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Analysis of two Equivalent Circuit Models for State of Charge Estimation using Kalman Filters

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Abstract—Battery State of Charge estimation is critical to improve performance, lifetime, and safety of batteries used in agricultural robotic systems. This paper analyzes two different Equivalent Circuit Models that can be used for State of Charge estimation of batteries. We analyze the RC model proposed by the National Renewable Energy Laboratory and the 2RC Thevenin model. The main objective of this study is to determine the advantages and limits of equivalent circuit models for online State of Charge tracking. We performed, for each model, an estimation of the parameters offline. Also, the Recursive Least Square technique was implemented for one of the models to analyze the effect of the online estimation in battery modeling. Then, we implemented and analyzed two Kalman Filters for State of Charge estimation: Linear Kalman Filter and Unscented Kalman Filter. Finally, we provide simulation results of models implemented using MATLAB that we evaluated using experimental data from tests performed on a Panasonic 18650PF Li-ion Battery.

Index Terms—Equivalent Circuit Model, Battery, SoC estimation, Kalman Filters, RLS, Parameter estimation.

I. INTRODUCTION

Agricultural robots can be used to increase the productivity of farms while reducing the environmental impact of agriculture. They offer several advantages over the heavy agricultural machine that uses fossil fuels. The implementation of robots in farms has several benefits, like improving precision farming, reducing soil damages due to heavy machinery, and reducing the use of pesticides. The advantages of robots can be used to produce more food, ensure sustainability and sufficient food supply to the growing population while replacing fossil fuels with green and efficient energy sources, as recommended by the Food and Agriculture Organization of the United Nations [1]. However, to spread the use of robots in farms, it is necessary to improve some aspects, like reducing battery charging time, increasing the operation time, and implementing efficient energy management strategies [2]. The Battery State of Charge (SoC) is critical information to solve the problems related to energy management of the robot and increase the presence of robots in farms.

The SoC of the battery provides information about the remaining energy in the battery and is a critical parameter to ensure the safe and efficient operation of the robot's battery. Also, using accurate information about the battery SoC, the battery's performance and lifetime can be increased.

Therefore, the SoC is essential to decide on the operation of the agricultural robots and could help to avoid battery damage [3]. For example, the good estimation of SoC can prevent over-discharge, overcharge, and unexpected shutdowns due to battery depletion. However, the battery SoC cannot be measured directly and must be estimated using operation information, like the voltage, current, temperature, and aging.

The SoC estimation for batteries is a complicated task. Although several methods for SoC estimation have been presented in the literature, improving the estimation accuracy is still a significant challenge [4]. One popular method is the Open Circuit Voltage (OCV) method. This method uses a lookup table with the relationship between the OCV and the SoC of the battery. However, the OCV method is unsuitable for mobile robotics applications because the battery should rest for an extended period to measure the battery's OCV. Another conventional and well-known method is the Coulomb counting method that consists of integrating the current that exits and enters the battery. However, several limitations exist, such as the need to know the initial battery SoC, the error introduced by the current sensor over time, and the error introduced by the battery maximum capacity that varies over time and under different operational conditions.

Recently, several SoC estimation techniques based on artificial intelligence have been presented. However, the amount of data needed and the computational cost to implement AI-based strategies are considerable. Several tests must be performed, the test times are long, and the battery life is reduced with each test. Therefore, the significant amount of data needed to perform an accurate SoC estimation makes these approaches impractical [5].

Several model-based methods have been presented recently to overcome the limitations of the conventional approaches. The SoC model-based methods represent the battery behavior using state equations and can accurately describe the behavior of the battery. Although an accurate battery model is essential to improve the accuracy of the SoC estimation, it is also important to keep the model as simple as possible to be implemented in embedded systems suitable for agricultural robots. One of the challenges of developing a battery model is to take into account the effect of the battery's temperature, aging, and hysteresis while keeping the model simple. One of the most popular modeling approaches for batteries is the Equivalent Circuit Model (ECM). ECM consists of a circuit containing several resistors, capacitors, and sources to model the dynamic behavior of the battery terminal voltage [6]. Two popular models are the 2RC Thevenin model and the RC model developed by the National Renewable Energy Laboratory (NREL) [7], [8].

Some SoC estimation methods have been implemented for mobile robotics applications. For example, in [9] the SoC of a battery-powered wheeled mobile robot is performed using an Unscented Kalman Filter (UKF). The proposed approach estimated the SoC of a battery with a voltage of 37 V and a capacity of 8 Ah. Another SoC estimation implementation for robotics is presented in [10] where the battery SoC is estimated with an H ∞ observer. The method was validated using an inspection robot powered by a battery with a voltage of 25 V and a capacity of 18 Ah. Although these methods showed accurate results in their intended applications, implementing them in another application like agricultural robots may affect their performance. Therefore, it is important to develop SoC estimation methods adapted to agricultural robots.

This paper aims to compare the two circuit models mentioned before to identify the advantages and limitations of each one. The results of this research will determine which model is more suitable for battery SoC estimation. Also, the SoC estimation will be performed using each model to compare their performance and accuracy. This paper introduces two model state-space representations and each model's parameter estimation process in Section II. Section III presents the results of the SoC estimation methods for the two models. Finally, the concluding remarks, current challenges, and proposed future work in SoC estimation are discussed in section IV.

II. BATTERY MODELING

An accurate battery model is needed to improve the performance of the Battery SoC estimation. In this section, the 2RC Thevenin and RC models are studied, and each model's parameter identification process is explained.

A. RC Model - Offline Parameter Estimation

The RC Model, shown in Fig. 1, consists of a terminal resistance Rt, a bulk capacitor Cb, a surface capacitor Cs, a bulk resistance Rb and a surface resistance Rs. The terminal resistance models the voltage drop when the battery has a load connected. Rs and Cs model the diffusion effects of the battery. Vcs is the voltage of capacitor Cs. Vo is the battery's terminal voltage and I is the battery's current. The bulk capacitor represents the battery storage capacity, and its voltage, Vcb, represents the battery Open Circuit Voltage (OCV). Therefore, the Vcb values can be used to estimate the SoC using the OCV vs. SoC curve of the battery. However, Vcb cannot be measured and must be estimated. The statespace expressions describing the model are the following:



Fig. 1. NREL RC Equivalent Circuit Model [8].



Fig. 2. HPPC Test data from [11].

$$\begin{bmatrix} \dot{Vcb} \\ \dot{Vcs} \end{bmatrix} = \begin{bmatrix} -\frac{1}{Cb*(Rb+Rs)} & \frac{1}{Cb*(Rb+Rs)} \\ \frac{1}{Cs*(Rb+Rs)} & -\frac{1}{Cs*(Rb+Rs)} \end{bmatrix} * \begin{bmatrix} Vcb \\ Vcs \end{bmatrix} + \begin{bmatrix} \frac{Rs}{Cb*(Rb+Rs)} \\ \frac{Rb}{Cs*(Rb+Rs)} \end{bmatrix} * \begin{bmatrix} I \end{bmatrix}$$
(1)

$$\begin{bmatrix} Vo \end{bmatrix} = \begin{bmatrix} \frac{Rs}{Rb+Rs} & \frac{Rb}{Rb+Rs} \end{bmatrix} * \begin{bmatrix} Vcb\\Vcs \end{bmatrix} + \begin{bmatrix} Rt + \frac{RsRb}{Rb+Rs} \end{bmatrix} * \begin{bmatrix} I \end{bmatrix}$$
(2)

The Hybrid Pulse Power Characterization (HPPC) test dataset provided in [11] was utilized to estimate the parameters of the RC model. The HPPC test shown in Fig. 2 is used to obtain the Battery OCV vs. SoC curve and the circuit components' initial values (Cb, Rb, Cs, Rs, and Rt). The HPPC test consists of pulses separated by a rest period that allows the battery to reach a stable state. By analyzing a single pulse, like the one shown in Fig. 3, the circuit model parameter values can be obtained using an approach similar to the one presented in [12].

The bulk capacitance Cb is derived from the capacitor energy equation by using the following equation:

$$Cb = \frac{2 * Q * V_{100\% SoC}}{V_{100\% SoC}^2 - V_{0\% SoC}^2}$$
(3)

Where Q is the Battery nominal capacity in ampere-second. $V_{0\% SoC}$ and $V_{100\% SoC}$ are the battery's voltage at 0% SoC and 100% SoC, respectively.



Fig. 3. Single pulse analysis.



Fig. 4. Relaxation period curve fitting.

The battery's total resistance can be found by obtaining the voltage variation $(V_2 - V_1)$ when a load or discharge pulse (I_{pulse}) is applied to the battery. Equation (4) shows the expression for the battery's total resistance.

$$R_{total} = \frac{V_2 - V_1}{I_{pulse}} \tag{4}$$

The values of Rs and Rb are equal and 75% of the Battery total resistance [12]:

$$Rb = Rs = 0.75 * R_{total} \tag{5}$$

The terminal resistance Rt is obtained by the following equation:

$$Rt = R_{total} - Rb/2 \tag{6}$$

The surface capacitor Cs is obtained by performing a custom curve fitting using MATLAB *fit* function to the first 500ms of the current pulse. Fig. 4 shows the curve fitting results. The following equation was used as *fittype*:

$$Vstart + (Vend - Vstart) * (1 - exp(-x/tau))$$
 (7)

Where V start is the value at the beginning of the relaxation period, Vend is the voltage after 500ms of relaxation. Using the time constant tau obtained from the curve fitting we can find the Cs using the following equation:

$$Cs = \frac{tau}{Rs} \tag{8}$$

Table I shows the initial parameters estimated offline for the RC model of the Panasonic 18650PF Li-ion battery.

 TABLE I

 INITIAL PARAMETER ESTIMATION FOR THE RC MODEL OF THE

 PANASONIC 18650PF LI-ION BATTERY

Value
107.64F
12217F
0.0157Ω
0.0157Ω
0.0131Ω

Since the model parameters do not change along the battery SoC, the performance of the model will be affected. Fig. 5(a) and Fig. 6 show the performance of the battery model. There is an offset between the estimated and measured battery voltage. However, the dynamic behavior of the model follows the dynamic behavior of the measured voltage.

B. RC Model - Online Parameter Estimation

The RLS method was implemented to improve the accuracy of the battery model using MATLAB and Simulink. The following transfer function of the model is used to define the Parameter Vector θ and the Regressors φ .

$$\frac{V_o(s)}{I(s)} = \frac{\left(\frac{Rb}{2} + Rt\right)s^2 + \frac{(Rb+Rt)(Cb+Cs)}{2\ Cb\ Cs\ Rb}s + \frac{1}{2\ Cb\ Cs\ Rb}}{s^2 + \frac{Cb+Cs}{2\ Cb\ Cs\ Rb}\ s}$$
(9)

The Regressors φ and the Parameter Vector θ are defined as follows:

$$\varphi(t) = -\int V_o(t) + I(t) + \int I(t) + \iint I(t) \quad (10)$$

$$\hat{\theta}_{est} = \begin{bmatrix} a_1 \ b_1 \ b_2 \ b_3 \end{bmatrix}^T$$
(11)

Where:

$$a_1 = \frac{Cb + Cs}{2 \ Cb \ Cs \ Rb} \tag{12}$$

$$b_1 = \frac{Rb}{2} + Rt \tag{13}$$

$$b_2 = \frac{(Rb + Rt)(Cb + Cs)}{2 \ Cb \ Cs \ Rb}$$
(14)

$$b_3 = \frac{1}{2 \ Cb \ Cs \ Rb} \tag{15}$$

This method can update the parameters of the Equivalent Circuit Model Online; then, the model can consider several factors that affect the battery, like aging and capacity fade. Also, there is no need to perform the extended HPPC test, reducing the testing time and the manual work to estimate the parameters online. Fig. 5(b) and Fig. 7 show that using RLS for parameter estimation, the performance of the model is improved.



Fig. 5. (a) Modeled battery voltage compared to measured voltage and voltage error using Offline Estimated Parameters, (b) modeled battery voltage compared to measured voltage and voltage error using RLS, (c) modeled battery voltage compared to measured voltage and voltage error using 2RC Thevenin Model with 68 SoC levels.



Fig. 6. Modeled battery voltage compared to measured voltage and voltage error using Offline Estimated Parameters.



Fig. 7. Modeled battery voltage compared to measured voltage and voltage error using RLS.

C. 2RC Thevenin Model

The 2RC Thevenin model consists of two RC network that models the diffusion effects of the battery. This model also has a terminal resistance and an SoC-dependent voltage source that models the battery OCV. In contrast with the RC model presented before, this model has a nonlinear behavior, and SoC cannot be estimated using Linear Kalman filters. Fig. 8 shows the 2RC Thevenin model. The following equations are the mathematical expressions that model the system:

$$\dot{SoC} = -\frac{\eta I}{Q} \tag{16}$$

$$\dot{V}_1 = -\frac{V_1}{R_1 C_1} + \frac{I}{C_1} \tag{17}$$

$$\dot{V}_2 = -\frac{V_2}{R_2 C_2} + \frac{I}{C_2} \tag{18}$$



$$V_o = V_{ocv}(SoC) - V_1 - V_2 - IR_t$$
(19)

Where η is the coulomb efficiency of the battery, *I* battery's current, *Q* is the battery capacity in ampere-second, and the



Fig. 9. (a) Simulation results compared to measured voltage and voltage error using 2RC Thevenin model and 30 SoC levels, (b) simulation results compared to measured voltage and voltage error using 2RC Thevenin model and 15 SoC levels, (c) simulation results compared to measured voltage and voltage error using 2RC Thevenin model and 15 SoC levels, (c) simulation results compared to measured voltage and voltage error using 2RC Thevenin model and 15 SoC levels, (c) simulation results compared to measured voltage and voltage error using 2RC Thevenin model and 15 SoC levels, (c) simulation results compared to measured voltage error using 2RC Thevenin model and 15 SoC levels, (c) simulation results compared to measured voltage error using 2RC Thevenin model and 15 SoC levels, (c) simulation results compared to measured voltage error using 2RC Thevenin model and 15 SoC levels, (c) simulation results compared to measured voltage error using 2RC Thevenin model and 11 SoC levels.

voltages of the first and second RC branches are V_1 and V_2 , respectively.

The parameters of this model were estimated offline using the approach proposed in [13]. Each pulse of the HPPC test is analyzed individually and represents a SoC level. Fig. 3 shows a single pulse of this test. A set of parameters can be extracted for each SoC Level. Since we have 68 pulses, we can have up to 68 SoC levels. The instant when the load is connected to the battery is utilized to estimate the model OCV V_{ocv} and terminal resistance R_t . The V_{ocv} is the voltage at the end of the relaxation period of the pulse, and R_t can be obtained as in (4) using (20).

$$R_t = \frac{V_2 - V_1}{I_{pulse}} \tag{20}$$

The relaxation period that starts when the load is removed and ends when the next pulse starts is used to obtain the time constants of the two RC networks. A MATLAB *fit* was used to get the two time constants values by using the *fittype* with the expression shown in (21).

$$Vo = V_{start} + V_1 e^{-\frac{t}{tau1}} + V_2 e^{-\frac{t}{tau2}} - V_1 - V_2$$
(21)

The resistances values of each RC network are estimated by analyzing the battery response during the pulse. The *lsqlin* MATLAB function was used to obtain these values.

The initial parameter estimation presented before was used to obtain a preliminary version of the model parameters. Using Simulink Design Optimization to analyze each pulse, the initially estimated parameters can be refined. The reason to make an initial estimation is to avoid local minima and facilitate the optimization of the parameters. The final model parameters are validated using the LA92 Drive cycle using the dataset from [11]. Fig. 5(c) compares the simulation results of the model using 68 SoC levels with the measured battery voltage. Also, this article aims to compare the performance of the 2RC Thevenin model using 30, 15, and 11 SoC levels. The results of each model are presented in Fig. 9(a), Fig. 9(b), and Fig. 9(c).

III. SOC ESTIMATION RESULTS

A. Offline estimation using RC model

The SoC of the battery can be estimated using the voltage across the bulk capacitor and comparing these values with the OCV vs. SoC Curve. To estimate the bulk capacitor voltage, we implemented a linear Kalman filter. Using the state space representation of the model, we can implement a Kalman filter using MATLAB and Simulink. Fig. 10(a) shows the performance of the SoC estimation using a Kalman Filter and the offline parameters of the model. The values of the error covariance matrix P, process error covariance Q, and measurement error covariance R are:

$$P_0 = \begin{bmatrix} 1 & 0\\ 0 & 1 \end{bmatrix}; Q = \begin{bmatrix} 0.008 & 0\\ 0 & 0.088 \end{bmatrix}; R = 1000$$
(22)

We can see how the SoC estimation error increases at low SoC levels. Using the Vcb as the battery OCV has provided good performance at high SoC levels; however, the SoC estimation performance at different SoC levels is not good. This model could be improved by making the circuit parameters dependent on the SoC. Also, we compared the same model using a different value of measurement error covariance. The Fig. 11 shows the results of the SoC estimation using another R=10; we can observe how a bad initialization of the filter parameters can affect the accuracy of the estimation.

B. Online estimation using RC model

Fig. 10(b) shows the results of the SoC estimation using online parameter estimation. The estimation is heavily affected since the parameters are changing constantly. In order to improve the performance of this filter, we need to implement a system that can tune the Kalman Filter. Also, we should



Fig. 10. (a) Estimated SoC compared to Reference SoC (Coulomb Counting Method) and error using offline parameter estimation and Kalman Filter, (b) estimated SoC compared to reference SoC and error using online parameter estimation and Kalman Filter, (c) estimated SoC compared to reference SoC and error using 2RC Thevenin model with 68 SoC levels.



Fig. 11. Estimated SoC compared to Reference SoC (Coulomb Counting Method) and Relative error, R = 10.

consider using the Extended or Unscented Kalman Filters (EKF or UKF) which are adapted for nonlinear systems.

C. 2RC Thevenin Model

Since the model based on the state-space representation is nonlinear, the state space representation can be expressed as:

$$\dot{x} = f(x, u) \tag{23}$$

$$y = h(x, u) \tag{24}$$

$$\begin{bmatrix} \dot{SoC} \\ \dot{V}_1 \\ \dot{V}_2 \end{bmatrix} = \begin{bmatrix} -\frac{\eta I}{Q} \\ -\frac{V_1}{C_1} + \frac{I}{C_1} \\ -\frac{V_2}{C_2} + \frac{I}{C_2} \end{bmatrix}$$
(25)

$$V_o = V_{ocv}(SoC) - V_1 - V_2 - IR_t$$
(26)

Using equations (25) and (26), we can obtain the state estimates and the output estimation of the model. Furthermore, we can easily implement a State of Charge estimator using the



Fig. 12. Estimated SoC compared to reference SoC (Coulomb Counting Method) and error using 2RC Thevenin model with 11 SoC levels.

Unscented Kalman Filter (UKF) Block from MATLAB and Simulink. The Fig. 10(c) shows the performance of the SoC estimation using a UKF to estimate the SoC of the battery.

It is important to notice that the number of parameters affects the State of Charge estimation. Fig. 12 shows the model's performance using 11 SoC levels.

We can see that with a reduction of the parameters, the accuracy of the SoC estimation is reduced. However, it is important to note that the filter's initial states and parameters were not modified. Then, we could expect a good SoC estimation if we tune the UKF initial parameters for each model.

IV. CONCLUSION

This paper presented a study of two Equivalent Circuit Models used for SoC estimation. We observed that the SoC estimation using the 2RC Thevenin model offers a better performance. Also, the results of this study show how parameter estimation for the 2RC Thevenin model can improve the accuracy of the SoC estimation. However, the parameters for the 2RC model were estimated offline and did not consider the battery's temperature and aging effects.

Online parameter estimation techniques improve the model performance while reducing the number of tests needed to build the model. This study implemented an online parameter estimation for the RC model using RLS, avoiding the need for an extended HPPC test to find the model parameters. Also, online parameter estimation can be performed using metaheuristic optimization algorithms, Kalman Filters, and Artificial Intelligent techniques. However, for agricultural robots application, special attention should be given to the complexity of the parameter estimation process because new techniques should be implemented in embedded systems. Therefore, future work on battery modeling should focus on estimating the parameters online while keeping the parameter estimation process simple.

Also, we have seen the importance of the initial parameters of the Kalman filters. Having a wrong initialization will reduce the accuracy of the SoC estimation. Therefore, finding an optimal way to tune the Kalman filters is critical to use these approaches to estimate the battery's SoC.

Several tests must be performed using agricultural robots in farms to validate the performance of these methods. Although some methods have shown good performance under simulations and laboratory environments, validation under actual farm conditions is needed to confirm the effectiveness and accuracy of the SoC technique.

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