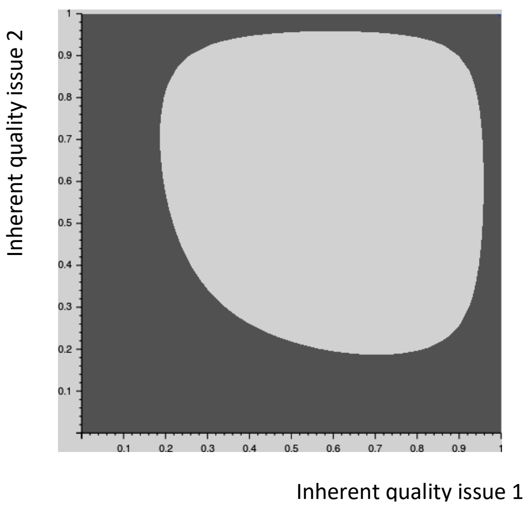

Figure 1. Difference expected judgment < 5 % of the amount at stake (black area).

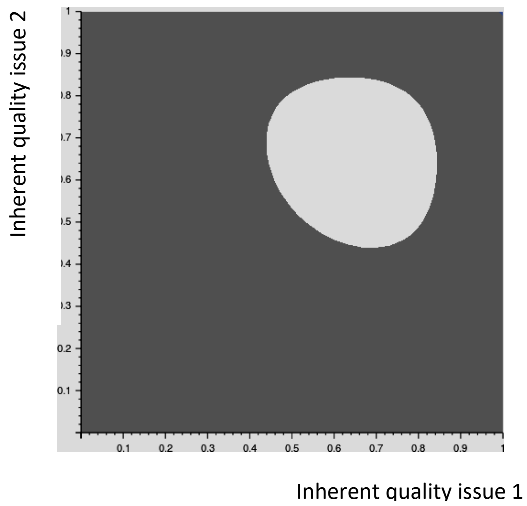

Figure 2. Difference expected judgment < 10% of the amount at stake (black area).

4.2. Litigation expenditures regarding issue 1

Result 2: *The litigation expenditures regarding each issue may be either lower or higher when the product rule applies than when it does not.*

Interestingly, expenditures are higher under the product rule for cases in which the inherent quality of the first issue is relatively large. Figure 3 illustrates this. Intuitively, for these cases, applying the product rule causes the marginal value of the expenditures to be larger because they may partially compensate for the relative weakness of the second issue. Such compensation is not possible when the product rule is not applied. For example, suppose there is an 80 percent chance that the court will consider the posterior probability of issue 2 to be 0.3, and a 20 percent chance that it will assess this probability at 0.6. Suppose further that the plaintiff can make an investment with respect to issue 1 which may bring the court's posterior assessment of this issue from 0.8 to 0.9. When the product rule does not apply, this investment may have little benefit, but when the product rule applies, making this investment is the only way to still have a chance to win the trial (i.e., if the court considers the posterior probability of issue 2 to be 0.6, the combined posterior probability increases from 0.8×0.6=0.48 to 0.9×0.6=0.54).

For cases in which the inherent quality of the first issue is somewhat lower or weaker, expenditures are lower when the product rule is used. Here, the marginal value of expenditures is often lower under the product rule. For example, suppose that an additional investment by the plaintiff could bring the court's posterior assessment regarding issue 1 from 0.6 to 0.7. When the product rule is not used, this expenditure has value as long as the court assesses the posterior probability of issue 2 to exceed 0.5. When the product rule is used, however, this expenditure only has value if the court assesses the posterior probability of issue 2 to be at least 0.714.

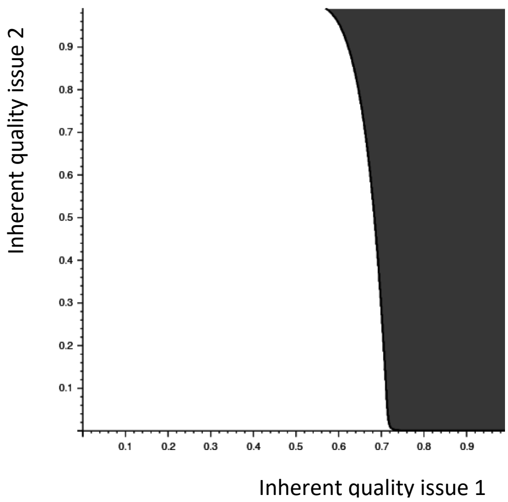

Figure 3. Expenditures of issue 1 are larger under the product rule (black area).

The situation regarding issue two is completely symmetrical to the previous case. The litigation expenditures regarding the second issue may be either lower or higher when the product rule applies than when it does not. Expenditures are higher when the product rule is used for cases in which the inherent quality of the second issue is relatively large.

4.3. Total litigation expenditures

Result 3: *For the majority of cases (not taking into account the decision to file), total expenditures are smaller when the product rule is used than when it is not. Cases where expenditures are larger under the product rule are those for which the inherent quality of at least one of the issues is quite large.*

The black area in Figure 4 represents cases for which total expenditures are larger under the product rule, while the black area in Figure 5 represents the cases for which the difference in total expenditures is larger than 5 % of the amount at stake.

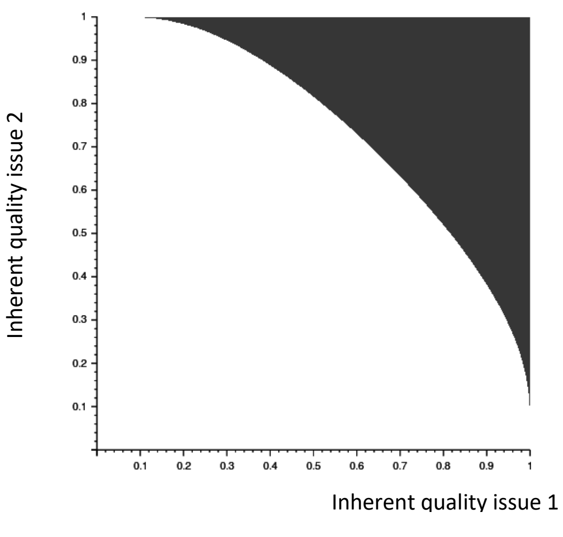

Figure 4. Total expenditures are larger under the product rule (black area)

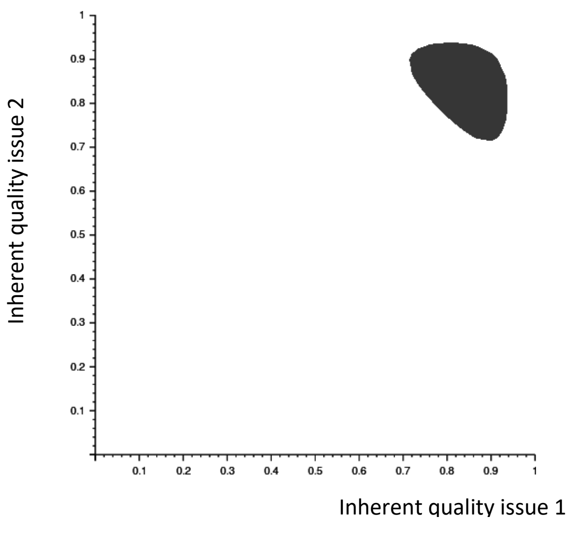

Figure 5. Difference in total expenditures > 5% amount at stake (black area).

4.4. Expected value of the plaintiff

Result 4: *For strong cases, the plaintiff's expected value is typically lower under the product rule. For many weak cases, especially the weakest ones, the plaintiff's expected value is larger under the product rule.*

With respect to strong claims, this result follows from the analysis above, given that for these types of cases the expected judgment is lower and the expenditures are often higher. For the weakest cases, the reduction in expenditures outweighs the reduction in the expected judgment. In Figure 6, the dark area represents the cases for which the plaintiff's expected value is larger when the product rule is used.

Note however that these cases will generally not be filed because the plaintiff's participation constraint is not satisfied—i.e., these cases have negative expected value. Of course, some of these cases may be filed anyway if the plaintiff has an exogenous benefit of going to trial-- e.g. to have his day in court, or because he has a long-term interest in precedent.

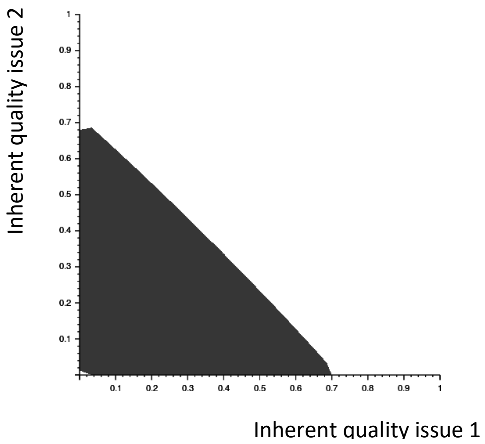

Figure 6. Plaintiff's expected value is larger under the product rule (black area).

The dark area in Figure 7 shows which cases are filed under the product rule, while Figure 8 shows which cases are filed when the product rule is not applied. Remember that the plaintiff should win a case whenever the inherent quality of each issue exceeds 1/2 (as discussed in section 3), and the defendant should win otherwise. When the product rule is suppressed, the plaintiff files all cases that he or she should win. This is not the case when the product rule is applied. Many cases which the plaintiff should win are not filed (e.g. $F_1 = F_2 = 0.6$). On the other hand, suppressing the product rule has the disadvantage that more cases are filed which the plaintiff should not win, e.g. the case in which $F_1 = 0.3$ and $F_2 = 0.8$ is filed when the product rule is suppressed but not when it is applied.

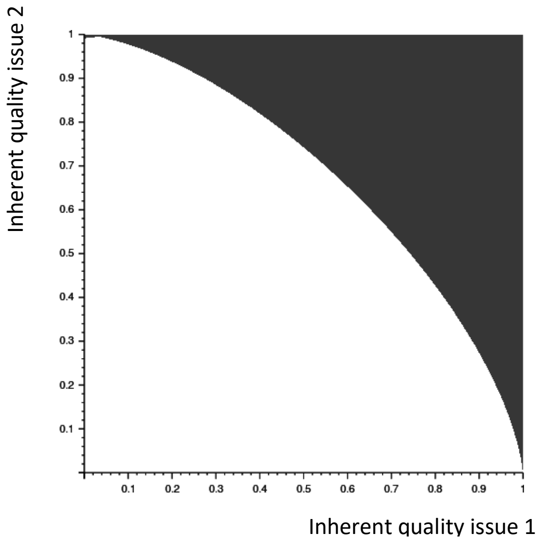

Figure 7. Cases filed under product rule (black area).

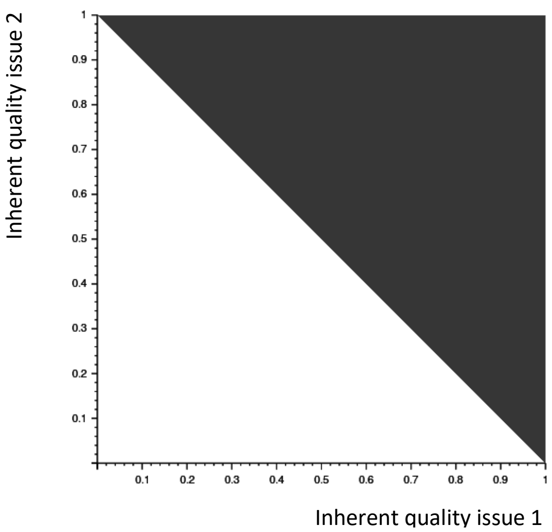

Figure 8. Cases filed with suppression of product rule (black area).

4.5. Ex ante incentives of the injurer

Result 5: *Although the influence of suppressing the product rule on deterrence is ambiguous, the positive effects are likely to outweigh the negative effects.*

The fact that the plaintiff is willing to file more cases that he or she should win when the product rule is not applied (and the fact that the expected judgment is typically larger with suppression when meritorious cases are filed under each system) provides stronger incentives for injurers to engage in efficient behaviour when the product rule is suppressed. In contrast, the fact that the plaintiff files more cases he or she should not win when the product is suppressed has the opposite effect, making the overall effect ambiguous. However, this second, undesirable effect of suppression is often very modest, while the negative effect of suppression on meritorious claims is quite large. For example, when $F_1=F_2=0.8$, the difference in the plaintiff's expected value without and with suppression equals about 15 percent of the amount at stake. For the case in which $F_1=0.3$ and $F_2=0.8$, the difference with and without suppression equals only 2 percent of the amount at stake. Figure 9 below shows (from different angles) the difference in the plaintiff's expected value with and without suppression. The x-axis represents F_1, the y-axis F_2, and the z-axis the difference in expected value (which ranges from - 1 percent of the amount at stake to 15 percent of the amount at stake). Clearly, the greatest differences exist for strong cases.[18]

[18] Note that the differences are slightly exaggerated because we subtracted negative values when the plaintiff's expected value was negative. So both the positive effect and the negative effect of suppressing the product rule is slightly exaggerated (when the claims have negative value, it concerns only a few percent of the amount at stake). The main result, that the difference is much greater for strong claims, remains valid if a value of zero would be subtracted.

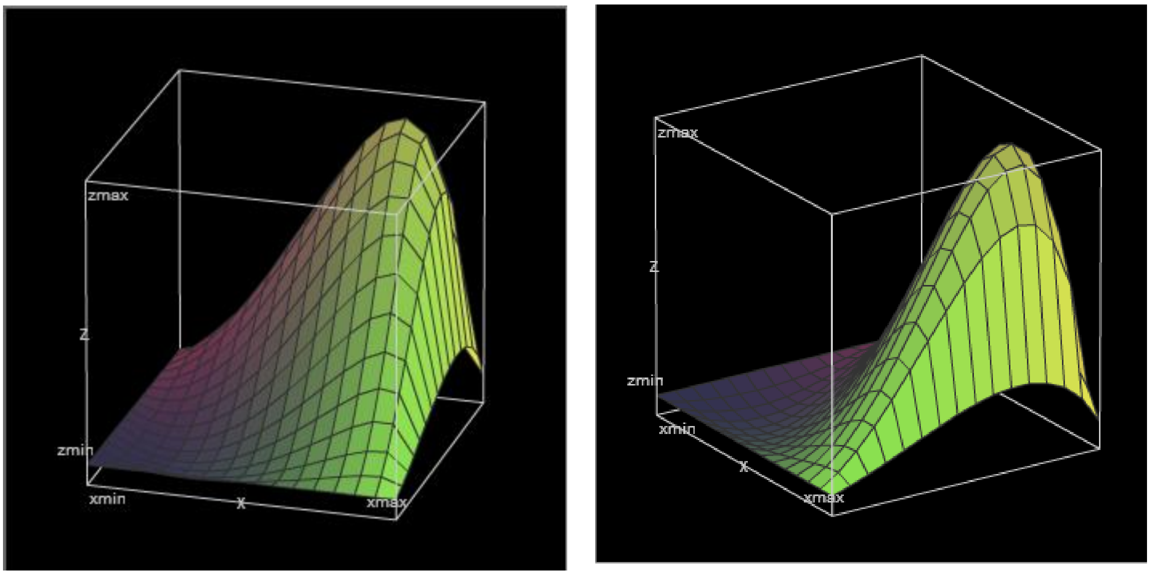

Figure 9. The difference between plaintiff's expected value without and with the product rule (the second graph shows a different angle).

5. Conclusion

In cases where the court must judge multiple legal questions in order to assign liability, the question arises as to whether it should judge them individually or jointly. In the presence of evidentiary uncertainty, this amounts to asking whether the probability of each element separately, or the product of the probabilities, must surpass the legal threshold in order for the plaintiff to win. This question has provoked a large literature debating the two approaches. In this paper, we have contributed to this debate by examining how the two rules affect the amount that plaintiffs and defendants invest in legal expenditures at trial. Our conclusions are as follows.

First, the expected judgment is always lower when the product rule is used, and this is especially so for relatively strong cases. Second, the plaintiff's expected value for strong cases is further decreased because for such cases litigation expenditures are often larger when the product rule is used. Third, while the plaintiff files all cases he or she should win when the product rule is suppressed, this is not so when the product rule is applied (the upside of this is of course that litigation costs are avoided). Fourth, for many of the weakest cases, the expected value of the plaintiff’s case is larger when the product rule is used. The principal argument in favor of the application of the product rule is that plaintiffs file more cases in which the quality of one issue is weak and the quality of the other issue is strong when the product rule is suppressed. However, the effect of this on the ex ante incentives of the injurer is relatively small.

We have provided a first analysis seeking to justify the suppression of the product rule from a rent seeking perspective. Future research could take several extensions into account. For example, we have assumed that the parties bear their own costs of litigation (the American rule of cost allocation applies). It could be interesting to extend the model to regimes in which the loser pays the winner's costs (the English rule of cost allocation). Also, our model investigates situations with two issues at stake. Future research could expand the number of issues. Finally, one could introduce the possibility for the parties to settle, including an agreement over the governing evidentiary rule.

Appendix

With the product rule, the equilibrium expenditures equal (proof on file with the authors):

$$X_1{**}=Y_1{**}=F_1(1-F_1)J+F_1(1-F_2)\frac{2F_2(1-F_1)(\ln(1-F_1)+\ln(1-F_2)+\ln 2)-2F_1F_2(1-F_1)-F_2(3F_1-1)-(2F_1-1)^2(1-F_1)(1-F_2)-2F_1(2F_1-1)(1-F_2)}{(F_1+(2F_2-1)(1-F_1))^2}J$$
$$-2F_1^2(1-F_2)\frac{2F_2(1-F_1)(\ln(1-F_1)+\ln(1-F_2)+\ln 2)-(2F_1-1)F_2-(2F_1-1)^2(1-F_2)}{(F_1+(2F_2-1)(1-F_1))^3}J$$

$$X_2{**}=Y_2{**}=F_2(1-F_2)J+F_1(1-F_1)\frac{2F_1(1-F_2)(\ln(1-F_2)+\ln(1-F_1)+\ln 2)-2F_1F_2(1-F_2)-F_1(3F_2-1)-(2F_2-1)^2(1-F_2)(1-F_1)-2F_2(2F_2-1)(1-F_1)}{(F_2+(2F_1-1)(1-F_2))^2}J$$
$$-2F_2^2(1-F_1)\frac{2F_1(1-F_2)(\ln(1-F_2)+\ln(1-F_1)+\ln 2)-(2F_2-1)F_1-(2F_2-1)^2(1-F_1)}{(F_2+(2F_1-1)(1-F_2))^3}J$$

Note that the limit cases (F_i=0 or F_i=1) lead to the results we should expect to find. For example, regarding the expenditures concerning the first issue, when F_2=1, only the first issue should matter. When we set F_2=1 in (12), we find that $X_1=Y_1=F_1(1–F_1)J$. When F_2=0, the outcome of the case is certain (the defendant will win), and it's no use investing in the first issue. Indeed, when we set F_2=0 in (14), we find that $X_1=Y_1=0$.

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