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Faculteit Bedrijfseconomische Wetenschappen

master in de handelswetenschappen

Masterthesis

Intermittent demand forecasting

Kevin Driesen

Scriptie ingediend tot het behalen van de graad van master in de handelswetenschappen, afstudeerrichting supply chain management

PROMOTOR :
Prof. dr. Inneke VAN NIEUWENHUYSE

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www.uhasselt.be
Universiteit Hasselt
Campus Hasselt:
Martelarenlaan 42 | 3500 Hasselt
Campus Diepenbeek:
Agoralaan Gebouw D | 3590 Diepenbeek

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This master thesis was written during the COVID-19 crisis in 2020-2021. This global health crisis might have had an impact on the (writing) process, the research activities and the research results that are at the basis of this thesis.

Intermittent demand forecasting

Kevin Driesen
Master Supply Chain Management
Faculty of Business Economics, Hasselt University

This master's thesis implements three different intermittent demand forecasting methods (Croston's method, the Syntetos Boylan Approximation and the Teunter Syntetos Babai method) and compares them to each other by using three different accuracy measures (Mean Error, Mean Squared Error and Mean Absolute Scaled Error). Intermittent demand may occur in many different industries and is very different from normal demand patterns. Intermittent demand products have many different periods where no demand occurs, it may take months before a period of positive demand occurs. Normal demand patterns deal with continuous positive demands. The analysis in this thesis is done based on a dataset obtained from Scania Parts Logistics.

Keywords: Intermittent demand, forecasting, Croston, Syntetos Boylan Approximation, Teunter Syntetos Babai, Excel

1 Introduction

The subject of this thesis is intermittent demand forecasting, i.e. demand that is characterized by infrequent demand occurrences: many periods have zero demand, while relatively few have positive demand. When demand occurs, amounts may vary (Aris A. Syntetos & Boylan, 2005). Demand forecasting is an important topic in supply chain management, because it gives the supply chain manager an estimate of the expected demand in future periods. An accurate demand forecast gives the opportunity to set an optimal inventory management for the desired customer service level (Aris A. Syntetos & Boylan, 2006). Both the variability of demand occurrence and the variability of demand size make intermittent demand more difficult to forecast, which occurs a lot in industries dealing with spare parts (e.g. electronics, automotive, high tech) (Li & Lim, 2018; Aris A. Syntetos & Boylan, 2006). Many different intermittent demand forecasting methods have been suggested in scientific literature (Babai, Dallery, Boubaker, & Kalai, 2019; Croston, 1972; Kourentzes, 2013; Li & Lim, 2018; Aris A. Syntetos & Boylan, 2005; R. H. Teunter, Syntetos, & Zied Babai, 2011). This thesis will compare the accuracy of three different forecasting methods applicable in Excel, using a real life intermittent demand dataset granted by Scania Parts Logistics (https://www.scania.com/partslogisticscenter/en/home.html).

Croston's method is a good place to start as it gave a new perspective on intermittent demand. Syntetos and Boylan (2001) identified limitations in Croston's method, saying the method is robust and the demand estimates are positively biased and suggest their theoretically unbiased method, the Syntetos

Boylan Approximation (further referred to as SBA). Teunter, Syntetos and Babai (2011) found another flaw to both Croston's method and the SBA, namely that these methods don't take the risk of obsolescence into account (i.e. Croston and the SBA do not recognize when an SKU is approaching the end of its product lifecycle). As a solution to both problems they introduced their method, the Teunter, Syntetos and Babai method (further referred to as TSB).

This thesis compares Croston's method, which is currently used in many software packages (e.g. SAP) for forecasting intermittent demand (Li & Lim, 2018), with two other methods: the SBA (2005) and the TSB method (2011). As the methods are built on each other, trying to improve previous flaws, they are linked to each other and comparing them will result in a clear and direct comparison.

More complex methods of intermittent demand forecasting are available (e.g. Neural Network forecasting, bootstrapping). Due to their complexity, data requirements and software (Kourentzes, 2013; Willemain, Smart, & Schwarz, 2004; Zhou & Viswanathan, 2011) they are not further investigated in this paper; they are only briefly mentioned in sections 3.2 and 3.3. Therefore, this thesis will stick to the aforementioned methods; Croston's method, the SBA and the TSB method.

The remainder of this thesis includes the following sections. In section 2, a problem statement is given to further understand the problem of intermittent demand together with the research objectives of this thesis. Section 3 gives an overview of literature related to intermittent demand forecasting. In section 4, the methodology used to compare the three forecasting methods is presented. Section 5 summarizes the empirical findings of this comparison. Finally, in section 6, the conclusions and main insights are provided.

2 Problem statement and research objectives

The majority of products have daily, weekly, monthly and even seasonal demand patterns making it relatively easy to forecast future demands. Products where demand has an intermittent nature do not have these predictable patterns. Therefore, they are called intermittent demand products, i.e. demand occurs sporadically: there are more often periods with zero demand than with positive demand, resulting in lengthy inter-demand intervals. Zero demand periods limit the available data, and the widely varying positive demand sizes make forecasting even more difficult (Aris A. Syntetos & Boylan, 2005).

Intermittent demand forecasting is an important tool for optimizing inventory management, specifically for industries dealing with spare parts (Hua, Zhang, Yang, & Tan, 2017). Good inventory management balances stock availability and costs of inventory. Customer demands not being met, may result in very high stock-out costs in the spare parts industry. Therefore, accurate forecasts of demand help to save costs (Turrini & Meissner, 2019). Accurate forecasting also prevents having too much stock laying around as intermittent demand stock keeping units (further referred to as SKU) may spend months in stock; it thus helps to decrease holding costs of the SKU. Finally, intermittent demand forecasting also grants an estimate to help optimally balancing customer service levels and SKU inventory costs. This is particularly important for high stock-out cost items and high holding cost items (Aris A. Syntetos & Boylan, 2006).

However, products have a certain lifetime. When they are at the end of their lifetime, there will be no further demand, turning leftover SKUs into dead stock, i.e. SKUs that have become obsolete and no longer have any value. In the case of intermittent demand SKUs it is very difficult to decide whether they actually have became dead stock or if they are just going through a lengthier inter-demand interval. In case of obsolescence, it is very important to decide to remove the dead stock and reduce unnecessary inventory costs (R. H. Teunter et al., 2011).

Another problem with intermittent demand is that the usual forecasting methods are not accurate enough. The conventional forecasting methods (e.g. moving average time series, exponential smoothing methods) use demand data to extract systematic patterns such as trends, levels and seasonal factors. As conventional forecasting methods don't capture the dual nature of inter-demand intervals and varying demand sizes, they cannot be used for intermittent demand forecasting (Li & Lim, 2018).

Intermittent demand SKUs include all kinds of intermediate or final goods (Li & Lim, 2018) and can be found at any level of the supply chain (Aris A. Syntetos & Boylan, 2006). They account for up to 60% of stock value in countless industries, e.g. electronics, automotive, spare parts, engineering and high tech (Li & Lim, 2018). Specifically the spare parts industry consists mostly of intermittent demand SKUs as service operations require a wide variation of spare parts. Syntetos, Babai and Gardner (2015) mention a survey by Deloitte in 2011, recording service operation revenues of many of the world's largest companies to be 26% of their combined revenue of 1.5 trillion dollars. As Deloitte's survey proves that intermittent demand items claim a quarter of the revenue and accounts for up to 60% of stock value in many organizations, improvements in managing intermittent demands can cause important cost decreases (Li & Lim, 2018). Intermittent demand items also have a greater risk of obsolescence: this can occur, for instance, when a newer and better product enters the market. Obsolete spare parts leave the producers with dead stock, resulting in a waste of invested money (Aris A. Syntetos et al., 2015).

The intermittent demand problem has been around for a long time. Croston's method was the first method proposed for intermittent demand forecasting in 1972 (Li & Lim, 2018). After Croston, the interest in the topic increased; the resulting literature is briefly discussed in section 3.

In this article, three intermittent demand forecasting (Croston, SBA, TSB) are compared and analysed in Excel using the following accuracy measures: mean error (further referred to as ME), mean square error (further referred to as MSE), mean absolute scaled error (further referred to as MASE). The ME was chosen to see if a method is positively or negatively biased for the used dataset. The MSE shows how big the forecasting errors were by squaring the errors. The MASE is used, because Hyndman and Koehler (2006) suggest it to be the best forecasting measure.

As discusses in the introduction, the three methods (Croston, SBA, TSB) are linked, as the SBA tries to improve Croston's method, while the TSB method tries to further improve the SBA. Therefore, the main goal of this thesis is to compare and analyse these three different intermittent demand forecasting methods to determine if the improvements made to Croston's method actually are improvements. In addition, this thesis is an example of the chosen intermittent demand forecasting methods implemented in Excel, as well as a comparison of the three methods, being helpful for any supply chain manager wishing to implement the methods on his/her own data.

3 Literature review

This section summarizes the scientific literature on intermittent demand forecasting methods. Subsection 3.1 presents several parametric forecasting methods suggested for intermittent demand. The bootstrapping method is explained is subsection 3.2. Finally, subsection 3.3 provides the neural network forecasting method for intermittent demand forecasting.

3.1 Parametric intermittent demand forecasting

A general approach to demand forecasting is the parametric approach. In this case, parametric means 'within assumed parameters'. Parametric forecasting methods extract information from existing demand data to forecast demands while staying within the assumed parameters, e.g. a fixed lead time, positive demand sizes follow a normal distribution, demand size and inter-demand intervals are independent (Aris A. Syntetos et al., 2015).

Examples of parametric methods are Simple Exponential Smoothing (further referred to as SES, (Gardner, 2006)) and the Holt-winters model (Gardner, 2006). Intermittent demand has traditionally been forecasted mainly with SES or Simple Moving Average (further referred to as SMA). However, SES is not a good method for forecasting intermittent demand patterns as it treats positive and zero demands in the same way, meaning that forecasts will be biased low before positive demand occurs and biased high after positive demand occurs (Aris A. Syntetos et al., 2015). Due to the increasing awareness of intermittent demand and its importance, new methods were suggested such as Croston's method and the corresponding improvement suggestions (e.g. SBA, TSB method, Syntetos' method) (Aris A. Syntetos & Boylan, 2005; R. H. Teunter et al., 2011; Zied Babai, Syntetos, & Teunter, 2014). More advanced methods, though still based on Croston's method, also gained scientific attention, such as the Aggregate-Disaggregate Intermittent Demand Approach (further referred to as ADIDA) and the inverse Aggregate-Disaggregate Intermittent Demand (further referred to as iADID) (Li & Lim, 2018). ADIDA and iADID are hierarchical forecasting (further referred to as HF) models and forecast demand of entire companies (all stores). HF models can be top-down or bottom-up. Top-down models start by forecasting the total demand of a company and continue by dividing this demand over the different stores. Bottom-up does the opposite: it starts by forecasting demand in stores and aggregates that to the total company demand (Kahn, 1998).

3.2 The Bootstrapping approach

Bootstrapping also uses historical demand data to forecast demand: it takes multiple random samples from the dataset and uses these samples to create a histogram. Mean and variance are then calculated from this histogram as a forecast for the upcoming period. In the scientific literature, multiple advanced bootstrapping methods have been suggested (e.g. Willemain, Smart and Schwarz method, Zhou and Viswanathan method), adding extra properties for more accurate forecasts for intermittent demand patterns (Aris A. Syntetos et al., 2015; Willemain et al., 2004; Zhou & Viswanathan, 2011). Bootstrapping methods generally bring more complexity to the process of forecasting as they require knowledge of coding the method: hence, bootstrapping demands a lot of time to implement. Syntetos, Babai and Gardner (2015) conclude that the added complexity makes the usage of bootstrapping questionable and find that simple parametric methods perform very well, considering the effort spent in their implementation, as opposed to bootstrapping methods. Furthermore, bootstrapping methods require much more computing power than parametric methods: this is not suitable when dealing large numbers of SKUs, as is often the case in the spare parts industry.

3.3 Neural network forecasting

Neural network (further referred to as NN) forecasting is another non-parametric intermittent demand forecasting method. Unlike the parametric and bootstrapping methods, NNs do not require human experts to provide a model. NNs are described as flexible, non-linear data driven models. They capture interactions between the positive demands and the demand intervals. As they require a lot of data to accurately predict the highly varying intermittent demand patterns, NNs are often seen as ‘data-hungry’ models (Kourentzes, 2013). Due to the complexity of these models and limited resources, they will not be analysed in this thesis. Readers interested in an application of NNs are referred to (Lolli et al., 2017), for a comparison of benchmark NNs to standard forecasting methods.

4 Methodology

Subsection 4.1 provides a table of notations used for the formulations of the methods in this paper. The three methods under study (Croston’s method, SBA and TSB) are clarified respectively in subsections 4.2, 4.3 and 4.4. The forecast accuracy measures used for the comparison are discussed in subsection 4.5. Finally, section 4.6 gives an insight on the dataset used.

4.1 Notations

D_t	Actual demand for an SKU in period t
D'_t	Demand forecasted in period t for period t + 1
S'_t	Smoothed demand size for an SKU in period t
I_t	Actual interval of periods since last positive demand in period t (Croston and SBA)
I'_t	Smoothed interval of periods since last positive demand in period t (Croston and SBA)
P'_t	Smoothed probability of demand occurrence in period t (TSB)
α	Smoothing parameter $0 \le \alpha \le 1$
β	Smoothing parameter $0 \le \beta \le 1$
n	Total amount of periods with both actual demand and forecasted demand
n_1	Total amount of periods with actual demand (MASE)

4.2 Croston's method

The most well-known and most often used parametric forecasting method for intermittent demand is the method suggested by Croston in 1972. Croston's method divides intermittent demand in two series: the average size of positive demand and the average inter-demand intervals, these series update when positive demand occurs. Both series are separately forecasted by SES, a forecasting method for regular demand patterns (Hasni, Babai, Aguir, & Jemai, 2019). Then Croston's estimate is based on the ratio between the two series (Li & Lim, 2018).

Croston's method is calculated as follows (Gardner, 2006; Zied Babai et al., 2014):

1) $D_t = 0 \rightarrow I'_t = I'_{t-1}\ ;\ S'_t = S'_{t-1}\ ;\ D'_t = D'_{t-1}$ (i.e. smoothed estimators are unchanged)
2) $D_t > 0 \rightarrow I'_t = I'_{t-1} + \alpha(I_t - I'_{t-1})$ (i.e. smoothed interval updates)
3) $D_t > 0 \rightarrow S'_t = S'_{t-1} + \alpha(D_t - S'_{t-1})$ (i.e. smoothed demand size updates)
4) $D_t > 0 \rightarrow D'_t = \frac{S'_t}{I'_t}$ (i.e. Croston's forecast updates)

Expression (1) dictates what happens to the estimates when actual demand is zero, i.e. none of the smoothing estimators are updated. Expression (2) shows the formula for updating the smoothed inter-demand interval estimator when actual demand is positive, while expression (3) formulates how the smoothed demand size updates in that case. Finally, expression (4) shows the formula for updating the forecast for the next period. Croston first suggested using the same smoothing parameter ɑ for I_t' and S_t', a more general variant suggested by others (e.g. Schultz, (1987)) uses different smoothing values as it may be beneficial (Zied Babai et al., 2014). Croston's original suggestion of using the same smoothing parameters for I_t' and S_t' is used in this paper. As for the value of the smoothing parameter, Croston found suggestions of keeping the smoothing value of ɑ between 0,1 – 0,2 (Croston, 1972). For research purposes, this thesis will include multiple values between 0 and 1.

Croston's method is the most often used forecasting method for intermittent demand, and has also been integrated in many software packages (e.g. SAP). Furthermore, Croston's method forms the base of many other intermittent demand forecasting methods, despite the associated positive bias in its estimates (Li & Lim, 2018). The bias in Croston's estimates was proved by Syntetos and Boylan (2001) as a first step to improve Croston's method. They found an error in the mathematical derivation of the expected estimate of demand, resulting in higher demand forecasts. However, this error contributed to the unexpected benefits of Croston's method in comparison with SES. Syntetos and Boylan (2001) then modified Croston's method to be unbiased: this method is known as the SBA and is further explained in subsection 4.2. Readers interested in the explanation of the mathematical error in Croston's method are referred to Syntetos and Boylan's paper (A. A. Syntetos & Boylan, 2001).

Furthermore, Teunter, Syntetos and Babai (2011) exposed another flaw to Croston's method, namely Croston's method not dealing with obsolescence issues. Obsolete products are less in demand and naturally gain more and more zero demand periods. Croston's method does not update with periods of zero demand, thus it does not recognize the obsolescence of products and will continue to forecast demand estimates based on previous positive demand averages. The authors solve this issue and the bias problem without adding complexity to Croston's method; we refer to this method as the TSB method, and it is further explained in subsection 4.3.

Apart from these well-known criticisms on Croston's method, Shenstone and Hyndman (2005) challenge Croston's assumptions that positive demand sizes follow a normal distribution, inter-demand intervals follow a geometric distribution and that positive demand sizes and inter-demand intervals are independent of each other (Aris A. Syntetos et al., 2015). Interested readers are referred to Shenstone and Hyndman's paper (Shenstone & Hyndman, 2005).

4.3 Syntetos and Boylan Approximation (SBA)

Croston's method shows theoretical superiority over more simplistic forecasting methods (e.g. moving average time series, exponential smoothing methods). However, as mentioned above, Syntetos and Boylan (2001) investigated Croston's method and showed that forecasts were biased due to a mathematical error. After proving the bias in Croston's method, they suggested a modification to Croston's method that multiplies Croston's estimate by a factor $(1 - \frac{\beta}{2})$, with β being a smoothing parameter; the goal was to obtain unbiased estimates, without adding complexity. This method is now known as the Syntetos and Boylan Approximation (SBA).

The SBA is calculated with the following formula (Gardner, 2006; Zied Babai et al., 2014):

1) $D_t = 0 \rightarrow I'_t = I'_{t-1}$; $S'_t = S'_{t-1}$; $D'_t = D'_{t-1}$ (i.e. smoothed estimators are unchanged)
2) $D_t > 0 \rightarrow I'_t = I'_{t-1} + \beta(I_t - I'_{t-1})$ (i.e. smoothed interval updates)
3) $D_t > 0 \rightarrow S'_t = S'_{t-1} + \alpha(D_t - S'_{t-1})$ (i.e. smoothed demand size updates)
4) $D_t > 0 \rightarrow D'_t = (1 - \frac{\beta}{2})\frac{S'_t}{I'_t}$ (i.e. SBA forecast updates)

Just like Croston's method, when actual demand is zero, the SBA does not update any smoothing estimators as shown in expression (1). Expression (2) shows the formula for updating the inter-demand interval smoothed estimator when actual demand is positive. Expression (3) formulates how the smoothed demand size updates when actual demand is positive. Finally, the actual SBA forecast is formulated in expression (4). Note that the SBA differs from Croston's method by using two different smoothing parameters for I'_t and S'_t in expression (2 and 3) as suggested by Schultz (1987) and in the final demand estimate D'_t by multiplying a smoothing factor $(1 - \frac{\beta}{2})$ to Croston's estimate expression (4) (Zied Babai et al., 2014). Note that the β used in expression (2 and 4) are the same.

Syntetos and Boylan (2001) performed a statistical test on the improvement of accuracy between SBA and Croston's method. Their comparison was based on the accuracy measure Mean Absolute Percentage Error. They mention the improvement to be statistically significant at a significance level of 0.01, i.e. the SBA is more accurate. They also state that Croston's method is only accurate under Croston's stated assumptions (demand sizes follow a normal distribution and there is mutual independence between inter-demand intervals and demand sizes), which are not in line with the behaviour of intermittent demand in reality.

4.4 Teunter, Syntetos and Babai's method (TSB)

Teunter, Syntetos and Babai (2011) report that the multiplication factor added in the SBA to remove the bias in Croston's method actually overcorrects Croston's estimate. This makes the SBA negatively biased, and in some cases even more biased than Croston's method. They refer to other papers for the proof of this negative bias (R. Teunter & Sani, 2009; Wallström & Segerstedt, 2010).

Teunter, Syntetos and Babai (2011) find that Croston's method has two flaws. Firstly, Croston suggested a method that has a positive bias, as already explained in the paper by Syntetos and Boylan (2001). The second flaw is that Croston's method does not account for obsolescence issues, as it does not update the estimated demand after periods of zero demand. Croston's method only updates when positive demand occurs, i.e. longer periods of zero demand due to product obsolescence are not interpreted as a signal of obsolescence.

The newly suggested TSB method is unbiased, unlike Croston's method which is positively biased and the SBA where overcompensation resulted in a negative bias. On top of that, the TSB method resolves the issues with product obsolescence by working with a probability of demand occurrence instead of demand intervals and by updating this probability when zero demand occurs, i.e. longer periods of zero demand may indicate product obsolescence. The TSB method resolves these without adding much complexities to the formulas (R. H. Teunter et al., 2011).

The TSB method is calculated as follows (Zied Babai et al., 2014):

1) $D_t = 0 \rightarrow P'_t = P'_{t-1} + \beta(0 - P'_{t-1})$ (i.e. smoothed demand occurrence probability updates)
2) $D_t = 0 \rightarrow S'_t = S'_{t-1}$ (i.e. smoothed demand size is unchanged)
3) $D_t = 0 \rightarrow D'_t = P'_t S'_t$ (i.e. TSB forecast updates)
4) $D_t > 0 \rightarrow P'_t = P'_{t-1} + \beta(1 - P'_{t-1})$ (i.e. smoothed demand occurrence probability updates)
5) $D_t > 0 \rightarrow S'_t = S'_{t-1} + \alpha(D_t - S'_{t-1})$ (i.e. smoothed demand size updates)
6) $D_t > 0 \rightarrow D'_t = P'_t S'_t$ (i.e. TSB forecast updates)

The TSB method is very different from Croston's method and the SBA as it is the only method that also updates when zero demands occur. The TSB method also uses a probability of demand occurrence factor instead of the inter-demand intervals. The probability factor is shown in expression (1 and 4), formulating how demand occurrence probability updates when actual demand is respectively zero or positive. Expression (2) shows that the smoothed demand size remains unchanged when actual demand is zero. As the probability of demand occurrence factor updates when actual demand is zero, this automatically updates the TSB forecast too; see the TSB forecast formula in expression (3). The smoothed demand size estimator is the same as in Croston's method and the SBA, which is formulated in expression (5). Finally, expression (6) shows the formula for the TSB forecast when actual demand is positive (Zied Babai et al., 2014). Note that there is no difference between expression (3 and 6), because expression (1 and 4) already include the impact of actual demand occurring.

4.5 Forecast accuracy metrics

Intermittent demand patterns have, as mentioned in the intermittent demand subsection, many zero demand periods. Some accuracy measures (e.g. mean absolute percentage error and median absolute percentage error) are calculated with fractions where actual demand is used as the denominator. In the case of zero demand, that would result in infinity. Therefore, these relative measures are not usable to measure intermittent demand forecasting accuracy (Aris A. Syntetos & Boylan, 2005).

The most simplistic accuracy measure is the ME. When ME is negative, the forecasting method is negatively biased and mostly forecasts values lower than actual demand. When ME is positive, the forecasting method is positively biased and mostly forecasts values higher than actual demand. ME is calculated as follows:

$$ME = \frac{\sum_{t=1}^{n} (D_t' - D_t)}{n}$$

Another easy to compute metric, which is based on the ME, is the MSE. Here, the total sum of all squared errors is divided by the total amount of forecasts (Nikolopoulos, Syntetos, Boylan, Petropoulos, & Assimakopoulos, 2017). The squaring of errors negates the visibility of any bias and punishes big forecast errors to a larger extent in the accuracy measure. It also makes the resulting MSE strictly positive. The MSE is calculated as follows:

$$MSE = \frac{\sum_{t=1}^{n} (D_t' - D_t)^2}{n}$$

Hyndman and Koehler (2006) suggested the MASE as the standard measure for comparing forecast accuracy, as it provides a scale-free forecast accuracy measurement that always has a meaningful outcome, i.e. when MASE is lower than 1, the errors from the used method are on average smaller than the naïve forecasting method, e.g. period 4 had an actual demand of 20 units, naïve forecasting method then forecasts a demand of 20 units in period 5 (Hyndman & Koehler, 2006). The MASE cannot be negative, and is calculated according to the following formula (Zied Babai et al., 2014):

$$MASE = \frac{(1/n) \sum_{t=1}^{n} |D_t' - D_t|}{(1/n_1 - 1) \sum_{i=2}^{n_1} |D_i - D_{i+1}|}$$

4.6 Dataset

The dataset used for this thesis is provided by Scania Parts Logistics (https://www.scania.com/partslogisticscenter/en/home.html), the logistical heart of the entire Scania Group that organizes Scania's distribution of spare parts worldwide. A total of 50 different SKUs is used for the comparison.

The SKUs were exported from the system of Scania Parts Logistics to an Excel document. Table 1 shows that the demand patterns of the 50 SKUs have a minimum of 5 months with positive demand and a maximum of 14 months with positive demand within a time period of 48 months. The rest of the months, there is no demand for the SKUs. Table 2 shows the minimum and maximum demand size of the 50 SKUs (excluding zero demand periods). An average demand of 4,9 shows that demand sizes were rather low in comparison to the maximum demand size of 101. There still were some SKUs with high demand sizes, even though the average may suggest low demand sizes.

Positive demand months	
Average	8,7
Minimum	5
Maximum	14

Table 1: Positive demand months data

Positive demand size	
Average	4,9
Maximum	101
Minimum	1

Table 2: Positive demand size data

Table 3 is the demand pattern of SKU 10094, an example of a demand pattern with low demand sizes and a total of 8 positive demand occurrences within a time period of 48 months. Table 4 is the demand pattern of SKU 35528, an example of a demand pattern with high demand sizes and a total of 13 positive demand occurrences within a time period of 48 months. Periods with no demand have been left out of table 3 and 4.

Date	Period	Actual demand	Periods since last demand
1/02/2017	3	5	3
1/04/2018	17	2	14
1/05/2018	18	1	1
1/09/2018	22	1	4
1/04/2019	29	3	7
1/06/2019	31	1	2
1/07/2019	32	1	1
1/08/2019	33	1	1
	Total	**15**	

Table 3: SKU 10094 demand pattern

Date	Period	Actual demand	Periods since last demand
1/01/2017	2	45	2
1/03/2017	4	4	2
1/06/2017	7	3	3
1/07/2017	8	4	1
1/09/2017	10	16	2
1/10/2017	11	20	1
1/02/2018	15	52	4
1/03/2018	16	15	1
1/07/2018	20	20	4
1/08/2018	21	10	1
1/09/2018	22	13	1
1/11/2018	24	15	2
1/03/2019	28	15	4
	Total	**232**	

Table 4: SKU 35528 demand pattern

5 Results

Section 5.1 explains what the parameter values are for this case. Section 5.2 compares the accuracy measures ME, MSE and MASE between the different methods, as averages over 50 SKUs. Section 5.3 discusses the results when the parameters are optimized by using the Excel solver option, which is done for 10 different SKUs separately.

5.1 Parameters

As mentioned in section 4, the forecasting methods use smoothing parameters α and β. For research purposes, the values of α and β will range from 0 to 0,4 in steps of 0,05. The forecasting methods are implemented on data from 50 different SKUs with different amounts of demand occurrences and low or highly varying demand sizes. This is done to ensure a broad view of different situations. For closer inspection, the parameters of the methods are optimized for 10 different SKUs separately in the optimization section 5.3.

5.2 Accuracy measures

Note that the conclusions drawn from section 5.2 are based on averages of 50 different SKUs. This means that conclusions may not fit for multiple SKUs in particular and the parameters that seem optimal are by no means optimal parameters for every SKU specifically. Averages are also impacted by possible outliers, therefore it is important to know that conclusions are only correct for the dataset used in this thesis.

5.2.1 Mean error

Table 5 shows the average mean errors from 50 SKUs for each combination of α and β. The ME shows if a method over- or underestimates demand. Table 5 shows that Croston's method indeed mostly overestimates, as mentioned in the literature review. However, the SBA does not seem to be underestimating, as we would expect based on the literature. Although the SBA still overestimates just like Croston's method, for different combinations of α and β the SBA forecasts are more accurate than Croston's method. The TSB method seems to be following a diagonal between α and β indicated by the yellow marked cells in table 5. The parameter combinations α and β below the yellow diagonal show that the TSB method mostly tends to underestimate. Above the yellow diagonal the TSB method tends to overestimate. However, it is clear that the TSB method is more accurate than Croston's method and the SBA as the ME values are much closer to 0. Note that combinations of α and β above the parameters of this thesis (max α of 0,40 and max β of 0,40) may contain better values for ME. The lowest values of the ME for Croston's method and the SBA are indicated by the orange and red cells respectively.

5.2.2 Mean squared error

Table 6 shows the average mean squared errors from 50 SKUs for each combination of α and β. The lower the MSE, the more accurate a forecasting method is. The MSE also shows that the SBA and the TSB method are more accurate than Croston's method, just as the ME already insinuated. However, the MSE shows that the SBA makes lower forecasting error than the TSB for higher values of α and β. This may be the result of outliers when the values of α and β are high. Nevertheless, based on the lowest MSEs, the TSB method performs better than the SBA. Note that combinations of α and β above the parameters of this thesis (max α of 0,40 and max β of 0,40) may contain better values for MSE. The lowest values of MSE for Croston's method, the SBA and the TSB method are indicated by the orange, red and yellow cells respectively.

5.2.3 Mean absolute scaled error

Table 7 shows the mean absolute scaled errors from 50 SKUs for each combination of α and β. As mentioned in section 4.5, the MASE is used to prove if a forecasting method is better than the naïve forecasting method. When the MASE is less than 1, the forecasting method is better than the naïve forecasting. Table 7 shows that the TSB method is better than naïve forecasting when β is around 0,25 or higher. The α does not affect the MASE too much for the TSB method. Croston's method and the SBA seem to always be worse than naïve forecasting as their MASEs are always more than 1. However, when checking the minimum MASEs it is clear that there are cases where Croston's method and the SBA do perform better than naïve forecasting, and get a MASE less than 1. Section 5.3 proves this by giving a closer inspection on some SKUs in particular and optimizing their parameters. Note that combinations of α and β above the parameters of this thesis (max α of 0,40 and max β of 0,40) may contain better values for MASE. The lowest values of MASE for Croston's method, the SBA and the TSB method are indicated by the orange, red and yellow cells respectively.

5.3 Optimized parameters

Excel is widely used across the globe and has many features; one of those features is the Excel solver. When working with formulas that require variables, the Excel solver can optimize the variables within set boundaries so that the output of the formula is optimized (either maximized or minimized). The solver can also be set to change the variables so that the formulas result in a chosen value, e.g. an output of 0. This feature can be handy for many occasions; for this thesis the Excel solver is used to set the smoothing parameters to an optimized value between 0 and 1 for the three different accuracy measures. The first optimized set is where the ME reaches a minimum in its absolute value. The second optimized set is where the MSE reaches a minimum. Finally, a third optimized set is reached when MASE is at its minimum. This process is done for 10 different SKUs. The results of these optimizations can be found in the appendix.

Optimizing parameters to obtain the minimum absolute value of ME, MSE or MASE causes very diverse combinations of α and β, which makes sense following the explanations of the accuracy measures in section 4.5. However, it was still rather unexpected how big these differences in parameters α and β actually were, as there were SKUs where α and/or β went from 0 to 1 depending on the optimized accuracy measure (e.g. table 11a and table 11b).

As already concluded in section 5.2, Croston's method obviously performs worse than the SBA and the TSB method on any SKU and any accuracy measure optimization. However, looking at SKUs 14102, 14764, 15434, 35435 and 35528 it seems that the TSB method does not always perform better than SBA, which is the opposite of what was found during the literature study. This may be caused by the demand patterns in these SKUs. Note that the TSB method does not perform that much worse than SBA in these cases. For the other 5 SKUs where the TSB method does perform better than SBA, the TSB method outperforms SBA almost as much as SBA outperforms Croston's method. This indicates that SBA was an improvement on Croston's method and that the TSB method was an improvement on SBA as also found in the literature study.

When analysing the optimized parameters in comparison to the averaged accuracy measures, it became clear that trying to take the optimal averages of the accuracy measure and their corresponding values of α and β is not very effective. Optimizing the parameters for SKUs separately gave very different values of α and β. Some optimized parameters came close to the optimal parameters from section 5.2. However, the respective accuracy measure values were very different to the averaged accuracy measures values. Because of this it was difficult to compare optimized parameters with the optimized accuracy measures and their respective optimal parameters.

6 Conclusions and insights

The results indicate that the SBA is in fact an improvement on Croston's method as the SBA achieves better accuracy measure values. The literature review mentioned that Croston's method overestimates, which is true according to the ME. However, the SBA was said to underestimate forecasts, which is not confirmed in our experiments. The TSB method does not always outperform the SBA, but both methods are clearly an improvement on Croston's method. Although not mentioned in the literature review, the TSB method seems to have a slight negative bias for different combinations of α and β and a slight positive bias for other combinations of α and β creating a diagonal where the ME approaches 0. However, the ME is not as far away from 0 in comparison to the MEs of Croston's method and the SBA. The MASE shows that Croston's method and the SBA perform worse than the naïve forecasting method for all values of α and β within the boundaries of this thesis (max α of 0,4 and max β of 0,4) for section 5.2. On the other hand the TSB starts performing better than the naïve forecasting method when the value of β reaches 0,25 or higher according to the MASE. Again, it should be kept in mind that the boundaries of this thesis may prevent the visibility of other combinations of α and β where the TSB method performs better than the naïve forecasting method according to the MASE.

Note that these results and conclusions are based on the findings of research done on spare parts SKUs with forecasting methods applied in Excel. There may also be different conclusions to be made when α and β combinations go further than the boundaries set for this thesis (max α of 0,4 and max β of 0,4) in section 5.2. For future research, it is possible to conduct an experiment on the difference in difficulty and performance between applying these methods in Excel or an application of these methods in coding software designed for forecasting and the usage of a bootstrapping method.

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Appendix

Average Mean Error

Average Mean Error									
Method	**β**	**α = 0,05**	**α = 0,10**	**α = 0,15**	**α = 0,20**	**α = 0,25**	**α = 0,30**	**α = 0,35**	**α = 0,40**
Croston	/	0,457	0,434	0,431	0,440	0,457	0,480	0,507	0,538
SBA	***0,05***	0,424	0,458	0,489	0,516	0,541	0,564	0,585	0,604
TSB	***0,05***	-0,009	0,015	0,035	0,053	0,068	0,081	0,093	0,103
SBA	***0,10***	0,336	0,368	0,395	0,420	0,441	0,461	0,479	0,496
TSB	***0,10***	-0,043	-0,019	0,000	0,017	0,032	0,044	0,055	0,064
SBA	***0,15***	0,277	0,307	0,332	0,354	0,374	0,392	0,408	0,423
TSB	***0,15***	-0,066	-0,042	-0,023	-0,006	0,007	0,019	0,030	0,039
SBA	***0,20***	0,234	0,262	0,287	0,307	0,326	0,342	0,356	0,370
TSB	***0,20***	-0,080	-0,057	-0,038	-0,022	-0,009	0,003	0,012	0,021
SBA	***0,25***	0,202	0,229	0,252	0,272	0,289	0,304	0,318	0,330
TSB	***0,25***	-0,090	-0,067	-0,049	-0,033	-0,020	-0,009	0,000	0,009
SBA	***0,30***	0,177	0,203	0,225	0,244	0,260	0,274	0,287	0,299
TSB	***0,30***	-0,097	-0,074	-0,056	-0,041	-0,028	-0,017	-0,008	0,000
SBA	***0,35***	0,156	0,181	0,203	0,221	0,237	0,250	0,262	0,273
TSB	***0,35***	-0,101	-0,079	-0,061	-0,046	-0,034	-0,023	-0,014	-0,007
SBA	***0,40***	0,138	0,163	0,184	0,201	0,217	0,230	0,242	0,252
TSB	***0,40***	-0,104	-0,083	-0,065	-0,050	-0,038	-0,028	-0,019	-0,011

Table 5: Mean error averages from 50 SKUs.

Average Mean Squared Error

Average Mean Squared Error									
Method	**β**	**α = 0,05**	**α = 0,10**	**α = 0,15**	**α = 0,20**	**α = 0,25**	**α = 0,30**	**α = 0,35**	**α = 0,40**
Croston	/	22,148	22,083	22,176	22,366	22,629	22,960	23,358	23,835
SBA	***0,05***	22,045	22,182	22,375	22,621	22,922	23,274	23,677	24,127
TSB	***0,05***	21,261	21,294	21,350	21,420	21,504	21,600	21,710	21,833
SBA	***0,10***	21,795	21,890	22,026	22,199	22,409	22,655	22,936	23,251
TSB	***0,10***	21,174	21,222	21,294	21,379	21,474	21,576	21,687	21,807
SBA	***0,15***	21,701	21,775	21,881	22,015	22,175	22,363	22,577	22,816
TSB	***0,15***	21,193	21,256	21,347	21,451	21,565	21,685	21,813	21,949
SBA	***0,20***	21,679	21,743	21,832	21,944	22,076	22,228	22,401	22,595
TSB	***0,20***	21,265	21,341	21,449	21,572	21,704	21,843	21,989	22,142
SBA	***0,25***	21,696	21,753	21,834	21,932	22,047	22,179	22,327	22,492
TSB	***0,25***	21,368	21,458	21,583	21,724	21,874	22,030	22,194	22,364
SBA	***0,30***	21,730	21,784	21,861	21,953	22,059	22,178	22,310	22,457
TSB	***0,30***	21,497	21,601	21,742	21,900	22,068	22,242	22,422	22,610
SBA	***0,35***	21,769	21,823	21,899	21,989	22,090	22,203	22,327	22,464
TSB	***0,35***	21,645	21,764	21,922	22,098	22,283	22,475	22,673	22,878
SBA	***0,40***	21,805	21,861	21,938	22,029	22,130	22,241	22,363	22,496
TSB	***0,40***	21,812	21,946	22,121	22,315	22,519	22,729	22,944	23,166

Table 6: Mean squared error averages from 50 SKUs.

Average Mean Absolute Scaled Error

Average Mean Absolute Scaled Error									
Method	**β**	**α = 0,05**	**α = 0,10**	**α = 0,15**	**α = 0,20**	**α = 0,25**	**α = 0,30**	**α = 0,35**	**α = 0,40**
Croston	***/***	1,413	1,368	1,344	1,331	1,325	1,325	1,329	1,337
SBA	***0,05***	1,392	1,390	1,390	1,390	1,391	1,391	1,392	1,393
TSB	***0,05***	1,077	1,077	1,076	1,076	1,077	1,077	1,078	1,079
SBA	***0,10***	1,330	1,329	1,328	1,328	1,329	1,329	1,330	1,330
TSB	***0,10***	1,038	1,037	1,036	1,036	1,036	1,036	1,037	1,037
SBA	***0,15***	1,288	1,287	1,286	1,286	1,287	1,287	1,287	1,287
TSB	***0,15***	1,019	1,017	1,016	1,015	1,015	1,016	1,016	1,017
SBA	***0,20***	1,257	1,256	1,255	1,255	1,255	1,255	1,256	1,256
TSB	***0,20***	1,008	1,006	1,004	1,003	1,003	1,003	1,003	1,004
SBA	***0,25***	1,234	1,233	1,232	1,232	1,231	1,231	1,232	1,232
TSB	***0,25***	1,000	0,998	0,996	0,995	0,995	0,994	0,994	0,995
SBA	***0,30***	1,216	1,214	1,213	1,213	1,213	1,213	1,213	1,213
TSB	***0,30***	0,994	0,992	0,990	0,989	0,988	0,988	0,988	0,988
SBA	***0,35***	1,202	1,199	1,198	1,198	1,197	1,197	1,197	1,197
TSB	***0,35***	0,989	0,987	0,985	0,985	0,984	0,983	0,983	0,983
SBA	***0,40***	1,190	1,188	1,186	1,185	1,185	1,185	1,185	1,185
TSB	***0,40***	0,985	0,983	0,982	0,981	0,980	0,980	0,979	0,979

Table 7: Mean absolute scaled error averages from 50 SKUs.

SKU 10094

10094					
Croston's method		**Syntetos Boylan Approximation**		**Teunter Syntetos Babai's method**	
Optimal α	0,358	Optimal α	1,000	Optimal α	0,980
Optimal β	/	Optimal β	0,211	Optimal β	0,610
ME	0,46	ME	0,21	ME	0,00
MSE	1,11	MSE	0,94	MSE	1,16
MASE	1,66	MASE	1,32	MASE	0,92

Table 8a: SKU 10094 absolute value of ME minimized.

10094					
Croston's method		**Syntetos Boylan Approximation**		**Teunter Syntetos Babai's method**	
Optimal α	0,237	Optimal α	0,582	Optimal α	0,157
Optimal β	/	Optimal β	0,000	Optimal β	0,000
ME	0,48	ME	0,26	ME	-0,03
MSE	1,08	MSE	0,93	MSE	0,78
MASE	1,66	MASE	1,33	MASE	0,96

Table 8b: SKU 10094 MSE minimized.

10094					
Croston's method		**Syntetos Boylan Approximation**		**Teunter Syntetos Babai's method**	
Optimal α	0,297	Optimal α	0,914	Optimal α	0,911
Optimal β	/	Optimal β	0,049	Optimal β	0,000
ME	0,46	ME	0,22	ME	-0,12
MSE	1,09	MSE	0,93	MSE	0,81
MASE	1,65	MASE	1,30	MASE	0,85

Table 8c: SKU 10094 MASE minimized.

Demand 10094	
Average size	1,875
Max size	5
Min size	1
Occurrences	8

Table 8d: SKU 10094 demand data.

SKU 10387

10387					
Croston's method		**Syntetos Boylan Approximation**		**Teunter Syntetos Babai's method**	
Optimal α	0,495	Optimal α	1,000	Optimal α	1,000
Optimal β	/	Optimal β	0,833	Optimal β	1,000
ME	0,62	ME	0,24	ME	0,03
MSE	1,85	MSE	1,51	MSE	1,49
MASE	2,38	MASE	1,81	MASE	1,09

Table 9a: SKU 10387 absolute value of ME minimized.

10387					
Croston's method		**Syntetos Boylan Approximation**		**Teunter Syntetos Babai's method**	
Optimal α	0,298	Optimal α	1,000	Optimal α	1,000
Optimal β	/	Optimal β	0,248	Optimal β	0,067
ME	0,67	ME	0,38	ME	0,21
MSE	1,69	MSE	1,33	MSE	1,20
MASE	2,54	MASE	2,04	MASE	1,81

Table 9b: SKU 10387 MSE minimized.

10387					
Croston's method		**Syntetos Boylan Approximation**		**Teunter Syntetos Babai's method**	
Optimal α	0,595	Optimal α	1,000	Optimal α	1,000
Optimal β	/	Optimal β	0,821	Optimal β	1,000
ME	0,63	ME	0,24	ME	0,03
MSE	2,04	MSE	1,51	MSE	1,49
MASE	2,37	MASE	1,81	MASE	1,09

Table 9c: SKU 10387 MASE minimized.

Demand 10387	
Average size	3,33
Max size	4
Min size	2
Occurrences	6

Table 9d: SKU 10387 demand data.

SKU 14102

14102					
Croston's method		**Syntetos Boylan Approximation**		**Teunter Syntetos Babai's method**	
Optimal α	0,482	Optimal α	0,379	Optimal α	1,000
Optimal β	/	Optimal β	0,852	Optimal β	1,000
ME	0,17	ME	0,01	ME	0,01
MSE	0,38	MSE	0,35	MSE	0,63
MASE	1,12	MASE	0,95	MASE	1,03

Table 10a: SKU 14102 absolute value of ME minimized.

14102					
Croston's method		**Syntetos Boylan Approximation**		**Teunter Syntetos Babai's method**	
Optimal α	0,251	Optimal α	0,000	Optimal α	0,295
Optimal β	/	Optimal β	0,431	Optimal β	0,074
ME	0,20	ME	0,09	ME	0,10
MSE	0,35	MSE	0,32	MSE	0,33
MASE	1,16	MASE	1,03	MASE	1,07

Table 10b: SKU 14102 MSE minimized.

14102					
Croston's method		**Syntetos Boylan Approximation**		**Teunter Syntetos Babai's method**	
Optimal α	0,409	Optimal α	0,368	Optimal α	0,461
Optimal β	/	Optimal β	0,785	Optimal β	0,791
ME	0,17	ME	0,01	ME	0,02
MSE	0,36	MSE	0,34	MSE	0,48
MASE	1,12	MASE	0,95	MASE	0,98

Table 10c: SKU 14102 MASE minimized.

Demand 14102	
Average size	1,3
Max size	2
Min size	1
Occurrences	10

Table 10d: SKU 14102 demand data.

SKU 14144

14144					
Croston's method		**Syntetos Boylan Approximation**		**Teunter Syntetos Babai's method**	
Optimal α	0,254	Optimal α	0,194	Optimal α	1,000
Optimal β	/	Optimal β	0,681	Optimal β	1,000
ME	0,75	ME	0,44	ME	0,03
MSE	2,25	MSE	1,69	MSE	2,70
MASE	1,83	MASE	1,52	MASE	1,00

Table 11a: SKU 14144 absolute value of ME minimized.

14144					
Croston's method		**Syntetos Boylan Approximation**		**Teunter Syntetos Babai's method**	
Optimal α	0,150	Optimal α	0,042	Optimal α	0,025
Optimal β	/	Optimal β	0,571	Optimal β	0,162
ME	0,79	ME	0,48	ME	0,26
MSE	2,13	MSE	1,63	MSE	1,37
MASE	1,85	MASE	1,52	MASE	1,17

Table 11b: SKU 14144 MSE minimized.

14144					
Croston's method		**Syntetos Boylan Approximation**		**Teunter Syntetos Babai's method**	
Optimal α	0,212	Optimal α	0,114	Optimal α	0,720
Optimal β	/	Optimal β	0,547	Optimal β	0,644
ME	0,76	ME	0,46	ME	0,06
MSE	2,18	MSE	1,65	MSE	1,85
MASE	1,83	MASE	1,50	MASE	1,00

Table 11c: SKU 14144 MASE minimized.

Demand 14144	
Average size	2,22
Max size	7
Min size	1
Occurrences	9

Table 11d: SKU 14144 demand data.

SKU 14258

14258					
Croston's method		**Syntetos Boylan Approximation**		**Teunter Syntetos Babai's method**	
Optimal α	0,531	Optimal α	1,000	Optimal α	1,000
Optimal β	/	Optimal β	1,000	Optimal β	0,670
ME	0,59	ME	0,23	ME	0,01
MSE	1,25	MSE	0,79	MSE	1,09
MASE	1,94	MASE	1,28	MASE	0,95

Table 12a: SKU 14258 absolute value of ME minimized.

14258					
Croston's method		**Syntetos Boylan Approximation**		**Teunter Syntetos Babai's method**	
Optimal α	0,313	Optimal α	0,657	Optimal α	0,305
Optimal β	/	Optimal β	1,000	Optimal β	0,000
ME	0,62	ME	0,23	ME	0,09
MSE	1,23	MSE	0,79	MSE	0,69
MASE	1,97	MASE	1,28	MASE	1,01

Table 12b: SKU 14258 MSE minimized.

14258					
Croston's method		**Syntetos Boylan Approximation**		**Teunter Syntetos Babai's method**	
Optimal α	0,496	Optimal α	1,000	Optimal α	1,000
Optimal β	/	Optimal β	1,000	Optimal β	0,389
ME	0,59	ME	0,23	ME	0,01
MSE	1,25	MSE	0,79	MSE	0,88
MASE	1,93	MASE	1,28	MASE	0,94

Table 12c: SKU 14258 MASE minimized.

Demand 14258	
Average size	2,2
Max size	5
Min size	1
Occurrences	5

Table 12d: SKU 14258 demand data.

SKU 14764

14764					
Croston's method		**Syntetos Boylan Approximation**		**Teunter Syntetos Babai's method**	
Optimal α	0,243	Optimal α	0,106	Optimal α	1,000
Optimal β	/	Optimal β	0,715	Optimal β	1,000
ME	0,16	ME	0,00	ME	0,01
MSE	0,45	MSE	0,44	MSE	0,80
MASE	1,06	MASE	0,95	MASE	0,98

Table 13a: SKU 14764 absolute value of ME minimized.

14764					
Croston's method		**Syntetos Boylan Approximation**		**Teunter Syntetos Babai's method**	
Optimal α	0,125	Optimal α	0,000	Optimal α	0,000
Optimal β	/	Optimal β	0,247	Optimal β	0,033
ME	0,18	ME	0,06	ME	0,06
MSE	0,44	MSE	0,41	MSE	0,04
MASE	1,06	MASE	0,95	MASE	0,97

Table 13b: SKU 14764 MSE minimized.

14764					
Croston's method		**Syntetos Boylan Approximation**		**Teunter Syntetos Babai's method**	
Optimal α	0,179	Optimal α	0,000	Optimal α	0,000
Optimal β	/	Optimal β	0,640	Optimal β	0,133
ME	0,17	ME	-0,01	ME	0,02
MSE	0,44	MSE	0,42	MSE	0,41
MASE	1,05	MASE	0,92	MASE	0,95

Table 13c: SKU 14764 MASE minimized.

Demand 14764	
Average size	1,31
Max size	2
Min size	1
Occurrences	13

Table 13d: SKU 14764 demand data.

SKU 15434

15434					
Croston's method		**Syntetos Boylan Approximation**		**Teunter Syntetos Babai's method**	
Optimal α	0,717	Optimal α	0,601	Optimal α	1,000
Optimal β	/	Optimal β	0,000	Optimal β	1,000
ME	0,16	ME	0,00	ME	0,01
MSE	1,43	MSE	1,43	MSE	3,07
MASE	0,96	MASE	0,79	MASE	0,97

Table 14a: SKU 15434 absolute value of ME minimized.

15434					
Croston's method		**Syntetos Boylan Approximation**		**Teunter Syntetos Babai's method**	
Optimal α	0,374	Optimal α	0,669	Optimal α	0,264
Optimal β	/	Optimal β	0,799	Optimal β	0,000
ME	0,17	ME	-0,01	ME	0,07
MSE	1,42	MSE	1,40	MSE	1,40
MASE	0,97	MASE	0,82	MASE	0,87

Table 14b: SKU 15434 MSE minimized.

15434					
Croston's method		**Syntetos Boylan Approximation**		**Teunter Syntetos Babai's method**	
Optimal α	0,727	Optimal α	1,000	Optimal α	1,000
Optimal β	/	Optimal β	0,643	Optimal β	0,000
ME	0,16	ME	-0,11	ME	-0,90
MSE	1,43	MSE	1,44	MSE	1,52
MASE	0,96	MASE	0,71	MASE	0,71

Table 14c: SKU 15434 MASE minimized.

Demand 15434	
Average size	2,25
Max size	7
Min size	1
Occurrences	8

Table 14d: SKU 15434 demand data.

SKU 35435

35435					
Croston's method		**Syntetos Boylan Approximation**		**Teunter Syntetos Babai's method**	
Optimal α	0,824	Optimal α	0,661	Optimal α	1,000
Optimal β	/	Optimal β	0,779	Optimal β	0,570
ME	0,69	ME	0,00	ME	0,00
MSE	19,29	MSE	17,03	MSE	21,29
MASE	1,22	MASE	0,96	MASE	0,96

Table 15a: SKU 35435 absolute value of ME minimized.

35435					
Croston's method		**Syntetos Boylan Approximation**		**Teunter Syntetos Babai's method**	
Optimal α	0,419	Optimal α	1,000	Optimal α	0,000
Optimal β	/	Optimal β	0,307	Optimal β	0,038
ME	0,97	ME	0,08	ME	0,29
MSE	17,89	MSE	16,00	MSE	16,45
MASE	1,32	MASE	0,97	MASE	1,12

Table 15b: SKU 35435 MSE minimized.

35435					
Croston's method		**Syntetos Boylan Approximation**		**Teunter Syntetos Babai's method**	
Optimal α	0,794	Optimal α	1,000	Optimal α	1,000
Optimal β	/	Optimal β	0,817	Optimal β	0,155
ME	0,69	ME	-0,28	ME	-0,19
MSE	18,90	MSE	16,89	MSE	16,83
MASE	1,22	MASE	0,84	MASE	0,88

Table 15c: SKU 35435 MASE minimized.

Demand 35435	
Average size	9,14
Max size	20
Min size	2
Occurrences	7

Table 15d: SKU 35435 demand data.

SKU 35528

35528					
Croston's method		**Syntetos Boylan Approximation**		**Teunter Syntetos Babai's method**	
Optimal α	0,692	Optimal α	0,648	Optimal α	0,904
Optimal β	/	Optimal β	0,788	Optimal β	0,570
ME	3,04	ME	0,00	ME	0,00
MSE	122,47	MSE	109,95	MSE	155,13
MASE	1,09	MASE	0,89	MASE	0,90

Table 16a: SKU 35528 absolute value of ME minimized.

35528					
Croston's method		**Syntetos Boylan Approximation**		**Teunter Syntetos Babai's method**	
Optimal α	0,644	Optimal α	0,000	Optimal α	0,000
Optimal β	/	Optimal β	0,775	Optimal β	0,097
ME	3,05	ME	-0,31	ME	0,65
MSE	122,41	MSE	106,31	MSE	111,92
MASE	1,09	MASE	0,85	MASE	0,90

Table 16b: SKU 35528 MSE minimized.

35528					
Croston's method		**Syntetos Boylan Approximation**		**Teunter Syntetos Babai's method**	
Optimal α	0,970	Optimal α	0,000	Optimal α	0,000
Optimal β	/	Optimal β	1,000	Optimal β	0,440
ME	3,18	ME	-0,82	ME	-0,35
MSE	126,71	MSE	107,35	MSE	128,89
MASE	1,06	MASE	0,80	MASE	0,82

Table 16c: SKU 35528 MASE minimized.

Demand 35528	
Average size	17,85
Max size	52
Min size	3
Occurrences	13

Table 16d: SKU 35528 demand data.

SKU 36987

36987					
Croston's method		**Syntetos Boylan Approximation**		**Teunter Syntetos Babai's method**	
Optimal α	0,314	Optimal α	0,563	Optimal α	0,712
Optimal β	/	Optimal β	0,313	Optimal β	0,646
ME	0,00	ME	0,00	ME	0,00
MSE	315,31	MSE	320,09	MSE	362,42
MASE	1,17	MASE	1,18	MASE	1,06

Table 17a: SKU 36987 absolute value of ME minimized.

36987					
Croston's method		**Syntetos Boylan Approximation**		**Teunter Syntetos Babai's method**	
Optimal α	0,092	Optimal α	0,191	Optimal α	0,000
Optimal β	/	Optimal β	0,000	Optimal β	1,000
ME	-2,50	ME	-1,65	ME	-4,04
MSE	304,04	MSE	301,37	MSE	291,04
MASE	0,95	MASE	1,01	MASE	0,76

Table 17b: SKU 36987 MSE minimized.

36987					
Croston's method		**Syntetos Boylan Approximation**		**Teunter Syntetos Babai's method**	
Optimal α	0,000	Optimal α	0,000	Optimal α	0,000
Optimal β	/	Optimal β	0,173	Optimal β	1,000
ME	-3,84	ME	-3,93	ME	-4,04
MSE	306,96	MSE	309,59	MSE	291,04
MASE	0,83	MASE	0,83	MASE	0,76

Table 17c: SKU 36987 MASE minimized.

Demand 36987	
Average size	39,86
Max size	92
Min size	12
Occurrences	7

Table 17d: SKU 36987 demand data.