Faculteit Bedrijfseconomische Wetenschappen

master in de handelswetenschappen

Masterthesis

Forecasting modellen voor nieuwe producten

Ivo Van Brempt

Scriptie ingediend tot het behalen van de graad van master in de handelswetenschappen, afstudeerrichting supply chain management

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2018
2019

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Diffusion models in new product forecasting

Ivo Van Brempt
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When forecasting the demand of a new product, historical data are unavailable for managers to predict future sales or adoption of the product. A broad range of models has been developed with the purpose of tackling this problem. This dissertation discusses a sub-branch of quantitative forecasting techniques, namely diffusion models. The main interest of diffusion models is forecasting the adoption of innovative durable products or innovations as a whole. Adoption of a new product can be defined as a customer taking up the product or buying the product for the first time. The basic premise of the diffusion models is that adoption of products follows a sigmoidal trend. In the beginning, few people adopt the product. This initial period is followed by a spike in the adoption rate. Eventually the market is saturated, and growth slows down. The three most significant models that express the sigmoidal growth curve are the Bass, logistic and Gompertz model. Various extensions of the first two functions have been proposed. The extensions mainly deal with incorporating more flexibility into the basic models. Despite the popularity of diffusion models in the research on adoption of new products, they also have received a great deal of criticism. The lack of distinctiveness between the models, the inseparable characteristic of the need for data and the lack of practical business cases are major concerns impacting the general applicability of diffusion models. For the purpose of testing the ability of the three basic models to describe the diffusion pattern of an innovation, the functions are fitted for mobile subscription data of five different European countries. If the actual data showed little trend fluctuations, the models provided a good fit. However, the data also demonstrated various fluctuations the diffusion models couldn't account for. Furthermore, the empirical analysis also established that even for European countries, diffusion patterns can differ significantly.

Keywords: new products, innovation, diffusion, growth models, forecasting

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1. Introduction

When companies wish to launch a new product, an obvious question arises: "What will be the future demand of this product?" An accurate forecast is of great importance for multiple branches of a business. A few examples of how an adequate forecast can aid a company include: better financial planning, the avoidance of major stockouts/overstock and the planning of advertising campaigns. However, when forecasting the demand of a new product, a problem occurs. Unlike products already on the market, new products lack historical data. Traditional methods, which often use past sales as input for the required forecast, are therefore unusable (Wright & Stern, 2015).

Because of the importance of an accurate forecast for new products, extensive research has been performed in the search of finding valuable methods. The literature distinguishes three main categories of new product forecasting: judgmental methods, market research and quantitative techniques (Kahn, 2006). This dissertation focuses on a specific branch of the quantitative methods, namely diffusion models. Diffusion is a concept that is not exclusively used in forecasting. The underlying mathematical reasoning also underpins the modeling of growth of bacterial cells and the prediction of mortality rates. Consequently, the models are often interchangeable among different disciplines.

The interchangeability has fostered the use of a wide variety of diffusion models in the research on new product forecasting. However, the abundance of models can cause the user to doubt his choice of model. Furthermore, the applicability of diffusion models in practical business cases has been questioned (Kahn, 2006).

This essay aims to explain the basic diffusion models and their notable extensions. In addition, the dissertation seeks to provide an analysis for which purposes diffusion models can and cannot be used when forecasting demand for a new product. Two valuable reasons are given to support the focus on 'basic' diffusion models. First, a manager will be more inclined to apply a model if the manager understands its underlying premise (Fader & Hardie, 2005). Secondly, the basic models are still being used today and have proven to be valuable contemporary diffusion models (Goodwin et al., 2014).

The remainder of this article is structured as follows: Section 2 elaborates on the methodology used for the research of this dissertation. In Section 3, a classification is given of new product types. In Section 4, the premise of diffusion is discussed as well as the models that try to capture the phenomenon. An examination of the diffusion models in practice is given in Section 5. In the penultimate segment, Section 6, the growth curves are fitted for real world data. The aim of this section is to demonstrate what a diffusion model can and cannot account for in a practical case. In the ultimate section, Section 7, the results of the essay are summarized.

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2. Methodology

A broad range of diffusion models has been developed over the years. For this reason, criteria for classification of the models are established, that aid in determining which models are relevant for inclusion in the analysis. For the basic models, three criteria are set to select a model for further examination in this dissertation. The first criterion states that the model must be referred to in multiple papers. A model lacking follow-up research, is therefore not included. Second, the model must have been the subject of recent research in papers published after 2010. A lot of models were developed before the year 2000, but they are still referenced to and used in current research. The third and last condition determines that the growth functions must have been utilized in various comparative empirical analyses. Furthermore, variants of basic models are also analyzed. The selection of variants of the basic models depends only on the first two criteria. Due to the frequent non-existence of comparative analyses, the third criterion is not applied for the selection of variants of basic models.

Based on these pre-determined criteria, the classification yields three basic models and various notable extensions. Two specific extensions are analyzed in the same manner as the basic models, as their development led to a broad range of other extensions (Meade & Islam, 2006). For the three basic models and the two extensions, the influence of a change in their parameters is graphically demonstrated. This is done by changing a parameter with the other parameters held constant. To accurately depict the influence of the different parameters, the values were chosen based on previous research (Mahajan et al., 1990; Trappey & Wu, 2008).

The critique section is built upon recurrent themes in review papers of diffusion models. Furthermore, arguments for the critique section readily flow from the analysis of the mathematical underpinnings of diffusion models.

For the empirical section, data were carefully chosen. In order to test the different growth models, data must be accurate and complete. Flaws in the regression analysis can then be contributed to the growth models rather than inaccurate data. For this reason, data were extracted from The World Bank Group database. This database provides unequivocal data-sourcing and explanatory notes on interpretation of the data.

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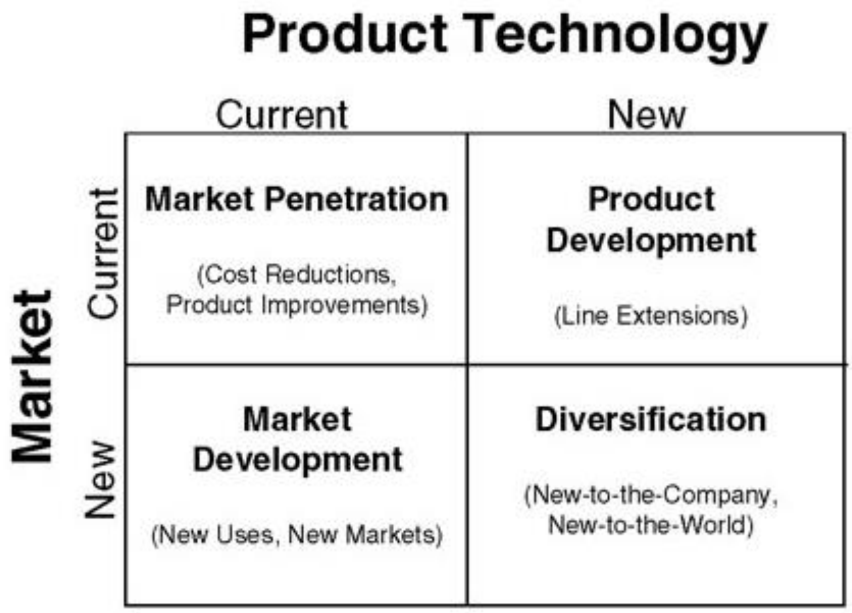
3. Newness of a product

Prior to the discussion of different models forecasting future demand of new products, a definition of newness or novelty of a product should be given. Extending the Ansoff matrix, Kahn (2006) proposes that novelty can be defined along a market axis (a product entering a new market) and a product axis (the level of technological innovation of the product). Hence, a product-market matrix can be populated depending on the degree of entering a novel market and the degree of technological innovation as demonstrated in Figure 1.

Figure 1 New product-market matrix (Kahn, 2006)



Although this matrix model by Kahn presents an intuitive and straightforward way of classifying products, other characteristics of newness of a product can modulate market performance. For example, Van Trijp & Van Kleef (2008) conclude in a review article that uptake of a new product will be enhanced if it induces an experience of meaningful differentiation in consumers. Stated otherwise, meaningful differentiation represents another dimension of novelty of a product, albeit one that can only be described in a qualitative way. Yet another important characteristic is the durability of a product. Within the scope of forecasting, durable products are defined as goods that remain usable over a longer time period or may even yield a benefit over time. Non-durable goods will demonstrate a short or gradual life span (e.g., bread, cosmetics) (Cooper, 1994).

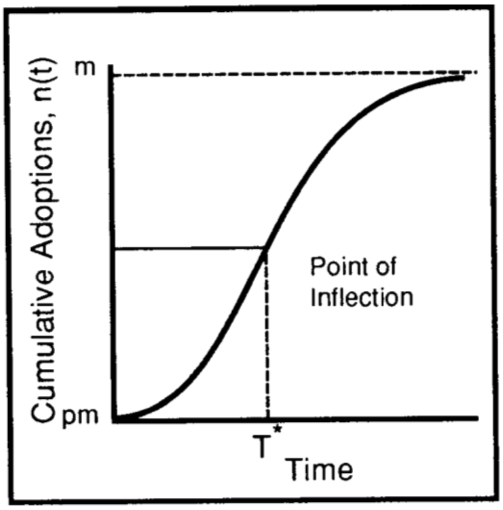
In general, diffusion models have been extensively applied in forecasting demand for new durable products (e.g., HDTV & electrical cars) and technological innovations (e.g., telephony & broadband) (Goodwin et al., 2014; Meade et al., 2006). Both types of applications must be classified in Kahn's matrix as new-to-the world. These are products/innovations that create a market that previously did not exist (Kahn, 2006).

For predicting future demand of the other categories in Figure 1 and non-durable products, approaches such as forecasting by analogy and test marketing have been proven as more acknowledged tools (Goodwin et al., 2014). This illustrates that assumptions about the qualitative novelty (product matrix, durability, differentiation) of a product should be scrutinized first in order to apply the appropriate model. For example, the best quantitative model could ineffectively predict the future demand of a new product, if the consumers' feeling of differentiation or added value is not accounted for.

4. Diffusion models

Before differentiating the various diffusion models, their underlying shared premise must be elucidated. This is the assumption that cumulative product adoptions, like several other phenomena (see Section 1), follow an S-shaped pattern (Radas, 2005). A distinction needs to be made between cumulative sales and cumulative adoptions. Adoption of a new product can be defined as a customer taking up the product or buying the product for the first time. However, Norton and Bass (1987) state that for durable products cumulative sales and adoption curves follow the same pattern, as few repeat purchases are made. Figure 2 depicts an S-shaped curve of cumulative adoptions (n(t)) over time (Time), where the inflection point T* is the point of the curve where the curvature changes sign. Stated differently, if (and only if) the first derivative *f′* of the S-shaped function *f* has an isolated minimum or maximum at T*,* then (T, f(T)) is an inflection point of *f* (the S-shaped function *f* being a differentiable function) (Jaakkola, 1995). The inflection point is of importance because it indicates the moment when the new product is getting adopted at the fastest rate or when the demand for the new product is the highest (Mahajan et al., 1990).

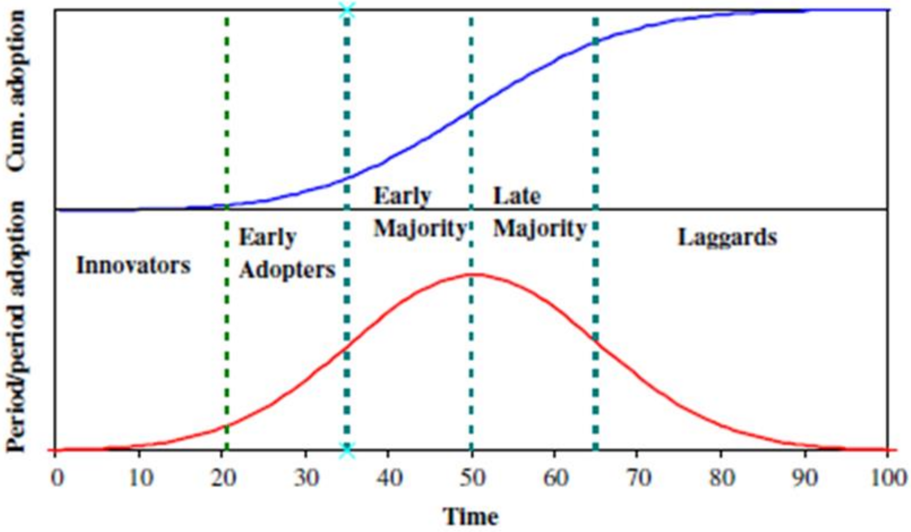
Figure 2 Cumulative product adoptions (Mahajan et al., 1990)



Rogers (1983) states that "diffusion is defined as the process by which an innovation is communicated through certain channels over time among the members of a social system" (Rogers, 1983, p. 5). The channels through which the innovation is communicated is one of diffusion theory's main topics of interest (Mahajan et al., 1990). The participants in an innovation/durable new product diffusion process can be categorized as innovators, early adopters, early majority, late majority, and laggards (Rogers, 2003; Dearing & Cox, 2018). Stated otherwise, individuals have a different threshold to adopt a product. Rogers (1962) states that the adoption thresholds for participants of an innovation are normally distributed, therefore creating the S-curve for cumulative adoption.

Figure 3 illustrates the bell-shaped curve of normal distribution for the different thresholds (with period by period adoption representing the speed of the adoption) and the S-shaped curve of cumulative adoptions.

Figure 3 Classification of adopters (Meade & Islam, 2006)



Except for the Bass model, which classifies adopters as innovators or imitators, the mathematical diffusion models do not differentiate between the participants in an adoption process. They assume that initial growth happens at a slow pace followed by a rapid one, and then finally plateaus. The patterns of cumulative adoption described by the formal models occur irrespectively of the heterogeneity of the participants 'threshold' in the adoption process (Kucharavy & De Guio , 2011). Hence, an S-shaped curve of a diffusion model can trace cumulative adoptions in a homogenous population, in contrast to the distribution of adaptor thresholds as described by Rogers (1983) above.

In the following sections the three basic formal diffusion models and their extensions will be discussed. First the Bass model is elaborated on, followed by the logistic model and lastly, the Gompertz model. The Bass model has undoubtedly figured most prominently in the research on diffusion of new products. It is specifically developed for the diffusion of new products, unlike the logistic and the Gompertz model.

4.1 Bass model

This section covers the Bass model and its extensions. In Section 4.1.1, the basic model is depicted. Section 4.1.2 discusses how the basic model's parameters can be estimated. Next, in Section 4.1.3, the Bass model is extended for succeeding generations of technology. Section 4.1.4 and Section 4.1.5 demonstrate how the market environment can play a role in the Bass model. Lastly, in Section 4.1.6, an extension of the Bass model is examined that illustrates how companies adopt new technologies.

4.1.1 Basic Bass model

The Bass model describes how new products get adopted in a population. The equation discloses how the population that already adopted the new product interacts with potential adopters. In order to create a diffusion model for new technologies, Bass (1969) makes a series of assumptions. First, Bass assumes that the moment an individual decides to adopt a new product is related to the number of previous adopters. The second assumption expresses the binarity of an adopter: you either have adopted the product or you have not. The third and final assumption states that there are two types of adopters: there are either innovators or imitators. The former adopt early by the influence of mass media and their decision to adopt is independent of other people's choices. The latter are influenced by word of mouth communication (Mahajan et al., 1990). Considering these assumptions, Bass formulated the diffusion process as:

$$\frac{g(t)}{1-G(t)} = p + qG(t) \qquad (1)$$

Using Function (1), Bass (1969) illustrates the linear relationship between prior adopters and potential adopters. The relationship is formulated as follows: "The probability of a current adoption (g(t)/ (1 – G(t)) is a linear function of the number of prior adoptions" (Radas, 2005, p. 36). In equation (1) p and q are constant parameters. p represents the coefficient of innovation, whereas q represents the coefficient of imitation (Liang et al., 2015). G(t) represents the cumulative fraction of potential adopters at time t. Consequently, (1-G(t)) represents the cumulative fraction of potential adopters who have not adopted the product yet at time t. The probability density function of adoptions is represented by g(t) (Massiani & Gohs, 2015). In other words, g(t) expresses the rate at which the adoption of the new product happens (Liang et al., 2015). Because g(t) = dG(t)/dt, Function (1) can be rewritten as (Radas, 2005):

$$\frac{dG(t)}{dt} = (p + qG(t))(1 - G(t)) \qquad (2)$$

If we solve for G(t) with the initial condition G(0) = 0 we get the non-linear cumulative distribution function:

$$G(t) = \frac{p\,(\,1-e^{-\,(p+q)t})}{p+qe^{-(p+q)t}} \qquad (3)$$

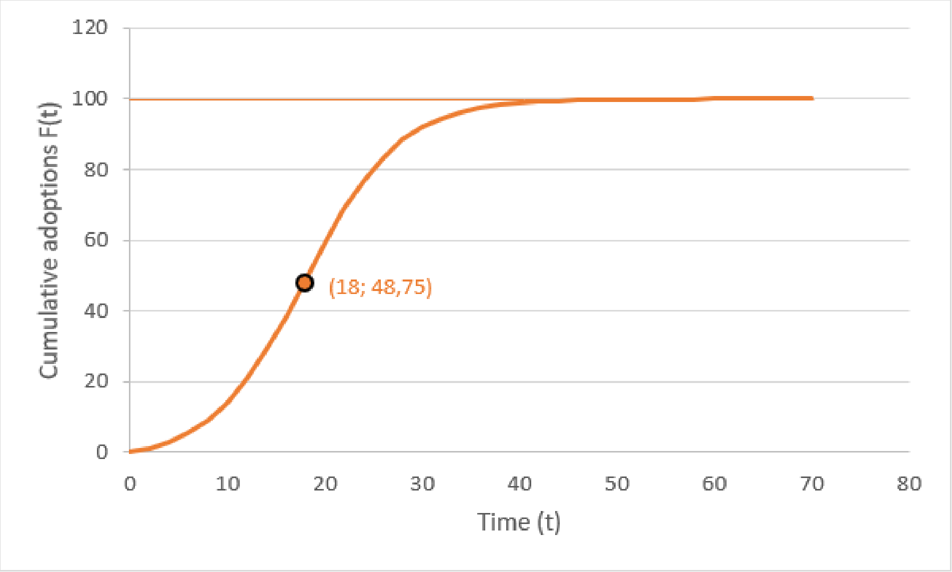
In order to find the cumulative adoptions of a product at a time t, a market upper limit (M) needs to be defined. This is because G(t) represents the cumulative fraction of adoptions and not the cumulative adoptions F(t) itself. To find F(t), we need to multiply G(t) by the market upper limit of adopters M (Satoh, 2001):

$$F(t) = M * \frac{p\,(\,1-e^{-\,(p+q)t})}{p+qe^{-(p+q)t}} \qquad (4)$$

The inflection point of Function (4) occurs at $\left(-\frac{1}{(p+q)} * \ln\left(\frac{p}{q}\right);\ M * \left(\frac{1}{2} - \frac{p}{2q}\right)\right)$ (Mahajan et al., 1990).

In Figure 4, Function (4) is plotted for M = 100, p = 0.005, q = 0.2. The inflection point is found at (18;48,75).

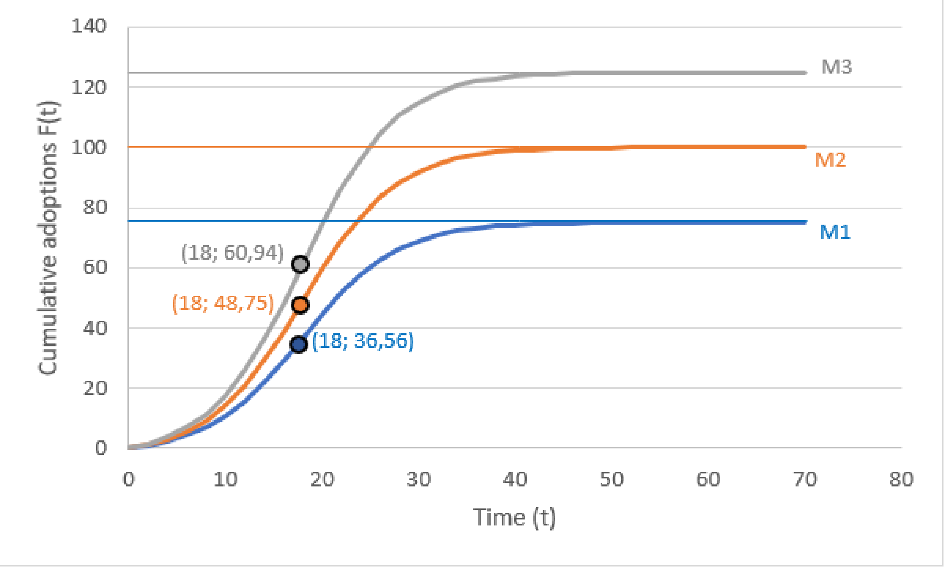
Figure 4 Basic Bass model



Function (2) allows to explain the sigmoidal shape of the Bass function, as displayed in Figure 4. M is not of importance for this explanation. Consequently, the probability density function is used. Initially, when nobody has adopted the product yet, G(t) will be equal to zero. This means that qG(t), the rate of imitation, equals zero. The speed of diffusion dG(t)/dt is then only influenced by people who are adopting independently of others. This causes the slow start of the growth curve, as seen in Figure 4. Later as G(t) grows, qG(t) will become of greater importance. Because q is multiplied by G(t), we can derive that the larger the number of adopters is, the larger the rate of imitation (qG(t)) will be (Colapinto et al., 2014). In this phase, dG(t)/dt will reach its maximum value which corresponds to the inflection point in Figure 4. In the final stage, the maximum of potential adopters has been reached almost completely. (1-G(t)) will be close to 0, which means that the speed of adoption will decrease. The S-curve flattens out again.

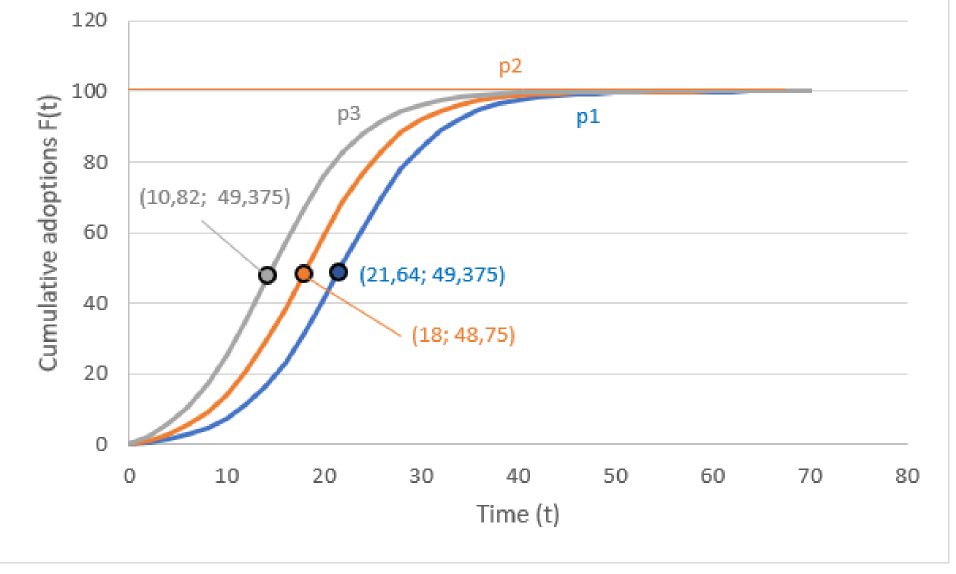
To illustrate the influence of the individual parameters in the basic Bass model, parameters M, p and q will respectively be changed with other parameters held constant. The parameters applied in Figure 4 are used as parameters of reference. First M is changed for: M1 = 75, M2 = 100, M3 = 125, see Figure 5. A change in M leads to an equal shift in the upper horizontal asymptote of the Bass function. The time of inflection is independent of the change in M. However, the cumulative adoptions at the time of inflection will increase as M increases.

Figure 5 Influence of parameter M in the basic Bass model



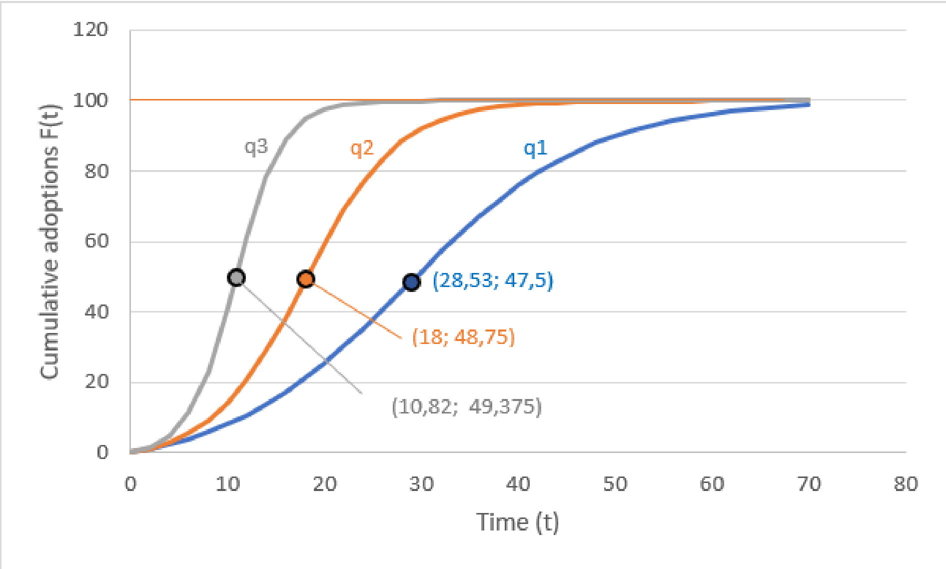
Next, parameter p is changed for p1 = 0.0125, p2 = 0.025, p3 = 0.1, see Figure 6. As mentioned previously, in the beginning the adoption only depends on the innovation coefficient. An increase or decrease in the value of coefficient p will consequently lead to a change in the initial growth rate. This occurrence can be observed in Figure 6. The initial growth rate of the Bass curve with a relatively low coefficient of innovation (p1) is lower than the intitial growth rate of a Bass curve with a high coefficient of innovation (p3).

Figure 6 Influence of parameter p in the Bass model



Lastly, the influence of the parameter of imitation (q) is demonstrated. q is changed for: q1 = 0.1, q2 = 0.2, q3 = 0.4, see Figure 7. A change in the value of q implies a change in the rate of imitation. As more people adopt the product, parameter q will become of more importance. Initially, not many people have adopted the product. Thus, when a rise or decline in the value of q occurs, the initial rate of growth will not be affected drastically. Once more people have adopted the product, the change in q becomes apparent. A lower value for q causes the diffusion to happen at a slower rate (q1) than in the case of a higher value for q (q3) (Jackson, 2010).

Figure 7 Influence of parameter q in the Bass model



4.1.2 Parameter estimation of the basic Bass model

Extensive research has been performed in the search of finding feasible parameter estimation methods for the Bass model. There are three main categories of estimation methods for growth models: management judgement techniques, diffusion of analogous products and using data of early time-periods of the new product (Mahajan et al., 1990).

Management judgement techniques are qualitative methods used to estimate the different coefficients. Mahajan and Sharma (1986) suggest the estimation of three key variables by managers: the upper market limit (M), the time of inflection and the cumulative adoptions at the time of inflection. However, this method is not feasible because the inflection point is a crucial output of the model, rather than an input (Mahajan et al., 1990). Lawrence and Lawton (1981) propose an alternative algebraic estimation technique that also requires the estimation of three variables: the market upper limit (M), the number of adoptions in the first period and the sum of the coefficient of innovation and imitation (p + q). These variables are then estimated by managers or using market surveys.

Another way the parameters of the Bass model can be estimated is using the diffusion data of similar products. The growth models are then fitted for another product of which the actual diffusion data are known. However, as a new product will possess different characteristics, diffusion by analogy is often used in combination with management judgement (Mahajan et al., 1990). An example where two methods are combined is when p + q in the Lawrence and Lawton method would be estimated using the data of similar products.

Many different statistical techniques have been proposed to estimate the parameters of the Bass model when little data are available. When first describing his model, Bass (1969) used the ordinary least square technique (OLS) to determine the value of the parameters p, q and M. Because the data-input is discrete, the differential equation must be discretized when applying the OLS method. By using an ordinary forward difference equation, Function (4) can be discretized (Satoh, 2001):

$$F(t_i) - F(t_{i-1}) = pM + (q - p)F(t_{i-1}) - \frac{q}{M}F^2(t_{i-1}) \quad (5)$$

If X(i) = $F(t_i) - F(t_{i-1})$, pM = α_1, (q – p) = α_2 and -q/M = α_3, Function (5) can be rewritten as:

$$X(i) = \alpha_1 + \alpha_2 F(t_{i-1}) + \alpha_3 F^2(t_{i-1}) \quad (6)$$

Now, regression analysis can be used to determine $\hat{\alpha}_1$, $\hat{\alpha}_2$ and $\hat{\alpha}_3$ and subsequently the parameters $\hat{p}$, $\hat{q}$ and $\hat{m}$:

$$\hat{p} = \frac{-\hat{\alpha}_2 + \sqrt{\hat{\alpha}_2^{\,2} - 4\hat{\alpha}_1\hat{\alpha}_3}}{2} \quad (7)$$

$$\hat{q} = \frac{\hat{\alpha}_2 + \sqrt{\hat{\alpha}_2^{\,2} - 4\hat{\alpha}_1\hat{\alpha}_3}}{2} \quad (8)$$

$$\hat{m} = \frac{-\hat{\alpha}_2 - \sqrt{\hat{\alpha}_2^{\,2} - 4\hat{\alpha}_1\hat{\alpha}_3}}{2\hat{\alpha}_3} \quad (9)$$

Besides the benefit of being easy to implement, the OLS technique also has some disadvantages (Schmittelein & Mahajan, 1982). The expected multicollinearity between variables $F(t_{i-1})$ and $F^2(t_{i-1})$ can decrease the reliability of the estimated parameters in the model (Alin, 2010). Another disadvantage is that because discrete data are used to describe a continuous model, the use of the OLS method causes a time-interval bias. This is because in theory the left-hand side of equation (6) should be the derivative of F(t) and not the difference between $F(t_i)$ and $F(t_{i-1})$. Before the point of inflection, the right-hand side of this equation will overestimate the derivative. After the point of inflection, it will underestimate the derivative (Hong et al., 2016).

To overcome these shortcomings, Srinivasan & Mason (1986) propose the use of the nonlinear least squares estimation method (NLS). After conducting an empirical comparison of the different forecasting methods, they have found the nonlinear least squares method to be the most accurate. Despite the overall superiority of the NLS method, the technique also has some downsides. First, the estimation method is less straightforward than the OLS. Second, the outcome of using NLS may differ according to the chosen starting values of the parameters. Last, the procedure might not find a global optimum (Satoh, 2001).

4.1.3 Norton-Bass model

The Norton-Bass model is an extension of the basic Bass model that incorporates the influence of succeeding generations of technologies on each other. The model presumes that the adoption of a specific generation of technology is not only determined by its own diffusion, but also by the possibility for adaptors to shift from earlier generations of a product to newer ones (Jiang & Dipak, 2012). Therefore, the Norton-Bass model is frequently referred to as a combination of a substitution model and a Bass model. In addition to the assumptions of the basic Bass model, Norton and bass (1987) make three supplementary assumptions regarding the Norton-Bass model. First, every generation of technology is introduced before the prior one has reached its maximum of potential adopters (M). Secondly, the prior technology will always cease to exist before the next one does. Lastly, all potential adopters of the previous generation are possible adopters for the following generation. In the case of two succeeding generations of technologies, the model is written as follows:

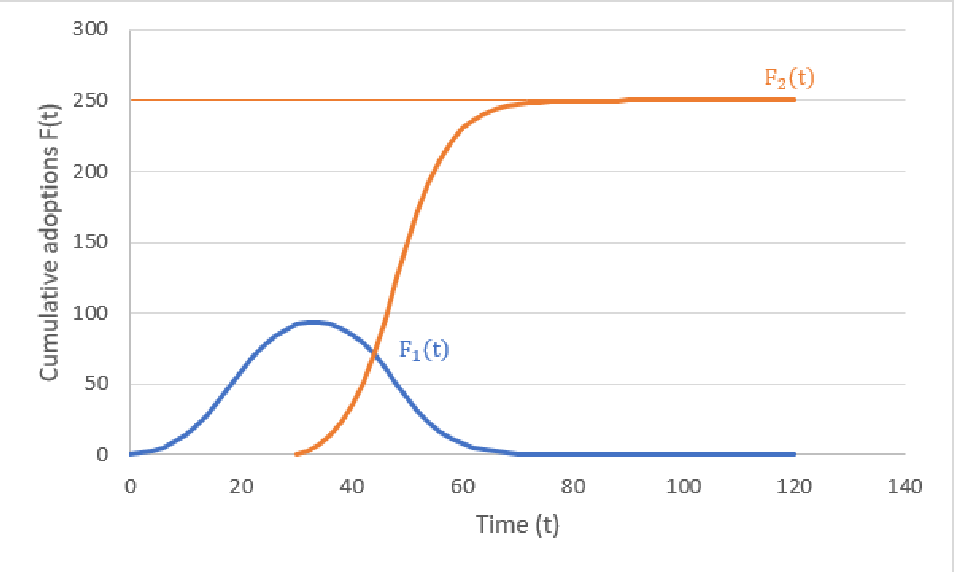
$$F_1(t) = M_1 G_1(t)[1 - G_2(t - \tau_2)] \text{ for } t > 0 \quad (10)$$

$$F_2(t) = G_2(t - \tau_2)[M_2 + M_1 G_1(t)] \text{ for } t > \tau_2 \quad (11)$$

where $F_i(t)$ equals the number of cumulative adopters of the *i*th generation of technology at time t. G_i is calculated as in Function (3) but with unique values for parameters M, p and q for each generation of technology. τ_i is the moment in time when the *i*th generation is introduced. If the *i*th generation has not been introduced yet, $F_i(t - \tau_i) = 0$.

In Figure 8 Function (11) and Function (12) are plotted for M_1 = 100, p_1 = 0.005, q_1 = 0.2, M_2 = 150, p_2 = 0.005, q_2 = 0.2, τ_2 = 30.

Figure 8 Norton-Bass model

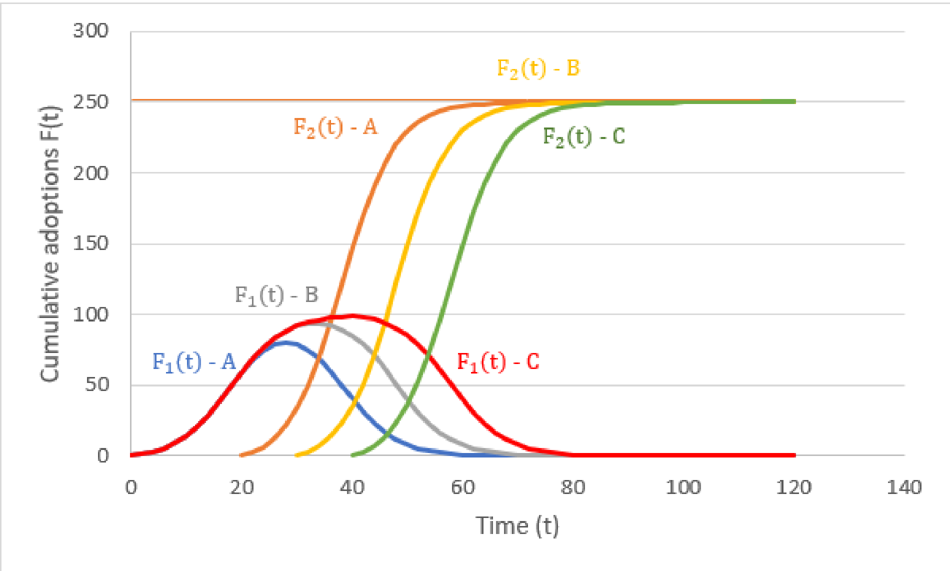


During the introduction phase of generation 1 of the new product the diffusion happens as in the regular Bass model for cumulative adoptions. Once generation two is introduced, current adopters and potential adopters of the first-generation product will start shifting to the second generation (Jiang & Dipak, 2012). When $M_1G_1(t)$ becomes smaller than $M_1G_1(t)G_2(t-\tau_2)$ in Function (10), the number of cumulative adopters will start to decline. $F_2(t)$ then grows because of its own adoptive component $G_2(t-\tau_2)m_2$ and the adopters who shift from the first generation of technology to the second one $M_1G_1(t)G_2(t-\tau_2)$. In general, an *i*th product generation will gain adopters from its previous generation and lose adopters to its succeeding generation.

The third assumption made by Norton and Bass (1987) regarding the model becomes evident in Figure 8. $F_2(t)$ reaches its market upper limit at 250 ($M_1 + M_2$), which demonstrates that all adopters of the previous generation are possible adopters for the following generation.

The parameters p, q and M impact the Norton-Bass model in the same manner as the Bass model. In Figure 9 the influence of the additional parameter τ_i is exemplified with other parameters held constant: τ_2(A) = 20, τ_2(B) = 30, τ_3(C) = 40.

Figure 9 Influence of parameter τ_i in the Norton-Bass model



A change in the time of introduction of the succeeding product generation (τ_i) causes a displacement along the x-as of the curve of the succeeding generation ($F_2(t)$) as well as a displacement of the curve of the prior generation ($F_1(t)$). When the time of introduction of the succeeding generation is delayed ($F_2(t)$ − C), more people will already have adopted the product of the prior generation ($F_1(t)$ − C) before shifting to the second generation. The cause can simply be derived mathematically. Because of the time shift $M_1G_1(t)$ will be larger than $M_1G_1(t)G_2(t-\tau_2)$ for a longer period.

An important drawback of the Norton-Bass model is that the model is not capable of estimating the cross-generational repeat adopters of the product. This is because, "when counting the number of adopters who substitute an old generation with a new generation, the NB model does not differentiate those who have already adopted an earlier generation and those who have not" (Jiang & Dipak, 2012, p. 1888).

4.1.4 Generalized Bass model

Over the years, many models have been proposed to incorporate explanatory variables in the Bass model. The models have mainly focused on the determination of the influence of price and advertisement. Bass, Krishnan and Jain (1994) created a generalized model that incorporates these explanatory variables into the Bass model. The general model is an extension of Function (1):

$$\frac{g(t)}{1-G(t)} = (p + qG(t)) * x(t) \qquad (12)$$

x(t) is then a function of marketing-mix variables (e.g., price and advertising):

$$x(t) = 1 + \mu_1 \frac{[P(t) - P(t-1)]}{P(t-1)} + \mu_2 \text{Max}\left\{0, \frac{[A(t) - A(t-1)]}{A(t-1)}\right\} \qquad (13)$$

P(t) is the price level at time t, μ_1 measures the increase in the rate of diffusion deriving from a 1 % increase in price, A(t) is the advertisement level at time t and μ_2 measures the increase in the rate of diffusion deriving from a 1 % increase in advertising. However, Function (13) is an example by Bass, Krishnan and Jain (1994) of how marketing mix variables can be incorporated in the generalized Bass model. Many other marketing variables can be added into the equation. The generalized Bass model can also be written in a closed form solution (Guidolin & Mortarino, 2010):

$$F(t) = M * \frac{p\,(1 - e^{-(p+q)\int_0^t x(t)dt})}{p + qe^{-(p+q)\int_0^t x(t)dt}} \qquad (14)$$

Important to note is that x(t) in Function (14) does not alter the upper market limit (M), the coefficient of innovation (p) or the coefficient of imitation (q). Rather than modifying its internal parameters, x(t) influences the shape of diffusion by anticipating or delaying adoptions. Note that for x(t) =1, there is no influence of marketing variables resulting in the regular Bass model (Guidolin & Mortarino, 2010).

4.1.5 Bass model in an international environment

When a new product is introduced in several countries at a different time, the struggle for data is partially resolved. Diffusion data of countries where the product is introduced earlier can be used to forecast the diffusion of the new product in countries where a later introduction occurs. The general presumption is that a delayed introduction gives people in the relevant country time to asses the new product and understand the benefits of using the novel product. If the potential adopters in the lagging country asses that the product is received positively in the leading countries, they will be more inclined to adopt the new product upon introduction in their country. As a result, the initial rate of adoption will be higher compared to the leading countries (Meade & Islam, 2006).

Nevertheless, adopters from different countries display different traits in terms of innovativeness, imitativeness and market potential. Many attempts have been made in trying to find determinants that influence these parameters. For example, Lee (1990), Meade and Islam (2002) found that gross domestic product plays an important role in the rate of product adoption. Further research includes the work of Hofstede (1984), who categorized typical national characteristics for identifying different cultures. These attributes where then tested as regression coefficients for explaining the values of the parameters of the Bass model (Meade & Islam, 2006). The predominance of national characteristics can in some cases cancel out the benefit of lagging countries. In the research of Talukdar et al. (2002) this predominance is illustrated. Their empirical findings demonstrate new products getting adopted at a slower rate in developing countries than in the preceding developed countries.

4.1.6 Geroski model

The Geroski model is a mixed information model that depicts the adoption of new technologies by firms (Geroski, 2000). The model is an extension of the Bass model in the way that, analogous to the Bass model, the equation expresses that the subject is influenced by an external source and by imitation. The mixed information model's basic assertion is that some firms adopt a superior technology earlier than other firms because they have earlier access to the information of a new technology. This information can be transmitted via two different sources. The first one is the is the central source, which spreads the information of the new technology to a certain percentage of potential adopters each time period. The rate at which the central source broadcasts information is independent of potential adopters. The other way information about the new technology diffuses, is through word of mouth. The main source of information is then previous users of the new technology, which are essentially the current adopters in Bass model (Chu et al., 2009).

By combining the two mentioned sources of information, Geroski (2000) noted the following differential equation:

$$\frac{dF(t)}{dt} = \gamma_1(M - F(t)) + \gamma_2 F(M - F(t)) \quad (15)$$

In Equation (15), F is the market upper limit or maximum potential adopters, F(t) is the number of adopters at time t, γ_1 expresses the information transmission by the central source in percentage and γ_2 represents the information spread through word of mouth in percentage. Like other diffusion models, Function (15) then also displays a sigmoidal function.

4.2 Logistic model

The models discussed in this section are all either a variation or an extension of the logistic function. In section 4.2.1, the basic model is explained. The following segment, Section 4.2.2, discusses three methods on how to estimate the logistic model's parameters. In Section 4.2.3, the Richard model is reviewed, which allows for a more flexible formulation. Logistic substitution models are discussed in Section 4.2.4, whilst simultaneously formulating a linearized version of the logistic function.

4.2.1 Basic logistic model

The basic logistic model is a growth model that was first introduced by Verhulst in 1838. Over the years, different versions have been used in a variety of disciplines, including the diffusion of new products. The basic logistic model for new product forecasting can be written as (Chu et al., 2009):

$$F(t) = \frac{M}{1 + e^{-\beta(t-c)}} \quad (16)$$

In Function (16) F(t) represents the cumulative adopters of the new product at time t, M is the market upper limit of adopters, β determines the rate of the diffusion and c is a time shift parameter. In this case parameter c also denotes the time of inflection. In many applications of the logistic model, Function (16) is rewritten as (Mohammad & Greg, 2014):

$$F(t) = \frac{M}{1 + Ae^{-\beta t}} \qquad (17)$$

A is a then a constant that replaces $e^{\beta c}$. The point of inflection of the basic logistic curve is reached when half the market is captured by the new product or F(t) = M/2 at time c. This causes the symmetry of the logistic growth curve (Sharif & Islam, 1980). In Figure 10, Function (16) is plotted for M = 100, c = 30 and β = 0.25. The inflection point is found at (c; M/2), in this case (30; 50).

Figure 10 Basic logistic model

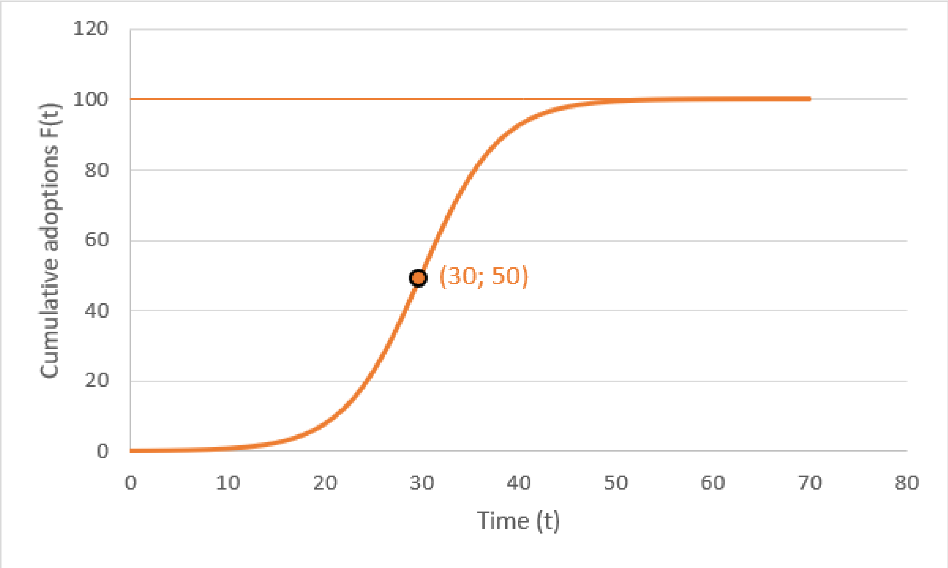
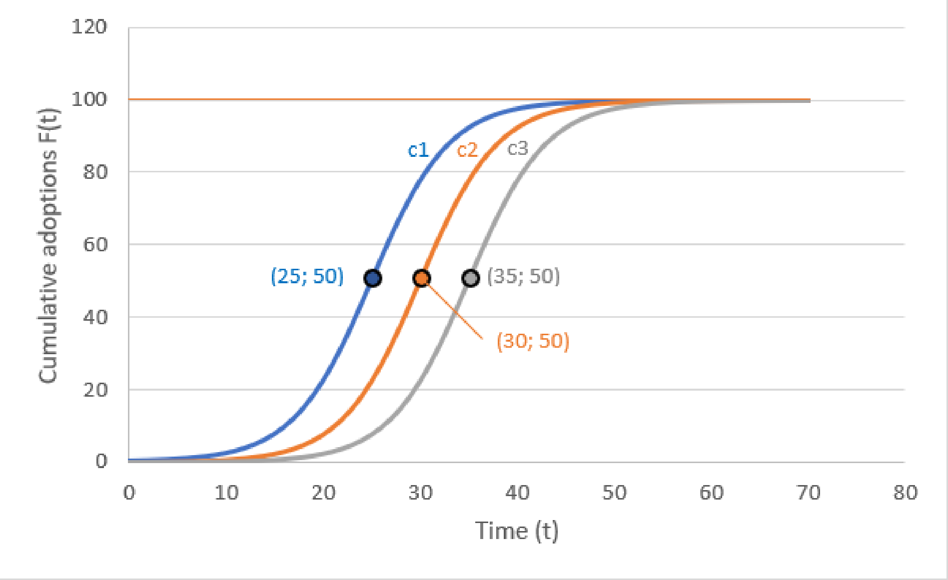


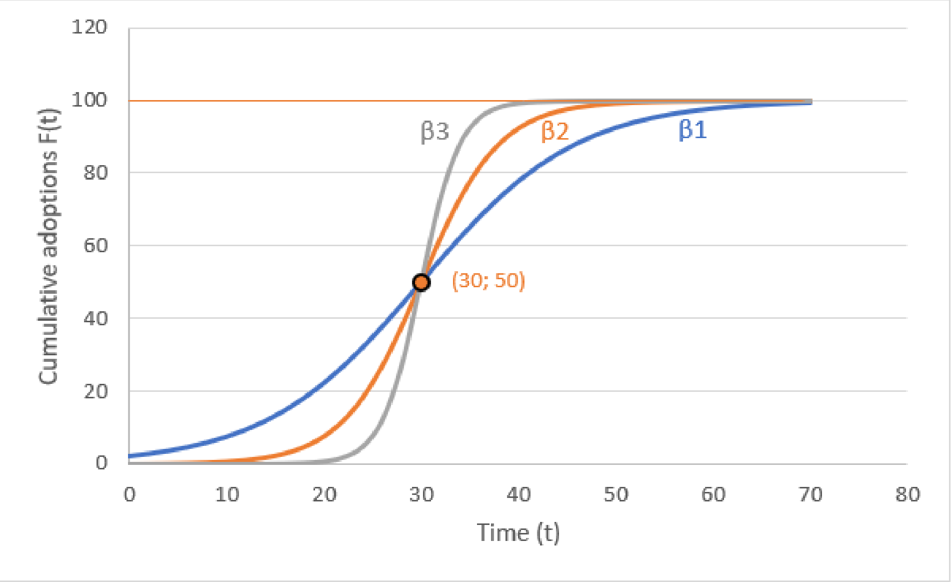
Figure 10 demonstrates the symmetric characteristic of the basic logistic curve. The curve after the inflection point is the reverse graphical image of the curve before the inflection point. To illustrate the influence of the individual parameters in the basic logistic model, parameters c and β will respectively be changed with other parameters held constant. A shift in the upper market limit M will have the same impact as described in Section 4.1.1: a shift of the upper horizontal asymptote of the growth function. Therefore, a change in M will not be discussed in this paragraph. The parameters applied in Figure 10 are used as parameters of reference. First, c is changed for values: c1 = 25, c2 = 30, c2 = 35, see Figure 11. A change in c displaces the logistic curve along the x-axis, causing the time of inflection to change accordingly. The value F(t) at which half of the cumulative adoptions are achieved, remains the same: M/2 = 50.

Figure 11 Influence of parameter c in the basic logistic model



Finally, the influence of the parameter β in the basic logistic model is tested: β_1= 0.125, β_2= 0.25, β_3 = 0.5, see Figure 11. Parameter β influences the rate of diffusion. In the case of a low value for β (β1) the logistic curve initially grows at a relatively fast pace, but the rate of growth does not increase rapidly. A high value for β (β3) initially causes an initial slow diffusion but the growth rate increases quickly. Once the inflection point is passed, the logistic curves with a higher β pass the ones with a lower β in terms of total cumulative adoptions F(t). Therefore, β can also be referred to as a delay factor, because the value of β determines whether the initial growth phase starts at a high or a low rate. A low delay factor causes little delay and vice versa (Morrison, 1996).

Figure 11 Influence of parameter β in the basic logistic model



4.2.2 Parameter estimation of the basic logistic model

Similar to the Bass model, three estimation techniques can be used to determine the parameters of the logistic model: management judgment techniques, diffusion of analogous products and statistical procedures when limited data is available.

The first method requires three inputs from the user of the logistic model: M, c, and β. These parameters can be found by performing surveys, market research and graphical evaluations of the projected diffusion. According to Morrison (1995), the diffusion rate (β) is the most difficult parameter to estimate. Therefore, as many variables as possible must be considered when estimating this factor. When management judgment is used to determine the logistic curve, Bonett (1987) refined the model so that the parameters can be estimated by the user via more meaningful growth characteristics.

The second method applies regression analysis on diffusion data of analogous products to estimate the parameters. This method is particularly useful in the case of products that are substitutes of each other, see Section 4.2.4.

Lastly, when limited data are available, statistical procedures such as nonlinear least squares can be used. Initial data of diffusion of the new product are used to estimate the unknown logistic curve. These methods are more extensively discussed in Section 4.1.2.

4.2.3 Richard model

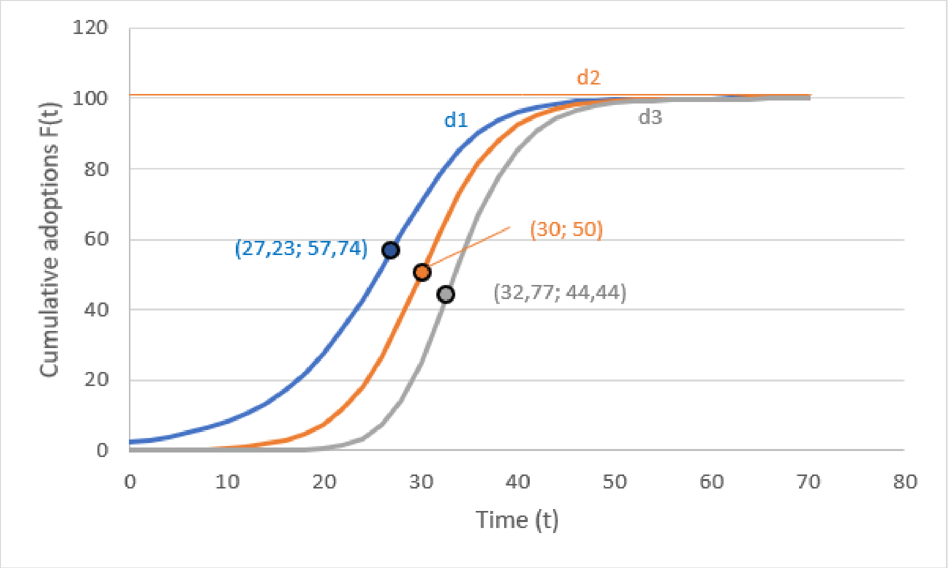
In the basic logistic model, the point of inflection occurs when half of the market is captured. Hence, the growth model displays a symmetric curve. In addition, the logistic model is not applicable when the cumulative adoptions are estimated to start at a fast rate (Sokele, 2008). Because of these characteristics the basic logistic model is often criticized for being too rigid (Bewley & Fiebig, 1988). The Richard model can overcome these shortcomings by allowing the point of inflection of the logistic model to be flexible rather than being fixed at F(t) = M/2. The Richard growth model is defined by the following function (Sokele, 2008):

$$F(t) = \frac{M}{\left[1 + e^{-\beta(t-c)}\right]^{d}} \quad (18)$$

F(t) represents the cumulative adoptions of the new technology, M is the upper market limit of adopters and β determines the rate of the diffusion. c and d influence the position of the inflection point, with time of inflection = c + $\frac{\ln(d)}{\beta}$ (Sokele, 2008). If d = 1, the basic logistic function is achieved (Function (16)).

The influence of parameters M, β, c is discussed in Section 4.2.1. Changing c will have the same influence as changing c in Function (16), although in this instance the time of inflection will change with an additional factor $\frac{\ln(d)}{\beta}$. To demonstrate the influence of parameter d, the example of the basic logistic curve is used where M = 100, c = 30, β = 0.25. d is then changed for values: d1= 0.5, d2 = 1, d3 = 2, see Figure 12.

Figure 12 Influence of parameter d in the Richard model



By changing parameter d in the Richards curve, the point of inflection gets affected and therefore the symmetry of the displayed curve as well. A relatively low value for d (d1) causes a rapid initial growth. The inflection point is then located earlier in comparison to Richard curves with a higher value for d (d2, d3), but will occur at a greater level of cumulative adoptions.

The addition of parameter d causes for a more flexible logistic function. However, an important drawback of the Richards model is that the additional parameter d causes the need for data of an extra time period of the new technology (Mahajan et al., 1990).

4.2.4 Substitution models

Substitution models depict advancing technology as a set of competitive substitution processes (Bhargava, 1995). This means that the rate of market penetration of the new technology is dependent on the old technology it replaces. An important characteristic of the models outlined below is that they can be written as linear functions of time. This trait makes them suitable for use in practice (Norton & Bass, 1987).

The most basic and often-used substitution model is the Fisher-Pry model (Fisher & Pry, 1971). There are three assumptions that need to be considered. First, a technological advance can be considered as a substitute of another technology if it satisfies the same need as the prior technology. The second assumption states that once a new technology has started substituting an old one, it will proceed to fully substitute this technology (Norton & Bass, 1987). The third and final assumption is that the rate at which the new technology replaces the old one, is influenced by the remaining amount of the old technology left to be substituted (Fisher & Pry, 1971). With these assumptions taken into account, the authors created the following basic differential equation:

$$\frac{dX(t)}{dt} = s(1 - X(t))X(t) \tag{19}$$

In Function (19), X(t) is the fraction (value between 0 and 1) of adoptions of the new technology at time t, 1- X(t) is the fraction of potential adopters still using the old technology and s is the rate of substitution. In order to find the market share of the newest innovation, the variable t_0 is introduced (Norton & Bass, 1987). This is the time at which half of the potential adopters have substituted the old by the new technology. In other words: $X(t_0) = ½$. Now, Function (19) can be solved for the fraction of market substitution by the new technology X(t) (Peterka, 1977):

$$X(t) = \frac{1}{1 + e^{-s(t-t_0)}} \qquad (20)$$

Fisher and Pry (1971) defined a more convenient function to facilitate the use of the substitution model. The authors altered Function (20) so that the left-hand side of the equation represents the ratio of market share of the new technology to that of the prior technology:

$$\frac{X(t)}{1-X(t)} = e^{s_0+st} \text{ or } \ln\left(\frac{X(t)}{1-X(t)}\right) = s_0 + st \qquad (21)$$

where s_0= -st_0. If Function (21) is plotted using a logarithmic scale, a linear curve will be achieved. The parameters of the model can then be estimated through linear regression, which makes the model easier to apply in practice (Young, 1993).

Fisher and Pry (1971) tested their model for 17 different competitive substitutions. They found that the model closely matched the existing data of the various substitutions. According to Norton and Bass (1987) this is because the evolution of market share of technologies usually shows a lot of regularity.

In a practical case of Kawamato (2010), the Fisher-Pry model is tested in a modern environment. The model was used to project the penetration of the Home Broker tool among individuals investing in the Brazilian stock market. This tool makes it easier for individuals to execute their orders on the stock market. To measure the fraction of people using the Home Broker tool, the following Fisher-Pry equation was utilized:

$$H(t) = \frac{K}{1 + e^{-s(t-t_0)}} \qquad (22)$$

H(t) is the fraction of investors using Home Broker, while K is the maximum number of users of the technology and equals 1. As such, the model assumes that eventually all relevant customers will adopt Home Broker. To estimate the substitution parameter s and the time to reach half of the maximum potential adoptions, they used data of the prior technology. To analyze the accuracy of their forecast Kawamato (2010) compared the adoption of the new technology with the prediction of the Fisher-Pry model. They found the Fisher-Pry forecast to be accurate.

By modifying the Mansfield model, Blackman came up with a substitution model that incorporates the upper limit of adoption in the Fisher-Pry model (Norton & Bass, 1987). Blackman (1971) created the following linearized function to describe the substitution process of new technologies:

$$\ln\left(\frac{F(t)}{M - F(t)}\right) = s_0 + st \tag{23}$$

where F(t) represents the cumulative adoptions at time t and M is the upper limit of adoption. The other parameters are equal to the ones of Function (21). It is worth noting that just like in the Fisher-Pry model, the assumption is made that technological substitution only takes place between the two most recent technologies (Norton & Bass, 1987).

If s is taken to represent a general growth parameter β, Function (23) then equals the basic logistic model. Therefore, Function (16) can also be estimated via linear regression. The drawback is that upon using this regression technique, M must be estimated beforehand (Young, 1993).

Because the discussed substitution models are basic logistic functions, the influence of the parameters is the same as depicted in paragraph 4.2.1.

4.2 Gompertz model

The Gompertz function is a growth model created by Gompertz to forecast mortality rates in 1825. Analogous to the logistic model, the Gompertz model has been repeatedly used in the research on diffusion of technological products. The growth model is expressed as (Chu et al., 2009):

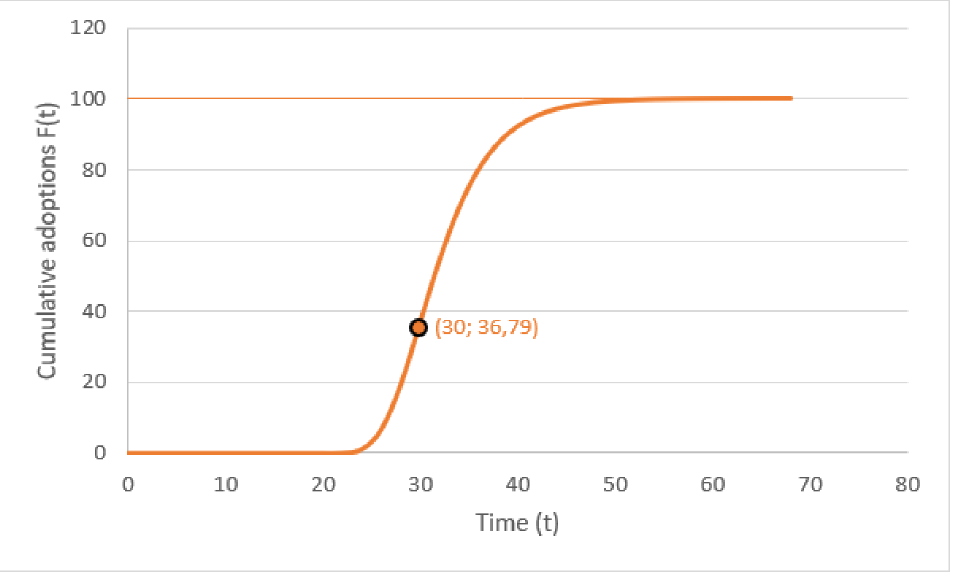
$$F(t) = M * e^{-e^{-\beta(t-c)}} \tag{24}$$

where F(t) represents the cumulative adoptions of the new technology, M is the upper market limit of adoption, β determines the rate of the diffusion and c is a time shift parameter which also denotes the time of inflection. The Gompertz growth model uses the same parameters as the basic logistic model and is therefore also often written in the form (Mohammad & Greg, 2014):

$$F(t) = M * e^{-Ae^{-\beta t}} \tag{25}$$

with a constant A = $e^{\beta c}$. However, unlike the basic logistic curve, the point of inflection is not reached when half the market is captured (F(t) = M/2), but when F(t) = M/e at time of inflection c (Goshu & Koya, 2013). This is approximately at 37 percent of the market limit of adoption, which causes the cumulative adoption curve to be non-symmetrical. In Figure 13, Function (24) is plotted for M = 100, c = 30, β = 0.25 with the inflection point occurring at (30; 36,79).

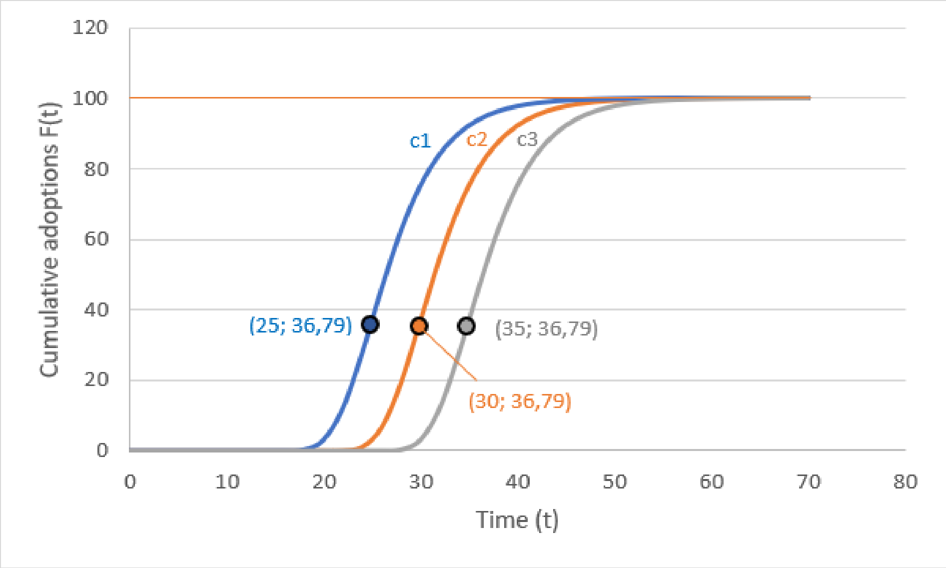
Figure 13 The Gompertz model



When analyzing Figure 13, concluding that diffusion in a Gompertz model starts at a slower rate than in the logistic model is wrong. This is because parameters β and c are parameters that must be estimated (e.g., by nonlinear least square). For example, the Gompertz curve could have a lower estimated value for c, causing a shift more to the left in comparison to the logistic curve. The main differentiating characteristic of the Gompertz curve is that the point of inflection (in terms of F(t)) is reached earlier than in the logistic function (Trappey & Wu, 2008).

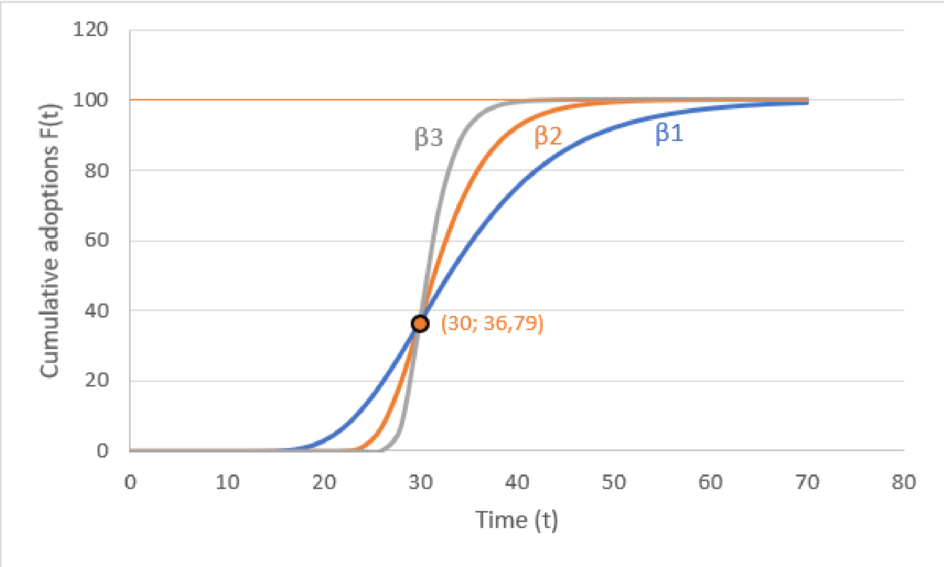
To illustrate the influence of the individual parameters in the Gompertz model, parameters c and β will be changed respectively with other parameters held constant. M is simply a horizontal asymptote which will shift accordingly to its change. First c is changed with other parameters held constant for c1 = 25, c2 = 30, c3 = 35, see Figure 14. Analogous to the logistic model, a change in parameter c displaces the growth model along the x-axis. The cumulative adoptions F(t) at the inflection point remain constant, but the time of inflection changes.

Figure 14 Influence of parameter c in the Gompertz model



Next, growth parameter β is changed for $\beta_1 = 0.125$, $\beta_2 = 0.25$, $\beta_3 = 0.5$, see Figure 15. A change in parameter β causes, similar to the basic logistic model, a curve stretching. The lower the value of β (β1), the more the curve linearizes. In this instance, the diffusion will happen at a steadier rate and it will take longer to reach its horizontal asymptote M. The opposite effect occurs when β takes on a higher value (Morrison, 1996).

Figure 15 Influence of parameter β in the Gompertz model



Similar to the logistic curve, the Gompertz model can be written in a linearized version. As seen in Section 4.2.4, the advantage of a linear model is that the parameters can be estimated using linear regression when the value of M is predetermined. Thus, Function (24) with $\beta_0 = -\beta c$ can be transformed into (Young, 1993):

$$\ln\left(-\ln\left[\frac{F(t)}{F}\right]\right) = \beta_0 + \beta_1 t \tag{26}$$

Analogous to the other growth models, three estimation techniques can be considered when determining the parameters of the Gompertz function: management judgement, diffusion of similar products and statistical procedures when little data are available. Because of the similarity of the parameter estimation techniques, the methods will not be discussed in this section.

5. Applicability of diffusion models

The applicability of diffusion models in practical business cases is often questioned (Kahn, 2006). Section 5 aims to demonstrate the difficulties and opportunities of new product forecasting using diffusion models. In Section 5.1 an overview is given of important flaws of diffusion models. Section 5.2 illustrates for which business purposes diffusion models can be used.

5.1 Criticism on the use of diffusion models in new product forecasting

An often-mentioned flaw of the diffusion models is inherent to the category the models belong to. Quantitative models need data in order to provide accurate forecasts. The accuracy of diffusion models depends greatly on the number of data points used to estimate the growth model's parameters. In fact, some criticism goes as far as stating that parameters can only be estimated correctly if the data includes the maximum of the non-cumulative adoption curve (the inflection point in the cumulative adoption curve) (Mahajan et al., 1990). Other research suggests that parameters of diffusion models can be predicted precisely if six to ten measurements of actual diffusion data are available (Tseng & Hu, 2009). However, this would imply that "by the time sufficient observations have developed for reliable estimation, it is too late to use the estimates for forecasting purposes" (Mahajan et al., 1990, p. 9). Various extensions of the basic models have been proposed in trying to (partially) resolve the problem of data scarceness. For example, Moe and Fader (2002) incorporated advance purchase orders into diffusion models to forecast new product sales. By classifying innovators as individuals who place an advance order and imitators as the ones waiting until the product reaches a certain level of adoption, they successfully forecasted music album sales before its launch. Another model that deals with data shortage was proposed by Lee et al. (2018). The authors incorporate patent citation and web search traffic in the Bass model to more accurately forecast new product demand. These two examples demonstrate that the continuously increasing availability of data via different sources (e.g., internet) creates new possibilities for further research on diffusion models.

Goodwin et al. (2014) state that most empirical research has focused on fitting the various growth models to the diffusion data rather than using it as a forecasting tool. Because enough data are available, the empirical research reasonably concludes that the growth models can provide a good fit. Even though more datapoints can be used this way, the diffusion of a new product will always differ from its 'analogous one'. Fitting growth models in this manner can however be useful for new product forecasting using similar products. Market research and management judgement techniques can then be used to further optimize the forecast (Mas-Machuca et al., 2013).

Furthermore, the research experiences difficulties in the search of a superior growth model. Despite multiple endeavors, no favorable model comes out on top. Meade and Islam (2006, p. 534) believe that "for heterogeneous data sets, the evidence continues to point to the non-existence of a best forecasting diffusion model".

Another issue that occurs in the empirical research of diffusion models for new product forecasting, is the one-sidedness of the research. In recent literature, diffusion models have mainly been used for forecasting innovations rather than durable products. Although innovations are an important application, the literature suggests a broader use of the diffusion models. Put simply, there is disparity between theoretical research and empirical research (Buchele et al., 2016).

Lastly, the underlying premise of the diffusion models can be questioned. The reason why diffusion models are S-shaped is because they assume that information spreads (too) slowly (Geroski, 2000). In an ideal market the information of new technology should spread almost immediately and therefore not display an S-curve. Geroski (2000) proposes that policy makers can try and influence diffusion by providing additional information that speeds up the diffusion process as much as possible. If not, lagging adopters will miss out on earlier access to newer products. When the assumption is made that newer products are naturally superior to older versions, this would mean that consumers are disadvantaged.

5.2 Value of diffusion models in new product forecasting

A notable advantage that diffusion models have over judgmental methods and market research methods is that they can forecast time-series of a new product. Judgmental methods and market research are often used to predict first year adoption levels or total cumulative adoptions, but these techniques are unable to predict how the product gets adopted over time (Goodwin et al., 2014). This stems from the fact that managers and consumers have a hard time in predicting or judging trends (Timmers & Wagenaar, 1977). Knowing how the demand of a product evolves over time is of importance for a business as it enables the company to adjust its strategical decisions coherently.

Emerging economies are an important area where businesses can utilize diffusion models to predict time-series of new technologies. Companies that operate in industries where demand has saturated, can shift their attention to emerging markets. As mentioned in Section 4.1.5, diffusion data of countries where the product was launched earlier, can then be used to predict the diffusion in lagging countries. This method of forecasting can give the business a competitive advantage when entering a new market. Empirical research has mainly applied analyses as such in the telecommunication industry (Peres et al., 2010).

Social network services are another market where diffusion models are capable of giving companies an advantageous position. The growth of online social platforms can be forecasted by utilizing the growth models. A way to gather the input for the diffusion models is by search frequency analysis. This method uses data from search engines to detect the interest of users in a particular social media platform (Bauckhage & Kersting, 2014).

6. Fitting the growth models for mobile subscription diffusion in Europe

In this section the Bass (Function 4), logistic (Function 16) and Gompertz model (Function 24) are fitted for mobile phone subscription data in Europe. The aim of this section is to examine if the basic growth models provide a good fit for the analysed data and if there are differences in the diffusion patterns of the analysed countries. Section 6.1 explains how certain companies could use a similar analysis as an aid in forecasting. In Section 6.2 the used methodology is elaborated. In section 6.3 the results are displayed and interpreted.

6.1 Case description

"Mobile cellular telephone subscriptions are subscriptions to a public mobile telephone service that provide access to the PSTN (public switched telephone network) using cellular technology" (The World Bank, 2017). Many different telecommunication companies base their strategy on providing mobile telephone subscriptions to customers. However, in various developed countries the demand for mobile subscriptions has saturated (Chang et al., 2014). A telecommunication company can consider multiple strategies in this case. It can focus on differentiating itself by developing new technologies or offering a lower price, granting the company an advantageous position. Another strategy, in which diffusion models can play a significant role, is focusing on markets where the saturation point has not been reached yet (Feldmann, 2002). Companies can then use growth models to determine the lifecycle of the saturated markets and use the achieved model to identify opportunities in non-saturated markets.

6.2 Procedure for fitting the growth models

Mobile subscription data between 1980 and 2017 of five different European countries are used to analyze the different growth models. The data are retrieved from the World Bank Group, which utilizes the information as a measure for economic development (The World Bank, 2017). The y-axis represents the amount of mobile subscriptions per 100 people and the x-axis is the timescale. The diffusion of mobile phone subscriptions of the analyzed European countries often starts in different years. Therefore, a timescale of 30 periods is chosen, with 0 being the starting point of the diffusion in a specific country. The analyzed time-periods of the European countries are then as follows: Spain (1985-2014), Portugal (1988-2017), Norway (1980-2009), Germany (1984-2013), Denmark (1981-2010).

The growth models are fitted using the nonlinear least squares technique in SPSS. The statistical program is able to perform nonlinear regression using the Levenberg–Marquardt algorithm. To compare the individual forecasting performances of the Bass, logistic and Gompertz model, two measures of accuracy are used. The first method is the mean squared error (MSE). The lower the value of the MSE, the more accurate the growth model represents the actual data. The measure is given by the formula (Wang & Bovik, 2009):

$$MSE = \frac{1}{n}\sum_{t=0}^{n}(A(t) - F(t))^2 \qquad (27)$$

where A(t) is the actual value at time t, F(t) is the value at time t predicted by the growth model and n is the amount of observations. The other used measure of accuracy is the mean absolute error (MAE) (Willmott & Matsuura, 2005):

$$MAE = \frac{1}{n}\sum_{t=0}^{n}|A(t) - F(t)| \quad (28)$$

where A(t) and F(t) are equal to the parameters in Function (27). Analogous to the MSE, lower values correspond to a more accurate model.

6.3 Results

The results of the regression analysis are presented in Table 1 and Table 2. In Table 1 the estimated parameters for the different growth models can be found.

Table 1 Parameters of the fitted growth models

	BASS			LOGISTIC			GOMPERTZ		
	M	p	q	M	β	c	M	β	c
SPAIN	107,78	7,5E-05	0,59	107,78	0,59	15,17	109,92	0,40	14,15
PORTUGAL	115,85	5,6E-04	0,58	115,83	0,58	11,94	116,91	0,41	10,82
NORWAY	111,77	3,1E-04	0,38	111,73	0,38	18,79	121,24	0,22	17,46
GERMANY	119,98	1,6E-04	0,47	119,96	0,47	17,07	123,38	0,31	15,78
DENMARK	122,67	3,4E-0.4	0,37	122,59	0,37	19,14	135,30	0,21	17,84

The coefficients of innovation (p) turn out to be very small in the Bass model. The low value for p implies that in the five tested European countries the diffusion of mobile phone subscriptions almost solely depends on ‘imitators’. Note that at the same time the values of the coefficient of imitation (q) and of the upper limit of adoptions (M) are (almost) equal to the values of the growth parameter (β) in the logistic model. This is because the Bass model resembles the logistic model when p approaches zero (Witzany, 2017, p. 161). When comparing the logistic and the Gompertz model, the difference in c becomes apparent. The values of c in the Gompertz model are for every analysed country lower than the logistic model’s values for c. This indicates that the Gompertz model estimates the diffusion pattern to reach the time of inflection earlier than in the case of the logistic model. Furthermore, the former model judges the value of the upper limit of adoptions (M) higher than the latter model.

Table 2 displays the MSE and the MAE for the growth models of the different countries. If for a growth model the MSE and MAE are shaded green, that particular model most accurately fitted the actual diffusion values of a particular country. If MSE and MAE are shaded orange, the two measures of accuracy give different growth models as being the most representative of the actual values.

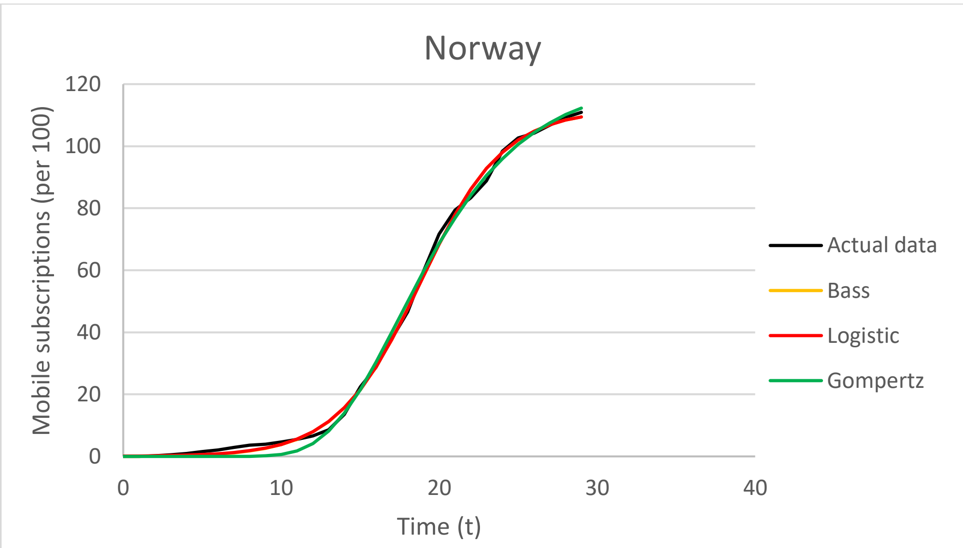
Table 2 Measures of accuracy of the fitted growth models

	BASS		LOGISTIC		GOMPERTZ	
	MSE	MAE	MSE	MAE	MSE	MAE
SPAIN	10,71	2,17	10,72	2,18	6,04	1,65
PORTUGAL	23,35	3,25	23,42	3,27	23,37	2,89
NORWAY	2,30	1,16	2,27	1,14	4,37	1,69
GERMANY	37,23	4,32	37,26	4,34	33,81	4,16
DENMARK	5,12	1,76	5,13	1,75	6,21	2,00

Analyzing Table 2, it becomes clear that different growth models excel at representing the actual diffusion in the tested countries. In Norway the logistic model has the lowest values for MSE and MAE. The Gompertz function seems to best represent the diffusion in Spain and Germany. For Portugal the MSE of the Bass model is slightly smaller than the MSE of the Gompertz function but its MAE is higher. In table 2 is illustrated that the MSE and MAE of the Bass and logistic model are almost equal. As mentioned before, this is because of the low value of p in the Bass model.

The results can also be graphically presented. A graphical evaluation aids in understanding what a diffusion model can/cannot account for and what the individual parameters signify in a practical case. The diffusion of Norway and Germany is graphically depicted. The figures of the other countries are found in Appendix. First, the diffusion of mobile subscriptions in Norway is analyzed, see Figure 16.

Figure 16 Diffusion of mobile subscriptions in Norway

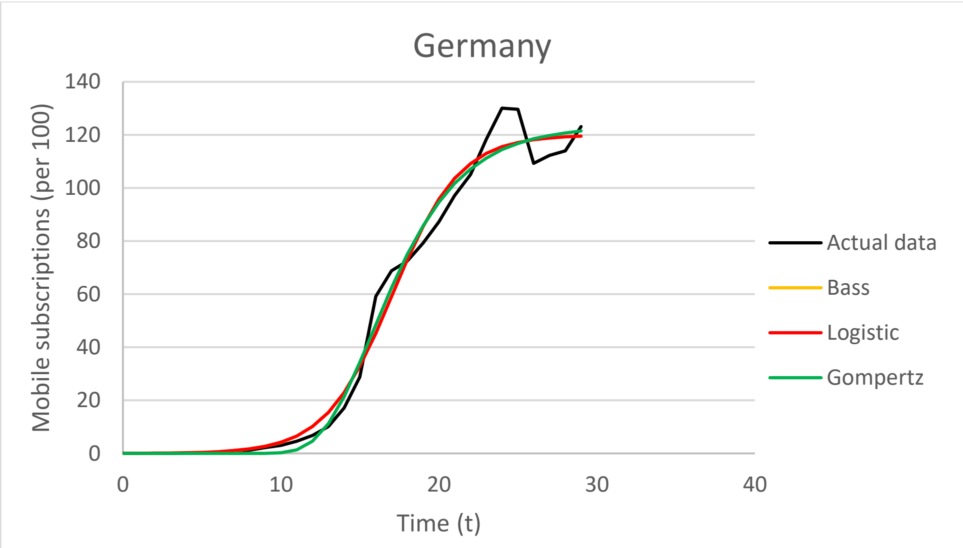


The diffusion of mobile subscriptions in Norway can be accurately fit by the growth models. This is because the data show little irregularities. The logistic curve provides the best results. However, as explained previously, the value of p in the Bass model is so low that there is little difference between the logistic and the Bass model. The Gompertz curve provides the worst fit for the diffusion of mobile phone subscriptions in Norway.

Compared to the other growth models, the rate of diffusion is relatively low (e.g., Norway β = 0,38 compared to Portugal β = 0,58). A possible cause is that the diffusion started the earliest in Norway (1980). Inhabitants of countries where a later introduction occurred, like Portugal (1988), had more time to assess the benefits of a mobile phone subscription. Therefore, once started, the diffusion happened at a higher rate in Portugal. The explanation above is based on Section 4.1.5.

Next, the diffusion in Germany is analysed, see Figure 17.

Figure 17 Diffusion of mobile subscriptions in Germany



The diffusion curve of actual values in Germany exhibits a lot of irregularity towards the end of the analysed time period. Around the 20-year mark the actual diffusion curve steepens again. The tested growth models cannot account for this because they assume that once the inflection point has been passed, the growth rate will diminish. Furthermore, the basic Bass, Gompertz and logistic function are unable to justify the decline after the steep increase. This is a result of the models not allowing a negative growth rate.

A few remarks need to be made regarding the achieved results. First, when a certain growth model provides a better fit for a country in a predetermined time interval, it does not mean that this growth model is better for that specific country (see Section 5.1). It's possible that for other time intervals, different models will prevail. Therefore, different growth curves should be utilized that allow for updates as time progresses. Secondly, our empirical results show that even for European countries, growth rates and upper adoption limits can differ significantly. If diffusion data from other countries are used for forecasting purposes, the different characteristics of the country that might affect the diffusion should be taken into account (as mentioned in Section 4.1.5).

Finally, this empirical exercise uncovered some irregularities even though growth curves are only used to fit data and not to forecast mobile phone subscriptions. When forecasting, the accuracy will go down as less data are available. It's therefore crucial, as mentioned in Section 5.1 for the manager to be involved in determining the values of the different parameters.

Ivo Van Brempt
Diffusion models in new product forecasting
Promoter: Prof. Dr. Inneke Van Nieuwenhuyse

7. Conclusion

Diffusion models provide a simple, yet elegant answer to the question of how new products are adopted in a population. With the aid of basic differential equations, a time-series of the lifecycle of innovative new products can be forecasted. Three different models dominate the literature in the search of finding the appropriate model: The Bass model, the logistic model and the Gompertz model. Although only the Bass model is developed specifically for diffusion of new products, the three models are all able to illustrate the typical sigmoidal pattern of new product diffusion: a slow start followed by a rapid increase that ultimately plateaus. The three models have not been found superior to one another in any particular situation.

Variations of the basic models try to solve specific problems associated with the basic diffusion models. Extensions try to incorporate more flexibility, external factors, multiple generations of technologies and other confounding factors. Although many modifications have been proposed, the basic models still hold their ground compared to the more complex versions. This is mainly due to the biggest problem that remains unsolved by most variations: how to deal with the lack of enough supporting data to accurately parametrize the model?

As comprehensive as the models were initially presented, theoretical and empirical research has narrowed down to the diffusion of innovations as a whole, rather than the diffusion of innovative new products. Applications of the model focus on fitting the curves, rather than forecasting demand. This poses the question of how managers should utilize diffusion models in practice. For telecommunication applications the literature proposes that companies can use diffusion models to determine where demand hasn’t saturated yet and consequently discover potential opportunities. However, the empirical research of this dissertation demonstrates that multiple difficulties have to be taken into account.

Diffusion models can provide useful insights for a company, albeit with certain precaution. If diffusion models are fitted for analogous products to forecast the adoption of a new product, the company should be aware of the different circumstances and characteristics of the products. In addition, future demand should not be forecasted solely based on growth models using little data to estimate its parameters. This can lead to risky interpretations and possibly unfavourable outcomes.

Further research should be conducted in three different areas. First, frameworks should be composed allowing managers to use diffusion of analogous products more easily during forecasting of adoption. This should give the company a roadmap on how to tackle new product demand forecasting using growth models. Second, more research is needed on diffusion models that incorporate external information into diffusion models. As discussed in Section 5.1, measures like pre-sales and web traffic can help in determining the parameters of the diffusion models. The last proposition of further research is one that extends beyond business purposes. As discussed in Section 6.1, telecommunications are also a measure of economic welfare in a country. Diffusion models could be useful for policy makers to predict the evolution of economic prosperity.

Acknowledgements

The literature on the subject of my dissertation was often challenging and diverse. I would like to thank my promoter, Professor Doctor Van Nieuwenhuyse for offering insightful comments on how to present complicated models in a legible manner. In order to verify that mathematical concepts were interpreted correctly, I could call upon the knowledge of Elien Auwers, a graduate in mathematics and Eva Verschueren, a PHD in mathematics. To ensure the preciseness of my criticism, I could appeal to Joe Toth, Supply chain manager at Johnson & Johnson. I would like to thank my dad, Ronald Van Brempt, as well, who offered advice on the correct wording in my dissertation. Lastly, I would like to express my gratitude to Doctor Van Gils, who performed a final review of my thesis.

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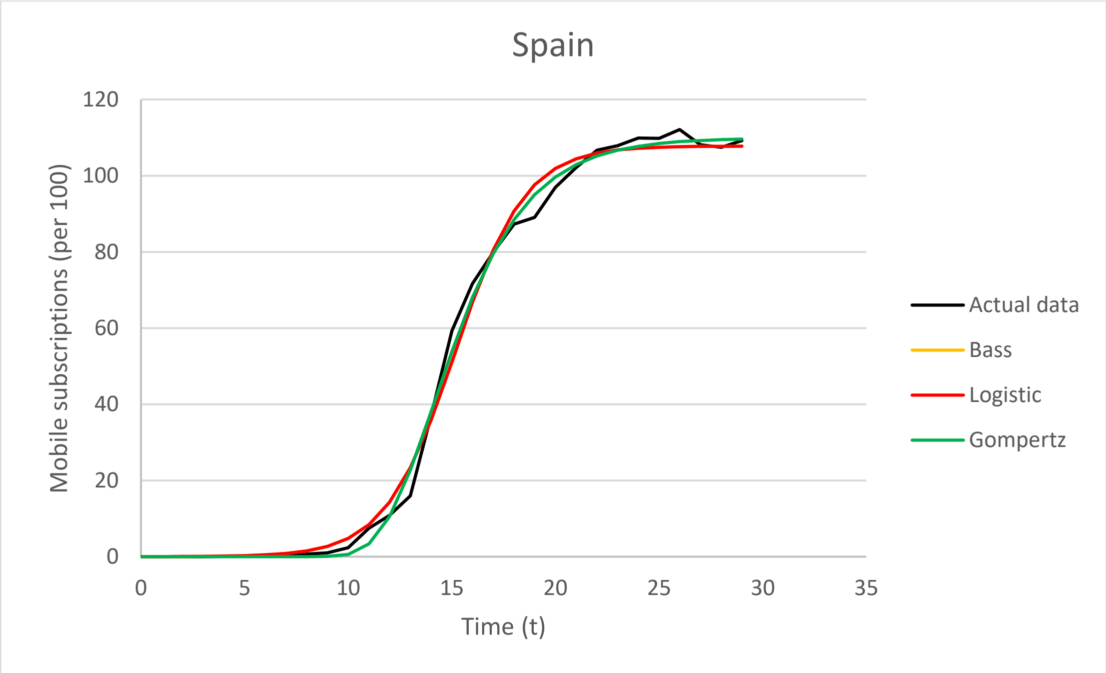
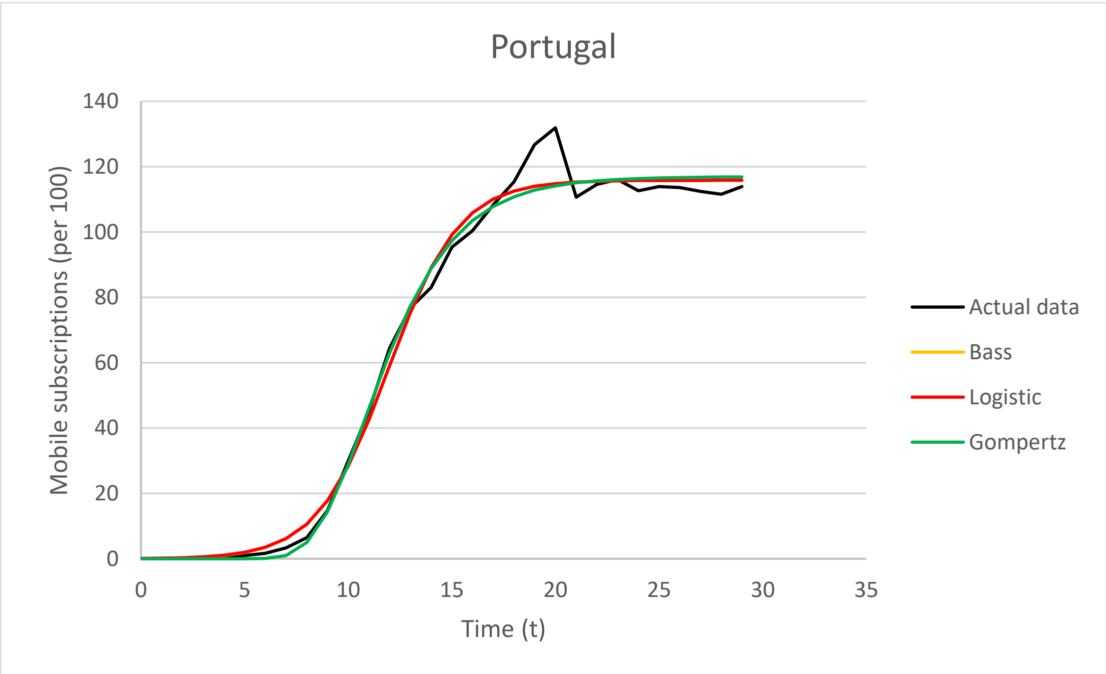
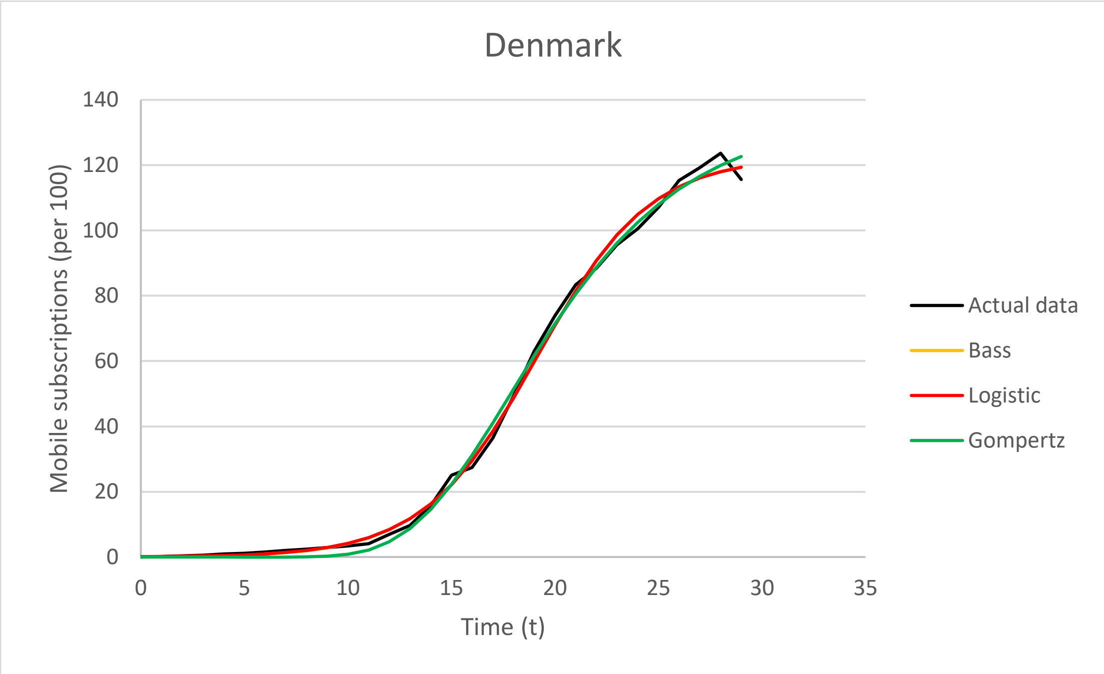
Appendix

Fitting the diffusion models for mobile subscription data in Europe






Denmark
Mobile subscriptions (per 100)
140
120
100
80
60
40
20
0
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5
10
15
20
25
30
35
Time (t)
Actual data
Bass
Logistic
Gompertz