First high-quality draft genome of Ochrobactrum haematophilum P6BS-III, a highly glyphosate-tolerant strain isolated from agricultural soil in Argentina

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A NOTE ON NON-UNIQUE ENHANCEMENTS

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ABSTRACT. We give an easy example of a triangulated category, linear over a field k, with two different enhancements, linear over k, answering a question of Canonaco and Stellari.

In their recent survey paper on enhancements for triangulated categories, Canonaco and Stellari pose the following question:

Question ([1, Question 3.13]). Are there examples of triangulated categories, linear over a field k, with non-unique k-linear enhancement?

Below we give such an example. In fact, we simply observe that the topological graded field example (see [6, §2.1]) can be made to work in the algebraic case.[1]

Example. Let $K = k(x_1, \cdots, x_{n+1})$ and $F = K[t, t^{-1}]$, where $n > 0$ is even, t has cohomological degree n, and K is concentrated in degree zero. Since all homogeneous elements in F are invertible, we call F a graded[2] field. Let $0 \neq \eta \in \mathrm{HH}_k^{n+1}(K)$. Then, by Lemma 1 below, $\tilde{\eta} = \eta \otimes d/dt$ is a non-zero element of $\mathrm{HH}_k^{n+2}(F)^{-n}$. Since, also by Lemma 1, $\mathrm{HH}_k^s(F) = 0$ for $s > n+2$, we may construct a minimal A_∞ structure $(0, m_2, 0, \ldots, 0, m_{n+2}, m_{n+3}, \ldots)$ on F such that the class of m_{n+2} is $\tilde{\eta}$ (see [4, Lem. B.4.1]). Let F_η be the resulting A_∞-algebra and let $f_1 \in \mathrm{Aut}_k(F)$. One checks using [4, Lem. B.4.2]) or directly that $\tilde{\eta} \circ f_1$ is the first obstruction against extending f_1 to an A_∞-isomorphism $f : F \to F_\eta$. Since $\tilde{\eta}$ is non-trivial, the same is true for $\tilde{\eta} \circ f_1$ and so F and F_η are not A_∞-isomorphic.

As in [5], we see that the triangulated category $\mathrm{Perf}(F_\eta)$ of right perfect F_η-modules is equivalent, as a graded category, to the category of graded F-vector spaces of finite rank. Since the latter category is semi-simple, it has only one

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[1] Another, more complicated, but also potentially more interesting, example is presented in [3]. The techniques in [3] are based on an ingeneous direct manipulation of solutions to the Maurer-Cartan equation. However, the authors feel that as it stands, the arguments are not fully complete. In particular, while it is possible to “remove strict units” ([3, §4.4]) from solutions to the Maurer-Cartan solution, the required A_∞-isomorphism will in general be more complicated than the proof suggests. This makes the verification of “Condition (1)” in [3] more delicate (if at all possible) and therefore the same is true for the claim that the constructed functor is exact. We are currently discussing these points with the author.

[2] Throughout all graded notions are interpreted in the “super” sense.

triangulated structure compatible with the graded structure. Hence $\mathrm{Perf}(F)$ and $\mathrm{Perf}(F_\eta)$ are equivalent as trianguled categories.

On the other hand, $\mathrm{Perf}(F)$ and $\mathrm{Perf}(F_\eta)$ have canonical A_∞-enhancement given by the A_∞-categories of twisted complexes $\mathrm{Tw}(F)$ and $\mathrm{Tw}(F_\eta)$ (see [4, Ch. 7]). We claim that $\mathrm{Tw}(F)$ and $\mathrm{Tw}(F_\eta)$ are not A_∞-equivalent. Indeed, any A_∞-equivalence between them would have to send the indecomposable (right) F-module F_F to an object in $\mathrm{Tw}(F_\eta)$ which is A_∞-isomorphic to $\Sigma^u(F_\eta)_{F_\eta}$ for some u. Hence $\mathrm{Tw}(F)(F_F, F_F) \cong F$ and $\mathrm{Tw}(F_\eta)(\Sigma^u(F_\eta)_{F_\eta}, \Sigma^u(F_\eta)_{F_\eta}) \cong F_\eta$ would have to be A_∞-isomorphic A_∞-algebras (since they are both minimal). This is not the case as we have established above.

Remark. There is nothing special about the particular pair (K, η) we have used. The chosen (K, η) simply allows for the most trivial argument for the existence of an A_∞-structure on F with the given m_{n+2}.

We have used the following basic lemma:

Lemma 1. *Let $F = K[t, t^{-1}]$ be as above. The Hochschild cohomology of F is given by $\mathrm{HH}^*_k(F) \cong \mathrm{HH}^*_k(K) \otimes_k \mathrm{HH}^*_k(k[t, t^{-1}])$. Moreover $\mathrm{HH}^i(k[t, t^{-1}]) = 0$ for $i > 1$ and the derivation d/dt represents a non-trivial element of $\mathrm{HH}^1(k[t, t^{-1}])$.*

Proof. By the next lemma we only have to understand $\mathrm{HH}^*(k[t, t^{-1}])$. Since t has even degree, F is graded commutative and so the claim follows from the graded version of the HKR theorem. ☐

Lemma 2.[3] *Let A, B be graded k-algebras such that the graded tensor product $B^e := B \otimes_k B^\circ$ is noetherian. Then $\mathrm{HH}^*(A \otimes_k B) \cong \mathrm{HH}^*(A) \otimes_k \mathrm{HH}^*(B)$.*

Proof. Let $Q^\bullet$ be a resolution of B by finitely generated graded projective B-bimodules, and let $P^\bullet$ be an arbitrary resolution of A by graded projective A-bimodules. Then $P^\bullet \otimes_k Q^\bullet$ is a graded projective resolution of $A \otimes_k B$ and we have

$$\begin{aligned}\mathrm{HH}^*(A \otimes_k B) &= H^*(\mathrm{Hom}_{A^e \otimes_k B^e}(P^\bullet \otimes_k Q^\bullet, A \otimes_k B)) \\ &\cong H^*(\mathrm{Hom}_{A^e}(P^\bullet, A) \otimes_k \mathrm{Hom}_{B^e}(Q^\bullet, B)) \\ &\cong \mathrm{HH}^*(A) \otimes_k \mathrm{HH}^*(B).\end{aligned}$$ ☐

References

[1] Alberto Canonaco and Paolo Stellari, *A tour about existence and uniqueness of dg enhancements and lifts*, J. Geom. Phys. **122** (2017), 28–52. MR3713870

[2] Petter Andreas Bergh and Steffen Oppermann, *Cohomology of twisted tensor products*, J. Algebra **320** (2008), no. 8, 3327–3338. MR2450729

[3] Hiroshige Kajiura, *On A_∞-enhancements for triangulated categories*, J. Pure Appl. Algebra **217** (2013), no. 8, 1476–1503. MR3030547

[4] K. Lefèvre-Hasegawa, *Sur les A_∞-catégories*, Ph.D. thesis, Université Paris 7, 2003.

[5] J. Lurie, *Chromatic homotopy theory (252x)*, Course Notes, Lecture 24.

[6] Stefan Schwede, *The stable homotopy category has a unique model at the prime 2*, Adv. Math. **164** (2001), no. 1, 24–40. MR1870511

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[3] This lemma is stated in [2, Thm 4.7] without any hypotheses on A, B. However, the proof in [2] (essentially the proof we have given) requires some kind of finiteness hypothesis which seems to have been inadvertently omitted. Indeed the result is false if A, B are fields of infinite transcendence degree over k.

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