

A simplified kinematic approach for the shear strength of fiber-reinforced concrete deep beams

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1 **A simplified kinematic approach for the shear**
2 **strength of fibre-reinforced concrete deep beams**

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1 **Abstract:** Deep beams with shear span-to-depth ratios $a/d \leq$
2 2.5 are used to resist large shear forces due to their ability to
3 develop direct strut action. To further enhance the shear
4 strength and crack control of such members, researchers have
5 studied the use of fibre-reinforced concrete (FRC). However,
6 while this solution is promising, there is a need for rational me-
7 chanical models capable of predicting the shear strength of
8 FRC deep beams in a sufficiently simple manner. This paper
9 proposes such a model based on first principles: kinematics,
10 equilibrium and constitutive relationships. The proposed model
11 simplifies an earlier two-parameter kinematic theory (2PKT)
12 for the complete shear behavior of FRC deep beams, to predict
13 the shear strength and components of shear resistance in a
14 straightforward manner. The new simplified method (S2PKT)
15 is validated by comparing the predicted results to 22 tests from
16 the literature, as well as to FEM and 2PKT predictions. It is
17 shown that the proposed simplified kinematic approach predicts
18 well the shear strength with an average experimental-to-
19 predicted shear strength ratio of 1.12 and a coefficient of varia-
20 tion of 12.9%. Furthermore, the model is used to discuss the
21 effect of shear span-to-depth ratio and fibre volumetric ratio on
22 the shear strength of FRC deep beams.

23 **Keywords:** deep beams, shear, fibre-reinforced concrete, kin-
24 ematic model

1 **1 Introduction**

2 Deep beams with small shear span-to-depth ratios ($a/d < 2.5$)
3 can carry high shear forces by means of strut action, and thus
4 are widely used as transfer girders, cap beams, pile caps and
5 other heavily loaded members. Researchers have investigated
6 the possibility of further enhancing the shear strength of such
7 members with the use of steel fibres [1-3], which bridge the
8 critical shear cracks and act as an additional shear-carrying
9 mechanism. However, while fibre-reinforced concrete (FRC)
10 deep beams exhibit improved shear-carrying capacity and
11 greater post-peak ductility than reinforced concrete, exactly
12 quantifying these beneficial effects remains a challenging prob-
13 lem. This is in large part due to the difficulty in extending the
14 strut-and-tie method [4] (Fig. 1a) for reinforced concrete (RC)
15 deep beams to FRC members, as the this method neglects the
16 tension in the concrete while enhanced behavior in tension is
17 the main advantage of FRC.

18 An alternative modeling approach for FRC deep beams that
19 considers the effect of steel fibres based on compatibility of
20 deformations, constitutive relationships and equilibrium was
21 recently developed by Mihaylov et al. [5]. This approach is an
22 extension of a two-parameter kinematic theory (2PKT) for RC
23 deep beams [6], see Fig. 1b. It has been demonstrated that this
24 kinematics-based approach is capable of modeling the complete
25 shear response of FRC deep beams, including their post-peak

1 behavior [5], see Fig. 1c. As evident from Fig. 1c, the kinemat-
2 ics-based approach accounts for five shear-carrying mecha-
3 nisms in FRC deep beams; critical loading zone V_{CLZ} , aggregate
4 interlock V_{ci} , transverse shear reinforcement V_s , dowel action of
5 the longitudinal reinforcement V_d , and steel fibres V_f . While it
6 provides excellent shear strength predictions and complete
7 pre-peak, peak and post-peak load-deformation behavior of
8 FRC deep beams, this approach requires an iterative computa-
9 tion procedure and does not lend itself to simple calculations.
10 The main aim of this paper is to simplify this approach to com-
11 pute only the peak resistance of FRC deep beams in a straight-
12 forward manner with negligible compromise on the accuracy of
13 the predictions. The main assumption used to derive the simpli-
14 fied model can be inferred from inspecting Fig. 1c and results
15 from other similar analyses: the peak resistance of FRC deep
16 beams coincides with that of the critical loading zone (CLZ).
17 Hence, it is assumed that the failure of the beam is triggered by
18 the crushing of the CLZ. This is a reasonable assumption to
19 make for FRC deep beams, as the CLZs of such D-region
20 members typically carry a significant portion of the shear, and
21 the crushing of which is brittle in nature leading to a sudden
22 drop in shear capacity. Similar simplification [7] has been made
23 for the kinematic model for complete shear behavior of FRC
24 coupling beams [8].
25 In addition to the derivation of the simplified kinematic model

1 (hereafter abbreviated as S2PKT), this paper presents compari-
2 sons with 22 tests as well as with nonlinear finite element simu-
3 lations. Furthermore, the predictions of the simplified method
4 for shear strength of FRC deep beams are compared with those
5 of the kinematic model for complete shear behavior.

6 **2 Kinematics of deep beams**

7 Figure 2 summarizes the compatibility equations of the two-
8 parameter kinematic theory for reinforced concrete deep beams
9 which have been derived earlier [6]. It is assumed that the shear
10 failure in short shear spans occurs along a straight crack that
11 extends from the support to the load along the diagonal of the
12 shear span (critical diagonal crack). The important defor-
13 mations in the crack include the average crack width w , crack
14 slip s , and the strain in the stirrups ε_v halfway along the crack.
15 As evident from Eqs. 5–7 in Fig. 2c, these deformations are
16 expressed as functions of degrees of freedom (DOFs) $\varepsilon_{t,avg}$ and
17 Δ_c .

18 The DOFs of the kinematic model are illustrated in Fig. 2a. The
19 first DOF is the average strain $\varepsilon_{t,avg}$ along the cracked length
20 of the bottom reinforcement, which is assumed equal to the
21 strain in the section with maximum bending moment as charac-
22 teristic of direct strut action in deep beams. This strain is asso-
23 ciated with a flexural deformation pattern (see top diagram in
24 Fig. 2a) where the critical crack opens but does not undergo
25 sliding displacements. In contrast, the second DOF Δ_c , which is

1 associated with a shear deformation pattern (see bottom dia-
2 gram in Fig. 2a), results in both opening and sliding displace-
3 ments. More specifically, Δ_c is the vertical displacement be-
4 tween the crack faces associated with crushing of the concrete
5 in the CLZ at the top of the crack. The size of the CLZ is de-
6 pendent on a key geometric property, namely the effective
7 width of the loading plate l_{b1e} which can be determined with
8 Eq. 1 in Fig. 2b.

9 Other geometric properties that are necessary to describe the
10 kinematic model are the angle of the critical diagonal crack α_1 ,
11 the cracked length along the bottom longitudinal reinforcement
12 l_t , and the length along the bottom reinforcement l_k where the
13 reinforcement works in double curvature. These geometric
14 properties are expressed with Eqs. 2–4 in Fig. 2b. The angle of
15 the crack coincides with the angle of the diagonal of the shear
16 span α (from center of support plate to center of loading plate),
17 except in the transition from deep to slender beams where α_1 is
18 fixed at 35° . Angle α_1 is used to evaluate the cracked length l_t ,
19 which is in turn necessary for the evaluation of the deflection of
20 the shear span Δ expressed with Eq. 8 in Fig. 2c.

21 The complete deformation pattern of the beam is obtained as a
22 linear combination of the two basic patterns governed by DOFs
23 $\varepsilon_{t,avg}$ and Δ_c . To determine the values of the DOFs at peak
24 capacity, the kinematic equations are combined with equilibri-
25 um conditions and constitutive relationships for the mecha-

1 nisms of shear resistance across the critical crack.

2 **3 Mechanisms of shear resistance**

3 As mentioned earlier, FRC deep beams carry shear through five
4 mechanisms of shear resistance: 1) diagonal compression in the
5 critical loading zone V_{CLZ} , 2) aggregate interlock across the
6 critical diagonal crack V_{ci} , 3) tension in the transverse shear
7 reinforcement V_s , 4) dowel action of the bottom longitudinal
8 reinforcement V_d and 5) shear carried by the steel fibres V_f (see
9 Fig. 3). The first four mechanisms have been derived elsewhere
10 [6, 9] and are expressed with Eqs. 9-12 in Fig. 3. The shear
11 contribution of steel fibres V_f is discussed in more detail below.

12 The steel fibres enhance the shear behavior of FRC deep beams
13 mainly in two ways: 1) they act as an additional shear-carrying
14 mechanism by transferring tension across the critical diagonal
15 crack and 2) they improve the post-peak ductility of the CLZ in
16 compression [10] due to the confinement effect of the fibres.

17 However, as the second effect mainly impacts the post-peak
18 behavior of the beam and has an insignificant impact on its
19 peak resistance (strength), it is neglected here for the sake of
20 simplicity. Therefore, Eq. 9 in Fig. 3 for the shear carried in the
21 CLZ is adopted directly from the 2PKT for RC deep beams.
22 This expression was derived based on the assumption that the
23 CLZ is at failure under principal compressive stresses inclined
24 at angle α (Fig. 2) [6].

25 The tensile behavior of steel fibres across cracks has been stud-

1 ied by a number of researchers [11-14], particularly for the case
2 of pure opening of the crack without slip displacements (Mode
3 I fracture). It is understood that the relationship between normal
4 stress across the crack and opening of the crack can be repre-
5 sented by the superimposition of two stresses (Fig. 4): tension
6 transferred directly between the two crack faces (tension sof-
7 tening of concrete), and tension carried by the steel fibres
8 which are anchored on each side of the crack. The tension sof-
9 tening of concrete is neglected here as the kinematic model for
10 deep beams assumes a fully-cracked member. The tensile stress
11 transferred by straight fibres can be computed by a variable
12 engagement model proposed by Voo and Foster [11]:

$$\sigma_f = \frac{0.396\sqrt{f'_c}\rho_f l_f}{d_f} \frac{\tan^{-1}\left(\frac{w}{d_f/3.5}\right)}{\pi} \left(1 - \frac{2w}{l_f}\right)^2 \quad (13)$$

13 where $0.396\sqrt{f'_c}$ is the bond stress between the concrete and
14 the fibres, ρ_f is the volumetric ratio of steel fibres, l_f is the
15 length of the fibres, d_f is the diameter of the fibres, and w is
16 the crack width. However, it must be noted that Eq. 13 only
17 considers the frictional bond behavior of steel fibres and does
18 not take into account the mechanical anchorage effect of
19 hooked-end steel fibres. Thus, for the shear carried by the
20 hooked-end fibres, it is recommended to use the simplified di-
21 verse embedment model proposed by Lee et al. [14] which ex-
22 plicitly considers both effects, while for straight fibres the sim-
23 pler model given in Eq. 13 can be used.

1 To use Eq. 13 to model the critical diagonal crack in FRC deep
2 beams, it is necessary to consider that the crack does not under-
3 go a pure opening, but a combination of opening and slip dis-
4 placements (mixed Mode I and II fracture). Therefore, based on
5 experimental observations that the vertical displacements in the
6 crack w_v dominate the crack kinematics near shear failure [6],
7 it is proposed to replace w in Eq. 13 with w_v , and to assume
8 that the tensile force in the fibres is aligned with w_v . The verti-
9 cal crack displacement halfway along the critical crack is de-
10 rived directly from the kinematic model by applying small-
11 displacement kinematics:

$$w_v = 0.5\varepsilon_{t,avg}l_k \cot \alpha_1 + \Delta_c \quad (14)$$

12 where the two terms of this equation are associated with the
13 two DOFs of the model. Thus, with an expressed fibre stress
14 $\sigma_f(w_v)$, the shear contribution of the fibres across the critical
15 crack is:

$$V_f = \sigma_f(w_v)bd / \sin \alpha_1 \quad (15)$$

16 where b is the width of the section, d is the effective depth of
17 the section, α_1 is the angle of the critical crack, and $d / \sin \alpha_1$ is
18 the length of the crack.

19 As all shear-carrying mechanisms are expressed in terms of the
20 two DOFs of the kinematic model, it is necessary to compute
21 the values $\varepsilon_{t,avg}$ and Δ_c at peak resistance in order to predict
22 the shear strength of FRC deep beams.

1 **4 Shear strength prediction**

2 Degree of freedom Δ_c at peak resistance is derived by follow-
3 ing the same assumption used in the derivation of V_{CLZ} , that the
4 failure of FRC deep beams is triggered by the crushing of the
5 CLZ. As the CLZ is at crushing, the strain along its bottom
6 inclined face is assumed equal to -0.0035, and the strain along
7 its top horizontal face is neglected. By also defining the geome-
8 try of the CLZ in terms of the effective width of the loading
9 plate l_{b1e} (Eq. 1) and the angle of the critical crack in the vicin-
10 ity of the loading plate (Eq. 2), Δ_c has been expressed as [6]:

$$\Delta_c = 0.0035 \times 3l_{b1e} \cot \alpha \quad (16)$$

11 To predict the second DOF $\varepsilon_{t,avg}$, it is necessary to consider
12 the equilibrium between the internal and external forces acting
13 on the deep beam. This is performed by equating the summa-
14 tion of all the shear resistance mechanisms expressed in terms
15 of one unknown $\varepsilon_{t,avg}$ (Eq. 9-12 and 15), with the shear de-
16 mand expressed from the tension in the longitudinal reinforce-
17 ment, also given in terms of $\varepsilon_{t,avg}$. Assuming that the longitu-
18 dinal reinforcement remains linear elastic and the tension
19 stiffening effect of the concrete around the reinforcement is
20 negligible near failure, the force in the longitudinal reinforce-
21 ment is:

$$T = E_s A_s \varepsilon_{t,avg} \quad (17)$$

22 where E_s is the Young's modulus of steel and A_s is the area of
23 the bottom longitudinal reinforcement.

1 This force is used to evaluate the shear demand V by using the
2 moment equilibrium of the shear span:

$$V = Tz/a \quad (18)$$

3 where $z \approx 0.9d$ is the approximate lever arm of the longitudi-
4 nal internal forces at the section with the maximum moment,
5 and a is the shear span.

6 The solution procedure of finding $\varepsilon_{t,avg}$ that satisfies equilibri-
7 um between the shear capacity and the shear demand at failure
8 is summarized in the flowchart in Fig. 5. Although the solution
9 procedure is iterative, the convergence is very fast as demon-
10 strated in the step-by-step calculation given in the following
11 section.

12 **5 Example of S2PKT calculations**

13 The S2PKT calculation procedure will be illustrated by provid-
14 ing the steps taken in computing the shear strength of specimen
15 B7 by Mansur and Ong [2], with reference to the flowchart
16 given in Fig. 5. Specimen B7 had an a/d ratio of 1.23 and a
17 fibre volumetric ratio $\rho_f = 0.5\%$. The effective depth of the
18 member was $d = 463$ mm, the flexural reinforcement ratio was
19 $\rho_l = 1.93\%$, and the stirrup ratio was $\rho_v = 0.47\%$. The com-
20 plete set of material and geometric properties of specimen B7
21 are given in the database of shear critical FRC deep beams in
22 Table 1.

23 The first step of the procedure is to compute the geometry of
24 the kinematic model. For a shear force-to-applied point load

1 ratio $V/P = 1$ (symmetrical three-point bending), the effective
2 length of the loading plate l_{b1e} (Eq. 1) is 80 mm, and hence, the
3 angle of the critical diagonal crack $\alpha_1 = \alpha$ is 41.3° (Eq. 2). The
4 cracked length along the bottom reinforcement l_t (Eq. 3),
5 which is needed to express the deflection of the beam, is 528
6 mm. The dowel length of the bottom longitudinal reinforce-
7 ment l_k (Eq. 4) is 63 mm.

8 The next step of S2PKT calculations is to compute DOF Δ_c at
9 peak capacity. As mentioned before, the proposed simplified
10 kinematic method assumes that the peak capacity of an FRC
11 deep beam coincides with that of the CLZ. Therefore, from Eq.
12 16, DOF Δ_c at the crushing of the CLZ is 0.96 mm.

13 The solution procedure now requires an iteration, which com-
14 mences with an initial estimate of the shear strength V – for
15 example $0.1bdf'_c \approx 140$ kN. At this load, the force in the lon-
16 gitudinal reinforcement T from Eq. 18 is 192 kN. Then, the
17 second DOF $\varepsilon_{t,avg}$ is computed from Eq. 17 as 0.0012. Once
18 both DOFs are known, it allows the computation of the defor-
19 mations along the critical diagonal crack; crack width w (Eq. 5)
20 and average strain in the stirrups ε_v (Eq. 7), which are respec-
21 tively 0.78 mm and 0.0055. From Eq. 9, the shear carried by
22 the CLZ V_{CLZ} is calculated as 75 kN. This value will stay con-
23 stant during the iterations as it is governed by DOF Δ_c , which
24 remains unchanged. The shear carried by aggregate interlock
25 V_{ci} (Eq. 10), which depends on the crack width, is 43 kN. The

1 shear strength component V_s depends on ε_v , and is calculated
 2 from Eq. 11 to be 62 kN. Note that as ε_v exceeds the yield
 3 strain of the steel, the stirrups are yielding at this level of load-
 4 ing. Furthermore, applying Eq. 12 for the dowel action of the
 5 flexural reinforcement which depends directly on DOF $\varepsilon_{t,avg}$,
 6 shear component V_d is evaluated at 27 kN. The final shear-
 7 strength component V_f (Eq. 15), which depends on the vertical
 8 displacement of the critical crack w_v (Eq. 14), is calculated to
 9 be 15 kN. Thus, at the end of the first iteration, the calculated
 10 shear capacity is $\sum V_i = 75 + 43 + 62 + 27 + 15 = 222$ kN,
 11 and the difference between the assumed shear strength and the
 12 calculated shear capacity is $|V - \sum V_i| = |140 - 222| = 82$
 13 kN. As the two shear forces differ significantly, the iterations
 14 continue with the calculated shear capacity $\sum V_i$ as the new es-
 15 timate of shear strength. After only 3 additional iterations, the
 16 error $|V - \sum V_i|$ decreases monotonically to only 1 kN, and
 17 therefore the solution can be considered converged. This fast
 18 convergence is not limited to the selected example only, but it
 19 is typical of the proposed solution procedure. Finally, at equi-
 20 librium between the internal and external forces acting on the
 21 deep beam, the predicted shear strength for this specimen is
 22 208 kN.

23 The described solution procedure of the kinematic model equa-
 24 tions performed for specimen B7 is illustrated graphically in
 25 Fig. 6. The plot shows the variation of the components of shear

1 resistance V_d , V_s , V_{ci} , V_{CLZ} and V_f , the shear capacity $\sum V_i$ (thick
2 black line) and the shear demand (dashed line), with increasing
3 average tensile strain in the bottom longitudinal reinforcement
4 $\varepsilon_{t,avg}$. As evident from the plot, the shear capacity decreases
5 slightly with increasing $\varepsilon_{t,avg}$. This is due to the diminishing
6 aggregate interlock as the critical diagonal crack widens, and
7 the loss of dowel action as the dowels are subjected to increas-
8 ing axial tension. Component V_{CLZ} , which provides the largest
9 contribution to the shear capacity, remains unchanged as it is
10 governed by DOF Δ_c only, and not DOF $\varepsilon_{t,avg}$. The iterative
11 calculations performed earlier, which started with an estimated
12 value of the shear strength of 140 kN, are illustrated with the
13 spiraling set of arrows. As evident from the plot, the arrows
14 spiral towards the solution where the shear capacity and de-
15 mand curves intersect, and thus the shear forces are in equilib-
16 rium. For this beam, the obtained shear strength experimental-
17 to-predicted ratio is $V_{exp}/V_{pred} = 220 \text{ kN}/208 \text{ kN} = 1.06$.

18 **6 Comparisons with tests and FEM simulations**

19 Similar analyses were performed for 22 tests from three exper-
20 imental studies reported by Cho and Kim [3], Mansur and Ong
21 [2], and Mansur and Alwist [1]. Table 1 lists the properties of
22 the test specimens, the measured shear strengths V_{exp} and the
23 predictions of the proposed simplified kinematics-based ap-
24 proach V_{S2PKT} . The shear span-to-depth ratio a/d of the beams

1 vary from 0.31 to 1.85, the effective depth d from 168 mm to
2 624 mm, the compressive strength of the FRC f'_c from 25.7
3 MPa to 86.1 MPa, the longitudinal reinforcement ratio ρ_l from
4 0.81% to 2.82%, the transverse reinforcement ratio ρ_v from 0%
5 to 1.26% and the fibre volumetric ratio ρ_f from 0% to 1.5%.
6 The tests contained both straight and hooked-end fibres, with
7 the length of fibres l_f varying from 30 mm to 36 mm and the
8 diameter of fibres d_f from 0.4 mm to 0.6 mm. Table 1 also in-
9 cludes the shear strength predictions from the 2PKT method for
10 complete shear response of FRC beams V_{2PKT} , as well as pre-
11 dictions V_{FEM} obtained with nonlinear finite element program
12 VecTor2 based on the Disturbed Stress Field Model (DSFM
13 [15]). The DSFM is a smeared rotating crack model that origi-
14 nates from the Modified Compression Field Theory [16] for
15 reinforced concrete elements subjected to shear. To model the
16 behavior of steel fibres across cracks, the DSFM incorporates
17 the diverse embedment model [12, 14]. The finite element
18 analyses were performed as part of this study following the
19 modeling procedure described elsewhere for short FRC cou-
20 pling beams [8]. It should be noted that this modeling approach
21 is significantly more complex than the 2PKT as it requires con-
22 siderable time for modeling and computations, as well as sig-
23 nificant expertise to use properly.

24 For all 22 tests in Table 1, the proposed simplified kinematic
25 approach produces an average shear strength experimental-to-

1 predicted ratio of 1.12 with a coefficient of variation (COV) of
2 12.9%. For the same tests, the respective values obtained from
3 the nonlinear FEM analyses are an average of 1.04 and a COV
4 of 13.4 %. It is therefore evident that, as compared to the far
5 more complex nonlinear FEM model, the proposed simplified
6 kinematic approach produces slightly more conservative shear
7 strength predictions with less scatter, while being rational and
8 simple enough to be calculated by hand or spreadsheet. Figure
9 7a and 7b display the variation of the experimental-to-predicted
10 ratios for all 22 tests plotted as functions of a/d and ρ_f , respec-
11 tively. The largest scatter in the predictions is observed for the
12 beams by Cho and Kim [3] without shear reinforcement and
13 without fibres. It should be noted that these specimens are the
14 smallest in the database with an effective depth of only 168
15 mm, meaning that the relative size of aggregates and voids in a
16 concrete section is larger, and thus the shear strength of these
17 specimens is more sensitive to random variations in the path of
18 the shear cracks. As shown in a previous study [9], the addition
19 of shear reinforcement and/or fibres tends to control the cracks
20 better and reduce this sensitivity.

21 To further illustrate the advantages of the proposed simplified
22 kinematic approach, comparisons were made with the shear
23 strength predictions obtained with the kinematic model for
24 complete shear behavior of FRC deep beams [5]. Figure 8a and
25 8b plot the ratio of the shear strengths calculated with the com-

1 plete and simplified kinematic approaches as functions of a/d
2 and ρ_f , respectively. It is evident from these plots that the pro-
3 posed simplified kinematic approach is able to nearly replicate
4 the shear strength predictions of the kinematic model for com-
5 plete shear behavior of FRC deep beams, for the entire range of
6 a/d and ρ_f values without any apparent bias. As it can be ex-
7 pected from a simplified model, it results in slightly more con-
8 servative predictions than the complete model from which it is
9 derived.

10 **7 Effect of test variables**

11 Validated in this manner, the simplified kinematic approach is
12 used to systematically study the effect of span-to-depth ratio
13 a/d and the volumetric ratio of fibres ρ_f on the shear strength
14 of FRC deep beams. For the purpose of comparison, the tests
15 conducted by Mansur and Ong [2] were selected.

16 **7.1 Effect of shear span-to-depth ratio a/d**

17 Figure 9 shows the effect of a/d on the measured (shown as
18 black dots) and predicted shear strengths (shown as a thick
19 black line) for six FRC deep beams (beams B1, B2, B3, B4, B5
20 and B9 in Table 1). The experimental points show a decrease in
21 shear strength as a/d increases from 0.31 to 1.85, and the sim-
22 plified kinematic approach captures well this trend, albeit with
23 slightly conservative predictions. The predicted components of
24 shear resistance show that the global trend of decreasing shear

1 strength with increasing a/d is mainly due to the diminishing
2 of components V_{ci} and V_{CLZ} . As the shear span a increases for a
3 constant d , the flexural strains $\varepsilon_{t,avg}$ also increase resulting in a
4 wider critical diagonal crack and diminishing aggregate inter-
5 lock resistance V_{ci} . Also, at smaller a/d the angle of the critical
6 diagonal crack is steeper, resulting in a larger CLZ and higher
7 V_{CLZ} , while at higher a/d the critical diagonal crack becomes
8 flatter, resulting in a smaller and weaker critical loading zone.
9 A flatter and longer critical diagonal crack crosses more stir-
10 rups and fibres, and therefore results in larger shear compo-
11 nents V_s and V_f . The contribution of the dowel action of the
12 longitudinal reinforcement to the shear resistance V_d also de-
13 creases with increasing a/d and diminishes completely once
14 the longitudinal reinforcement yields in tension.

15 It can be seen that the shear strength predictions made by the
16 simplified kinematic approach tend to become more conserva-
17 tive at very small shear span-to-depth ratios. This is because of
18 the inherently conservative simplification to neglect the en-
19 hancing effect of the steel fibres on the shear resistance of the
20 critical loading zone V_{CLZ} . As a/d decreases, the relative con-
21 tribution of V_{CLZ} increases, and so does the effect of the adopt-
22 ed simplifying assumption.

23 Also shown in Fig. 9 are the predicted shear strengths of the six
24 beams by nonlinear FEM analyses (shown as hollow triangles)
25 and by the kinematic model for complete shear behavior of

1 FRC deep beams (shown as hollow dots). It is evident from the
2 plot that the proposed simplified kinematic approach is able to
3 capture the effect of a/d equally well as the aforementioned
4 more complex methods.

5 **7.2 Effect of fibre volumetric ratio ρ_f**

6 Fig. 10 shows that the shear strength of FRC deep beams in-
7 creases with increasing amount of steel fibres. The four test
8 specimens used for comparison (beams B4, B6, B7 and B8 in
9 Table 1) had a shear span-to-depth ratio of 1.23, compressive
10 strength of the FRC of about 33 MPa, longitudinal reinforce-
11 ment ratio of 1.93%, and transverse reinforcement ratio of
12 0.47%. As the fibre volumetric ratio increases from 0 to 1.5%,
13 the measured shear strength increases by a factor of 1.27. The
14 predicted shear strengths from the proposed simplified kine-
15 matic approach agree well with the observed trend in the exper-
16 imental points, providing a slightly conservative estimate of the
17 shear capacity. The nonlinear FEM analyses and the kinematic
18 model for complete shear behavior of FRC deep beams provide
19 equally adequate estimates of the shear strength, however in the
20 case of the former, the predictions are slightly unconservative
21 for the beams with fibre volumetric ratios of 0 and 0.5%. As
22 can be seen from the predicted shear strength components, all
23 shear resistance mechanisms remain nearly constant, except for
24 the contribution of the fibres which increases linearly with ρ_f .

1 **8 Concluding remarks**

2 This paper presented a simplified kinematic approach for eval-
3 uating the shear strength of FRC deep beams. The proposed
4 model identifies that the peak resistance of FRC deep beams
5 occurs simultaneously with that of the critical loading zone, and
6 uses that to simplify an earlier two-parameter kinematic model
7 (2PKT) for complete shear behavior of FRC deep beams.

8 This approach was validated with 22 tests from the literature
9 and its accuracy was compared to that of nonlinear FEM mod-
10 els and the original 2PKT. The simplified kinematic approach
11 (S2PKT) predicts the shear strength of the test specimens with
12 an average shear strength experimental-to-predicted ratio of
13 1.12 and a COV of only 12.9%. It was shown that the S2PKT,
14 which uses only two degrees of freedom to describe the defor-
15 mation patterns of FRC deep beams, predicts the shear strength
16 with similar accuracy and less scatter than the complex FEM
17 models with thousands of DOFs. Furthermore, it was demon-
18 strated that the proposed model replicates the shear strength
19 predictions of the 2PKT for the entire range of a/d and ρ_f val-
20 ues without any apparent bias. Hence, the S2PKT offers a quick
21 and rational method of computing the shear strength of FRC
22 deep beams without the need to perform a full load-
23 displacement analysis. In addition to that, the proposed simpli-
24 fied approach captures well the decreasing and increasing
25 trends of shear strength variation of FRC deep beams with in-

- 1 creasing a/d and ρ_f , and explains them in a rational manner
- 2 with the help of five components of shear resistance.

3 Notation

a	=	shear span
a_g	=	maximum size of coarse aggregate
A_s	=	area of longitudinal bars on flexural tension side
b	=	width of the section
d	=	effective depth of section
d_b	=	diameter of bottom longitudinal bars
d_f	=	diameter of fibres
E_s	=	Young's modulus of steel
f'_c	=	concrete cylinder strength
f_y	=	yield strength of longitudinal bars
f_{yv}	=	yield strength of stirrups
h	=	total depth of section
k	=	crack shape factor
l_0	=	length of heavily cracked zone at bottom of critical crack
l_{b1}	=	width of loading plate parallel to longitudinal axis of member
l_{b1e}	=	effective width of loading plate parallel to longitudinal axis of member
l_{b2}	=	width of support plate parallel to longitudinal axis of member
l_f	=	length of fibres
l_k	=	length of dowels provided by bottom longitudinal reinforcement
l_t	=	cracked length along bottom reinforcement
n_b	=	number of bottom longitudinal bars
P	=	applied concentrated load
s	=	crack slip
s_{cr}	=	distance between radial cracks along bottom edge of member
T	=	tensile force in bottom reinforcement
V	=	shear force
V_{ci}	=	shear resisted by aggregate interlock
V_{CLZ}	=	shear resisted by CLZ

V_d	=	shear resisted by dowel action
V_f	=	shear resisted by steel fibres
V_s	=	shear resisted by stirrups
w	=	crack width
w_v	=	vertical crack displacement
z	=	lever arm of longitudinal internal forces at section with maximum moment
α	=	angle of the diagonal of the shear span
α_1	=	angle of critical diagonal crack
ε_v	=	transverse web strain
$\varepsilon_{t,avg}$	=	average strain along bottom longitudinal reinforcement
ε_{yv}	=	yield strain of stirrups
σ_f	=	tensile stress transferred by fibres
$\sigma_{v,avg}$	=	average stress in stirrups along the critical crack
Δ	=	total deflection
Δ_c	=	shear distortion of critical loading zone
Δ_t	=	deflection due to elongation of bottom longitudinal reinforcement
ρ_f	=	volumetric ratio of fibres
ρ_l	=	ratio of bottom longitudinal reinforcement
ρ_v	=	ratio of transverse reinforcement

1

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1 **Table 1** Database of shear critical FRC deep beams

No.	Ref.	Beam Name	a/d	b (mm)	d (mm)	h (mm)	a (mm)	l _{b1} (mm)	V/P	f _c ' (MPa)	a _g (mm)	ρ _l (%)	f _y (MPa)	ρ _v (%)	f _{yv} (MPa)	ρ _f (%)	l _f (mm)	d _f (mm)	Fibre type	f _{yf} (MPa)	V _{exp} (kN)	V _{S2PKT} (kN)	V _{exp} /V _{S2PKT}	V _{2PKT} (kN)	V _{exp} /V _{2PKT}	V _{FEM} (kN)	V _{exp} /V _{FEM}																				
1	3	F30-0.0-13	1.43	120	168	200	240	30	1	34.4	13	1.32	399	0	623	0	-	-	H	1100	73.8	58	1.28	58	1.27	71	1.04																				
2		F30-0.5-13	1.43	120	168	200	240	30	1	25.7	13	1.32	399	0	623	0.5	36	0.6	H	1100	60.9	62	0.98	65	0.94	72	0.85																				
3		F60-0.0-13	1.43	120	168	200	240	30	1	54.3	13	1.32	399	0	623	0	-	-	H	1100	65.1	76	0.86	78	0.84	83	0.79																				
4		F70-0.0-19	1.43	120	168	200	240	30	1	65.3	13	2.82	456	0	623	0	-	-	H	1100	117.6	104	1.13	106	1.11	123	0.96																				
5		F70-0.5-19	1.43	120	168	200	240	30	1	70.5	13	2.82	456	0	623	0.5	36	0.6	H	1100	178.8	129	1.39	141	1.26	170	1.05																				
6		F70-1.0-19	1.43	120	168	200	240	30	1	67.3	13	2.82	456	0	623	1	36	0.6	H	1100	169.5	147	1.16	164	1.03	167	1.02																				
7		F70-1.5-19	1.43	120	168	200	240	30	1	67.3	13	2.82	456	0	623	1.5	36	0.6	H	1100	186.7	169	1.10	195	0.96	176	1.06																				
8		F80-0.0-16	1.43	120	168	200	240	30	1	74.1	13	2.00	442	0	623	0	-	-	H	1100	146.1	99	1.48	102	1.44	115	1.28																				
9		F80-0.5-16	1.43	120	168	200	240	30	1	82.4	13	2.00	442	0	623	0.5	36	0.6	H	1100	157.9	132	1.20	150	1.05	129	1.22																				
10		F80-0.0-19	1.43	120	168	200	240	30	1	85.2	13	2.82	343	0	623	0	-	-	H	1100	108.4	112	0.97	115	0.94	132	0.82																				
11		F80-0.5-19	1.43	120	168	200	240	30	1	86.1	13	2.82	343	0	623	0.5	36	0.6	H	1100	153.5	138	1.11	158	0.97	137	1.12																				
12	2	B1	0.31	90	463	500	145	80	1	35.7	10	1.93	440	0.42	375	1	30	0.56	S	-	375	326	1.15	348	1.08	323	1.16																				
13		B2	0.62	90	463	500	285	80	1	35.7	10	1.93	440	0.49	375	1	30	0.56	S	-	360	291	1.24	311	1.16	285	1.26																				
14		B3	0.93	90	463	500	430	80	1	35.5	10	1.93	440	0.48	375	1	30	0.56	S	-	291	252	1.16	267	1.09	262	1.11																				
15		B4	1.23	90	463	500	570	80	1	31.1	10	1.93	440	0.47	375	1	30	0.56	S	-	228	212	1.08	221	1.03	229	1.00																				
16		B5	1.85	90	463	500	855	80	1	31.5	10	1.93	440	0.49	375	1	30	0.56	S	-	183	184	0.99	190	0.97	185	0.99																				
17		B6	1.23	90	463	500	570	80	1	34.4	10	1.93	440	0.47	375	0	-	-	-	-	205	194	1.06	195	1.05	223	0.92																				
18		B7	1.23	90	463	500	570	80	1	33.8	10	1.93	440	0.47	375	0.5	30	0.56	S	-	220	208	1.06	213	1.04	229	0.96																				
19		B8	1.23	90	463	500	570	80	1	33.2	10	1.93	440	0.47	375	1.5	30	0.56	S	-	260	229	1.14	240	1.09	249	1.05																				
20		B9	1.51	90	463	500	700	80	1	29.5	10	1.93	440	0.48	375	1	30	0.56	S	-	224	198	1.13	203	1.10	226	0.99																				
21		B10	1.51	90	463	500	700	80	1	30.1	10	1.93	440	1.26	375	1	30	0.56	S	-	290	322	0.90	333	0.87	226	1.28																				
22	1	WO-1/1	0.58	80	624	650	360	100	1	40	10	0.81	418	0.43	304	1	30	0.4	H	-	345	316	1.09	351	0.98	334	1.03																				
																						Avg.																				1.12		1.06		1.04	
																						COV																					12.9%		12.9%		13.4%

2 Notation: a = shear span; b = section width; d = effective depth; h = full depth; l_{b1} = width of loading plate parallel to beam axis; V/P = shear-force-to-applied-point-load ratio; f_c' = concrete cylinder
3 strength; a_g = maximum size of coarse aggregates; ρ_l = ratio of flexural reinforcement; f_y = yield strength of flexural reinforcement; ρ_v = stirrup ratio; f_{yv} = yield strength of stirrups; ρ_f = fibre volume ratio;
4 l_f = fibre length; d_f = fibre diameter; S = straight fibres; H = hooked fibres; f_{yf} = yield strength of fibres; V_{exp} = measured shear strength; V_{S2PKT} = predicted shear strength from simplified kinematic ap-
5 proach; V_{2PKT} = predicted shear strength from 2PKT for complete shear behavior; V_{FEM} = predicted shear strength from nonlinear FEM analysis; COV = coefficient of variation.

6 Note: The bold numbers in the database are assumed values as the original values were not provided by the authors of the publication.

No.	Caption
Figure 1	Models for deep reinforced concrete beams; (a) Strut-and-tie model [4], (b) Two-parameter kinematic theory [6] and (c) Predicted response of an FRC deep beam without stirrups ($V_s=0$) based on the 2PKT [5]
Figure 2	2PKT for deep beams [6]
Figure 3	Fig. 3. Mechanisms of shear resistance in deep beams [5], [8]; f'_c = concrete compressive strength; k = crack shape factor; b = section width; a_g = maximum diameter of coarse aggregates; $\sigma_{v,avg}$ = average stress in the stirrups along the critical crack; ρ_v = stirrup ratio; α_1 = angle of the critical diagonal crack; E_s = modulus of elasticity of stirrups; ϵ_{yv} = yield strain of stirrups; f_{yv} = yield strength of stirrups; n_b = number of bottom longitudinal bars; d_b = diameter of longitudinal bars; l_k = length of bar-dowels; f_y = yield strength of longitudinal bars; ϵ_y = yield strain of longitudinal bars
Figure 4	Tension behavior of FRC (adapted from Voo and Foster [11])
Figure 5	Flow chart of the S2PKT solution procedure
Figure 6	Solution of S2PKT equations (test B7 by Mansur and Ong [2])
Figure 7	Variation of experimental-to-predicted strength ratios for 22 tests with; (a) span-to-depth ratio and (b) fibre volumetric ratio (Avg. = 1.12 and COV = 12.9% for 22 tests)
Figure 8	Variation of 2PKT complete behavior-to-simplified method strength prediction ratios for 22 tests with; (a) span-to-depth ratio and (b) fibre volumetric ratio (Avg. = 1.07 and COV = 5.70%)
Figure 9	Effect of a/d ratio (tests by Mansur and Ong [2])
Figure 10	Effect of fibre volumetric ratio (tests by Mansur and Ong [2])