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Faculteit Bedrijfseconomische Wetenschappen

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Masterthesis

The newsvendor problem: A review

Stan Bellu

Scriptie ingediend tot het behalen van de graad van master handelsingenieur, afstudeerrichting operationeel management en logistiek

PROMOTOR :
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The newsvendor problem: A review

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The newsvendor problem has become indispensable for optimising inventory management for products with short life cycles. Due to the ever-increasing competition and the scarcer resources, finding the optimal solution to maximise the profit (or minimise the costs) has never been more critical. Any deficiency in the supply leads to enormous consequences in the supply chain. This paper studies the traditional problem and several extensions in pricing policies (markup pricing and clearance pricing), buyer risk (Conditional Value-at-Risk, shortage costs and emergency shipments), supply uncertainty, information value, and newsvendor networks to gain insights into the impact on the optimal order quantity. In the case of markup pricing, the newsvendor is searching for the ideal increase in price to cover the costs while maximising its profit. It is found that under the clearance pricing, the newsvendor has to balance the order quantity without damaging the brand. Buyer risk also influences the inventory strategy. Typically, a risk-averse investor prefers stockout over excess inventory. However, the general conclusion only holds in some cases when accounting for shortage costs (generally higher inventory level) and emergency shipments (typically lower inventory level). In addition, supply uncertainty and the availability of information may not be forgotten as they potentially have a severe impact on the optimal order quantity. Lastly, newsvendor networks are used for interconnected or multi-location systems where multiple newsvendors operate within a more extensive network or supply chain. These networks involve coordinating and managing inventory across multiple locations to meet customer demand efficiently. A retailer must understand his environment to construct a model that considers relevant variables to balance interpretability and complexity.

Keywords: Newsvendor problem, Buyer risk, Conditional value at risk (CVaR), Price policy, Newsvendor networks, Machine Learning, Inventory control, Supply uncertainty

1. Introduction

The newsvendor problem is a classic problem in inventory management that revolves around determining the optimal quantity of a product to order or to produce in a single selling season, given uncertain customer demand. The name "newsvendor problem" originated from the scenario of a newspaper vendor who needs to decide how many newspapers to buy each day to meet the demand (Dai & Meng, 2015). The fundamental challenge in the newsvendor problem is balancing the trade-off between stocking too few or too many items. On the one hand, if the vendor purchases a quantity lower than the actual demand, they will lose potential sales and profit. On the other hand, if the vendor buys an excessive amount, they will incur costs associated with remaining inventory and potentially have to discard unsold products, leading to waste and decreased salvage value (Poormoaied & Hosseini, 2021).

The problem becomes more complex when uncertain demand cannot be accurately predicted. In real-world situations, customer demand for products can vary due to changing market conditions, consumer preferences, and external events. This uncertainty makes it challenging for vendors to make optimal stocking decisions. In addition, the newsvendor problem has gained significance during the last decades since competition has heavily increased. It started as a simple, single-period problem applicable to several products but has become a key towards efficiency and profitability (Alwan, Xu, Yao, & Yue, 2016).

The newsvendor problem aims to maximise the vendor's profit or minimise the expected costs. To achieve this, the vendor must find the stocking quantity that balances the potential lost sales (due to understocking) and the costs associated with excess inventory (due to overstocking) (Qin, Wang, Vakharia, Chen, & Seref, 2011). Efficient inventory management, as addressed by the newsvendor problem, is crucial for businesses to meet customer demand, minimise costs, and improve profitability. By understanding and solving the newsvendor problem, companies can optimise their stocking decisions, reduce inventory holding costs, minimise lost sales, and enhance overall supply chain performance. The problem has now evolved towards a heavily studied problem regarding optimising inventory management of products with short life cycles under uncertain demand (Punia, Singh, & Madaan, 2020).

Over the years, researchers and practitioners have developed various approaches and techniques to tackle the newsvendor problem. These include mathematical models, statistical forecasting methods, optimisation algorithms, and decision-support systems (Qin et al., 2011). The objective is to provide vendors with tools and strategies to make informed decisions about stocking quantities, considering the uncertainty in customer demand. The applications are relevant to the newspaper industry and any business dealing with perishable or time-sensitive products. For example, it applies to industries such as fashion, food, electronics, and seasonal products (Poormoaied & Hosseini, 2021).

This literature study will present a structured overview of recent developments within the newsvendor problem. The goal is to provide related insights, opinions and limitations understandably so that managers and other stakeholders understand the newsvendor problem and all it entails. The remainder of this paper is organised as follows: Section 2 elaborates on the traditional newsvendor problem and sets the basis for the more complex scenarios. Section 3 briefly explains the methodology used for writing this review. Section 4 comprises the influence of specific pricing policies (markup and clearance pricing). Section 5 entails several situations (shortage costs and emergency shipments) considering the risk retailers are willing to take. Section 6 investigates the impact of supply uncertainty and the value of information on the optimal order quantity. Section 7 gives a general overview of the contents and advantages of newsvendor networks, and the last section discusses the results and indicates some points of attention and directions for future research.

2. The traditional newsvendor problem

Before diving into the intricacies of several extensions of the newsvendor problem, the traditional newsvendor problem (NVP), also called the single period problem (SPP), will be thoroughly explored as it sets the tone regarding the rest of the thesis. This straightforward situation, with a particularly clear-cut solution, is a vital component of the literature regarding inventory theory (Petruzzi & Dada, 1999) (Khanra, Soman, & Bandyopadhyay, 2014).

The traditional newsvendor problem refers to a decision-making scenario in which a retailer must determine the optimal order quantity for perishable products. The newsvendor faces random demand for a product that becomes outdated after a certain period (Petruzzi & Dada, 1999) (Khouja, 1999). The newsvendor problem has played a crucial role in establishing the theoretical basis of stochastic inventory management. Variations of this problem have been applied to analyse decision-making scenarios, including capacity control and product allocation, all operating on the fundamental assumption of uncertain demand (Güler, Körpeoğlu, & Şen, 2018).

The initial model consists of a risk-neutral newsvendor (i.e., he only cares about maximising the expected profits, not about the inherent uncertainty of the profits) looking for the optimal quantity of newspapers to buy at a regular purchase price (Dai & Meng, 2015). The newsvendor is a price taker, meaning he cannot influence the price he sells to the end customers. Moreover, it is assumed that the newsvendor faces an exogenous (influenced by external factors or variables outside the control of the decision-maker or the system being analysed), stochastic (i.e., it exhibits randomness or variability over time) demand. He only has one purchasing opportunity, and he places the expected profit-maximising order well before the selling period (Vipin & Amit, 2017). The problem’s objective is to maximise profit and minimise costs by constructing an appropriate order strategy to satisfy customer demand (X. Xu, Wang, Dang, & Ji, 2017). Hence, the order quantity is the deciding factor in obtaining the highest profit, as it accounts for the expected overstocking cost due to unsold inventory and the net value of the salvage price (Mitra, 2018).

Demand is implicitly or explicitly assumed to be independent and identically distributed in each sales period to solve the traditional problem. This results in a relatively straightforward answer of the optimal order quantity to generate the maximum profit. The profit function can be written as follows (X. Xu, Meng, Ji, Dang, & Wang, 2016):

$$\prod(q, D) = (p * \min(q, D)) - c * q + r(q - D)^{+} \qquad (1)$$

Let $\prod(q, D)$ be the profit generated by the chosen order quantity and demand, D stands for the demand for the product, p is the unit sales price, q represents the newsvendor’s ordered quantity, c is the wholesale price per unit of product, and r is the salvage value (the remaining value of the product in case it is not sold in the foreseen period). Min(q, D) represents the minimum of the order quantity and the demand (which is the quantity sold), and $(q\text{-}D)^{+}$ represents the positive value of the difference between the order quantity and the demand (which means the unsold inventory that incurs holding costs). If this last term is negative, it automatically equals zero. The goal is to combine all these factors to maximise profit by finding the optimal order quantity (X. Xu et al., 2016).

This formula assumes the demand is continuous and follows a known probability distribution. Moreover, it assumes the order quantity is a constant variable, which can take any (positive) value. This operational problem can be modelled as a single-period, single-item inventory decision-making problem and solved using various methods, including optimisation techniques. The solution for such a problem considers the demand distribution, the purchasing costs, the cost of lost sales, the costs of holding the product, and the selling price (Qin et al., 2011). By determining the optimal pricing policy, the retailer considers two key factors: the cost of overstocking and understocking. Thus, the pricing policy in the newsvendor problem aims to strike a balance between these two costs. The optimal amount to order should be set according to the following optimality condition (Khouja, 1999) (Qin et al., 2011):

$$F(Q^*) = \frac{P+S-C}{P+S-V} = \frac{p+B-v}{p+B-g} = \frac{C_u}{C_u + C_o} \qquad (2)$$

The left equation is retrieved from Khouja (1999), and the middle and right equations are found in Qin (2011). All equations represent the optimality condition. Let F(Q*) represent the cumulative distribution function of the demand up to order quantity Q where the asterisk (*) denotes optimality, P and p represent the selling price per unit, S and B are the shortage penalty cost, C and v stand for the cost per item, V and g substitute for the salvage value. The numerator in the formulas equals C_u (cost of underage), which is the cost incurred when demand exceeds the available inventory, resulting in lost sales or unfulfilled customer orders. C_o (cost of overage) refers to the expenses incurred each time an order is placed, or replenishment is made, such as order processing and transportation. The fraction $C_u/(C_u+C_o)$ allows us to assess the relative importance of holding cost per unit compared to the total cost per unit. It quantifies the proportion of the total cost attributed to holding costs. On the one hand, if $C_u/(C_u+C_o)$ is close to 1, the holding cost per unit is a significant portion of the total cost. This suggests that minimising the holding cost by optimising inventory levels is crucial for cost control. On the other hand, if $C_u/(C_u+C_o)$ is close to 0, it implies that the ordering cost per order dominates the total cost. In such cases, reducing the ordering cost or streamlining the ordering process becomes a priority for cost optimisation (Khouja, 1999; Qin et al., 2011).

In the simple newsvendor problem, forecasting methods, such as exponential smoothing and regression analysis, can be used. However, the traditional ordering policy falls short in terms of recognising important real-life factors that may influence the optimal order quantity, such as the risk profile of the newsvendor (risk-averse vs risk-neutral), pricing policies (markup pricing and clearance pricing) and the more complex real-world settings where multiple retailers try to outperform their competitors using more advanced techniques (newsvendor networks) (Qin et al., 2011), which will be discussed later in this paper.

Ordering too much inventory undisputedly leads to obsolete items (which may, at best, be sold at a salvage value) (Poormoaied & Hosseini, 2021), whereas not ordering enough entails a loss in sales. Therefore, it is in the vendors' best interest to predict the demand accurately to prevent over- or understocking and pursue the expected profit-maximising (EPM) order quantity (X. Xu et al., 2018). If the demand conceived is less than the required demand, the demand will be satisfied, and the leftover inventory holds a value equal to the salvage price. When the demand cannot be satisfied, which means there is excess demand, the newsvendor loses out on any opportunity regarding additional profit. So, in the classic model, the newsvendor neither influences the price nor can he put in any marketing effort to impact the market demand, and the model balances marginal expected profit against the risk of overstocking to reach the critical quantile of the demand distribution (Güler et al., 2018) (Dai & Meng, 2015) (Qin et al., 2011) (X. Xu et al., 2017).

In the following sections of this literature review, some variants of the traditional newsvendor problem will be studied. The manuscript is organised as follows. Section 3 first details the methodology used for this review. Next, section 4 discusses pricing policies, particularly markup pricing (section 4.1) and clearance pricing (section 4.2). Section 5 focuses on the complexities related to buyer risk, in particular, the general explanation of buyer risk and risk-neutral behaviour (section 5.1), the impact of shortage costs (section 5.2) and emergency shipments (section 5.3) on the optimal order quantity. Section 6 examines the effects of supply uncertainty (section 6.1) and the value of information (section 6.2) on the optimal order quantity. In section 7, newsvendor networks are discussed, focusing on insights to optimise the model. Section 8 discusses the conclusions and future research directions.

3. Methodology

This literature review is based on journal articles from Q1 and Q2 journals (see Fig. 1 below), based on the impact factor ranking on Web of Science. The journal impact factor is an often-used quality metric for journals, with Q1 indicating the highest quality and Q4 the lowest quality. Texts that were not classified as journal articles, such as conference papers etc., or did not have a ranking, have been excluded.

The following databases were used: Web of Science, Ebscohost and Google Scholar. The search terms included newsvendor problem, newsvendor networks, buyer risk, risk-neutral, risk-averse, Conditional Value-at-Risk (CVaR), emergency shipments, shortage costs, price policy, markup pricing, clearance pricing, machine learning, newsvendor networks, supply uncertainty, and the influence of information in the title and abstract. Another review paper on the newsvendor problem was published in 2011 (Qin et al., 2011). Therefore, this thesis consists primarily of articles published afterwards.

After collecting a substantial number of articles (43), the abstract, introduction and conclusion were read to obtain a general overview of the potential contribution to this review. Within these articles, the cascade principle has been applied in case a subject needs more clarification, and these articles have been included in the bibliography.

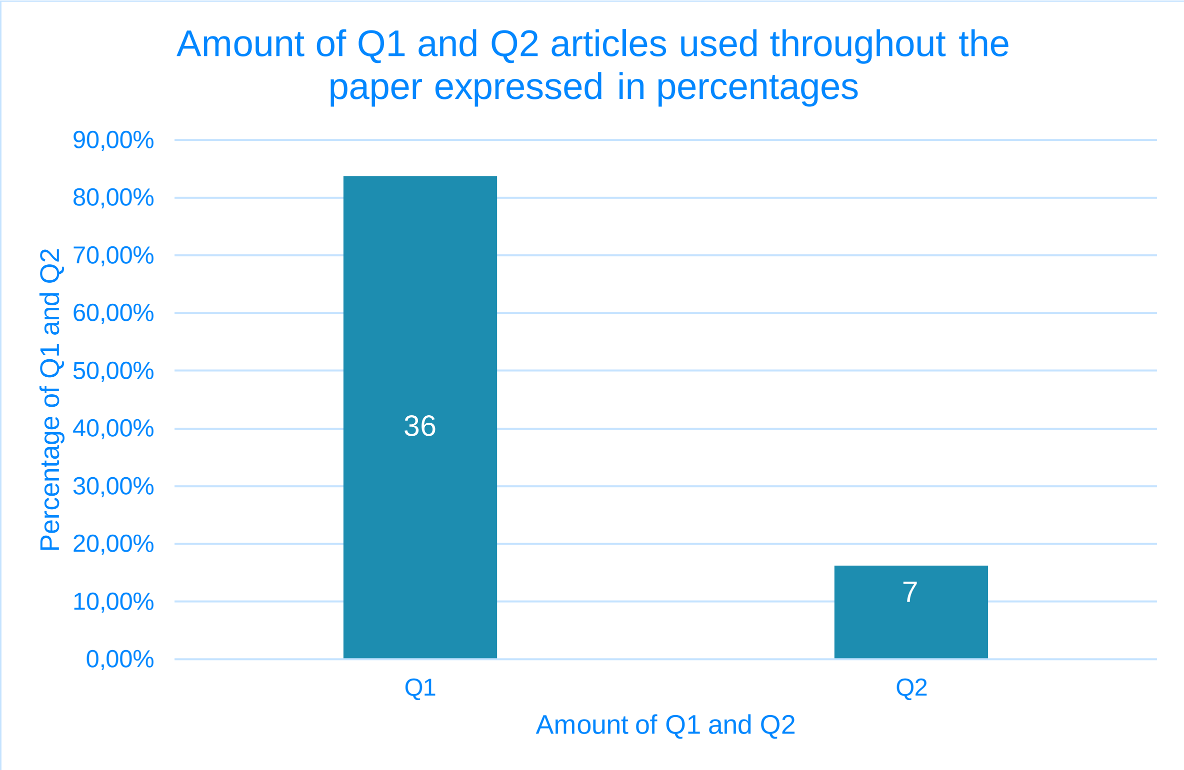

Figure 1: Percentage of Q1 and Q2 articles used in this literature study

4. Impact of pricing policy on optimal order quantity

This section will investigate a first extension of the newsvendor problem: pricing policy. Pricing policy in the newsvendor problem refers to the decision-making process of setting the selling price to the final customers for a product in uncertain demand. As stated in section 2, the newsvendor problem is an inventory management problem that tries to optimise its order quantity to maximise the expected profit by balancing overstocking costs versus understocking costs. The selling price of the product directly influences the profit generated from sales. By setting the selling price appropriately, the retailer can affect the demand for the product. Expensive products might prevent potential customers from buying the product. This results in lower demand but additionally in decreased possibility of overstocking. However, reduced prices can persuade more customers to purchase the item. This increases demand yet amplifies the risk of understocking. Therefore, pricing policies can be perceived as a valuable resource to control profit. The impact of a successful pricing strategy can be substantial, as the price is one of the critical determinants of demand (Canyakmaz, Özekici, & Karaesmen, 2022).

However, it is also possible that a retailer obtains a discount when ordering the items from the manufacturer. An example of such a price decrease in pricing policy is the quantity discount, i.e., based on the number of units ordered. By cost saving during the acquiring process, the retailer will more likely offer a reduced price to its customers, leading to a lower unit sales price (Qin et al., 2011). In general, three common types of quantity discounts are widely adopted: Linear quantity discount

(LD), All-units discount (AU) and incremental units quantity discount (IU). The first discount scheme results in a price decrease per unit the retailer orders (the more items ordered, the higher the discount on the total price, without an initial threshold). The second one refers to the retailer buying more than a prespecified threshold so that the cost per unit decreases significantly (after reaching a certain amount, the price per unit declines drastically at a fixed percentage rate). The third one leads to a discount after reaching a certain break point, which becomes higher per unit bought (the more items ordered, the more the percentage increases) (Khouja, 1999). While price break quantities can be candidates for the global optimum for an all-units quantity discount, this will not be the case when an incremental quantity discount is applied. Some industries also have to deal with input price volatilities, which constrain and determine the selling price to the customer.

The optimal pricing policy in the newsvendor problem is typically derived through mathematical modelling and optimisation techniques. It considers factors such as the cost structure, estimated demand distribution, and the desired service level (probability of meeting demand). By analysing these factors, businesses can determine the price that maximises expected profit, considering the trade-off between overstocking and understocking costs. It's important to note that the pricing policy in the newsvendor problem is closely related to the broader pricing strategy of a business. Other considerations, such as market competition, customer perception, and long-term business goals, may influence the pricing decision alongside the optimisation approach specific to the newsvendor problem.

In the rest of this section, markup pricing will be investigated, particularly dollar and percentage markup. Next, clearance pricing, also known as salvage pricing, will be explored thoroughly.

4.1 Markup pricing

In the newsvendor problem, markup pricing refers to a strategy used by the retailer to set the selling price of a product. Markup pricing contracts have been widely applied in many industries. Under these contracts, a retailer charges a margin over the wholesale price the supplier asks to guarantee a financial buffer. Markup pricing is one method that the retailer can apply to regulate the product’s selling price. Using markup pricing enables the retailer to adjust the selling price by including a fixed percentage or amount to the cost of the product. This markup is intended to cover the expenses incurred by the retailer and provide a profit margin. It determines how a firm makes its own decisions regarding ordering and pricing decisions when selling a product with random fluctuating input prices (Y.-Y. Wang, Wang, & Shou, 2013). Canyakmaz, Özekici and Karaesmen (2022) consider a single-period, single-item setting in which the stochastic behaviour of the input price is seen as a general continuous price process. Moreover, the individual customer arrival process is price dependent. This means that a specific demand is closely related to the price level, which allows the authors to investigate the impact of a volatile market on the optimal order quantity. This leads to insights into the implications of market price volatility on expected sales revenue and profit (Canyakmaz et al., 2022).

To apply markup pricing in the newsvendor problem, the retailer first estimates the cost of acquiring each product unit from the supplier. This cost may include the purchase price, transportation costs, and any other associated expenses. Then, the retailer adds a markup percentage or amount to the cost to determine the selling price. The specific markup percentage or amount used can vary depending on market conditions, competition, and the retailer's pricing strategy. The markup should be set to cover the costs and generate a reasonable profit margin, considering factors like demand uncertainty and the potential for too much inventory or stockouts. By using markup pricing in the newsvendor problem, the retailer aims to increase the profit while minimising the risk of stockouts or excessive inventory. It allows the retailer to consider the costs and potential profits associated with the product, helping managers make deliberate decisions about order quantities and pricing to optimise their overall profitability (J.-C. Wang, Wang, & Wang, 2013) (DeYong, 2020).

The dollar and percentage markup can result in the same selling price. However, the difference lies in how the markup is calculated. The dollar markup adds a fixed amount to the cost price, while the percentage markup adds a percentage of the cost price. Retailers can choose between dollar and percentage markup based on their pricing strategies, market conditions, and other factors. The choice between the two approaches can impact the profit margin and competitiveness of the product. Both percentage markups (%-markup, expressed as a percentage of the wholesale price) and dollar markups ($-markup, defined as an absolute margin that is added to the wholesale price) occur in academic research, although %-markup is most often adopted in business textbooks (J.-C. Wang et al., 2013). In general, when comparing $-markup contracts and %-markup contracts, the percentage markup contracts will influence the wholesale price much more since the impact of a different price is more significant.

In Wang et al. (2013), one retailer and two manufacturers interact on the market. All players are risk-neutral and play a non-cooperative game. Therefore, the two manufacturers are involved in a game where they simultaneously determine their wholesale price and production quantity. Moreover, the manufacturers provide substitution products at a constant production unit cost. The decision sequence under these assumptions is as follows. Firstly, the retailer offers a markup contract specifying the profit margin over the amount every manufacturer would charge. Secondly, each manufacturer responds by choosing a corresponding price and production quantity. Lastly, the demand is realised, and payments will be made between retailer and manufacturers (J.-C. Wang et al., 2013).

To investigate the ideal markup, the authors are examining a retailer who vends for a solitary product, and this product has an inherent commodity price value that undergoes continuous evolution over time. The retailer determines its inventory at the beginning of the season and receives orders immediately. The shape of the objective function tells us that an order-up-to-inventory policy is the best choice when a specific markup alpha (ɑ) in both markup strategies is given. By analysing the optimal order-up-to level, it can be understood how the customer demand process affects the price (Canyakmaz et al., 2022).

4.1.1 Deterministic vs stochastic demand situation

Before investigating Wang's (2013) model, the difference between deterministic and stochastic demand must first be explained. The critical difference is the level of uncertainty. Deterministic demand assumes a constant and predictable demand, while stochastic demand acknowledges that demand is subject to random fluctuations and uncertainty (J.-C. Wang et al., 2013). When making decisions related to inventory management, production planning, pricing, or ordering, understanding the nature of demand is crucial. In a deterministic demand situation, the findings in a %-markup contract are made in two sequential steps. In the first step, the retailer decides the retail percentage margin over the wholesale prices. In the second step, given this retail percentage margin, each manufacturer simultaneously selects the wholesale price and, consequently, the order quantity. The optimal quantity equilibrium is found by following these steps to maximise the retailer's profit. Given the retail percentage margin, the manufacturer selects the wholesale price to maximise his profit (J.-C. Wang et al., 2013) (Canyakmaz et al., 2022). The formula used in this paper is very mathematical. However, the retail margin also applies to the formula presented in section 2 by adjusting the price as it needs to account for the markup.

In Wang's model, the authors investigated the impact of price sensitivity (beta β) and level of manufacturer differentiation (gamma γ) on several parameters (retail percentage margin, wholesale price, selling price, order quantity and retailer/manufacturer profit) using a different markup system. Price sensitivity refers to the degree to which customer demand for a product is changed by transforming its selling price. This concept arises when determining the optimal order quantity. When customers are immensely price-sensitive, the retailer could pick a larger order quantity and decide upon a reduced selling price to stimulate demand. On the other hand, if customers are less price-sensitive, the retailer might choose a higher order quantity and set a higher selling price to maximise profit. Manufacturer differentiation refers to the situation where multiple manufacturers or suppliers offer the same product but with different characteristics or attributes since each manufacturer might have unique features, quality levels, or service levels associated with their product, such as supplier costs, lead times and reliability (J.-C. Wang et al., 2013).

Some exciting insights arise when investigating the effect of beta (price sensitivity parameter) and gamma (level of manufacturer differentiation) on the profit. On the one hand, as for an increase/decrease in β, the same effect occurs when implementing a %-markup or a $-markup, keeping all other variables constant. A decline in β pushes the demand (order quantity) to increase and all other parameters (retail percentage margin, wholesale price, selling price, retailer/manufacturer profit) to decrease. On the other hand, an increase in γ entails, for the most part, a contrasting effect, considering all parameters in the dollar markup and the retail percentage margin, price, manufacturer's profit and quantity in case of percentage markup. However, a change in gamma does not affect the wholesale price and retailer's profit. The opposite effect occurs when the parameters change, and the adaptation is reversed (J.-C. Wang et al., 2013).

In reality, it is challenging to acquire an optimal solution for any dollar markup contract when manufacturers are asymmetric (manufacturers who possess different levels of power, resources, capabilities, or market position compared to their counterparts in the supply chain) due to the depth of the variables (numerous factors and considerations need to be considered since the variables can be complex and interconnected). However, newsvendor networks, further explained in section 7, have the potential to solve the equation accurately. When manufacturers are symmetric ($c_1 = c_2$), lower wholesale prices can be achieved since the demand becomes more elastic, putting pressure on manufacturers to set lower prices. This leads to lower retail prices and, consequently, higher order volumes, which moderate the double marginalisation effect and raise the profit for the whole distribution channel (J.-C. Wang et al., 2013) (Canyakmaz et al., 2022).

Under the stochastic demand situation, a similar approach can be used. In the first step, the retailer decides the retail margin, and in the second step, given this retail margin, each manufacturer optimises his wholesale price and production quantity (J.-C. Wang et al., 2013). In Figure 2, the difference in influence per type of markup policy is illustrated for a given example. By implementing a dollar markup, the manufacturer continuously acquires the same margin on his production. At the same time, a retailer stands much closer to the customer and can quickly adapt to demand uncertainty. Consequently, raising the retail price potentially reduces the expected demand as fewer customers will be interested in buying the product (J.-C. Wang et al., 2013).

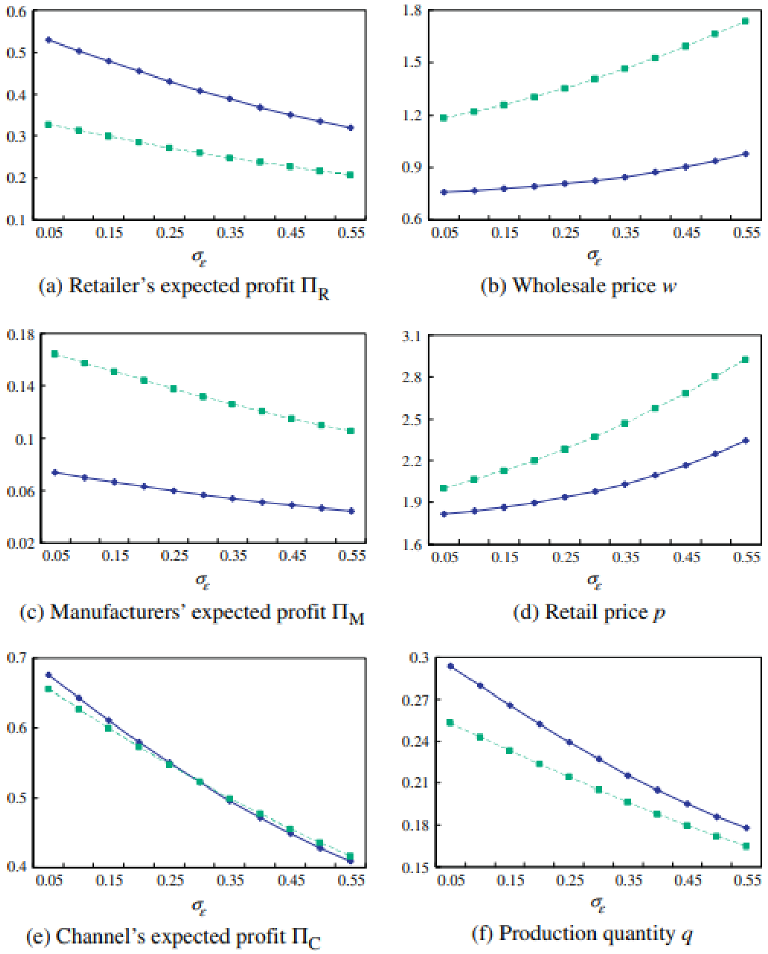

Figure 2: Effect of sigma where alpha = 1, c1 = c2 = 0.5, c0 = 0.15, beta = 3 and gamma = 1; "---" for percentage markup and "---" for dollar markup contracts (J.-C. Wang et al., 2013)

Subgraph (e) shows the expected profit of the supply chain (manufacturer + retailer); in this example, the type of markup has little influence. However, the impact varies for all other factors (subgraphs a-d and f) (J.-C. Wang et al., 2013). A scenario where a manufacturer and retailer choose a different markup contract has not been investigated in this paper.

4.2 Clearance pricing

Another form of pricing policy is clearance pricing, which comes into play when the selling season ends and the retailer is left with unsold inventory. The retailer can choose to reduce the price of the remaining units to stimulate demand and increase the likelihood of selling goods. It aims to incentivise customers to purchase the product, even though the season will end soon. By implementing clearance pricing, the retailer seeks to minimise losses associated with excessive stock (Mitra, 2018). While the clearance price may result in lower profit margins on the remaining units, the retailer can recover some revenue and avoid potential disposal costs. In the fundamental newsvendor problem and most extensions, a standard hypothesis has been that the clearance price for unsold units at the end of the period, also known as *salvage value*, is a given parameter. Most operations management literature assumes an exogenously set clearance price (independent of the other variables in the system). It uses the traditional single-period newsvendor model to establish the optimal order quantity. Post-season discounted or clearance sales are the most widespread way to clear leftover inventory (Avittathur & Biswas, 2017).

Clearing pricing deserves attention since retailers need to be cautious when determining discounts, as a disproportionate markdown of the remaining inventory potentially leads to profit reduction and (even worse) brand dilution. Over the past few decades, the markdown trend indicates that ordering too many items and, consequently, being stuck with high clearance inventory is becoming common in retailing. The marginal profit can even become negative when the salvage price falls below the marginal cost. In Avittathur (2017), the authors do not expressly state the exact level. However, when looking at the provided examples, the profit will decrease if the marginal cost of an extra unit exceeds the salvage value, which intuitively seems correct. In certain circumstances, the retailer preferably clears part of the leftover inventory by selling it at a salvage price and disposing of the remaining units for free (Avittathur & Biswas, 2017) (Mitra, 2018). The reasons to opt for free disposal are cost savings, image and reputation, efficiency and simplicity, and opportunity cost. Firstly, cost savings occur when the items require transaction costs, handling costs, or potential returns that still have to be paid for even though the products are sold for a low selling price. Secondly, price levels are often linked to image and reputation, as selling products for a low price might create the perception of low value or poor quality amongst customers, resulting in a negative impact in the long term perspective. Furthermore, disposing of items simplifies the clearance process (which can be linked to cost savings, allowing the retailer to focus on other aspects). Lastly, selling items against reduced prices generates little profit. In such cases, the resources, time, and effort spent selling the items for a low price could be better utilised elsewhere. Disposing of items frees up these resources for more productive activities. Nonetheless, it is essential to understand that the disposal of items needs to be carefully considered since various factors (specific circumstances, the costs involved, the potential impact on the brand, and the overall business strategy) have an effect in the long run. In some situations, selling items for a low price might still be a preferable option, especially if there is a possibility of recovering some revenue or other intentions for the inventory (Avittathur & Biswas, 2017) (Yu, Zhu, & Wang, 2013).

The basic clearance pricing model has different periods for clearance sales after the regular selling season. Demands are defined as endogenous (the level of demand that arises from within an economic system itself), stochastic and potentially correlated (Mitra, 2018). The amount of units sold during the clearance period is a function of the residual inventory and end-of-season demand. The following model implements order quantity and salvage price as decision variables and incorporates a single-period newsvendor problem with an endogenous end-of-period demand (i.e., influenced by the relationships and interactions within the system itself (Avittathur & Biswas, 2017)). It is assumed that the order quantity at the beginning of the regular selling season is decided independently from the clearance sale. Moreover, the excessive stock level is considered the initial inventory for the first clearance sale. Consequently, the decision maker determines the salvage price depending on the resources and information at hand, such as the leftover inventory and the end-of-period demand function (Mitra, 2018).

Three models are developed depending on whether the demands are exogenous or endogenous and deterministic or stochastic. The authors developed these three models to investigate the influence of clearance pricing on profit. The first model discusses the newsvendor problem when the single-period demand is exogenous and stochastic, and the end-of-period demand is endogenous and deterministic. The second model differs from the scope of the first model by assuming that the end-of-period demand is stochastic instead of deterministic. Finally, the third model assumes that both the demand functions are endogenous and stochastic. In the first two models, the newsvendor operates in a perfectly competitive market during the regular period. Hence, the retailer is a price-taker in the regular period, while the retailer is a price-setter at the end of the period. The third model simulates that the newsvendor is a monopolist and price setter in both the regular period and the end-of-period. The models consider the end-of-season demand parameter indicating market potential, the end-of-season demand parameter indicating the sensitivity of demand to price (price elasticity), the variable cost per unit, the mean seasonal demand, and the seasonal demand variance. All variables are inspected separately, meaning that when one variable changes, all others are kept constant (ceteris paribus) (Mitra, 2018).

For all models, it is seen that the optimal expected profit is very sensitive to the cost per unit, mean seasonal demand and market potential. Market potential indicates the maximum level of sales or revenue that a company or industry can generate, and it is estimated based on various factors such as the size of the target market, the demographic characteristics of the target customers, their purchasing power, and the overall economic conditions. Intuitively, the influence of price sensitivity can be confirmed since the expected profit and order quantity depend on the total number of units sold. When seasonal demand or market potential is high, the profit rises accordingly. Moreover, if the cost per unit or sensitivity of demand to price decreases, the profit and the optimal order quantity rise. However, when the seasonal demand variance increases, the order quantity increases but profit declines due to the required safety stock level. The opposite is also true in all models. For example, if the market potential decreases, the total profit shrinks with the optimal order quantity (Mitra, 2018).

5. Buyer risk

In the context of the newsvendor problem, buyer risk refers to the risk that the retailer is willing to take when making inventory ordering decisions. In reality, a retailer does not possess perfect information about future demand. Therefore, the possibility arises that the ordered quantity does not align with the actual demand. On the one hand, if the retailer orders insufficient products, he probably faces a stockout, which leads to missing sales and unhappy customers. On the other hand, if the retailer orders items abundantly, they could end up with excess inventory, ultimately leading to holding costs and markdowns (Ma, Zhao, Xue, Cheng, & Yan, 2012) (Qin et al., 2011). In the traditional newsvendor problem, the retailer is considered risk neutral and strives to maximise profit. Risk preference and (in)direct factors are closely related since understanding individual risk preferences and considering both direct and indirect factors is essential for effective decision-making under uncertainty. It helps in assessing and managing risks appropriately. Direct factors include all elements that directly affect the cost or revenue associated with the inventory decision. Examples are the cost of ordering inventory, the product's selling price and the unsold inventory's salvage value. Indirect factors comprise the elements that affect the demand for the product and, therefore, indirectly, the inventory decision. Examples are market demand, seasonal variations and market competition (Dai & Meng, 2015).

5.1 Risk-averse behaviour: terminology

Risk aversion refers to the retailer's tendency to avoid or minimise the potential negative consequences of uncertain customer demand. In some situations, a risk-averse retailer in the newsvendor problem is more concerned about the potential costs and losses associated with stockouts rather than the costs of holding excessive inventory. They prioritise avoiding running out of stock and disappointing customers over the risk of having too much inventory (X. Xu et al., 2017). Therefore, a newsvendor needs to hedge (a risk management technique used to minimise or offset potential losses) against potential risks for the loss-averse newsvendor, which has developed into an interesting case. The main question of this section is how to balance loss aversion and find the optimal order quantity (X. Xu et al., 2018). To explain the impact of risk-averse behaviour on the optimal order quantity, the term Conditional Value-at-Risk (CVaR) is essential to understand. Conditional Value-at-Risk measures the expected losses beyond a certain confidence level, accounting for the worst-case scenarios. By incorporating CVaR into the newsvendor problem, decision-makers can evaluate the potential downside risks and make more informed decisions that consider both the expected profit and the risk of stockouts (Dai & Meng, 2015) (Murarka, Sinha, Thakur, & Tiwari, 2019).

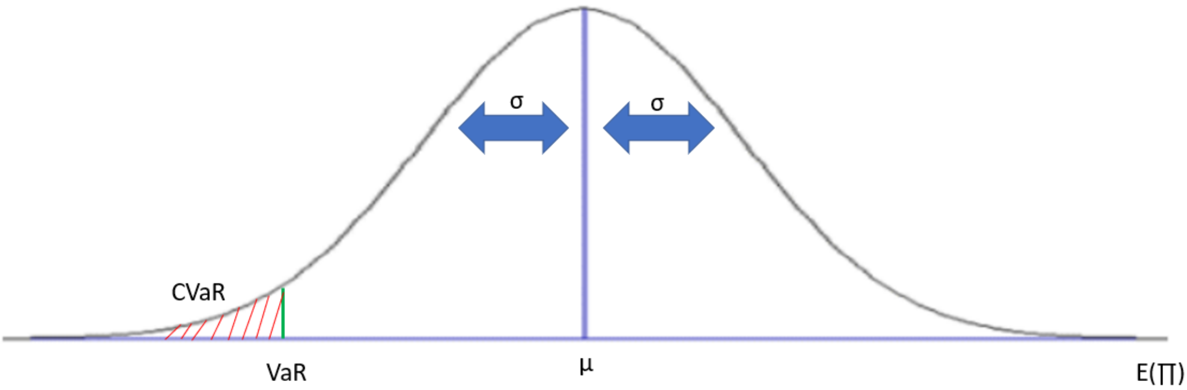

Figure 3: Graphical illustration of the CVaR concept

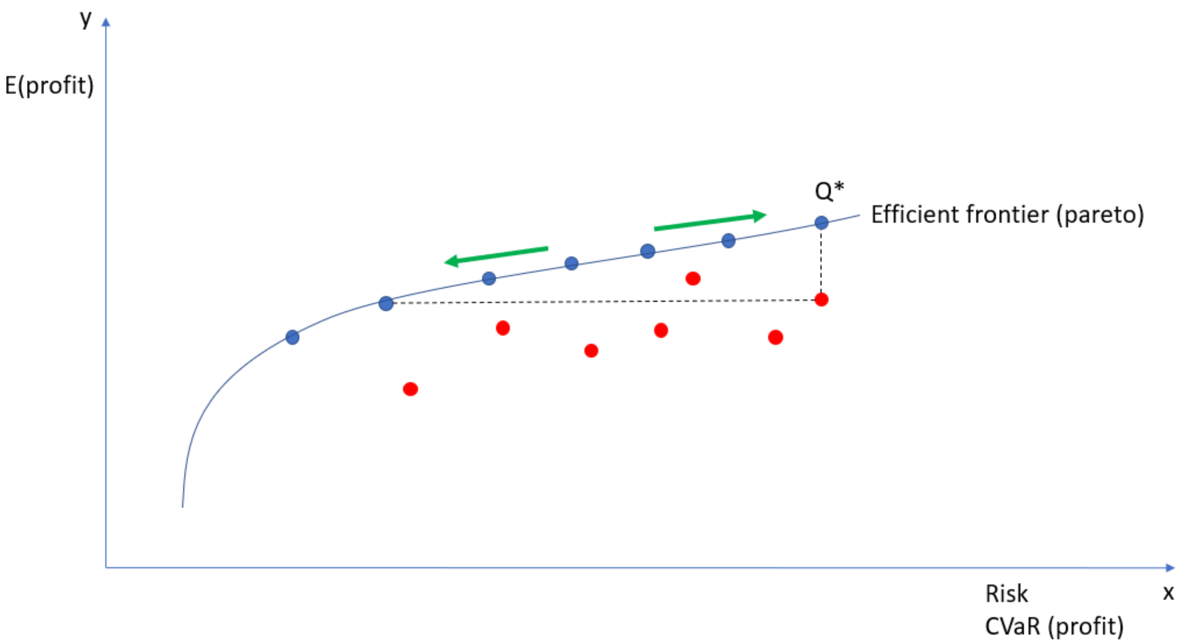

Figure 4: Graphical illustration of efficiency under CVaR in case of profit

In Figures 3 and 4, the concept of Conditional-Value-at-Risk is graphically illustrated. In Figure 3, a normal probability function is shown, where μ represents the average expected profit, σ stands for the standard deviation of the expected profit, and the red area under the graph equals the CVaR (lower tail) depending on the chosen level (typically 5%). The area can grow or shrink depending on the retailer's threshold. The VaR (Value-at-Risk) is closely related to CVaR and refers to the maximum amount a retailer is expected to lose at a specified confidence level. In Figure 4, an efficiency function is illustrated. On the x-axis, a measure of risk (CVaR) is shown. The further right on the axis, the more risk the decision-maker takes. On the y-axis, the expected profit is projected. Generally, the higher the risk the retailer takes, the higher the expected profit. The blue graph represents the efficient frontier of expected profit, and the blue dots represent a possible Q* (optimal order quantity under specific conditions). In contrast, the red dots are best avoided at all costs since these do not equal an optimal solution. Out of the red dot, a vertical projection must be made on the blue graph to find the optimal order quantity to acquire the optimal solution under the same amount of risk.

Intuitively, it is difficult to assess whether a loss-averse newsvendor keeps smaller inventory levels to minimise the risk of potential losses from overstocking or keeps a larger inventory to prevent understocking. Retailers may prioritise avoiding excessive inventory and associated holding costs, even if the risk of stockouts increases. This decision aligns with the desire to minimise potential losses. However, if the variance of the loss-averse utility is substantial, the chance of deviating from the optimal expected loss-averse function will be severe (Ma et al., 2012). So, the expected utility maximisation measure is insufficient to grasp the outcome for some newsvendors fully. Therefore, it is essential to consider the CvaR (X. Xu et al., 2018) (Murarka et al., 2019). The results show a renowned trade-off between the risk and expected returns. Risk-averse retailers generally keep a lower inventory level, allowing them to sell the stock quickly. However, it can lead to lost sales and reduced customer satisfaction. Missing out on potential sales in the product categories with higher volatility increases with the increase in risk aversion. A higher risk for loss due to stockout results in increased order quantity for products with higher demand volatility. Under the CvaR measure, it is possible to derive the loss-averse newsvendor's optimal order quantities (X. Xu et al., 2016) (X. Xu et al., 2017).

Closely related to the CvaR is the fill rate or service level, which equals the percentage of demand satisfied directly from stock-on-hand and represents the desired probability of not running out of stock (e.g. 95% or 99%). To incorporate risk aversion into the decision-making process of the newsvendor problem, retailers often set a target service level or fill rate. By selecting a high service level, the retailer aims to reduce the risk of stockouts and ensure high customer satisfaction. The retailer may incorporate safety stocks into their ordering decision to achieve the desired service level. Safety stocks act as a buffer to absorb demand variability and provide protection against stockouts. By maintaining additional inventory as safety stocks, the retailer reduces the risk of not meeting customer demand, even during higher-than-expected periods. However, it is essential to note that risk aversion in the newsvendor problem involves a trade-off. Increasing safety stocks to mitigate the risk of stockouts incurs additional holding costs, as excessive inventory needs to be stored and managed (Güler et al., 2018) (X. Xu et al., 2018). Therefore, the retailer must balance risk aversion (reducing stockouts) and cost efficiency (minimising excess inventory costs). The specific approach to incorporate risk aversion into the newsvendor problem may vary based on the retailer's risk preferences, industry norms, and the product's nature. By accounting for risk aversion in the newsvendor problem, retailers can take more informed inventory decisions that align with their risk preferences and ultimately enhance customer satisfaction and profitability (Dai & Meng, 2015).

5.2 The impact of shortage cost on the optimal order quantity

This section looks closer at the shortage cost under the CvaR objective and the influence on the optimal order quantity. Suppose the market demand is a random variable. Let p, c, r and s be the retail price, wholesale price, salvage price and shortage cost per product item, respectively. In this model, $p > c > r > 0$ and the combination between market demand (D) and order quantity (q) combine towards the total profit, and λ represents the loss-aversion coefficient. The larger the loss aversion, the more loss-averse the retailer is (Poormoaied & Hosseini, 2021) (X. Xu et al., 2018). This results in a more careful approach towards keeping stock during the season and a smaller order quantity at first glance. This model calculates the optimal order quantity for the loss-averse newsvendor. A distinction has been made in the model between a setting with and without shortage costs (Petruzzi & Dada, 1999) (X. Xu et al., 2018).

If the newsvendor opts to order fewer units, there is a lower fill rate. In other words, an increase in the loss aversion coefficient leads to a decrease in the fill rate (X. Xu et al., 2018). As stated in the introduction of this section, the fill rate is closely related to the profit made during the selling season. By investigating the behaviours of the retailers, two conflicting targets act up in the form of loss aversion and fill rate. These two factors significantly influence the optimal order quantity and revenue. Moreover, the larger the confidence interval, the more risk-averse the newsvendor becomes. If the newsvendor strives for a higher utility, he needs to invest in a higher order quantity which bears a higher risk (X. Xu et al., 2018) (X. Xu et al., 2016).

Shortage costs from lost sales sometimes influence the profit of the loss-averse newsvendor (Abad, 2014). Based on the traditional formula (1), an extension is made to include the shortage cost (X. Xu et al., 2018):

$$\prod(q, D) = (p * \min(q, D)) - c * q + r(q - D)^{+} - s(D - q)^{+} \qquad (3)$$

Let ∏(q, D) be the expected profit generated by the combination of order quantity and demand, D the demand for the product, p the unit sales price, q represents the newsvendor's ordered quantity, c is the wholesale price per unit of product, r is the salvage value (the remaining value of the product in case it is not sold in the foreseen period), and s equals the shortage cost. Min(q, D) represents the minimum of the order quantity and the demand (which is the quantity sold), $(q\text{-}D)^{+}$ stands for max(q-D,0) (which represents the unsold inventory that incurs holding costs), and $(D\text{-}q)^{+}$ stands for max(D-q,0) (which represents all items that could not be sold due to understocking). Combining all these factors leads to the expected profit ∏(q, D), and the goal is to maximise this function by finding the optimal order quantity (X. Xu et al., 2018).

For example, the stock-out potentially leads to a loss of goodwill. Furthermore, the adverse impact of a stockout frequently extends to other items in the current order and future demand. Thus, the costs of stockouts should be considered in determining the optimal order quantity, and adding shortage cost for the lost sales is an efficient method to deal with such cases (Poormoaied & Hosseini, 2021). Therefore, the optimal order quantity for the risk-averse newsvendor maximising the CvaR objective is studied (X. Xu et al., 2018).

All elements are kept constant in the below graphs except the shortage cost (s). The diagram indicates q_s^{α}, the optimal order quantity under specific CVaR-criterium and shortage cost. It can be observed in Figure 5 that when the shortage cost is relatively small (equals 6), fewer order quantity is realised as the newsvendor becomes more loss-averse. Whereas Figure 6 illustrates that if the shortage cost becomes substantially large (in this case, 18), the opposite occurs

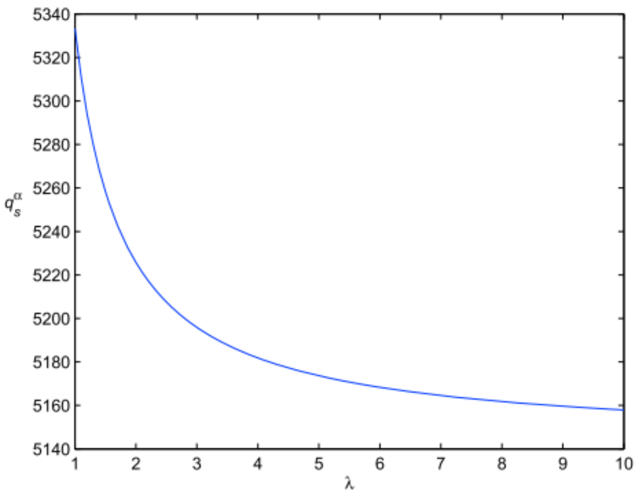

Figure 5: Influence of loss aversion coefficient λ on optimal order quantity q_s^α; p=10,c=6,r=1,s=6 and α = 0,5 (X. Xu et al., 2018)

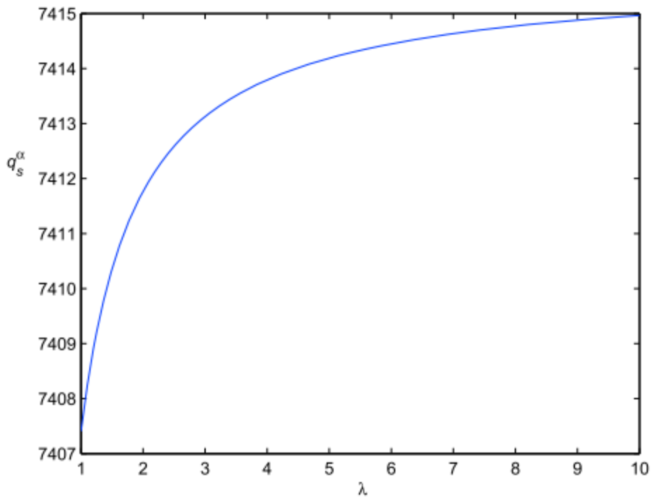

Figure 6: Influence of loss aversion coefficient λ on optimal order quantity q_s^α; p=10,c=6,r=1,s=18 and α = 0,5 (X. Xu, Chan, & Langevin, 2018)

(X. Xu et al., 2018). The same principle, as shown in Figure 4, can be applied following the green arrow to the right. The loss-averse newsvendor chooses a higher order quantity since excessive inventory weighs less than being unable to serve all customers (Petruzzi & Dada, 1999).

So, the optimal order quantity augments when the shortage cost increases. If the loss coming from excess orders is more impactful than the one coming from not being able to satisfy the customer demand (Abad, 2014). As seen in the example, the second scenario includes shortage costs, resulting in a different outcome. Thus, loss aversion and fill rate are closely related and compose conflicting targets. However, when shortage cost is included, this conclusion no longer holds when the shortage cost is higher than the marginal cost. In other words, the shortage cost encourages the loss-averse newsvendor to order more items to improve the fill rate. The higher the shortage cost, the higher the fill rate. Therefore, the shortage cost is essential in coordinating the newsvendor's loss aversion and fill rate targets (X. Xu et al., 2018).

5.3 The impact of emergency shipments on the optimal order quantity

Another scenario that deserves attention is the emergency shipments connected with the CvaR objective. Although the literature does not provide this connection, it is possible to create a relationship between the concepts by explaining the impact of emergency shipments on the optimal order quantity. In the context of the newsvendor problem, emergency shipments refer to an additional supply of goods that can be obtained in case of unexpectedly high demand or stockouts (M. Xu & Lu, 2013). Emergency shipments are only relevant when a retailer faces a situation where the initial order quantity is insufficient to meet the actual demand. These additional shipments can be arranged with the supplier or alternative sources to bridge the gap between the initial order and the unexpected request. The decision to use emergency shipments involves weighing the costs and benefits. On the one hand, emergency shipments incur additional expenses, such as expedited shipping fees or higher unit costs, if the supplier charges a premium for urgent orders. On the other hand, emergency shipments can help capture additional sales and avoid customer dissatisfaction, which may have long-term benefits for the retailer's reputation and customer loyalty (Khouja, 1999).

To optimise emergency shipments in the newsvendor problem, the retailer can consider several factors such as the cost of emergency shipments (transportation costs, any premium charged for expedited orders, and potentially higher unit costs), lead time (time required to receive an emergency shipment), probability of demand exceeding initial order (determine the level of risk associated with understocking), impact on customer satisfaction and lost sales cost (the value of emergency shipments in mitigating these costs), and inventory holding costs (comparing these costs with the expenses of emergency shipments). By considering all the previously mentioned factors, retailers can determine the optimal order quantity and whether or not to include emergency shipments in their inventory planning. The goal is to balance costs and service levels to maximise profitability and customer satisfaction. Therefore, emergency shipments are considered a powerful approach for mitigating the risk of shortages and increasing the resilience of storage systems (Khouja, 1999). A newsvendor model is studied where the retailer operates in a setting that allows the retailer to request one emergency shipment per selling season (Poormoaied & Hosseini, 2021). By this, the chance of imminent shortages diminishes, but the timing and size of the order play a substantial role in the matter. Hence, the model uses a time-weighted holding cost to compute the expected holding cost assuming that any unmet demand is not satisfied. Time-weighted holding cost means that having inventory ties up capital and incurs certain costs over time and can be calculated by multiplying the average inventory value, holding cost rate and time period. This concept is helpful in inventory optimisation as it allows businesses to assess the impact of inventory holding costs on their overall profitability. Therefore, a time-weighted holding cost will influence the timing of an emergency shipment (X. Xu et al., 2017). The goal is to determine the pre-season order quantity, the timing and the size of the emergency shipment to maximise the expected profit. The different structures will be studied to identify the optimal order quantity per situation (Poormoaied & Hosseini, 2021). Below the table, an overview is given of several papers that also discuss the topic of emergency shipments but with different assumptions:

Study	Control policy	Period	Item	Emergency shipment lead time	Demand	Number of emergency shipment deliveries per period	Execution shipments: Reactive or proactive?
(Poormoaied & Hosseini, 2021)	Periodic review	Single period	Single item	Zero lead time	Stochastic demand	Single emergency shipment	Proactive
(Poormoaied, Atan, de Kok, & Van Woensel, 2020)	Periodic review	Multi-period	Single item	Zero lead time	Poisson customer demand	Single emergency shipment	Reactive

(Chand, Li, & Xu, 2016)	Periodic review	Multi-period	Single item	Zero lead time	Stochastic demand	Single emergency shipment	Proactive
(Howard, Marklund, Tan, & Reijnen, 2015)	Continuous review	Multi-period	Single item	Positive lead time	Poisson customer demand	Single emergency shipment	Reactive
(Alvarez & van der Heijden, 2014)	Continuous review	Multi-period	Single item	Zero lead time	Poisson customer demand	Single emergency shipment	Reactive

Table 1: Overview papers that discuss emergency shipments in different settings

Any deficiency within the supply chain of raw materials and finished goods leads to customer dissatisfaction and vital costs for manufacturers and retailers in the short and long run. Before diving into the model, emergency shipments can be executed in two forms: reactive and proactive. On the one hand, the reactive approach includes all emergency shipments triggered when the inventory system indicates a stockout due to excess demand and backorder is necessary to satisfy demand. This approach aims to discover the optimal size of the emergency shipment. On the other hand, the proactive approach incorporates a schedule in that the time of placing the emergency shipment is predetermined (Poormoaied & Hosseini, 2021). Using this approach, the timing and size need to be optimally positioned. Instinctively, the unit purchasing cost associated with reactive emergency shipment is more significant than that associated with regular ordering as the urge for express order delivery and transportation (order beginning of the season = regular order) is larger (M. Xu & Lu, 2013). A conflict of interest arises as retailers prefer reactive emergency shipments with speedy deliveries, but the outside suppliers prefer proactive deliveries. The reason is that retailers desire zero lead times, but external suppliers wish to know beforehand, considering procurement efforts needed for each order (Poormoaied & Hosseini, 2021).

The model considers a newsvendor problem, a single-season inventory model with stochastic demand, where a regular order is placed at the beginning of the season. The ordering policy works as follows: The retailer can order twice per selling season, once at the beginning and one emergency shipment (Vipin & Amit, 2017). The order is at the start of the season (Q), and the retailer requests a second delivery when the inventory quantity (q) drops below a certain level (safety stock) (X. Xu et al., 2017). When an emergency shipment is requested at time t, the order will be delivered immediately; otherwise, no action is taken. Let T be the selling season length, and the inventory level is raised to q in case an emergency shipment happens. At the beginning of every season, decisions are made regarding the regular order quantity, the timing of the emergency shipment, and the order-up-to level of the emergency shipment. The product is always sold to the customer at price p regardless of whether the demand is satisfied during the season from stock or the emergency shipment. A shortage penalty (loss of goodwill) is billed when demand cannot be satisfied. The following assumption is made: $0 \leq r < c_o \leq c_e < p$ to remove unrealistic cases. In this assumption, c_0 represents the cost per item for a regular order, c_e stands for the cost per item for an emergency shipment, r substitutes for the salvage value, and p is the selling price per item. The goal is to determine the optimal value of the variables to maximise the profit (Poormoaied & Hosseini, 2021) (M. Xu & Lu, 2013). When implementing these variables in the formula of the traditional newsvendor problem (1), the following formula appears:

$$\prod(q, D) = (p * \min(q, D)) - c_o * q - c_e * q + r(q - D)^+ \ (4)$$

Intuitively, a well-timed emergency shipment triggers significant cost savings during the selling season since the fill rate is lower. Thus, the holding cost is more negligible concerning a whole period. The model compares a traditional situation to one including emergency shipments. It is noticeable that there is a trade-off between having the extra stock at the end of the period or running out of stock and not being concerned with holding costs or salvage value (Poormoaied & Hosseini, 2021). The effects of all different system parameters such as selling price, unit holding cost, the unit lost sales cost, unit regular order price, salvage value, emergency shipment price and all parameters together are further discussed in the paper but will not be studied in this section. Only the main findings are shortly presented (Poormoaied & Hosseini, 2021) (M. Xu & Lu, 2013):

- If the selling price is considerable or the item holding cost is low compared to the selling price, the optimal order quantity increases since it is more beneficial to have too much stock than to lose sales and vice versa. Therefore, it can be concluded that the selling price and holding cost greatly influence model performance.
- The optimal order quantity is highly sensitive to changes in the lost sales cost in the classical newsvendor model. In contrast, the emergency shipment model potentially reduces the chance of imminent shortages. Therefore, the order quantity is not highly affected.
- The order quantity decreases as the fixed ordering cost per emergency shipment increases.
- The higher the price per emergency shipment unit, the lower the order quantity, and the more this model converges to the traditional newsvendor problem.

The possibility of emergency shipments also changes the risk-return trade-off shown in Figure 3. Emergency shipments allow the retailer to react to extra demand, which implies that he can obtain a higher profit (he no longer loses sales) without taking additional risk (an emergency shipment results in a reaction to actual customer demand, so it does not increase the retailer’s risk of overstocking).

The impact of shortage costs and emergency shipments on the optimal order quantity is thus conflicting since shortage costs would encourage the retailer to keep a higher stock level. In contrast, emergency shipments entail the retailer lowering the risk by holding less inventory. This increases the complexity of the newsvendor problem, and therefore it would be interesting to conduct a new model that includes both variables, as this is currently unavailable.

6. Other extensions of the newsvendor problem

During the literature review, two topics were encountered outside of one of the earlier discussed extensions of the newsvendor problem. However, they were fascinating to investigate to get a clearer picture of the challenges a retailer has to deal with in his daily business. Therefore, in this section, a closer look is given at the effects of supply uncertainty on the optimal order quantity and the value of information.

6.1 Effects of supply uncertainty on the optimal order quantity

Typically, the newsvendor problem assumes that the demand for the product is uncertain, but the supply is known and fixed. However, in real-world scenarios, the newsvendor may also face uncertainty in the supply of the product. Several reasons, such as supplier delays, production issues, transportation problems, or unforeseen events like natural disasters, can cause uncertainty. Therefore, it is crucial to diminish the impact since this could lead to incomplete deliveries, as a supplier cannot guarantee a full delivery every time an order is placed (Ma et al., 2012). When supply uncertainty is present, the newsvendor needs to consider the potential variability in the supply while making the ordering decision. To address supply uncertainty in the newsvendor problem, the newsvendor can adopt different strategies such as safety stock, information sharing, multiple suppliers and contingency plans (DeYong, 2020).

This section focuses on the impact of supply uncertainty on the optimal decisions and expected profit with a price-dependent stochastic demand under the cost structures of both in-house production and the procurement case (M. Xu & Lu, 2013). This allows us to understand how supply uncertainty impacts newsvendor decisions and profits. In addition to the theory, several experimental results will be shown to prove the difficulty of newsvendor decisions under supply uncertainty (Käki, Liesiö, Salo, & Talluri, 2015).

In the traditional newsvendor problem, demand uncertainty is present, yet there is no supply uncertainty: the retailer always receives the amount ordered, regardless of how much he ordered. Supply chain uncertainty often leads to reduced performance (Van den Broeke, Boute, & Van Mieghem, 2018). The newsvendor problem characterises situations where a costly commitment (e.g. procurement order, production order, capacity plan etc.) must be made before demand is realised. When faced with supply uncertainty and the need to make ordering decisions, retailers often find it challenging to predict the future demand for the product accurately. In such situations, the pull-to-centre effect emerges. Rather than taking extreme positions by ordering significantly more or less than the expected demand, retailers tend to set their order quantities towards the middle ground between the expected demand and the optimal order quantity. However, it is essential to note that the pull-to-centre effect can lead to suboptimal decisions if the anticipated demand does not accurately reflect the actual demand distribution or if the optimal order quantity is not appropriately estimated (Käki et al., 2015) (Ma et al., 2012).

Supply uncertainty has a non-trivial impact on the optimal order quantity and can be divided into two categories: disruptional and operational uncertainty. Disruptional supply uncertainty refers to the potential risks and challenges that could arise due to the influence of external factors such as natural disasters and economic crises. These issues are unpredictable and challenging to handle (Käki et al., 2015). Operational supply uncertainty, however, includes the potential risks and challenges faced in a supply chain's regular, day-to-day business. This type of uncertainty potentially stems from unexpected fluctuations in demand, supplier delays, transportation disruptions or quality control problems (Dai & Meng, 2015) (Ledari, Pasandideh, & Koupaei, 2018). The main difference between the two is that factors within the control of supply chain participants cause operational supply uncertainty. In contrast, disruptional uncertainty is a result of external factors. In the rest of this section, operational uncertainty is the sole focus (Käki et al., 2015) (Y. Li, Chen, & Jia, 2013).

Understanding the challenges while determining the optimal order quantity and managing inventory levels effectively to reduce operational uncertainty is essential. As mentioned earlier, newsvendors can adopt various strategies to account for uncertainties in demand and supply. Examples of such strategies are new production technology, total quality control, and process improvements to improve the overall quality to determine a roadmap for the future. Hence, many interesting questions arise, such as the impact on a firm's profit and the impact of the yield rate (a measure that quantifies the proportion of received goods compared to the ordered quantity) on the order quantity (He & Lu, 2021). This model makes the following assumptions: the newsvendor faces a price-dependent and stochastic demand and must decide on its pricing and inventory decisions before the selling season (M. Xu & Lu, 2013).

As mentioned, two different settings impact the costing scheme: in-house production versus external procurement. The in-house costing scheme depends on the production input quality, while the external procurement only pays for the number of received goods. In the case of in-house production, yield uncertainty results from defective units produced during the production process. Yet, the manufacturer needs to pay for all the inputs used in production (this occurs, for instance, when remanufacturing returned products) (M. Xu & Lu, 2013). The effects of yield randomness on the optimal decisions and the expected profits can be calculated using stochastic comparisons. For the models, it is assumed that if the demand is not satisfied, then the associated cost equals e (cost for an emergency shipment), with the unit selling price $p > e$ (Poormoaied & Hosseini, 2021) such that the newsvendor has the incentive to use the emergency option (and thus, avoids lost sales). Therefore, for all examples in this section, $p > e$ and it is more beneficial for the firm to exercise this option (M. Xu & Lu, 2013).

Intuitively, a stochastically more significant yield rate should result in a smaller production quantity, as the production process uses resources more efficiently (i.e., without additional resources) and has lower waste. By maximising the yield rate, manufacturers can achieve cost savings, improve product quality, and enhance overall efficiency (M. Xu & Lu, 2013) (Mitra, 2018). However, this idea may not always be accurate when the optimal stocking factor (level of inventory or stock that a company maintains to meet customer demand and ensure product availability) and the associated costs, such as holding costs, opportunity costs and storage costs, are considered. If the optimal order quantity to maintain in reserve leads to lower overall costs, retaining a higher inventory level could be favourable. However, this depends on the most valued factor, profit or the stocking factor, chosen by the retailer. The impact on in-house production and procurement cases is similar (M. Xu & Lu, 2013) (Mitra, 2018).

Lastly, a model is examined wherein the impact of the pull-to-centre effect is reviewed. In this model, two groups will be compared, where the control group replicates a well-known newsvendor experiment, whereas the test group is confronted with additional supply chain uncertainty (Käki et al., 2015). The newsvendor optimum is a function of supplier cost and reliability (Ma et al., 2012). The model compares two products, one with a high-profit margin and the other with a low-profit margin. The authors find that the average orders of retailers in the test group are located between the average demand and the optimal order quantity since the average demand represents the average quantity of products customers are expected to ask for during a given time period. When retailers place orders, they consider the uncertain demand and aim to satisfy customer needs while avoiding high inventory costs. As a result, their order quantities tend to fall between the average demand and the optimal order quantity. This phenomenon shows up most clearly in the event of high supply uncertainty, whereas under moderate supply uncertainty, an amount close to the true optimum is found (Käki et al., 2015). Under the conditions of this model, the following general results are found: The interdependency (interconnectedness or mutual dependence between multiple variables, where the actions or changes in one variable can impact others) has a significant impact. Under the high-profit condition, the optimal expected profit is more important in the case of positive dependency (a situation where one entity relies on another entity for support, resources, or outcomes and implies a one-way relationship of reliance) compared to negative dependency. In the low-profit case, positive dependency influences the expected profit far more. Moreover, the optimal newsvendor decision is affected. The optimal order quantity is lower for the positive dependency, and this effect is enlarged in combination with the low-profit margin. The reason is the impact of excessive stock. In the low-profit case, the stock has a more considerable impact on profit than the high-profit margin (Käki et al., 2015) (X. Xu et al., 2017).

This section shows that by considering supply uncertainty and adopting appropriate strategies, the newsvendor can improve its inventory management and better meet customer demand while minimising costs. The actual order quantity would be expected to be far from the optimal quantity. However, this is not always the case. Solely the high-profit condition poses this problem as the retailers would love to take advantage of this opportunity. In the case of the low-profit margin, the optimal amount is still below the mean demand. Therefore, managers must be aware that supply uncertainty is a critical factor affecting decision-making in the newsvendor problem. Understanding the causes and implications of supply uncertainty is essential for developing effective inventory management strategies. By implementing appropriate techniques, newsvendors can mitigate the risks associated with supply uncertainty and improve their overall operational performance (Käki et al., 2015) (M. Xu & Lu, 2013).

6.2 The value of information

The value of information arises when additional knowledge about demand becomes available before the decision is made. This information could be in the form of more accurate demand forecasts, market intelligence, or insights about customer behaviour. The key idea is that having more information helps the retailer make better decisions, improving operational and financial outcomes. By incorporating additional information into the decision-making process, the retailer can make better-informed choices about the order quantity (Ma et al., 2012) (L. Li & Liu, 2020). This leads to improved decision accuracy, enhanced risk management and cost optimisation to gain an advantage over its competitors. Therefore, a natural extension of the traditional newsvendor problem involves the implementation of information asymmetry (which occurs when one party has access to information that the other party does not have or has limited access to). A retailer can acquire an advantage over its competitors by correctly implementing the obtained information (expertise, resources, privileged access) (Güler et al., 2018). In addition, the classical newsvendor problem strives for an optimal order quantity that maximises the expected profit. This expected profit maximisation (EPM) order quantity has been well studied and has become a useful managerial tool. In practice, however, the order quantity often differs from the EPM order quantity for different reasons. For instance, it may be based on subjective factors rather than objective information and logical reasoning. This phenomenon is referred to as decision bias. Managers must often make decisions without full knowledge of customer demand distribution (Clausen & Li, 2022). Furthermore, decision-makers may decide based on experience. Firms are also (potentially) differently informed about their cost and/or revenue structures (Qin et al., 2011) (Güler et al., 2018).

The newsvendor problem has been studied with asymmetric cost information (Güler et al., 2018). In this model, two retailers are considered amongst whom the customer demand is split, and both companies have private information regarding their costs. If one retailer cannot satisfy his part of the customer's demand, the demand spills over to the rival retailer.

In general, when two firms have the opportunity to fill up the lack of inventory of the other, they tend to grasp that opportunity (Ma et al., 2012). Moreover, the mere presence of strategic interactions incentivises all firms to order more. This may lead to the conclusion that competition always leads to higher total inventory in the industry (Güler et al., 2018), yet that is incorrect. In some situations, the buyer may be unwilling to commit to a fixed quantity and instead allocate a certain percentage of his total procurement to each supplier. To illustrate the value of information, a model is constructed where two firms offer perfect substitutes on the market. While this is not without loss of generality, the perfect substitutes assumption is well-justified in many industrial applications. As mentioned multiple times, a newsvendor must decide how many items it orders before observing the actual demand (Ma et al., 2012). When the opportunity arises, the competitor invests in spillover to sell more items potentially. The presence of these strategic interactions creates incentives. It can be concluded that when decision-makers strongly prefer a particular way of measuring performance, it becomes more obvious how well the model performs in terms of its quality or attractiveness. Moreover, the results illustrate a trade-off between the risk and expected returns. The analysis shows that there can be clear differences in order quantities since all firms understand the given information differently. In some scenarios, the spillover effect may be so substantial that the retailer will likely increase his fill rate (Murarka et al., 2019).

In summary, information can tremendously influence the chosen order quantity. When comparing the loss-averse and risk-neutral newsvendor, the value of information impacts the decision. The quantity the newsvendor selects depends on the specific risk preferences and the trade-offs between potential losses and gains.

7. Newsvendor networks

Effective inventory management is crucial for businesses to meet customer demand, minimise costs, and maximise profitability in today's dynamic business environment. While the classic newsvendor problem focuses on a single newsvendor facing uncertain demand, real-world scenarios often involve multiple newsvendors or retail locations operating within a broader network. These interconnected systems, known as newsvendor networks, present unique challenges and opportunities for optimising inventory management. Newsvendor networks refer to interconnected or multi-location systems where multiple newsvendors operate within a broader network or supply chain. These networks involve coordinating and managing inventory across multiple locations to meet customer demand efficiently (Huber, Müller, Fleischmann, & Stuckenschmidt, 2019; Mieghem & Rudi, 2002). In newsvendor networks, each location faces uncertain demand and independently makes inventory decisions. However, their decisions can have interdependencies and affect the network’s overall performance.

For example, excessive inventory at one location may result in redistribution to other locations. In comparison, stockouts at one location may lead to lost sales that could have been fulfilled from another site. Managing newsvendor networks aims to optimise inventory allocation and replenishment decisions to improve overall network performance and profitability. Key considerations include balancing inventory levels across locations, managing product allocation and transfers, and coordinating replenishment orders to minimise costs and meet customer demand effectively (Yu et al., 2013).

Thus, the interest of a newsvendor network lies within the coordination and optimisation of inventory decisions across all locations to ensure efficient management of stock levels, meet customer demand using demand forecasting and analytics, and supply chain performance evaluation (fill rates, costs, service levels, etc.) to maximise profitability within the broader network. Like the newsvendor problem, newsvendor networks require the retailer to decide the optimal order quantity before the selling season starts. The retailer faces a trade-off between the cost of understocking and overstocking, resulting in the well-known critical factor solution. These insights improve the coordination and collaboration among retailers, increasing efficiency and higher profit for the network (Mieghem & Rudi, 2002).

Implementing efficient inventory management strategies in newsvendor networks requires careful planning and consideration of various factors. Two key considerations need some further explanation. Firstly, it is crucial to integrate data that are essential for effective decision-making into the systems at all locations in the network. Businesses must establish robust data collection mechanisms, integrate inventory management systems, and ensure seamless data sharing across locations. This integration enables accurate demand forecasting, real-time visibility of inventory levels, and efficient coordination of replenishment orders. Secondly, an organisational structure and culture must be developed to support collaboration among locations, incentivise information sharing and coordination, and ensure effective communication channels. Such a collaborative culture facilitates knowledge sharing, joint problem-solving, and the effective implementation of inventory management strategies (Boute, Gijsbrechts, Van Jaarsveld, & Vanvuchelen, 2022).

When these factors are integrated correctly, the data can be exploited on three levels to generate value and make better decisions (Huber et al., 2019). The first level on which the data can be utilised is demand estimation. The available data potentially contain information about future demand that newsvendor networks can extract. Machine learning (ML) methods can process large datasets and provide forecasts based on historical demand data and other essential variables such as weekdays versus weekends, prices, weather, product ratings and seasonality. Moreover, more accurate decisions can be made if better forecasts are made. The second level entails the inventory decision. This decision is optimised based on the demand forecast and the historical forecast errors. To this end, it is essential to integrate the remaining uncertainty associated with the forecast, which is modelled through a demand distribution assumption. Nevertheless, if the demand distribution assumption is inaccurately represented, it can lead to suboptimal inventory strategies. On the third level, the first and second level (demand estimation and inventory decision) are combined into a single model that directly calculates the optimal decision integrating the inserted data of the other levels (Huber et al., 2019).

The advantages of newsvendor networks include tractability and effectiveness in producing insights into stochastic planning. Newsvendor networks naturally extend the multidimensional newsvendor model by incorporating inventory on hand and refining the processing formulation through activities and common constraints. This enables the newsvendor network to connect periods in a dynamic setting and to obtain a close-to-reality situation (Mieghem & Rudi, 2002) (Huber et al., 2019). Besides, it outperforms the standard approach of separate demand estimations per location since the ML technique can calculate all these jointly. However, due to the higher complexity of the model, some disadvantages are connected to implementing newsvendor networks. ML techniques are often black-box, which makes it more difficult to justify the resulting predictions. Yet, the advantages of increased forecast accuracy outweigh the issue of interpretability (Mieghem & Rudi, 2002) (Huber et al., 2019). Furthermore, the ML approach must enter a learning phase (during this phase, the ML algorithm is trained on historical data to learn patterns, make predictions, or optimise certain processes), where costly and cheap mistakes acquire identical attention. In the long run, the optimiser (the person in a firm that observes the results closely for potential errors) potentially becomes inattentive to the mistakes Machine Learning produces, which is dangerous for the firm (Neghab, Khayyati, & Karaesmen, 2022).

Based on the discussion mentioned above, three managerial insights can be drawn (Clausen & Li, 2022):

- Firstly, when confronted with an inventory management challenge where rich demand data is accessible, performing standard multiple-feature analyses can significantly enhance inventory performance compared to univariate models.
- Secondly, if the optimal predictor (most accurate and reliable factor or piece of information that can be used to predict or estimate a specific outcome or result) for the big data-driven model can be identified, it is often feasible to employ well-established estimation methods to address the challenges posed by the big data-driven problem.
- Thirdly, in situations where the optimal predictor is unknown, developing a machine learning model tailored/customised to the problem is still possible. However, the potency of such a model falls back on the complexity level and nature of the problem.

Recognising that complex models do not always generate more valuable results is essential. When the goal is only to estimate a relationship between a variable and several features, simpler models often suffice. Nonetheless, in most cases, multivariate data-driven models outperform univariate models. The ongoing challenge in the case of the newsvendor problem is to construct an understandable ML model for the more complex problem settings (Clausen & Li, 2022). Besides, the decision-maker must be cautious not to overfit the model to the data. Otherwise, proposed solutions/decisions can mislead (Ban & Rudin, 2019).

Table 2 presents several papers that have studied some real applications of newsvendor networks using ML techniques. Managing newsvendor networks requires a deep understanding of local demand patterns, cost structures, and supply chain dynamics. Businesses must balance centralised control and local autonomy, ensuring effective communication, collaboration, and information sharing. Companies can continuously refine and adapt their inventory management strategies to optimise their newsvendor networks and stay flexible in a rapidly evolving supply chain. Newsvendor networks represent a fascinating area of study and offer immense potential for businesses to optimise inventory management and enhance their competitive advantage (Mieghem & Rudi, 2002).

Study	Inventory model	ML technique	Demand function	Methodology	Regression type
(Neghab et al., 2022)	Newsvendor	DNN	Non-linear	ERM	Local regression
(Ban & Rudin, 2019)	Newsvendor	QR-ML + KO	Linear	ERM	Global regression
(Huber et al., 2019)	Newsvendor	ANN + DT	Non-Linear	ERM	Global regression
(Bertsimas & Kallus, 2020)	Multi-item	CART + RF	Non-linear	ERM	Local regression
(Punia et al., 2020)	Multi-item	QR-ML	Non-linear	ERM	?
(Clausen & Li, 2022)	Order-up-to-level (dynamic)	NN	Non-linear	ERM	Global regression

(Gijsbrechts, Boute, Van Mieghem, & Zhang, 2021)	Multi-item	DRL	Linear	ERM	Global regression
(Neghab et al., 2022)	Newsvendor	DRL + DNN	Non-linear	ERM	Local regression
(Shi, 2022)	Multi-item	SAA	Non-linear	ERM	Local regression

Table 2: Overview papers newsvendor networks; NN = Neural Network, QR = Quantile Regression, KO = Kernel weights Optimisation, ANN = Artificial Neural Networks, DNN = Deep Neural Networks, DRL = Deep Reinforcement Learning, DT = Decision Trees, CART = Classification And Regression Trees, RF = Random Forest, SAA = Sample Average Approximation; ERM = Empirical Risk Minimisation

8. Conclusions and directions for future research

This literature study explored the traditional newsvendor problem, its extensions and newsvendor networks through an extensive literature review. The problem plays a critical role in inventory modelling to maximise profit (or minimise costs) and remains relevant in today's supply chain management. A closer look was taken at pricing policies, buyer risk, the effects of supply uncertainty, the value of information and newsvendor networks.

It has been shown that the selling price is really important, as it influences the demand depending on the chosen price. A retailer can change the optimal order quantity by conducting market research to understand price sensitivity and manufacturer differentiation better. In this way, the retailer can positively influence his profitability. Also, the retailer has to decide the risk he is willing to take. It is shown that a risk-averse newsvendor usually orders less than the traditional newsvendor. However, when substantial shortage costs or emergency shipments are present, the equilibrium shifts under the CVaR-criterion. The newsvendor orders extra units when shortage costs exceed the marginal cost of an additional unit, while emergency shipments enable the newsvendor to keep a lower stock level as long as the cost of an emergency delivery does not exceed the selling price. Supply uncertainty contributes towards the decision-making process since it can influence the optimal order quantity in both directions: On the one hand, under the high-profit condition, it incentivises the newsvendor to keep safety stock in case the delivery is not fully received, even though the extra stock reduces the profit in case the items are not entirely sold. On the other hand, the optimal order quantity decreases under the low-profit condition as it does not encourage the retailer to keep safety stock. Therefore, managers must have accurate information to make the best decision. In newsvendor networks, it is in the interest of the newsvendors to optimise their coordination and inventory decisions across multiple locations to ensure efficient management of stock levels, meet customer demand and maximise profit.

In summary, it is crucial to understand the retailer's operating environment such that a model can be set up that considers all relevant variables. The current literature only reviews one variable at a time, while these factors interact continuously. Models that include combinations of variables are currently lacking and offer an excellent opportunity for further research. Moreover, it would increase the models' applicability in real life. This would enable supply chains to ameliorate their performance further.

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