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TILTING BUNDLES ON HYPERTORIC VARIETIES

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ABSTRACT. Recently McBreen and Webster constructed a tilting bundle on a smooth hypertoric variety (using reduction to finite characteristic) and showed that its endomorphism ring is Koszul.

In this short note we present alternative proofs for these results. We simply observe that the tilting bundle constructed by Halpern-Leistner and Sam on a generic open GIT substack of the ambient linear space restricts to a tilting bundle on the hypertoric variety. The fact that the hypertoric variety is defined by a quadratic regular sequence then also yields an easy proof of Koszulity.

1. INTRODUCTION

Below k is an algebraically closed ground field of characteristic 0. Let T be torus and let W be a symplectic representation of T. Then W is equipped with a canonical moment map[1] $\mu : W \to \mathfrak{t}^*$ with $\mathfrak{t} = \mathrm{Lie}(T)$. *Throughout we assume that the action of T is faithful* which implies that μ is surjective and flat.

Let $X(T)$ be the characters of T. For $\chi \in X(T)$ let $W^{ss,\chi}$ be the semi-stable part of W with respect to the linearization $\chi \otimes \mathcal{O}_W$. Recall that $X(T)_{\mathbb{R}}$ is equipped with a so-called secondary fan such that χ is in the interior of a maximal cone if and only if $W^{ss,\chi}/T$ is a (smooth) Deligne-Mumford stack [CLS11, Theorem 14.3.14]. We will call such χ *generic*. For generic χ, the DM stack $W^{ss,\chi}/T$ is a crepant resolution of the GIT quotient $W /\!\!/ T$ [ŠVdB17c, Lemma A.2 and its proof]. Building on the methods developed in [ŠVdB17a] Halpern-Leistner and Sam constructed a tilting bundle on $W^{ss,\chi}/T$ [HLS16]. See Theorem 3.2 below.

For $\xi \in \mathfrak{t}^*$ the *hypertoric variety* associated to the data (χ, ξ) is the GIT quotient $\mu^{-1}(\xi)^{ss,\chi} /\!\!/ T$. For χ generic, $\mu^{-1}(\xi)^{ss,\chi}/T$ is also a smooth Deligne-Mumford stack which is a crepant resolution of the hypertoric variety $\mu^{-1}(\xi) /\!\!/ T$ (see §4).

The following is our main result.

Theorem 1.1. Let χ be generic and let $\mathcal{T}$ be the tilting bundle on $W^{ss,\chi}/T$ constructed in [HLS16] (see Theorem 3.2).

[1]If $\langle -, - \rangle$ is the symplectic bilinear form on W, then one has $\mu(w)(v) = (1/2)\langle vw, w \rangle$ for $w \in W, v \in \mathfrak{t}$.

(1) For $\xi \in \mathfrak{t}^*$ the restriction $\mathcal{T}_\xi$ of $\mathcal{T}$ to $\mu^{-1}(\xi)^{ss,\chi}/T$ is a tilting bundle.
(2) Put $\Lambda = \mathrm{End}_{W^{ss,\chi}/T}(\mathcal{T})$, $\Lambda_\xi = \mathrm{End}_{\mu^{-1}(\xi)^{ss,\chi}/T}(\mathcal{T}_\xi)$. Then Λ_ξ is the quotient of Λ by the defining relations of $\mu^{-1}(\xi)$ which form a regular sequence.
(3) Λ_ξ is a "non-commutative crepant resolution" [VdB04] of $k[\mu^{-1}(\xi)/\!\!/T]$.
(4) Λ_0 is Koszul when graded using the dilation G_m-action on W.

The fact that $\mu^{-1}(\xi)^{ss,\chi}/T$ admits a tilting bundle with Koszul endomorphism ring when $\xi = 0$ was proved in [MW18] using the Bezrukavnikov-Kaledin method based on reduction mod p.

Remark 1.2. Λ is an NCCR for $W/\!\!/T$ (see Remark 3.3) but it is not Koszul, except in trivial cases. This follows from Proposition 4.1 applied in the same way as in the proof of Theorem 1.1(4). For example in the case of the conifold (G_m acting with weights $1, 1, -1, -1$ on a 4-dimensional representation) Λ has cubic relations. This is in fact expected as explained in the next remark.

Remark 1.3. From the fact that Λ_0 is Koszul one obtains in particular that it is quadratic. Using the explicit form of $\mathcal{T}$ (see Theorem 3.2) one then quite easily obtains a quiver presentation of Λ_0. See [MW18, Corollary 3.18]. This presentation of Λ_0 may be lifted to a quadratic non-homogeneous presentation of Λ as $\mathrm{Sym}(\mathfrak{t})$-algebra (with $|\mathfrak{t}| = 2$). This then yields a more complicated presentation of Λ as k-algebra, possibly involving extra quadratic generators and cubic and even quartic relations.

Similar results as those of McBreen and Webster have been announced by Tatsuyuki Hikita [Hik17].

2. Acknowledgement

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3. The tilting bundle on $W^{ss,\chi}/T$

Recall that a tilting bundle on an algebraic stack $\mathcal{Y}$ is a vector bundle $\mathcal{T}$ such that $\mathrm{Ext}_{\mathcal{Y}}^{>0}(\mathcal{T}, \mathcal{T}) = 0$ and such that $\mathcal{T}$ generates $D_{\mathrm{Qch}}(\mathcal{Y})$ in the sense that $\mathcal{T}^\perp = 0$.

For benefit of the reader we now describe explicitly the tilting bundle on $W^{ss,\chi}/T$ constructed in [HLS16]. The construction in fact only requires that W is *quasi-symmetric* [ŠVdB17a], i.e. the sum of weights of W on each line through the origin is zero.

Let $(\beta_i)_{i=1}^{2e}$ be the T-weights of W. Let $\varepsilon \in X(T)$. Put

$$\Sigma = \left\{ \sum_i a_i \beta_i \mid a_i \in]-1, 0] \right\} \subset X(T)_{\mathbb{R}}, \qquad \bar{\Sigma}_\varepsilon = \bigcup_{r>0} \bar{\Sigma} \cap (r\varepsilon + \bar{\Sigma}).$$

The following result indicates the combinatorial significance of the zonotope $\bar{\Sigma}$.

Proposition 3.1 ([HLS16, Proposition 2.1]). *Assume that W is quasi-symmetric. A character $\chi \in X(T)$ is generic if and only if it is not parallel to any face of $\bar{\Sigma}$.*

For $\varepsilon \in X(T)$ put $\mathcal{L}_\varepsilon = X(T) \cap (1/2)\bar{\Sigma}_\epsilon$. The following is one of the main results of [HLS16].

Theorem 3.2 ([HLS16, Corollary 4.2].)**.** Assume that W is quasi-symmetric and that $\chi \in X(T)$ is generic. Then for any generic $\varepsilon \in X(T)$

$$\mathcal{T} = \bigoplus_{\mu \in \mathcal{L}_\varepsilon} \mathcal{O}_{W^{ss,\chi}} \otimes_k \mu$$

defines a tilting bundle on $W^{ss,\chi}/T$.

Remark 3.3. If W is "generic" in the sense of [ŠVdB17a, Definition 1.3.4], i.e. the complement of $\{x \in W \mid Tx \text{ is closed and Stab } x = \{e\}\}$ has codimension ≥ 2 then $\Lambda = \mathrm{End}_{W^{ss,\chi}/T}(\mathcal{T}) = (\mathrm{End}_k(\bigoplus_{\mu \in \mathcal{L}_\varepsilon} \mu) \otimes_k k[W])^T$ is the NCCR of $W/\!\!/T$ constructed in [ŠVdB17a]. Assuming that T acts faithfully and that W is symplectic, one can always reduce to the case of generic W.

If W is not generic then all except for two T-weights of W (whose sum is 0) lie in a hyperplane. Let W' be a vector subspace of W spanned by the weight vectors whose weights lie in this hyperplane. Then $T = T' \times G_m$, where G_m acts trivially on W' and the action of T' is faithful on W'. This induces the decomposition of $X(T)$ and $\mathfrak{t}$. The corresponding projections onto $X(T')$ and $\mathfrak{t}'$ will be denoted by $'$. We have $W/\!\!/T = W'/\!\!/T' \times_k \mathbb{A}^1$, and for a generic χ also $W^{ss,\chi}/T = W^{ss,\chi'}/T' \times_k \mathbb{A}^1$. Moreover, $\mathcal{L}_\varepsilon = \mathcal{L}'_{\varepsilon'} \times \{m\}$ ($\mathcal{L}'_{\varepsilon'}$ with respect to (W',T')) for some $m \in \mathbb{Z}$. Repeating if necessary we reduce to the case that W is generic.

We also note that the corresponding hypertoric varieties do not change. Let μ, μ' correspond to the moment maps associated to W, W', resp. Then $\mu^{-1}(\xi)/\!\!/T = \mu'^{-1}(\xi')/\!\!/T'$, $\mu^{-1}(\xi)^{ss,\chi}/T = \mu'^{-1}(\xi')^{ss,\chi'}/T'$ (for a generic χ).

4. Proofs

We revert to the setting of the introduction, i.e. W is a symplectic representation of the torus T. Assume $\chi \in X(T)$ is a generic character. As explained in the introduction $W^{ss,\chi}/T$ is a DM stack which yields a (stacky) crepant resolution $\pi : W^{ss,\chi}/T \to W/\!\!/T$ of the Gorenstein variety [Sta83, Corolllary 13.3] $W/\!\!/T$. The fact that the stabilizers of the T-action on $W^{ss,\chi}$ are finite yields by the defining property of the moment map that $\mu|W^{ss,\chi}$ is smooth. Hence for every $\xi \in \mathfrak{t}^*$, $\mu^{-1}(\xi)^{ss,\chi}/T$ is also a smooth Deligne-Mumford stack.

Since T is commutative, $\mu^{-1}(\xi) \subset W$ is cut out by a T-invariant regular sequence of global sections (see [Vin86, Theorem 2] and [Sch95, Proposition 9.4]). In particular $\mu^{-1}(\xi)/\!\!/T$ is cut out by a regular sequence in $W/\!\!/T$ and the same regular sequence cuts out $\mu^{-1}(\xi)^{ss,\chi}/T$ in $W^{ss,\chi}/T$. It follows from this that $\mu^{-1}(\xi)/\!\!/T$ is Gorenstein and that π restricts to a crepant resolution $\pi_\xi : \mu^{-1}(\xi)^{ss,\chi}/T \to \mu^{-1}(\xi)/\!\!/T$.

Finally note that $\mu^{-1}(\xi)/\!\!/T$ is normal. For $\xi = 0$ this is shown in the proof of [BK12, Proposition 4.11]. For the benefit of the reader we give an elementary proof valid for general ξ. Using Remark 3.3 we first reduce to the generic case and in that case we will show that $\mu^{-1}(\xi)$ is normal (the quotient of a normal variety is normal). Since $\mu^{-1}(\xi)$ is Cohen-Macaulay, it suffices by Serre's normality criterion to prove that $\mu^{-1}(\xi)$ is regular in codimension 1. Thus, it is sufficient to prove that codimension in $\mu^{-1}(\xi)$ of the intersection of $\mu^{-1}(\xi)$ with the non-smooth locus of μ is ≥ 2. We will in fact verify that this codimension is ≥ 3!

By the defining property of the moment map, the non-smooth locus of μ is the locus of points in W whose stabilizer has dimension > 0. This is a union of T-invariant subspaces of W. Each such subspace W' is again a symplectic vector space with a faithful action of a quotient torus T/T' of T, where T' denotes the generic stabilizer of W' (hence by hypotheses $\dim T' > 0$) and moreover μ restricts to a moment map for the T/T'-action. Thus, as above, $W' \cap \mu^{-1}(\xi)$ is cut out by a regular sequence of length $\dim T/T'$ in W'. Hence the dimension of $W' \cap \mu^{-1}(\xi)$ is equal to $c := \dim W' - \dim T/T'$.

Choose a filtration $W = W_0 \supsetneq W_1 \supsetneq \cdots \supsetneq W_t = W'$ of T-invariant symplectic subspaces such that $\dim W_{i+1} = \dim W_i - 2$ and let T_i be the generic stabilizer of W_i. Write $c_i = \dim W_i - \dim T/T_i = \dim W_i \cap \mu^{-1}(\xi)$.

The dimension of T/T_i is equal to the rank of the sublattice of $X(T)$ spanned by the weights of W_i. Hence by genericity we have $\dim T/T_1 = \dim T$, and thus $c_1 = c_0 - 2$. Since $\dim T' > 0$ we have $t \geq 2$. In addition for $i \geq 1$ we have $c_{i+1} \leq c_i - 1$ ($\dim W_i$ goes down by 2 but $\dim T/T_i$ goes at most down by 1). So we get $c = c_t \leq c_0 - 3$ and we are done.

Proofs of Theorem 1.1(1) and 1.1(2). Let $i : \mu^{-1}(\xi)^{ss,\chi}/T \to W^{ss,\chi}/T$ be the inclusion. We have to prove that $\mathcal{T}_\xi = i^*\mathcal{T}$ is a tilting bundle on $\mu^{-1}(\xi)^{ss,\chi}/T$. We have
$$\operatorname{Ext}^*_{\mu^{-1}(\xi)^{ss,\chi}/T}(i^*\mathcal{T}, i^*\mathcal{T}) = \operatorname{Ext}^*_{W^{ss,\chi}/T}(\mathcal{T}, i_*i^*\mathcal{T}).$$
Now $\mu^{-1}(\xi)^{ss,\chi}$ is cut out in $W^{ss,\chi}$ by an invariant regular sequence. Tensoring the corresponding Koszul resolution of $i_*\mathcal{O}_{\mu^{-1}(\xi)^{ss,\chi}}$ with $\mathcal{T}$ we obtain a left resolution $K_\bullet$ of $i_*i^*\mathcal{T}$ which consists of direct sums of $\mathcal{T}$. Using the fact that $\operatorname{Ext}^{>0}_{W^{ss,\chi}/T}(\mathcal{T}, \mathcal{T}) = 0$ we obtain

$$(4.1) \qquad \operatorname{Ext}^*_{W^{ss,\chi}/T}(\mathcal{T}, i_*i^*\mathcal{T}) = H^*(\operatorname{Hom}_{W^{ss,\chi}/T}(\mathcal{T}, K_\bullet)).$$

Since $K_\bullet$ lives in degree ≤ 0 this implies $\operatorname{Ext}^{>0}_{\mu^{-1}(\xi)^{ss,\chi}/T}(i^*\mathcal{T}, i^*\mathcal{T}) = 0$. Since also tautologically $\operatorname{Ext}^{<0}_{\mu^{-1}(\xi)^{ss,\chi}/T}(i^*\mathcal{T}, i^*\mathcal{T}) = 0$ we obtain that the right-hand side of (4.1) is in fact a resolution of Λ_ξ, proving Theorem 1.1(2).

To finish the proof of Theorem 1.1(1) we have to check the generation property. Assume that $\operatorname{RHom}_{\mu^{-1}(\xi)^{ss,\chi}/T}(i^*\mathcal{T}, \mathcal{G}) = 0$ for $\mathcal{G} \in D_{\operatorname{Qch}}(\mu^{-1}(\xi)^{ss,\chi}/T)$. Then by adjunction $i_*\mathcal{G} \in \mathcal{T}^\perp$ and hence $i_*\mathcal{G} = 0$. It follows that $\mathcal{G} = 0$. $\qquad\square$

Proof of Theorem 1.1(3). Since π_ξ is a crepant resolution, it follows that $\Lambda_\xi = \pi_* \mathcal{E}nd_{\mu^{-1}(\xi)^{ss,\chi}/T}(\mathcal{T}_\xi)$ is an NCCR of $\mu^{-1}(\xi)/\!/T$ (see e.g. [ŠVdB17b, Corollary 4.3] and Corollary 4.7 in loc. cit. with its proof, together with Remark 3.3). $\qquad\square$

The proof of Theorem 1.1(4) will be based on the following more general criterion.

Proposition 4.1. Let Δ be a $\mathbb{N}$-graded homologically homogeneous k-algebra [BH84] (see also [VdB04, §3], [SVdB08, §2]) such that Δ_0 is semi-simple. Assume that Δ is finite as a module over a central subring R of Krull-dimension d. Then the following are equivalent.

(1) Δ is Koszul.
(2) The invertible Δ-bimodule $\omega_\Delta := \operatorname{Hom}_R(\Delta, \omega_R)$ (see [SVdB08, Proposition 2.6]) is generated as right module in degree d.

Moreover in (2) we may replace "right" by "left".

Proof. Note that by [SVdB08, Lemma 2.4(2), Proposition 2.6] $\omega_\Delta[d]$ is a "rigid dualizing complex" for Δ. Hence by the uniqueness of rigid dualizing complexes together with the proof of[2] [VdB97, Proposition 8.2(2)] we obtain as Δ-bimodules: $\omega_\Delta[d] = R\Gamma_{\Delta_{\geq 1}}(\Delta)^\vee$ where $(-)^\vee$ denotes the graded k-dual and $R\Gamma_{\Delta_{\geq 1}}$ is the derived functor of $\varinjlim_n \mathrm{RHom}_\Delta(\Delta/\Delta_{\geq n}, -)$. Moreover by [VdB97, Theorem 5.1] for $M \in D(\mathrm{Gr}(\Delta))$ we obtain as objects in $D(\mathrm{Gr}(\Delta^\circ))$

$$\mathrm{RHom}_\Delta(M, \omega_\Delta[d]) \cong R\Gamma_{\Delta_{\geq 1}}(M)^\vee .$$

Applying this with a graded simple Δ-module $\mathbb{S}$, concentrated in degree zero we find

$$\mathrm{RHom}_\Delta(\mathbb{S}, \omega_\Delta[d]) = \mathbb{S}^*;$$

i.e. $\mathrm{RHom}_\Delta(-, \omega_\Delta[d])$ restricts to the canonical bijection between the graded simple left and right Δ-modules, concentrated in degree zero.

We will now prove $(2) \Rightarrow (1)$. Since ω_Δ is invertible and generated in degree d we have that $\omega_\Delta \otimes_\Delta \mathbb{S} = \mathbb{S}^\dagger(-d)$ where $\mathbb{S}^\dagger$ is a graded simple Δ-module concentrated in degree zero. Hence we obtain

$$(4.2) \qquad \mathrm{RHom}_\Delta(\mathbb{S}, \Delta) = \mathrm{RHom}_\Delta(\omega_\Delta[d] \otimes_\Delta \mathbb{S}, \omega_\Delta[d]) = \mathbb{S}^{\dagger*}(d)[-d]$$

Let $P_\bullet = \cdots \to P_1 \to P_0$ be the minimal graded projective resolution of $\mathbb{S}$ over Δ. As Δ is homologically homogeneous, $P_\bullet$ is of length d, and by (4.2) $\mathrm{Hom}_\Delta(P_\bullet, \Delta)[d]$ is a projective resolution of $\mathbb{S}^{\dagger*}(d)$. Hence $P_d = P(\mathbb{S}^\dagger)(-d)$, where $P(\mathbb{S}^\dagger)$ is the graded projective cover of $\mathbb{S}^\dagger$.

Let f_i be the minimal degree of an element in P_i. We claim $f_{i-1} < f_i$. To see this note that the graded Jacobson radical of Δ is equal to $\Delta_{\geq 1}$ (as Δ_0 is semi-simple). Hence the image of $d_i : P_i \to P_{i-1}$ is in $\Delta_{\geq 1}P_{i-1}$.

An element x of degree f_i in P_i is not in $\Delta_{\geq 1}P_i$ and therefore it is not in the image of d_{i+1}. As $P_\bullet$ is acyclic it follows that $d_i(x) \neq 0$. Writing $d_i(x) = \sum_j l_j y_j$ with homogeneous $l_j \in \Delta_{\geq 1}$, $y_j \in P_{i-1}$, we see that some y_j must be non-zero. The fact l_j has strictly positive degree implies $\deg(y_j) < \deg(x)$ which proves our claim.

From the inequalities

$$d = f_d > f_{d-1} > \cdots > f_1 > f_0 = 0$$

we obtain $f_i = i$.

We now use a dual argument. Let f_i^* be the maximal degree of a generator of P_i. Using the dual resolution $\mathrm{Hom}_\Delta(P_\bullet, \Delta)(-d)[d]$ of $\mathbb{S}^\dagger$ (which is also minimal) we get that $-d + f_{d-i}^* = -i$, i.e. $f_i^* = i$. Therefore P_i is purely generated in degree i and so the resolution $P_\bullet$ is linear.

The implication $(1) \Rightarrow (2)$ is similar. From the nature of the minimal resolutions of the simples we get $\omega_\Delta \otimes_\Delta \mathbb{S} = \mathbb{S}^\dagger(d)$ for all $\mathbb{S}$. This implies that ω_Δ is right generated in degree d.

Since Koszulity is left right symmetric, the hypothesis of right generation in (2) may indeed be replaced by left generation. $\qquad \square$

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[2]The reference [VdB97] is concerned with the case $\Delta_0 = k$ but this hypothesis is not used in an essential way.

Proof of Theorem 1.1(4). Let $s = \dim T$, $2e = \dim W$ and $d = \dim \mu^{-1}(0)/\!/T$. Thus $d = 2e - 2s$.

By Theorem 3.2 have $\Lambda_0 = (\mathrm{End}_k(\bigoplus_{\mu \in \mathcal{L}_\xi} \mu) \otimes_k k[\mu^{-1}(0)])^T$. Hence Λ_0 is $\mathbb{N}$-graded and its part of degree zero is semi-simple.[3] Therefore by Proposition 4.1 we have to prove that ω_{Λ_0} is generated in degree d.

Put $R = k[W/\!/T]$, $R_0 = k[\mu^{-1}(0)/\!/T]$. We have

$$\omega_{R_0} = \mathrm{Ext}_R^s(R_0, \omega_R) = \mathrm{Ext}_R^s(R_0, R(-2e)) = R_0(2s - 2e) = R_0(-d)$$

where the first equality is the adjunction formula, the second equality follows from [Sta83, Corollary 13.3] and the third equality follows from the fact that R_0 is cut out from R by a quadratic regular sequence of length s. As Λ_0 is an NCCR by Theorem 1.1(3) we have $\mathrm{Hom}_{R_0}(\Lambda_0, R_0) = \Lambda_0$. It follows that $\omega_{\Lambda_0} = \mathrm{Hom}_{R_0}(\Lambda_0, \omega_{R_0}) = \Lambda_0(-d)$, finishing the proof. $\square$

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[3]This is the only place where we use the specific form of $\mathcal{T}$.