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Peer-reviewed author version

GEBOERS, Hans; DEPAIRE, Benoit & Straetmans, Stefan (2023) Peak-to-valley drawdowns: insights into extreme path-dependent market risk. In: Journal of Risk, 26 (2) , p. 65 -104.

DOI: 10.21314/JOR.2023.012
Handle: http://hdl.handle.net/1942/42691

Peak-to-Valley Drawdowns: Insights into Extreme Path-Dependent Market Risk

Abstract

In this paper, risk is studied from the perspective of peak-to-valley market drawdowns. The objective is to gain empirical insights into the drawdown behavior of various asset classes during several time intervals. While the existing literature on drawdown distributions has primarily focused on local drawdowns or consecutive daily drops in various asset classes, this paper focuses on extreme (cumulative) losses occurring over a daily, bi-weekly, monthly, quarterly, and yearly period. The typical investor is mainly concerned with significant negative downward movements, especially when several of these movements happen within a specific time frame. The drawdown measure studied herein embodies this path-dependent risk better than a typical daily standard deviation or Value at Risk estimate due to its cumulative and path-dependent nature. The drawdowns over different periods are analyzed for 25 assets linked to Equity Indices, Commodities, and Foreign Exchange. The tail observations of these drawdowns are fitted to the Power Law and the stretched exponential (Weibull). We find that the bulk of these observations is well-fitted by both distributions. Additionally, our analysis shows that the most extreme observations tend to fall between the Weibull and Pareto fits, suggesting these can be used to define a lower and upper boundary for modeling future drawdowns.

Keywords: Drawdown, Extreme risk, Asset allocation, Risk management.

1 Introduction

A drawdown equals the peak-to-bottom loss over a given period. Aside from using drawdowns (measured as retracements from previous highs) as a performance measure, they can also describe market behavior. While it is generally accepted that financial markets are risky and exhibit fat-tailed behavior (see, for example, Straetmans and Candelon (2013) and Tolikas (2014)), the odds of financial ruin often tend to be grossly underestimated. The accuracy of standard risk measures as tools for quantifying extreme downward risks has been questioned after the global financial crisis (Boucher et al., 2014). We motivate our empirical study by the need to be aware of these significant retracements and corrections over time by regulators, financial institutions, pension funds, and risk-takers in general. This should contribute to better risk management and a more sustainable financial ecosystem. Recognizing drawdown risks and their potential impact of drawdowns allows for proactive measures to mitigate risks, protect investments, and promote long-term stability in the financial industry. Ultimately, our study aims to contribute to the overall resilience and soundness of the financial system by emphasizing the significance of effective drawdown management.

Extreme price moves and significant drawdowns can be linked to several phenomena documented in the field of behavioral finance. One of the key features of drawdowns is that it measures the maximum loss versus a certain reference level. As investors are risk averse and reference points determine utility, the drawdown measure, which compares the current wealth to a previous high watermark, displays a remarkable connection with the concepts of prospect theory (Tversky and Kahneman, 1979). Significant market retracements are an essential feature of a typical boom and bust cycle in financial markets, in which initial over-optimism leads to bubbles. A phenomenon that could explain these cycles is the presence of overreaction and underreaction in stock markets, documented in several research papers (see e.g. Daniel et al. (1998), De Bondt and Thaler (1985), Hong and Stein (1999) and Cont and Bouchaud (2000)).

In this article, a drawdown is considered to be the maximum retracement from a previous high watermark over a specified investment horizon. The main advantage is that it gives an indication of a maximum loss and hence is a suitable indication for long-term risk. Several risk measures can be linked to this approach of defining a drawdown: the Conditional Drawdown measure (CDD) and the Conditional Expected Drawdown measures (Möller, 2018). The CDD measures proposed by Chekhlov et al. (2005) include the Maximum Drawdown and the Average Drawdown and are often used in practice and suitable for portfolio allocation and optimization. The CDD measure is also used as an input for the β_{CDD} measure described in Zabarankin et al. (2014) and Ding and Uryasev (2022). This measure captures how an instrument performed during periods of market drawdowns. The CED measure developed by Goldberg and Mahmoud (2017) allows for a study of the distribution of possible future drawdowns. This assessment of the possibility of future market drawdowns can help a risk-taker to form reasonable expectations.

Various statistical models are used in the literature to describe drawdown distributions. Johansen and Sornette (2002) studied consecutive daily drops (and labeled these as drawdowns) by looking at major financial indices, currencies, gold, and the twenty largest U.S companies. They found that the majority of these drawdowns follow the exponential distribution but do report the presence of outliers. Rebonato and Gaspari (2006) and Leal and Mendes (2005) did similar research on drawdowns for US bond futures and three stock indexes. An analytical result for the distribution of drawdowns from the previous peak, assuming a discrete multiplicative random walk, can be found in Maslov and Zhang (1999). Magdon-ismail et al. (2004) derive an analytical result for the expectation of the maximum drawdown.

The existing literature on drawdown distributions has primarily focused on local drawdowns or consecutive daily drops in various asset classes. Several statistical models have been proposed to describe these drawdowns, and they consistently demonstrate fatter tails than what would be expected from an exponential distribution, assuming normally distributed returns. However, there is a research gap in terms of examining peak-to-valley drawdowns during various time intervals for different asset classes.

In this study, we aim to fill this research gap by focusing on drawdowns from the peak, providing a comprehensive perspective on the drawdowns of various asset classes. By considering drawdowns from the peak, we capture the cumulative impact of negative returns over an extended period, offering insights into the long-term risk and potential loss potential associated with different investments. Examining drawdowns from the peak, considering different time horizons, allows us to investigate the persistence and duration of drawdowns, shedding light on the time it takes for an investment to recover from a significant decline. Furthermore, by analyzing the drawdown distribution from the peak, we can identify the presence of outliers or extreme events that may have a disproportionate impact on investment portfolios.

By addressing this underexplored area, our study contributes to a better understanding of drawdowns and their characteristics, providing valuable insights for risk management, portfolio optimization, and investment decision-making. The findings of our research can aid investors, fund managers, and financial analysts in assessing the potential risks and rewards associated with different asset classes and designing strategies to mitigate drawdown-related losses.

The remainder of the article is structured as follows: Section 2 describes the data and the applied methodology. Sections 3 and 4 present and discuss the empirical results. For each studied asset class, the drawdown data are described, modeled, and discussed. Section 5 concludes.

2 Methodology

This section first describes the different daily data sets used within the empirical analysis for Equity Indices, Commodities, and Foreign Exchange. Secondly, the drawdown process is illustrated for the S&P500 yearly drawdown data. The third part outlines a formal method to determine the tail of the drawdown distribution, along with an example of how the Power Law and Weibull distribution is used to model the tail behavior of the monthly S&P500 drawdown data.

2.1 Description of the data

This study focuses on liquid market data with a sufficiently long data history. The aim is to have a broad global view, covering several key asset classes relevant to investors and regulators. Table 1 shows an overview of all the price series that have been analyzed. The focus is on three broad asset classes: equity, commodities, and foreign exchange. For the equity class, several tradable country indices have been selected. For commodities, a selection ranging from metals to oil and grains is covered. Within this category, EU allowances and Bitcoin are also included. In terms of FX, we study the ten most frequently traded currency pairs. The granularity of the price data is daily so that daily, bi-weekly, monthly, quarterly, and yearly drawdowns can be considered.

Table 1: Asset class data.

Asset Class	Product	Description
I. Equity Indices	S&P500, Nasdaq, Nikkei225, CAC40, DAX, HSI, ASX, IBOVESPA	Specific indices, tradable via futures contracts, startdates = [1929,1972,1965,1991,1988, 1987,1992,1993], Source: Commodity Systems Inc
II. Commodities	Gold, Silver, Copper, Wheat, Sugar, Brent Oil, US Natural Gas, EU Carbon allowances, and Bitcoin	Daily futures prices starting between 1980 and 1990, except for US Natural Gas (1991), EU Carbon (2008) and Bitcoin (2013) Source: FRED and various exchanges
III. FX	EUR/USD, USD/JPY, GBP/USD, USD/CHF, CAD/USD, AUD/USD, NZD/USD, USD/CNY	Daily FX spot prices since 1972, except for EUR/USD starting in 1999. Source: FRED

[1] https://fred.stlouisfed.org/

2.2 Description of the drawdown process

Along the lines of Embrechts et al. (1997), who discuss the study of floodings in the Netherlands to come up with a reasonable height estimate for the construction of dikes, one can similarly study the "financial storm" experienced for a specific asset to come up with a reasonable risk assessment for a portfolio. This paper analyzes the data from table 1 by deconstructing the different time series into daily, bi-weekly, monthly, quarterly, and yearly intervals. For each interval, the maximum drawdown is measured[1]. The maximum relative drawdown is defined as follows: Let's consider a price process X during a time interval starting at time t_s and ending at time t_e.

$$MDD_X(t_s, t_e) = \sup_{t \in [t_s, t_e]} \left\{ \frac{M_t - X_t}{M_t} \right\}, \tag{1}$$

where $M_t = \sup_{s \in [t_s, t]} X_s$.

Figure 1 provides intuition for the concept of market drawdowns by analyzing the drawdowns for the S&P500 Index since 1928. The left panel in Figure 1 shows an application of the drawdown risk measure to the S&P500. In this case the time interval is the entire data set between 1928 and 2020. The grey-shaded areas highlight the periods in which an investor is in a drawdown state. Two dimensions appear from the chart; one being the magnitude dimension shown on a logarithmic scale, the other being the time dimension indicating how long the Index has been in a state of retracement. The right panel is created by splitting the time series into yearly intervals. For each interval the maximum drawdown is calculated and expressed in logarithmic returns. At first sight one can note big discrepancies over time and a few extremes appearing during periods of market turbulence such as the Great Depression or the Global Financial Crisis.

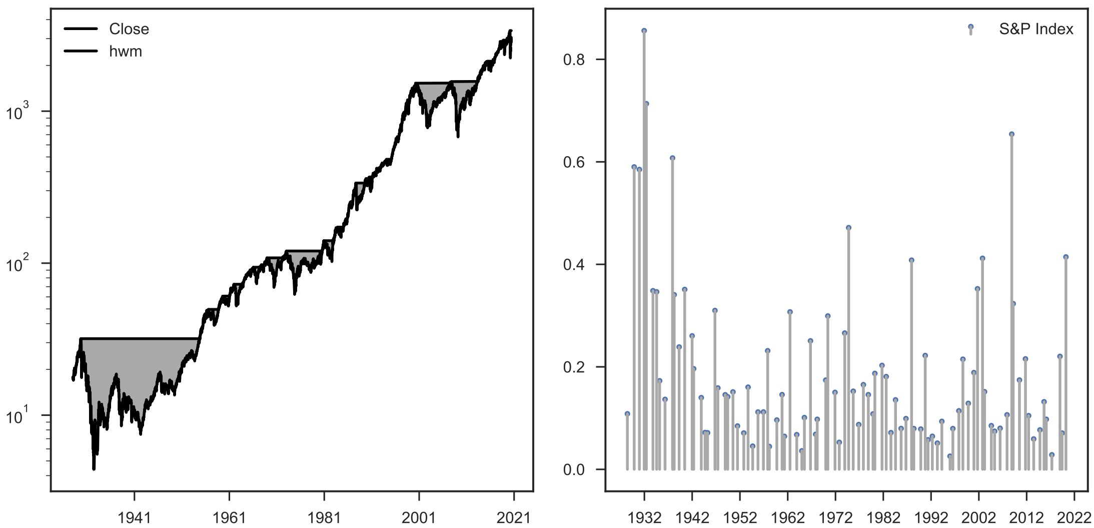

Figure 1: Left panel: Retracements from previous highs; the grey shaded area shows the periods where the index found itself at a lower level than the previous high. Two interesting drawdown dimensions appear: The magnitude of the drawdowns and the duration in time. Right panel: Yearly maximum drawdowns for the S&P500; the time series has been split into yearly periods, and for each year the maximum drawdown has been determined. We see big discrepancies over time; periods of market turbulence are characterized by significant maximum drawdowns, such as the period of the Great Depression and the Global Financial Crisis.

[1]For the daily data, the maximum drawdown equals the daily loss

An illustration of this method applied to yearly drawdowns for the S&P Index can be found in figure 2. Besides the maximum drawdown magnitude, one can also consider the time dimension: Assuming a drawdown starts at t_s; Time Under Water (TUW) refers to how long an investment subsequently remains under the level $X(t_s)$ before recovering to this level at time t_r. The time of maximum drawdown is defined to be t_{mdd} and indicates the specific date on which the recovery starts. The start time t_s and t_{mdd} are highlighted by the red shaded area in Panel (c).

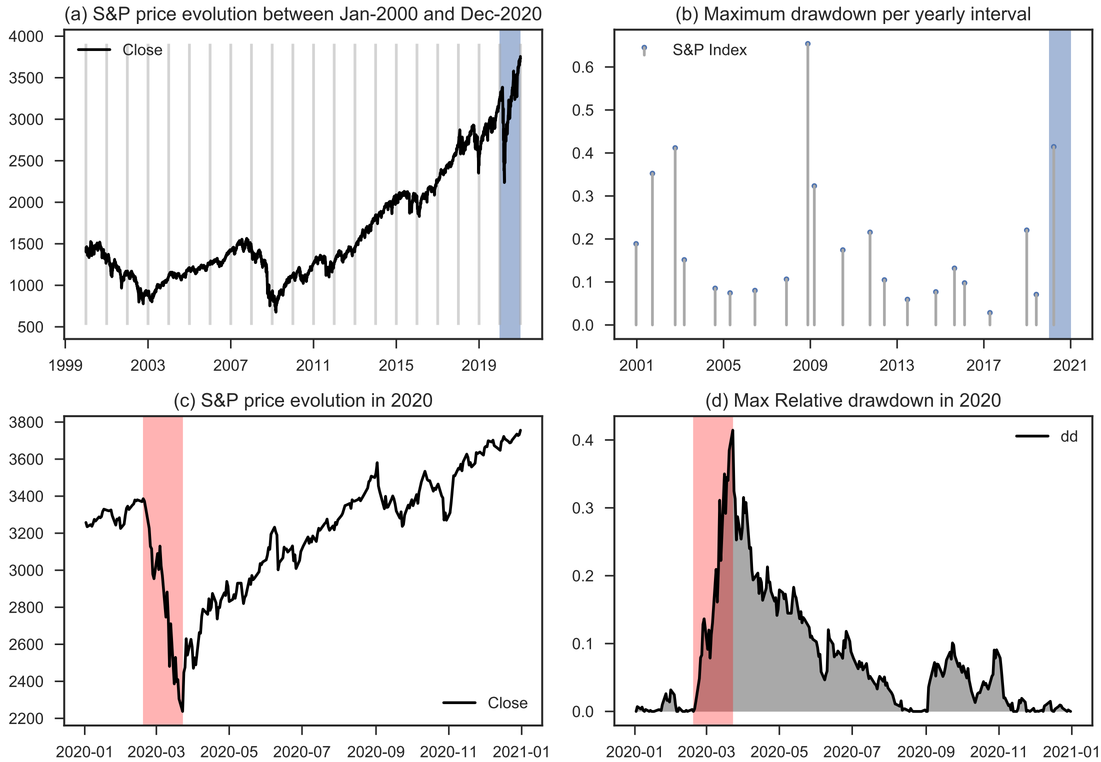

Figure 2: In Panel (a) the historical Time Series with daily closing prices between 2000 and 2020 are split into yearly intervals. 2020, the blue-shaded year, will be analyzed in the two lower panels. Panel (b) shows the maximum drawdown for each yearly interval. The stems coincide with the date of the maximum drawdown, which in 2020 occurs in March. Panel (c) shows the detail for the year 2020. The red-shaded area starts where the maximum drawdown period begins and ends at the point where the maximum drawdown is reached for 2020. The last panel shows the relative maximum drawdown as defined above.

2.3 Modeling the drawdown distribution

The literature considers several distributions for modeling empirical drawdown data such as the exponential, the Weibull, and Power Law distributions (see for example Maslov and Zhang (1999), Leal and Mendes (2005) and Johansen and Sornette (2001)). A first plausible candidate for modeling the tail of the drawdown distribution is a Power Law or Pareto distribution with a survival function defined by two parameters α and x_{min}.[2]

$$\overline{F}(x) = \Pr(X > x) = \begin{cases} \left(\frac{x}{x_{\min}}\right)^{-\alpha} & x \geq x_{\min}, \\ 1 & x < x_{\min}, \end{cases} \tag{2}$$

The modeling methodology is inspired by the principles outlined in Clauset et al. (2009) and Alstott et al. (2014). The first step in modeling the drawdown data is to identify the lower threshold x_{min}. The optimal value of x_{min} is derived in two steps. First, a power law fit is created starting from each unique value in the data set. This means that the initial fit covers all the drawdown observations. In the second step, the smallest observation is removed. This implies that at each step, the number of tail observations that are fitted becomes smaller.

After this first step, we restrict the x_{min} selection to those that yield a σ below a 0.25 threshold for the estimated shape parameter α. Subsequently, the one that results in the minimal Kolmogorov-Smirnov distance[3], D, between the data and the fit, is selected. The shape parameter α is estimated based on the method of maximum likelihood by means of the Hill estimator.

Figure 3 shows the application of this process to the monthly drawdown data of the S&P500. It shows the minimal Kolmogorov-Smirnov distance, D, is achieved at a value x_{min} of 6.19%. Another way to determine x_{min} is by visually inspecting the data to identify the point below which the data no longer appears to follow a Power Law. This method is more subjective and can vary depending on the observer.

It is important to acknowledge that not all assets have an equal number of observations or span the same time period when analyzing drawdowns for the different assets in table 1. The implications of such differences may be significant, especially when fitting drawdown data to a Pareto law or stretched exponential distribution. Having less extreme observations can make it more challenging to fit tail data to any distribution accurately, as the data may not capture enough extreme events that are critical for estimating tail parameters.

The main objective of this study is to provide valuable insights into the risk characteristics of the different assets. Therefore, the drawdown data for the individual assets are based on all available data. Data spanning different periods can make it challenging to compare drawdowns across assets accurately and can lead to biased estimates of the tail parameters of the distribution. For example, the amount of monthly drawdown observations for the S&P500 equals 1128, whereas

[2]The survival function of a power law distribution that is limited to values between $x_{\min}$ and $x_{\max}$ can be expressed as follows:

$$\overline{F}(x) = \left(\frac{x}{x_{\min}}\right)^{-\alpha}, \quad x_{\min} \leq x \leq x_{\max}$$

[3]The Kolmogorov-Smirnov distance (also known as the Kolmogorov-Smirnov statistic or simply the K-S distance) is a non-parametric statistical test that measures the maximum distance between the empirical distribution of a sample and a reference cumulative distribution function (CDF). Intuitively, it quantifies the difference between the empirical distribution of a sample and a hypothesized distribution. It is calculated by finding the largest vertical distance between the sample CDF and the reference CDF.

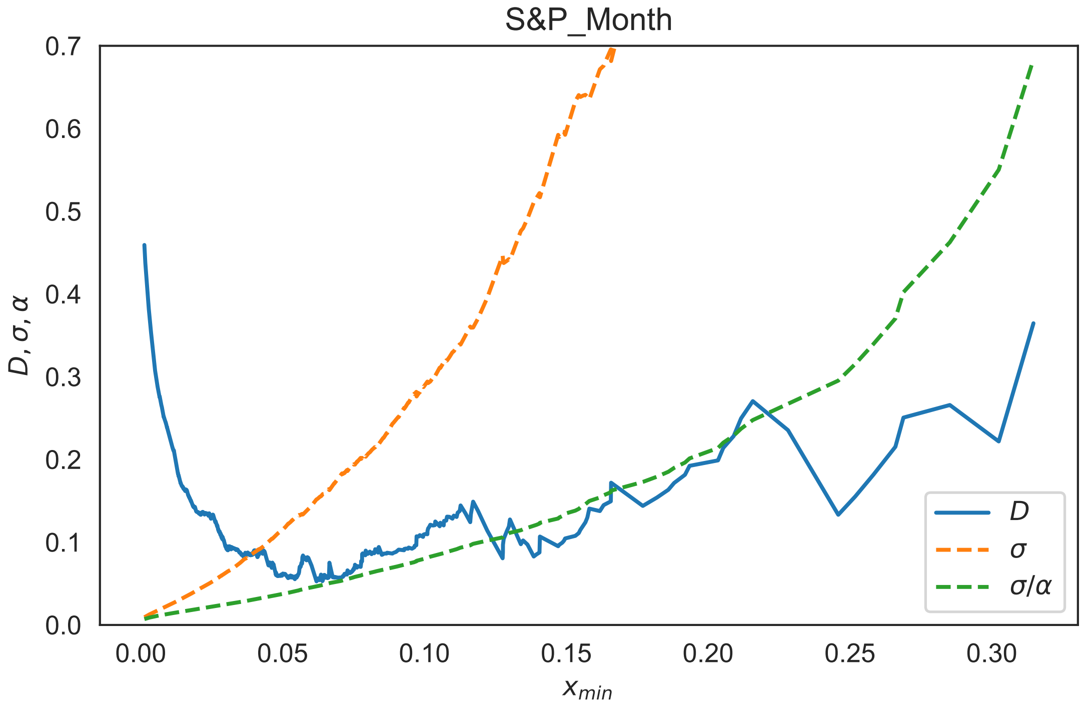

Figure 3: The Kolmogorov-Smirnov distance for the S&P500 monthly drawdown data.

the amount of monthly observations linked to Bitcoin is only 120.

If the objective would be to make an accurate analysis of the differences between different assets over time, one may consider adjusting the time periods. Nevertheless, different time spans do not entirely preclude asset comparison, as the presence of significant drawdowns over a short time period versus the absence of such events over a longer timeframe still indicates differences in risk profiles.

Besides using the Power law, the Weibull distribution or stretched exponential distribution as used in Johansen (2003) and Rebonato and Gaspari (2006) is considered to be a plausible candidate to model the drawdown data. Its strength lies in the fact that it is a generalization of the pure exponential. The survival function of a Weibull distribution with location parameter $x_{\min}$, scale parameter χ, and shape parameter z is given by:

$$\overline{F}(x) = \exp\left[-\left(\frac{x - x_{\min}}{\chi}\right)^z\right]$$

where $\overline{F}(x)$ is the probability that a random variable X is greater than x, given that it is greater than or equal to $x_{\min}$.

Both parameters χ and z, taken together, provide a concise characterization of the stretched exponential distribution. The parameter χ characterizes the typical size of the drawdown. The parameter z refers to the tail of the distribution. If the exponent z is smaller than 1, the stretched exponential distribution has fatter tails than the simple exponential. An exponent z greater than 1 has thinner tails than the simple exponential. Figure 4 shows the estimated distribution functions

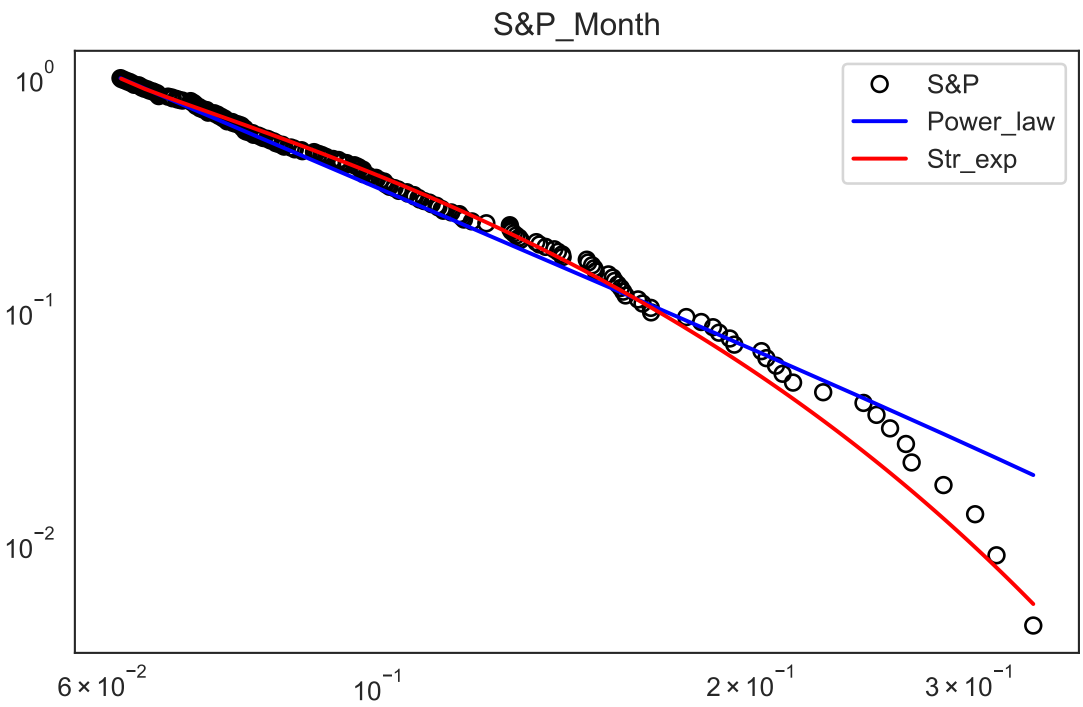

Figure 4: The estimated distribution functions for the monthly drawdown data of the S&P500

for the monthly drawdown data of the S&P500.

Finally, bootstrapping is applied by using 1000 re-sampled data sets to measure the uncertainty around the estimated parameters for the Pareto distribution and the Weibull distribution.

The choice to consider both the Weibull and Pareto distributions in the drawdown modeling methodology serves the purpose of capturing different characteristics of the data. The Weibull distribution and Pareto distribution are both plausible candidates, each offering unique insights into the tail behavior of the drawdown distribution.

3 Empirical results

This section analyzes the max drawdown data over different time intervals for each asset class. For each product within an asset class, the maximum drawdown characteristics for different time intervals have been derived. Besides providing an overview of the statistical results, the tail of the drawdown data is modeled using the Pareto and Weibull distribution as defined in Section 2. The results and the economic relevance of these drawdown observations are addressed in section 4.

3.1 Equity drawdown analysis

Figure 5 provides a graphical representation of the maximum drawdown data for each equity index through a boxplot. It provides a way to visualize the data's central tendency, spread, and skewness. The boxplot box represents the interquartile range (IQR), which is the range between the first and third quartiles (Q1 and Q3). The line inside the box represents the median. The first quartile, Q1, is the value greater than or equal to 25% of the observations, and the third quartile, Q3, is the value greater than or equal to 75% of the observations. The whiskers of the boxplot extend from the box to the smallest and largest observations that are still within 1.5 times the IQR of the box. Any observations outside this range are plotted as individual points. Table 2 complements figure 5 and shows the number of observations, the mean, the standard deviation, several percentiles, the maximum drawdown observed, and the skew and kurtosis of the observed data for each equity index.

The boxplots show that the scale of the drawdowns for the various equity indices is similar. Bi-weekly drawdowns above 20% have occurred for many indices, and the yearly maximum drawdowns for all major equity indices go to roughly 50%. Considering the time intervals, the most violent observations tend to occur on the daily, bi-weekly and monthly time frames.

A few observations stand out from these descriptive statistics. The empirical distribution of drawdowns is asymmetric and skewed to the right. This skewness may be due to a number of factors, such as market volatility, trading patterns, or investor behavior. It is clear from the different boxplots that there is significant dispersion in the data and a strong presence of extreme observations.

The tail of the drawdown data is modeled using the Pareto and Weibull distribution as defined in Section 2. After defining the optimal x_{min}, the parameters for both distributions are estimated. The estimates from the fit and their standard deviation obtained via bootstrapping can be found in table 3.

One notable observation is that the α power law coefficient is less than 2 for all Indices for both quarterly and yearly observations. An α coefficient below 2 indicates that the standard deviation of the distribution is undefined. The statistical significance is stronger for the yearly versus the quarterly observations.

Another observation is that the $\hat{z}$ estimate for daily, bi-weekly, monthly, and quarterly drawdowns is lower than 1. This suggests that the distribution of these maximum drawdowns belongs to the sub-exponential class, meaning that the tails of the distribution decay more slowly than an exponential distribution. The $\hat{z}$ score for yearly indices tends to be higher than 1, although not statistically significant.

Figure 6 zooms in on the monthly drawdowns for each equity index and shows how both distributions can capture the majority of drawdowns until a certain threshold.

Table 2: Descriptive statistics

Name	Period	Count	Mean	Std	p_50	p_75	p_90	p_95	p_99	max_dd	skew.	kurt.
S&P	Daily	10964	0.008	0.010	0.005	0.010	0.018	0.025	0.046	0.205	4.110	35.062
	2Week	2537	0.023	0.024	0.016	0.030	0.049	0.068	0.119	0.277	3.078	16.058
	Month	1128	0.043	0.040	0.032	0.054	0.084	0.116	0.211	0.337	2.851	11.701
	Quarter	376	0.081	0.066	0.062	0.095	0.164	0.228	0.342	0.427	2.241	5.924
	Year	94	0.165	0.118	0.134	0.211	0.324	0.444	0.515	0.575	1.435	1.785
Nasdaq	Daily	5668	0.009	0.010	0.006	0.012	0.022	0.029	0.047	0.123	2.863	14.206
	2Week	1376	0.025	0.028	0.017	0.033	0.058	0.078	0.129	0.253	2.542	10.140
	Month	612	0.047	0.044	0.034	0.061	0.098	0.130	0.227	0.356	2.487	9.007
	Quarter	204	0.090	0.073	0.064	0.113	0.185	0.255	0.356	0.364	1.834	3.380
	Year	51	0.192	0.126	0.151	0.234	0.360	0.477	0.520	0.538	1.261	0.883
Nikkei	Daily	6661	0.009	0.010	0.006	0.012	0.021	0.027	0.046	0.149	3.073	18.964
	2Week	1557	0.025	0.025	0.018	0.034	0.055	0.071	0.110	0.230	2.452	10.391
	Month	696	0.047	0.038	0.037	0.063	0.096	0.121	0.175	0.370	2.253	9.671
	Quarter	232	0.089	0.063	0.069	0.119	0.173	0.207	0.296	0.370	1.531	3.008
	Year	58	0.184	0.104	0.165	0.238	0.309	0.362	0.493	0.512	1.031	1.331
CAC40	Daily	3912	0.010	0.010	0.007	0.013	0.022	0.028	0.046	0.123	2.539	11.621
	2Week	862	0.029	0.025	0.023	0.039	0.060	0.076	0.123	0.202	2.007	6.242
	Month	384	0.052	0.039	0.044	0.069	0.093	0.128	0.219	0.313	2.332	8.894
	Quarter	128	0.097	0.065	0.078	0.113	0.170	0.256	0.309	0.386	2.051	4.635
	Year	32	0.193	0.113	0.158	0.236	0.380	0.410	0.466	0.481	1.085	0.412
DAX	Daily	4135	0.010	0.010	0.007	0.014	0.022	0.029	0.050	0.131	2.690	13.980
	2Week	940	0.029	0.027	0.020	0.039	0.063	0.081	0.136	0.205	2.170	6.646
	Month	420	0.052	0.043	0.040	0.067	0.099	0.136	0.218	0.304	2.278	7.126
	Quarter	140	0.098	0.075	0.075	0.116	0.178	0.284	0.381	0.388	2.054	4.416
	Year	35	0.201	0.127	0.151	0.267	0.383	0.454	0.510	0.524	1.017	0.235
HSI	Daily	4261	0.011	0.013	0.007	0.014	0.024	0.032	0.053	0.333	6.664	115.319
	2Week	971	0.032	0.032	0.023	0.042	0.068	0.090	0.144	0.344	3.405	21.126
	Month	432	0.060	0.051	0.047	0.079	0.116	0.154	0.219	0.442	3.046	15.706
	Quarter	144	0.116	0.085	0.093	0.141	0.233	0.270	0.399	0.520	1.869	4.428
	Year	36	0.240	0.135	0.219	0.292	0.442	0.473	0.573	0.601	0.930	0.189
ASX	Daily	3551	0.007	0.008	0.005	0.009	0.016	0.020	0.034	0.097	3.300	20.709
	2Week	809	0.021	0.019	0.016	0.027	0.042	0.051	0.093	0.195	3.032	16.293
	Month	360	0.037	0.031	0.029	0.047	0.068	0.084	0.174	0.294	3.489	19.516
	Quarter	120	0.069	0.052	0.056	0.086	0.124	0.153	0.281	0.365	2.691	10.985
	Year	30	0.139	0.091	0.115	0.166	0.214	0.301	0.441	0.472	2.195	6.109
IBOVESPA	Daily	3438	0.015	0.015	0.010	0.020	0.032	0.041	0.076	0.158	2.875	13.986
	2Week	779	0.041	0.038	0.032	0.052	0.084	0.114	0.188	0.320	2.690	11.034
	Month	348	0.075	0.060	0.059	0.092	0.140	0.178	0.316	0.409	2.493	8.593
	Quarter	116	0.137	0.097	0.113	0.184	0.242	0.311	0.498	0.569	2.032	5.204
	Year	29	0.282	0.140	0.241	0.321	0.475	0.561	0.609	0.613	0.978	0.213

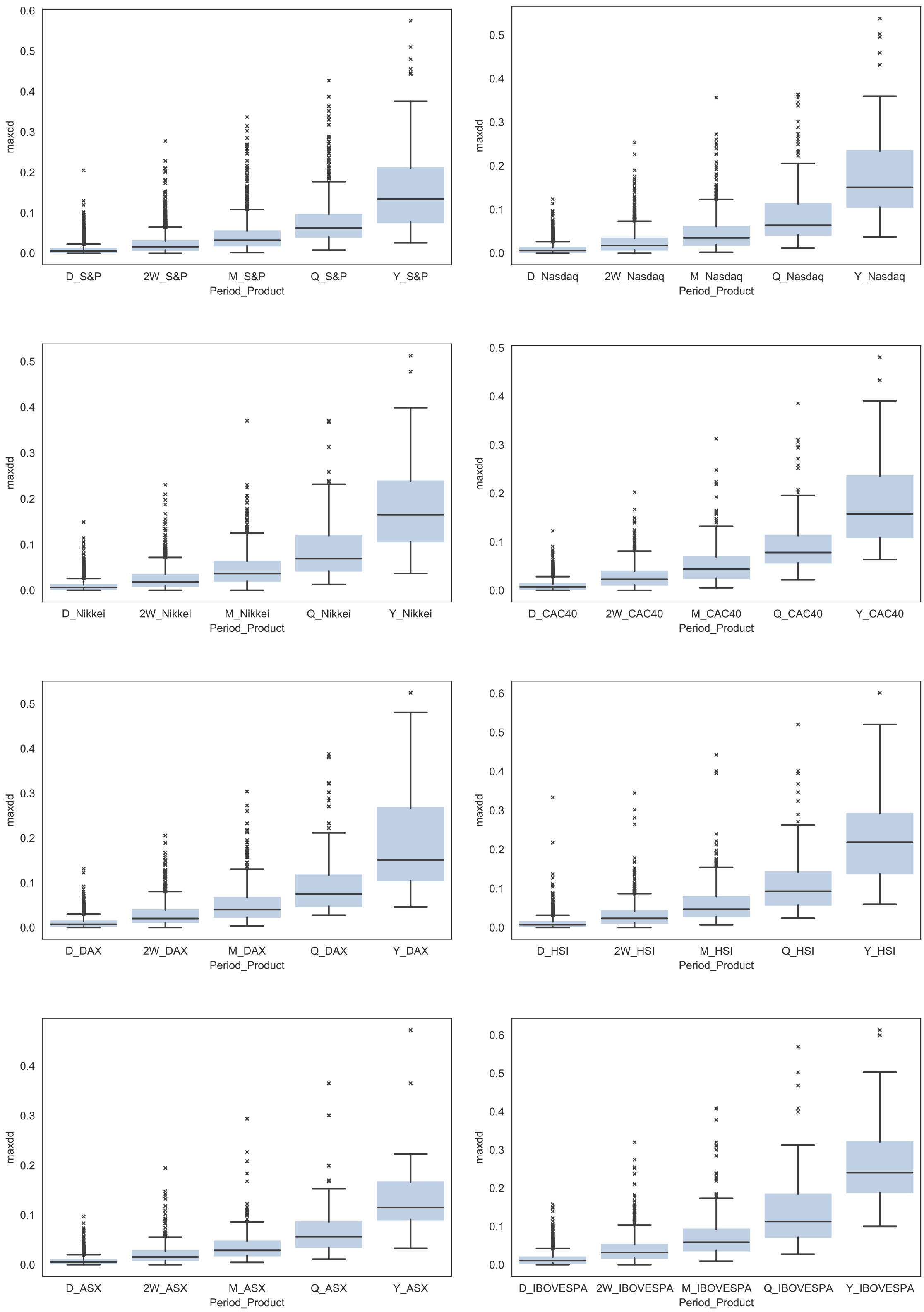

Figure 5: Boxplots for the various Equity Indices. The Y-axis shows the level of maximum drawdown for each of the daily, bi-weekly, monthly, quarterly, and yearly intervals.

Table 3: Fit of the tail maximum drawdown data to the Pareto and Weibull distribution. The cases where the Power Law $\hat{\alpha}$ 95% confidence interval is less than 2 are highlighted by a *.

					Pareto Distribution		Weibull Distribution			
Name	Period	Count	x_{min}	n_tail	$\hat{\alpha}$	$\sigma_{\hat{\alpha}}$	$\hat{z}$	$\sigma_{\hat{z}}$	$\hat{\chi}$	$\sigma_{\hat{\chi}}$
S&P	Daily	10964	0.028	444	2.862	0.122	0.866	0.032	0.013	0.001
	2Week	2537	0.063	160	3.083	0.227	0.841	0.046	0.027	0.003
	Month	1128	0.062	221	2.262	0.136	0.852	0.040	0.040	0.003
	Quarter	376	0.076	145	1.976	0.138	0.857	0.049	0.059	0.006
	Year	94	0.121	52	1.746	0.184	1.005	0.094	0.117	0.017
Nasdaq	Daily	5668	0.027	333	3.168	0.161	0.912	0.037	0.011	0.001
	2Week	1376	0.049	206	2.529	0.149	0.942	0.046	0.028	0.002
	Month	612	0.069	121	2.294	0.169	0.934	0.063	0.045	0.005
	Quarter	204	0.093	69	1.972	0.185	0.984	0.075	0.075	0.010
	Year	51	0.105	38	1.522*	0.181	0.975	0.128	0.126	0.022
Nikkei	Daily	6661	0.027	312	3.224	0.165	0.872	0.036	0.011	0.001
	2Week	1557	0.056	153	3.102	0.221	0.951	0.057	0.025	0.002
	Month	696	0.066	161	2.649	0.173	0.970	0.061	0.035	0.003
	Quarter	232	0.063	133	1.696*	0.105	0.971	0.073	0.062	0.006
	Year	58	0.104	43	1.477*	0.120	1.387	0.162	0.131	0.016
CAC40	Daily	3912	0.019	526	2.610	0.092	0.924	0.032	0.010	0.001
	2Week	862	0.043	193	2.654	0.167	0.899	0.048	0.022	0.002
	Month	384	0.036	240	1.746	0.082	0.959	0.055	0.034	0.002
	Quarter	128	0.065	86	1.947	0.178	0.893	0.061	0.053	0.007
	Year	32	0.086	27	1.277*	0.138	1.202	0.176	0.138	0.023
DAX	Daily	4135	0.023	408	2.868	0.125	0.886	0.038	0.011	0.001
	2Week	940	0.035	286	2.088	0.099	0.913	0.044	0.025	0.002
	Month	420	0.069	101	2.430	0.209	0.903	0.065	0.041	0.005
	Quarter	140	0.083	64	1.929	0.188	0.964	0.072	0.071	0.010
	Year	35	0.071	31	1.045*	0.101	1.197	0.158	0.158	0.024
HSI	Daily	4261	0.024	443	2.671	0.119	0.819	0.041	0.012	0.001
	2Week	971	0.045	223	2.312	0.136	0.840	0.049	0.028	0.002
	Month	432	0.080	107	2.584	0.213	0.913	0.074	0.045	0.005
	Quarter	144	0.100	65	1.978	0.189	0.938	0.089	0.080	0.011
	Year	36	0.108	31	1.311*	0.146	1.175	0.176	0.166	0.028
ASX	Daily	3551	0.019	230	3.046	0.190	0.833	0.040	0.008	0.001
	2Week	809	0.034	141	2.955	0.251	0.866	0.053	0.016	0.002
	Month	360	0.045	94	2.424	0.216	0.902	0.079	0.028	0.003
	Quarter	120	0.051	69	1.921	0.182	0.924	0.088	0.043	0.006
	Year	30	0.051	28	1.131*	0.119	1.149	0.253	0.100	0.018
IBOVESPA	Daily	3438	0.032	346	2.774	0.140	0.835	0.033	0.015	0.001
	2Week	779	0.046	246	2.107	0.112	0.852	0.041	0.033	0.003
	Month	348	0.087	99	2.325	0.207	0.879	0.062	0.055	0.007
	Quarter	116	0.104	66	1.981	0.206	0.913	0.076	0.083	0.012
	Year	29	0.120	27	1.296*	0.141	1.160	0.251	0.182	0.033

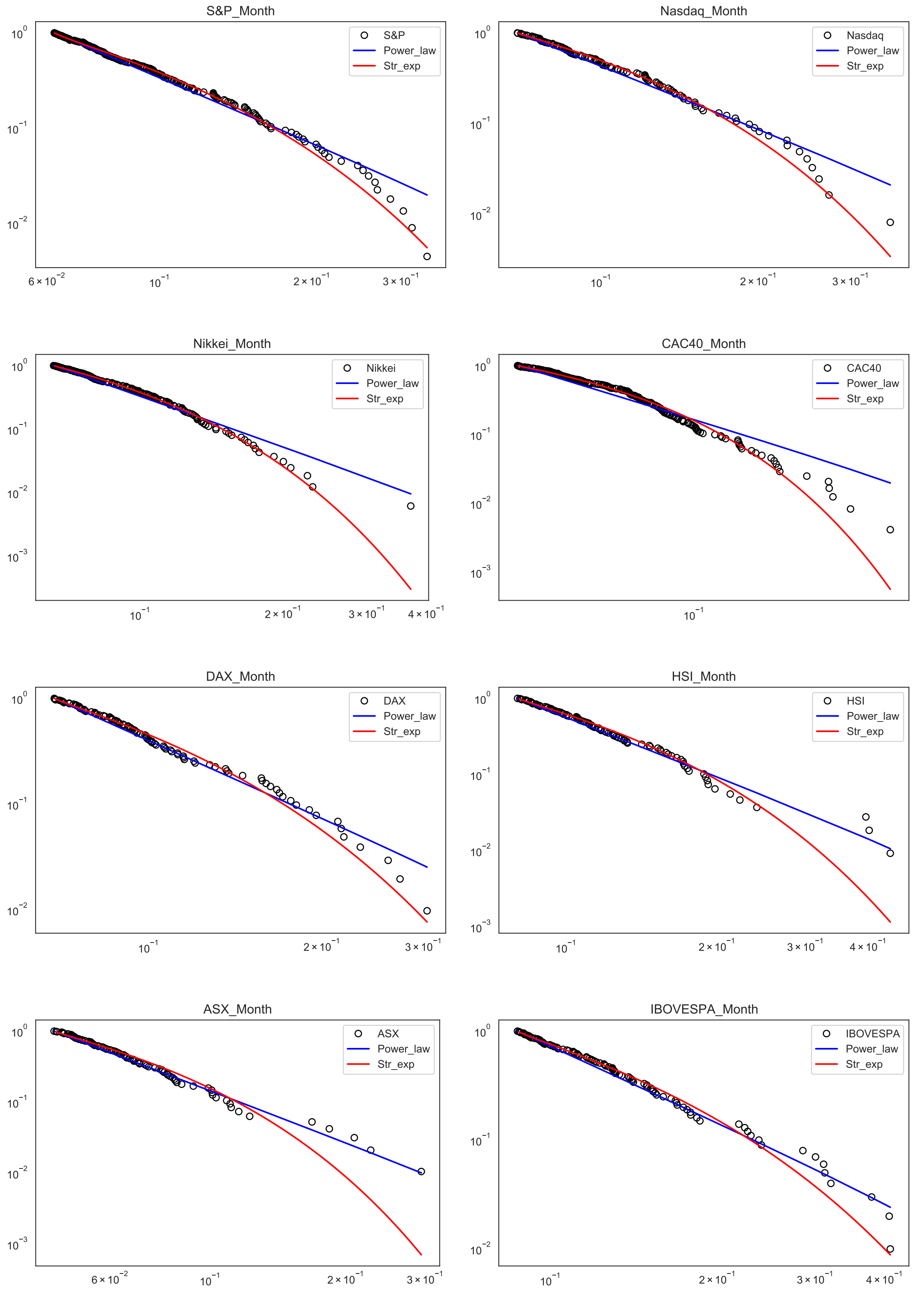

Figure 6: Fit of the drawdown data to the Pareto and Weibull distribution for the Equity Indices for the monthly maximum drawdowns.

3.2 Commodity drawdown analysis

Figure 7 and Table 4 provide a graphical representation and descriptive statistics of the maximum drawdowns of the analyzed commodity products. A first observation that stands out for the studied commodities is the size of the typical drawdowns and the outliers. Silver, Copper, Brent Oil, US Natural Gas, and Bitcoin have all experienced monthly drawdowns above 40%.

The largest average yearly and quarterly drawdowns occur for Bitcoin, followed by US Natural Gas and European Emissions prices. Unsurprisingly, this coincides with the highest optimal x_{min} obtained when fitting these drawdowns to the Pareto distribution: 67% for Bitcoin, 37.3% for US Natural Gas, and 35.7% for EU Carbon. The combination of a high x_{min} and a low number of yearly drawdown observations for these products leads to a small set of tail observations, which produces an α coefficient with a high margin of error that hence needs to be taken with a grain of salt.

Table 5 shows an overview of the estimated parameters. Except for these yearly drawdowns with a large margin of error, there is a noticeable pattern of declining α coefficients for the other commodity products as the time intervals lengthen. This observation aligns with what has been observed for the equity indices.

When considering the fit to the Weibull distribution, a similar pattern emerges as for the equity indices: the $\hat{z}$ scores tend to go up as the time intervals lengthen, whereas the scale factor $\hat{\chi}$ increases. Gold, Silver, Wheat, and Brent Oil show $\hat{z}$ scores, which are significantly below 1, for daily and bi-weekly observations. The other observations cannot be distinguished from a pure exponential based on the $\hat{z}$ estimates and their margin of error.

Figure 8 shows the monthly drawdowns and their fit to the Pareto and Weibull distribution. The majority of the drawdowns are well captured by both distributions. The outliers for the metals (Gold, Silver, and Copper) are better captured by the Power Law. Also, for Brent Oil, the Pareto distribution provides a remarkably good fit for all drawdowns. The outliers in Wheat, Sugar, and Natural Gas seem to be better captured by the Weibull distribution.

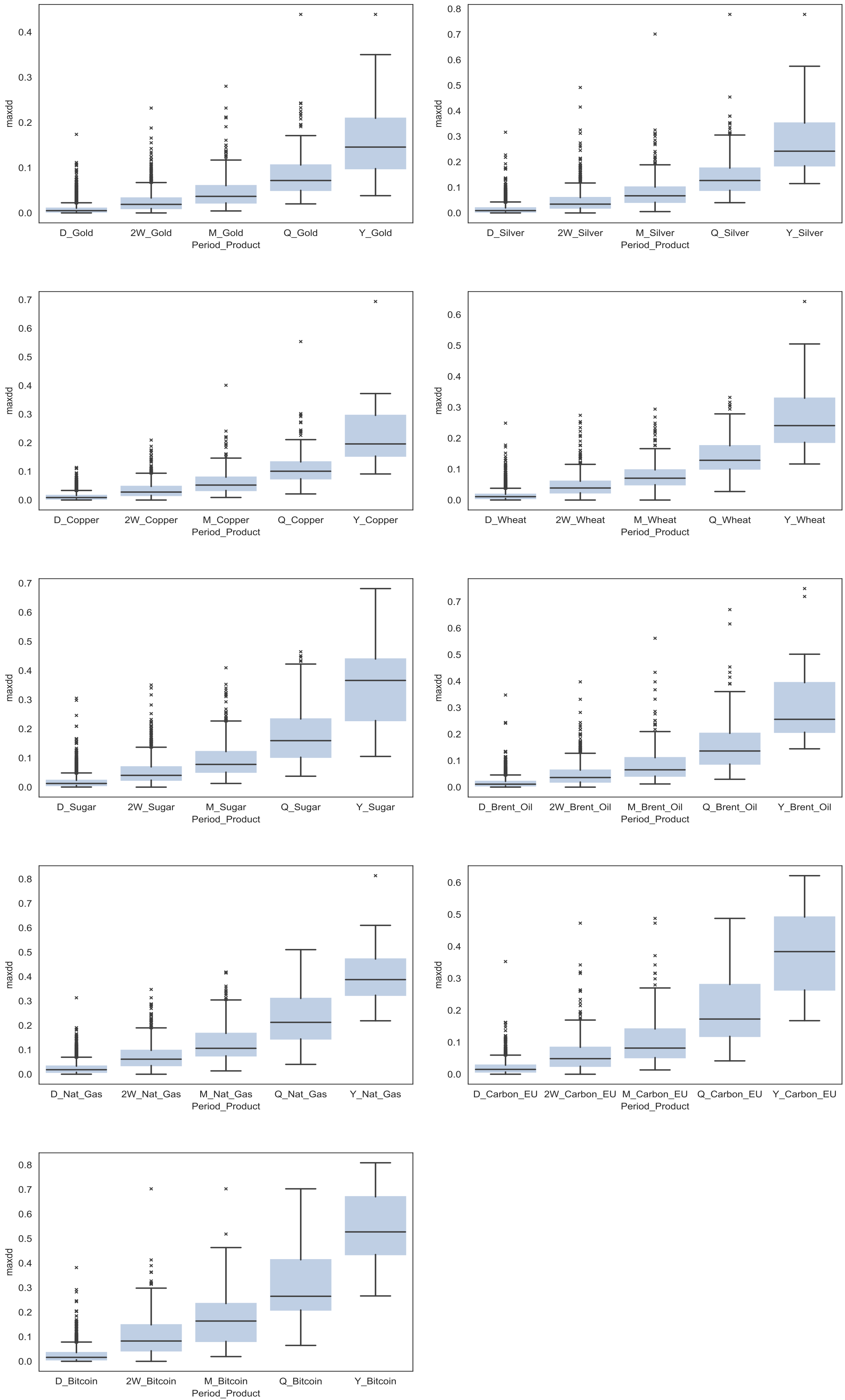

Figure 7: Boxplots for the various Commodities. The Y-axis shows the level of maximum drawdown for each of the daily, bi-weekly, monthly, quarterly, and yearly intervals.

Table 4: Descriptive statistics

Name	Period	Count	Mean	Std	p_50	p_75	p_90	p_95	p_99	max_dd	skew.	kurt.
Gold	Daily	5253	0.008	0.011	0.005	0.010	0.019	0.027	0.055	0.174	4.031	28.679
	2Week	1160	0.025	0.023	0.019	0.033	0.053	0.070	0.109	0.232	2.568	11.522
	Month	516	0.045	0.034	0.037	0.060	0.084	0.104	0.159	0.280	2.223	8.502
	Quarter	172	0.086	0.055	0.072	0.106	0.145	0.201	0.242	0.439	2.440	10.002
	Year	43	0.165	0.084	0.146	0.209	0.268	0.297	0.402	0.439	1.093	1.525
Silver	Daily	5498	0.015	0.019	0.009	0.020	0.037	0.053	0.079	0.316	3.612	27.608
	2Week	1160	0.046	0.042	0.035	0.060	0.090	0.112	0.191	0.492	3.382	21.793
	Month	516	0.080	0.059	0.067	0.101	0.138	0.183	0.298	0.701	3.435	25.472
	Quarter	172	0.146	0.092	0.127	0.176	0.234	0.313	0.401	0.778	2.723	13.217
	Year	43	0.282	0.133	0.242	0.352	0.430	0.503	0.693	0.778	1.561	3.397
Copper	Daily	4141	0.012	0.012	0.009	0.016	0.026	0.034	0.059	0.114	2.561	10.778
	2Week	917	0.036	0.028	0.028	0.047	0.070	0.089	0.141	0.210	1.879	5.256
	Month	408	0.062	0.041	0.052	0.080	0.109	0.137	0.205	0.401	2.426	12.291
	Quarter	136	0.116	0.069	0.101	0.133	0.200	0.242	0.300	0.554	2.594	11.627
	Year	34	0.228	0.117	0.196	0.296	0.338	0.371	0.588	0.694	1.993	6.437
Wheat	Daily	4332	0.014	0.015	0.011	0.018	0.028	0.037	0.072	0.249	4.136	34.084
	2Week	889	0.046	0.034	0.039	0.061	0.086	0.105	0.176	0.274	2.169	8.476
	Month	396	0.079	0.043	0.071	0.097	0.131	0.155	0.233	0.294	1.608	3.903
	Quarter	132	0.144	0.063	0.129	0.176	0.239	0.270	0.314	0.332	0.884	0.430
	Year	33	0.272	0.115	0.241	0.329	0.406	0.493	0.598	0.642	1.391	2.207
Sugar	Daily	5321	0.018	0.019	0.013	0.023	0.039	0.052	0.085	0.305	3.929	32.544
	2Week	1157	0.053	0.044	0.040	0.069	0.108	0.141	0.209	0.351	2.054	6.463
	Month	516	0.095	0.063	0.078	0.122	0.182	0.223	0.319	0.410	1.561	2.972
	Quarter	172	0.180	0.099	0.159	0.234	0.315	0.392	0.447	0.465	0.962	0.434
	Year	43	0.350	0.149	0.366	0.440	0.538	0.600	0.657	0.681	0.223	-0.788
Brent_Oil	Daily	4109	0.016	0.017	0.011	0.021	0.035	0.046	0.077	0.348	4.485	50.662
	2Week	917	0.048	0.042	0.036	0.064	0.096	0.127	0.199	0.397	2.399	10.259
	Month	408	0.084	0.063	0.065	0.111	0.153	0.188	0.329	0.562	2.663	12.353
	Quarter	136	0.160	0.107	0.136	0.203	0.280	0.368	0.559	0.670	1.959	5.530
	Year	34	0.309	0.145	0.256	0.394	0.463	0.578	0.740	0.749	1.561	2.556
Nat_Gas	Daily	4036	0.024	0.023	0.018	0.033	0.052	0.068	0.107	0.313	2.446	12.314
	2Week	863	0.073	0.053	0.062	0.098	0.144	0.173	0.245	0.347	1.401	2.590
	Month	384	0.128	0.076	0.106	0.167	0.229	0.283	0.354	0.419	1.174	1.325
	Quarter	128	0.235	0.112	0.213	0.311	0.386	0.441	0.497	0.510	0.485	-0.517
	Year	32	0.416	0.124	0.388	0.472	0.582	0.607	0.751	0.814	1.142	2.017
Carbon_EU	Daily	1837	0.022	0.022	0.015	0.028	0.048	0.065	0.105	0.353	3.594	30.836
	2Week	404	0.064	0.057	0.049	0.084	0.130	0.165	0.264	0.473	2.415	9.815
	Month	180	0.110	0.083	0.082	0.141	0.224	0.270	0.392	0.488	1.769	3.986
	Quarter	60	0.207	0.121	0.173	0.281	0.395	0.422	0.479	0.488	0.786	-0.474
	Year	15	0.379	0.147	0.384	0.492	0.562	0.594	0.616	0.621	0.010	-1.017
Bitcoin	Daily	1693	0.027	0.033	0.016	0.035	0.065	0.087	0.157	0.382	3.236	17.883
	2Week	269	0.107	0.089	0.083	0.149	0.212	0.264	0.372	0.703	2.060	7.812
	Month	120	0.177	0.119	0.164	0.235	0.341	0.407	0.508	0.703	1.300	2.646
	Quarter	40	0.313	0.154	0.265	0.413	0.519	0.593	0.663	0.703	0.674	-0.227
	Year	10	0.544	0.171	0.527	0.670	0.721	0.765	0.800	0.809	-0.080	-0.836

Table 5: Fit of the tail maximum drawdown data to the Pareto and Weibull distribution. The cases where the Power Law $\hat{\alpha}$ 95% confidence interval is less than 2 are highlighted by a *.

					Pareto Distribution		Weibull Distribution			
Name	Period	Count	x_{min}	n_tail	$\hat{\alpha}$	$\sigma_{\hat{\alpha}}$	$\hat{z}$	$\sigma_{\hat{z}}$	$\hat{\chi}$	$\sigma_{\hat{\chi}}$
Gold	Daily	5253	0.025	304	2.467	0.131	0.871	0.038	0.014	0.001
	2Week	1160	0.035	270	2.337	0.121	0.916	0.049	0.022	0.001
	Month	516	0.059	135	2.838	0.210	0.928	0.059	0.029	0.003
	Quarter	172	0.063	105	2.073	0.164	0.916	0.071	0.047	0.005
	Year	43	0.077	38	1.374*	0.123	1.209	0.191	0.107	0.015
Silver	Daily	5498	0.051	310	3.654	0.205	0.830	0.042	0.017	0.001
	2Week	1160	0.077	179	2.898	0.211	0.844	0.047	0.036	0.004
	Month	516	0.091	164	2.740	0.192	0.869	0.060	0.046	0.004
	Quarter	172	0.119	94	2.289	0.193	0.934	0.079	0.078	0.009
	Year	43	0.120	41	1.281*	0.095	1.416	0.164	0.188	0.023
Copper	Daily	4141	0.026	405	2.867	0.131	0.878	0.032	0.012	0.001
	2Week	917	0.058	159	3.112	0.212	0.959	0.051	0.025	0.002
	Month	408	0.067	145	2.668	0.180	0.959	0.072	0.036	0.003
	Quarter	136	0.084	87	2.138	0.178	0.965	0.088	0.061	0.007
	Year	34	0.097	32	1.298*	0.125	1.267	0.235	0.150	0.022
Wheat	Daily	4332	0.025	607	2.539	0.104	0.804	0.025	0.013	0.001
	2Week	889	0.057	255	2.843	0.159	0.871	0.044	0.027	0.002
	Month	396	0.065	232	2.470	0.127	0.981	0.048	0.038	0.003
	Quarter	132	0.108	90	2.374	0.170	1.175	0.071	0.069	0.007
	Year	33	0.117	32	1.294*	0.105	1.552	0.196	0.180	0.022
Sugar	Daily	5321	0.045	393	2.883	0.135	0.865	0.037	0.021	0.001
	2Week	1157	0.086	197	2.708	0.149	0.977	0.054	0.044	0.003
	Month	516	0.120	133	2.744	0.188	1.026	0.071	0.061	0.005
	Quarter	172	0.083	151	1.338*	0.065	1.111	0.076	0.118	0.009
	Year	43	0.144	40	1.200*	0.093	1.404	0.267	0.241	0.027
Brent_Oil	Daily	4109	0.032	535	2.726	0.103	0.856	0.035	0.016	0.001
	2Week	917	0.078	161	2.862	0.207	0.893	0.053	0.037	0.003
	Month	408	0.101	126	2.798	0.213	0.891	0.065	0.050	0.005
	Quarter	136	0.125	76	2.039	0.183	0.957	0.085	0.096	0.012
	Year	34	0.172	31	1.883	0.237	1.070	0.136	0.155	0.028
Nat_Gas	Daily	4036	0.063	256	3.575	0.215	0.865	0.044	0.022	0.002
	2Week	863	0.075	341	2.231	0.090	0.996	0.047	0.049	0.003
	Month	384	0.081	273	1.736*	0.073	1.022	0.048	0.077	0.005
	Quarter	128	0.153	92	1.771*	0.101	1.363	0.124	0.143	0.011
	Year	32	0.373	20	4.434	0.886	1.017	0.177	0.110	0.026
Carbon_EU	Daily	1837	0.024	563	1.866*	0.060	0.928	0.035	0.021	0.001
	2Week	404	0.059	170	1.831	0.104	0.974	0.061	0.052	0.004
	Month	180	0.093	79	1.776	0.140	1.037	0.092	0.087	0.010
	Quarter	60	0.120	44	1.580*	0.156	1.026	0.145	0.133	0.020
	Year	15	0.357	9	4.145	1.029	1.320	0.422	0.128	0.034
Bitcoin	Daily	1693	0.056	242	2.509	0.146	0.834	0.040	0.031	0.002
	2Week	269	0.114	97	2.076	0.155	1.045	0.094	0.086	0.009
	Month	120	0.143	69	1.999	0.158	1.160	0.103	0.116	0.013
	Quarter	40	0.157	35	1.470*	0.141	1.296	0.159	0.200	0.028
	Year	10	0.670	2	-	-	-	-	-	-

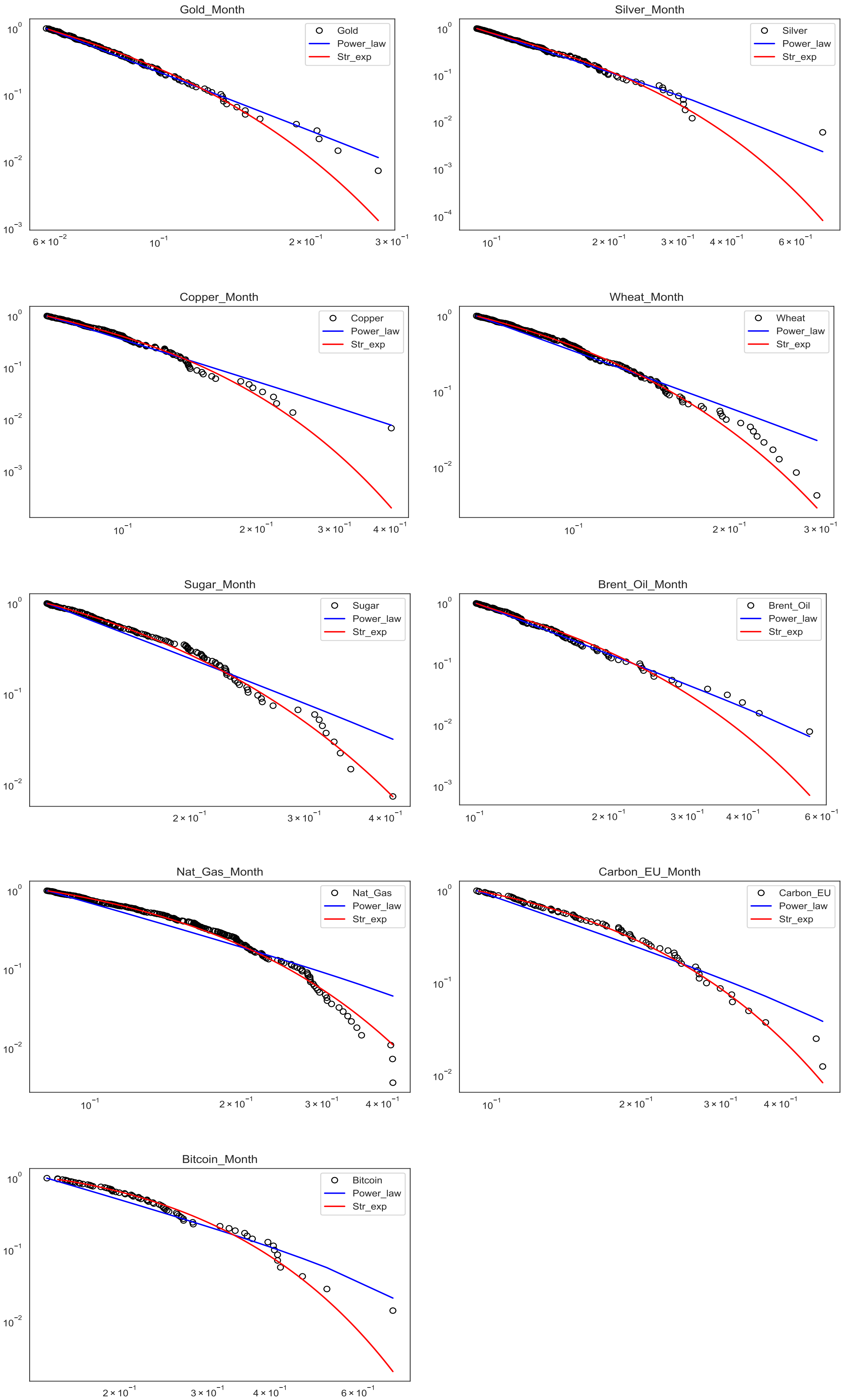

Figure 8: Fit of the monthly maximum drawdown data to the Pareto and Weibull distribution for the selected commodity prices.

3.3 FX drawdown analysis

Figure 9 and Table 6 provide an overview of the descriptive statistics linked to the empirical drawdown data for the selected FX indices. The magnitude of the FX drawdowns is smaller than for the previously analyzed equity indices and commodities. Similarly to the previously studied drawdowns, there is a presence of strong outliers for all studied time intervals.

The lower drawdowns for FX versus equity indices and commodities are generally confirmed by a lower x_{min} obtained for the Pareto Law fit. One $\hat{\alpha}$ coefficient that stands out from table 7 is the one for the daily losses in RMB, which is 1.353 ± 0.037. A more detailed analysis of the distribution fit shows that the fitted Pareto distribution fails to accurately model the most severe daily losses in this case.

Figure 10 shows the estimated fits for the monthly drawdowns of the selected FX pairs. Again, the bulk of the drawdowns is well described by both distributions, whereas the most extreme observations tend to fall between both fits.

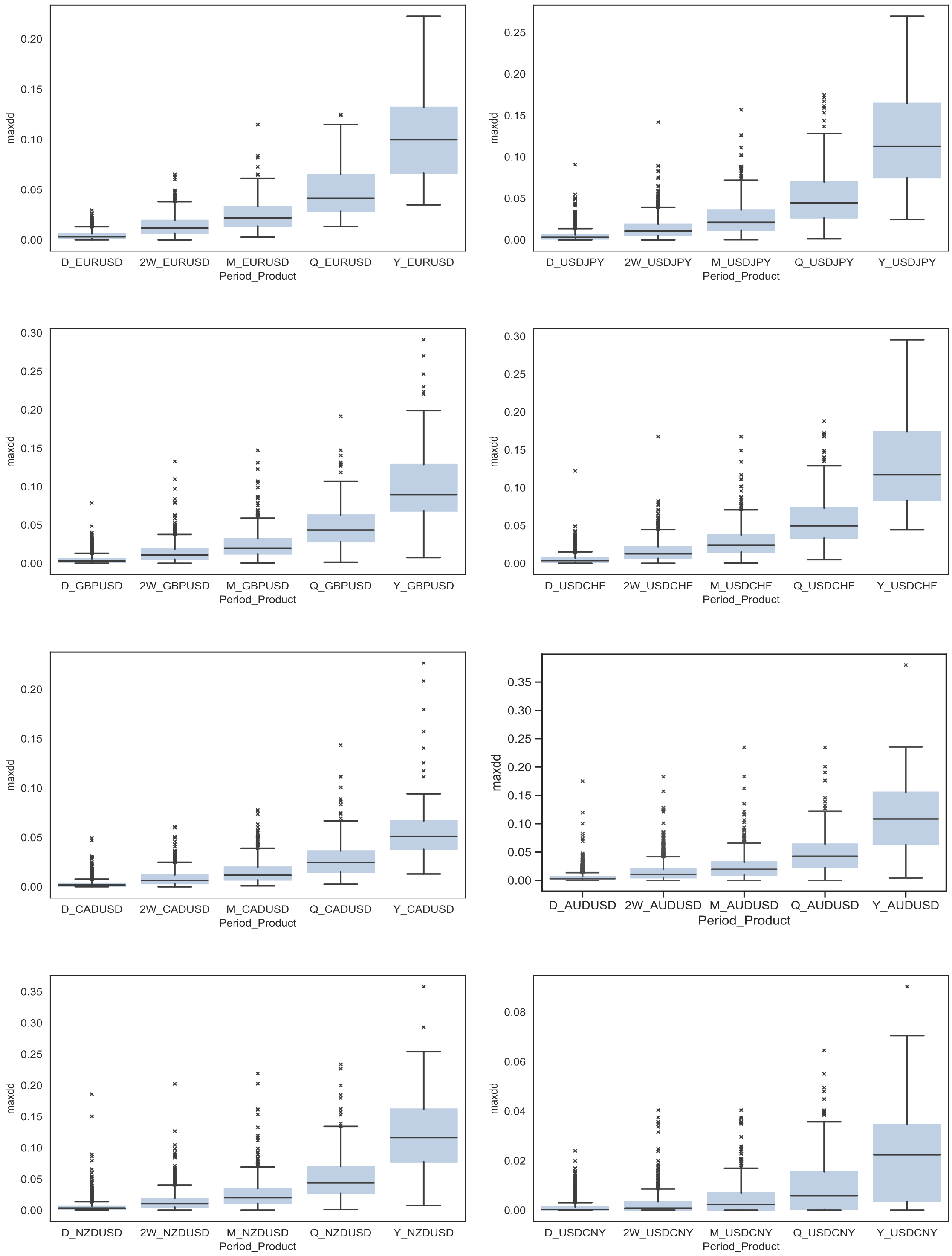

Figure 9: Boxplots for the most traded FX pairs. The Y-axis shows the level of maximum drawdown for each of the daily, bi-weekly, monthly, quarterly, and yearly intervals.

Table 6: Descriptive statistics for the selected FX pairs

Name	Period	Count	Mean	Std	p_50	p_75	p_90	p_95	p_99	max_dd	skew.	kurt.
EURUSD	Daily	3010	0.004	0.004	0.003	0.006	0.010	0.012	0.018	0.030	1.579	3.175
	2Week	647	0.014	0.010	0.012	0.019	0.027	0.035	0.047	0.065	1.486	3.115
	Month	288	0.025	0.016	0.022	0.033	0.045	0.057	0.074	0.115	1.534	4.110
	Quarter	96	0.049	0.027	0.042	0.065	0.081	0.104	0.125	0.125	1.053	0.673
	Year	24	0.107	0.051	0.100	0.132	0.173	0.196	0.217	0.223	0.529	-0.217
USDJPY	Daily	6246	0.005	0.005	0.003	0.006	0.010	0.014	0.024	0.091	3.289	25.217
	2Week	1403	0.014	0.013	0.011	0.019	0.030	0.038	0.056	0.142	2.318	10.968
	Month	624	0.026	0.020	0.021	0.036	0.052	0.065	0.099	0.157	1.776	5.184
	Quarter	208	0.053	0.035	0.044	0.070	0.094	0.121	0.171	0.175	1.299	1.946
	Year	52	0.124	0.064	0.113	0.164	0.223	0.239	0.260	0.270	0.581	-0.647
GBPUSD	Daily	6349	0.004	0.004	0.003	0.006	0.010	0.012	0.020	0.078	2.880	20.237
	2Week	1403	0.013	0.012	0.011	0.018	0.027	0.035	0.051	0.133	2.697	14.853
	Month	624	0.024	0.018	0.020	0.032	0.046	0.055	0.087	0.147	2.057	7.628
	Quarter	208	0.050	0.030	0.043	0.063	0.090	0.106	0.140	0.191	1.267	2.420
	Year	52	0.110	0.065	0.089	0.129	0.218	0.237	0.281	0.291	1.074	0.618
USDCHF	Daily	6417	0.005	0.005	0.004	0.007	0.011	0.015	0.023	0.122	3.721	47.858
	2Week	1403	0.016	0.013	0.013	0.022	0.033	0.040	0.060	0.167	2.353	14.246
	Month	624	0.030	0.021	0.024	0.037	0.058	0.068	0.102	0.167	1.875	5.978
	Quarter	208	0.059	0.036	0.050	0.073	0.110	0.136	0.170	0.188	1.182	1.172
	Year	52	0.136	0.069	0.117	0.174	0.233	0.273	0.290	0.296	0.715	-0.410
CADUSD	Daily	6384	0.003	0.003	0.002	0.004	0.006	0.009	0.014	0.049	3.346	24.906
	2Week	1403	0.009	0.008	0.007	0.012	0.019	0.023	0.034	0.061	2.146	7.834
	Month	624	0.015	0.012	0.012	0.020	0.030	0.036	0.057	0.078	1.807	4.790
	Quarter	208	0.029	0.021	0.025	0.036	0.053	0.067	0.111	0.143	1.983	5.934
	Year	52	0.064	0.046	0.051	0.067	0.125	0.167	0.217	0.226	1.982	3.810
AUDUSD	Daily	5878	0.005	0.006	0.003	0.006	0.011	0.014	0.025	0.175	8.123	152.795
	2Week	1403	0.015	0.016	0.011	0.020	0.031	0.041	0.074	0.183	3.480	21.776
	Month	624	0.024	0.023	0.019	0.032	0.048	0.064	0.116	0.235	3.154	17.866
	Quarter	208	0.050	0.039	0.043	0.064	0.101	0.122	0.190	0.235	1.658	3.839
	Year	52	0.115	0.070	0.108	0.155	0.191	0.215	0.306	0.380	1.077	2.734
NZDUSD	Daily	5909	0.005	0.007	0.003	0.006	0.011	0.015	0.027	0.186	8.265	160.014
	2Week	1403	0.015	0.015	0.011	0.019	0.032	0.040	0.064	0.202	3.396	25.144
	Month	624	0.027	0.025	0.020	0.035	0.054	0.068	0.119	0.219	2.917	14.197
	Quarter	208	0.054	0.041	0.044	0.070	0.109	0.133	0.199	0.234	1.682	3.536
	Year	52	0.125	0.071	0.117	0.162	0.203	0.242	0.325	0.358	0.804	1.408
USDCNY	Daily	3615	0.001	0.002	0.000	0.001	0.003	0.005	0.010	0.024	3.793	21.136
	2Week	1133	0.003	0.004	0.001	0.003	0.007	0.010	0.020	0.040	3.783	21.010
	Month	504	0.005	0.007	0.002	0.007	0.014	0.019	0.036	0.040	2.334	6.731
	Quarter	168	0.010	0.013	0.006	0.015	0.026	0.039	0.051	0.065	1.725	3.075
	Year	42	0.025	0.024	0.022	0.035	0.064	0.069	0.082	0.090	0.916	0.174

Table 7: Fit of the tail maximum drawdown data to the Pareto and Weibull distribution. The cases where the Power Law $\hat{\alpha}$ 95% confidence interval is less than 2 are highlighted by a *.

					Pareto Distribution		Weibull Distribution			
Name	Period	Count	x_{min}	n_tail	$\hat{\alpha}$	$\sigma_{\hat{\alpha}}$	$\hat{z}$	$\sigma_{\hat{z}}$	$\hat{\chi}$	$\sigma_{\hat{\chi}}$
EURUSD	Daily	3010	0.011	258	3.986	0.199	1.052	0.053	0.003	0.000
	2Week	647	0.020	157	3.041	0.202	0.978	0.056	0.009	0.001
	Month	288	0.022	148	2.242	0.139	0.968	0.073	0.014	0.001
	Quarter	96	0.032	66	1.811	0.154	1.016	0.112	0.028	0.004
	Year	24	0.035	23	1.006*	0.097	1.250	0.366	0.079	0.013
USDJPY	Daily	6246	0.014	327	3.280	0.170	0.856	0.042	0.005	0.000
	2Week	1403	0.025	211	2.887	0.166	0.951	0.059	0.012	0.001
	Month	624	0.036	161	2.788	0.182	0.950	0.061	0.018	0.002
	Quarter	208	0.052	90	2.380	0.185	1.037	0.084	0.033	0.004
	Year	52	0.065	43	1.497*	0.142	1.141	0.166	0.078	0.011
GBPUSD	Daily	6349	0.011	511	3.099	0.121	0.910	0.035	0.005	0.000
	2Week	1403	0.021	268	2.824	0.160	0.867	0.045	0.010	0.001
	Month	624	0.034	141	2.903	0.207	0.985	0.066	0.016	0.001
	Quarter	208	0.045	100	2.331	0.173	1.042	0.086	0.029	0.003
	Year	52	0.057	43	1.507*	0.154	1.061	0.125	0.069	0.010
USDCHF	Daily	6417	0.013	477	3.379	0.143	0.896	0.046	0.005	0.000
	2Week	1403	0.028	209	3.231	0.190	0.924	0.059	0.011	0.001
	Month	624	0.027	272	2.028	0.088	1.013	0.052	0.021	0.001
	Quarter	208	0.056	93	2.355	0.181	1.087	0.075	0.036	0.004
	Year	52	0.067	43	1.390*	0.116	1.272	0.177	0.092	0.012
CADUSD	Daily	6384	0.008	445	3.217	0.139	0.894	0.037	0.003	0.000
	2Week	1403	0.013	309	2.698	0.131	0.923	0.046	0.007	0.000
	Month	624	0.017	205	2.329	0.121	0.986	0.057	0.011	0.001
	Quarter	208	0.027	92	2.212	0.186	0.914	0.077	0.018	0.002
	Year	52	0.032	43	1.527	0.166	0.964	0.088	0.040	0.007
AUDUSD	Daily	5878	0.012	426	2.819	0.137	0.777	0.040	0.006	0.000
	2Week	1403	0.024	253	2.420	0.136	0.837	0.041	0.014	0.001
	Month	624	0.032	168	2.499	0.184	0.779	0.052	0.017	0.002
	Quarter	208	0.043	103	1.996	0.157	0.869	0.068	0.033	0.004
	Year	52	0.061	40	1.403*	0.119	1.182	0.228	0.081	0.011
NZDUSD	Daily	5909	0.013	431	2.665	0.125	0.793	0.037	0.007	0.000
	2Week	1403	0.023	279	2.410	0.119	0.905	0.051	0.014	0.001
	Month	624	0.038	139	2.512	0.193	0.828	0.058	0.021	0.002
	Quarter	208	0.044	103	1.882	0.144	0.922	0.065	0.037	0.004
	Year	52	0.074	40	1.583*	0.133	1.361	0.188	0.084	0.010
USDCNY	Daily	3615	0.002	746	1.353*	0.037	0.897	0.024	0.002	0.000
	2Week	1133	0.008	99	2.334	0.212	0.798	0.061	0.005	0.001
	Month	504	0.008	115	1.794	0.128	0.917	0.065	0.007	0.001
	Quarter	168	0.011	67	1.713	0.162	0.887	0.073	0.011	0.002
	Year	42	0.015	25	1.181*	0.120	1.295	0.175	0.027	0.004

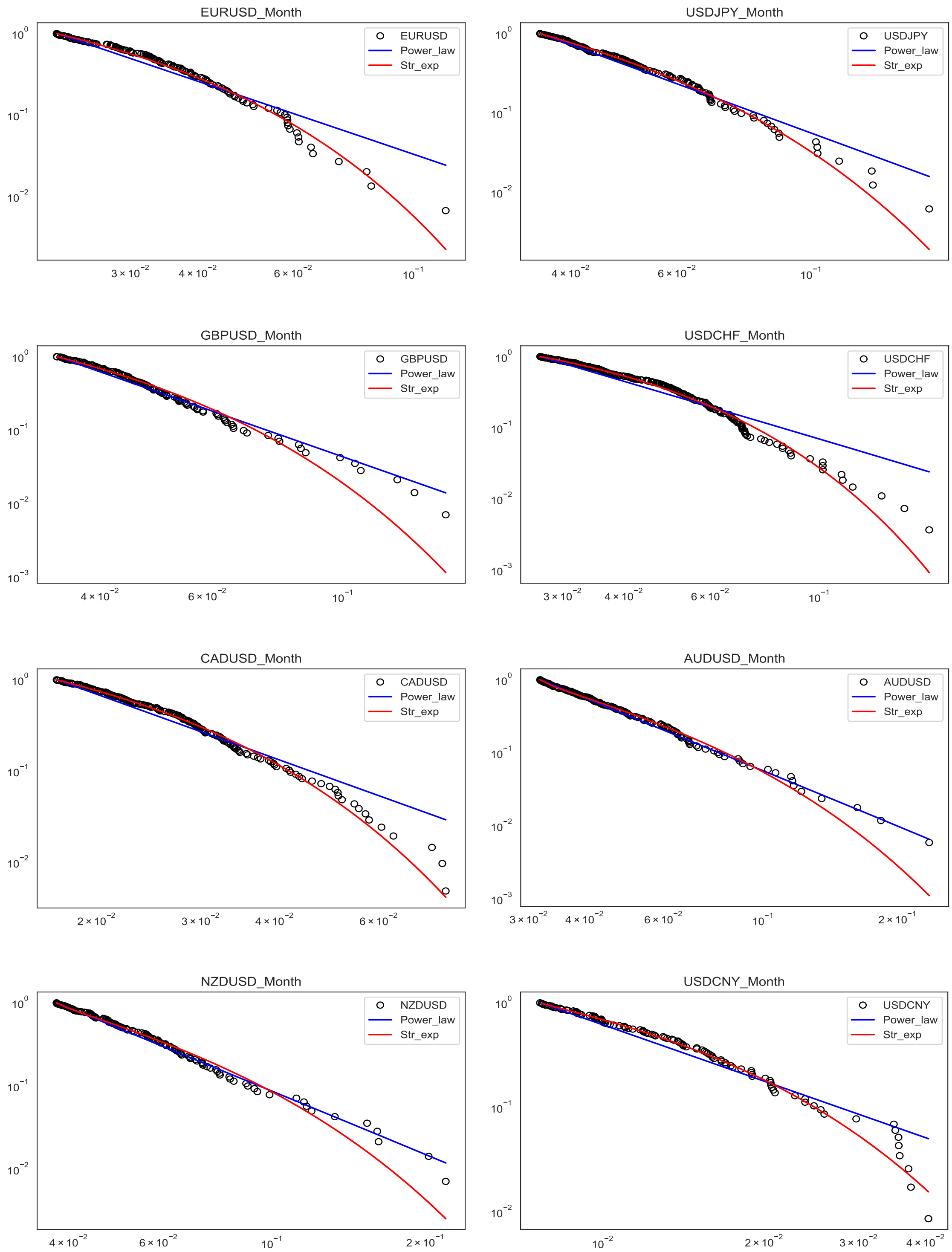

Figure 10: Fit of the monthly maximum drawdown data to the Pareto and Weibull distribution for the selected FX pairs.

4 Discussion

The results obtained for the Equity Indices highlight the significant risk of equity investments in all studied time intervals. The yearly maximum historical drawdowns for all major equity indices go to roughly 50%. However, this does not rule out the possibility of experiencing larger drawdowns than 50%. The findings equally suggest that the level of drawdown risk associated with these equity indices is comparable and that drawdowns are common across different markets. This finding has important implications for investors seeking to diversify their portfolios across multiple equity indices. It suggests that the risk of large drawdowns may not be reduced by simply investing in a broader range of indices.

The most extreme drawdowns tend to be underestimated by the stretched exponential, while most often, the risk of these drawdowns tends to be overestimated by the Pareto distribution. The stretched exponential provides a good fit for the S&P500 data, even for the most extreme drawdown observations. For several indices, like the ASX, Ibovespa, and HSI, the Power Law seems better equipped to capture the extremes. For the CAC40 and Nikkei, the most extreme drawdowns fall between the Power Law and the stretched exponential. The most extreme yearly drawdowns are not well captured by the Weibull distribution, suggesting that it may not be the best model for describing the behavior of extreme drawdowns over longer horizons.

There is a strong presence of outliers in the drawdown data. These outliers represent extreme drawdown events much larger than the typical drawdown observed in the data. Studying these outliers provides valuable insights into the factors that contribute to large drawdown events and may help investors to identify potential risks and vulnerabilities in their portfolios.

Certainly, the Global Financial Crisis (GFC) of 2008 provides an example of an outlier in terms of drawdown data. The drawdowns observed during the GFC were extreme outliers significantly impacting global financial markets. During the GFC, many equity indices experienced large drawdown events much larger than the typical drawdowns observed in the data. The S&P 500 Index experienced a maximum drawdown of approximately 50% during the crisis, representing a significant loss of value for investors. Several factors, including excessive risk-taking, unsustainable debt levels, and inadequate risk management practices, caused these events.

Within the instruments linked to Commodities, Silver stands out with a monthly drawdown greater than 70%. This drop, which occurred in March of 1980, is commonly called the "Silver Thursday" crash. The Hunt brothers, who had been attempting to corner the market on silver, failed to meet a margin call, and the price of silver subsequently plummeted.

The "Silver Thursday" crash in March 1980 serves as a striking example of the profound impact that the actions of a few individuals can have on the financial markets. Motivated by their belief that the value of silver would continue to rise, they accumulated massive holdings of the precious metal, driving up its price to unprecedented levels.

However, when the market turned against them, and they were unable to meet a margin call, panic ensued. The sudden realization that the Hunt brothers could not sustain their position led to a rapid and dramatic decline in the price of silver. The repercussions of the "Silver Thursday" crash echoed throughout the financial system, illustrating the interconnectedness and cascading effects that can arise from individual greed. Market participants and investors were swept up in the turmoil, leading to widespread panic selling and further amplifying the downward spiral of silver prices.

The "Silver Thursday" crash stands as a powerful reminder of how the actions of a few can create a vicious circle of market movements and destabilize entire sectors. It emphasizes the importance of risk management and the need for robust regulatory oversight to mitigate the potential impact of such outlier events.

Gold is often considered a safe haven investment due to its historical record of retaining value during economic or political uncertainty. It is a tangible asset not tied to any country's currency, making it a reliable store of value in times of inflation or currency fluctuations. This is confirmed by the typical size of the drawdowns, which is smaller than for the other analyzed commodities. Except for one outlier, the quarterly drawdowns have been less than 25%.

The outlier of 43.9% is again linked to a roller coaster ride for Gold prices. The rapid increase in gold prices during the end of 1979 and the first quarter of 1980 was not sustainable and eventually led to a significant correction. The main reason for the drop in gold prices in Q1 of 1980 was a combination of factors that changed the overall market dynamics. A tighter monetary policy, a stronger dollar, and reduced economic and political uncertainties led to the drop in gold prices in Q1 of 1980, reversing much of the price gains seen in the previous year.

The fact that gold typically experiences smaller drawdowns than other commodities supports its status as a safe haven. When stock markets or other commodities experience significant downturns, investors often turn to gold to diversify their portfolios and mitigate risk. However, the outliers show that it is not a risk-free investment and may not always perform as expected.

Bitcoin, even with its relatively short price history, experienced a remarkable event in which its value plummeted by 38.2% within a single day on March 12, 2020. This was a particularly volatile day for global financial markets, as concerns over the economic impact of the COVID-19 pandemic led to a widespread sell-off across a range of asset classes. The decline in Bitcoin prices on this day was particularly sharp, as investors, facing mounting uncertainty and panic in financial markets, rushed to liquidate their holdings, including Bitcoin and other cryptocurrencies, in favor of more traditional safe-haven assets like gold and the US dollar.

This significant daily drop is a noteworthy testament to the inherent volatility and rapid price movements associated with the cryptocurrency market. Despite its decentralized nature and distinct characteristics, Bitcoin was not immune to the swift and substantial fluctuations that can occur within this dynamic digital asset. The event serves as a vivid illustration of the risks and potential rewards inherent in investing in cryptocurrencies, emphasizing the need for cautious risk management and a comprehensive understanding of the market dynamics.

The largest daily drops for the studied FX indices occurred for the Australian and New Zealand Dollar versus the US Dollar in November 1976. In 1971, US President Richard Nixon announced a series of economic measures, including the suspension of the convertibility of the US dollar to gold, effectively ending the Bretton Woods system. This meant that the exchange rates of major currencies were no longer fixed to the US dollar and instead floated freely in the foreign exchange market. After the Bretton Woods system broke down in 1971, Australia abandoned its fixed exchange rate regime and adopted a floating exchange rate against the US dollar. To mitigate the volatility associated with this tie to the US dollar, Australia introduced a trade-weighted index (TWI) in September 1974, which measured the value of the Australian dollar against a basket of currencies. However, due to ongoing fluctuations in the exchange rate, the TWI valuation was periodically adjusted from November 1976.

A more recent extreme event is linked to the Swiss franc, which lost 12.2% against the US Dollar on the 15$^{\text{th}}$ of January 2015. On January 15, 2015, the Swiss National Bank (SNB) unexpectedly announced that it was abandoning its currency peg with the euro. The SNB had maintained a peg of 1.20 Swiss francs to the euro since 2011, which was intended to prevent the Swiss franc from appreciating too much against the euro and hurting Swiss exporters. When the SNB made the announcement, the Swiss franc surged in value against the euro and other major currencies.[4]

The Chinese Renminbi (RMB) has experienced relatively low drawdowns versus the US Dollar with a maximum yearly drawdown of 9%. It is worth noting that the RMB has experienced some fluctuations and volatility in recent years, particularly in response to global economic and political events, such as the US-China trade tensions and the COVID-19 pandemic. Nevertheless, compared to many other emerging market currencies, the RMB has been relatively stable, and its managed float exchange rate regime has played a significant role in limiting its volatility.

5 Conclusion

A key advantage of maximum drawdown processes is that they are able to describe extreme cumulative market moves occurring over a specified time frame. These drawdowns, therefore, offer a more natural path-dependent measure of market risk versus other measures based on fixed time scale distributions of returns, such as variance or Value-at-Risk. This paper analyzes the tail behavior of the maximum drawdowns for a range of liquid instruments from different asset classes for bi-weekly, monthly, quarterly, and yearly periods.

Summarizing the observations in the previous section, a few general themes stand out:

[4]In just a matter of minutes, the Swiss franc appreciated by more than 20% against the euro, which was a massive move in the foreign exchange market.

The maximum drawdowns seem to exist in two different regimes. Most drawdowns occur naturally in line with the ebb and flow of the typical behavior of financial markets. Another regime, where only a fraction of the maximum drawdowns occurs, could be linked to the extreme 'outlier' drawdowns. This behavior does not seem confined to the studied equity indices but is also prevalent in the analyzed commodity indices and foreign exchange rates.

For each modeled asset, several extreme 'outlier drawdowns' have been observed. The boxplots with the historical drawdowns show that the most violent observations occur over the daily, bi-weekly, and monthly time frames. The expression "markets go up on an escalator but down in an elevator" seems to hold true. The outliers in these distribution functions may indicate the presence of positive feedback mechanisms. The presence of behavioral biases, such as trend extrapolation and optimism, could create bubbles, which might subsequently be reversed in a period leading to significant drawdowns. For the longer time frames, such as the quarterly and yearly drawdowns, there might be a factor of mean-reversion coming into play, causing markets to move back into an equilibrium state. These 'outlier drawdowns' have significant implications for investors seeking to manage risk exposure and protect their portfolios against large losses. It also underscores the importance of diversification and risk management strategies that can help investors to mitigate the impact of large drawdown events.

The tail behavior of the drawdowns was estimated using a Pareto Law and a Weibull distribution, also labeled a stretched exponential. From a purely visual examination of the extreme tail events, it seems that commodities adhere more to the Power Law than FX and the equity indices, where the most extreme events seem to fall below the fitted line by the estimated Power Law. For equity indices, one could expect some rational lower bound after a certain drawdown. For certain commodities, on the other hand, extreme drawdowns can happen during oversupply or after a previous boom cycle.

When comparing the performance of the Pareto and Weibull distributions in capturing extreme drawdown events, the findings suggest that both models have limitations. The stretched exponential (Weibull) tends to underestimate the most extreme drawdowns, while the Power Law (Pareto) often overestimates the risk of these events. This indicates that neither distribution is a perfect fit for all asset classes and highlights the challenges of accurately modeling extreme drawdowns. The varying performance of the different distributions across asset classes further emphasizes the importance of tail risk analysis tailored to each specific market.

A small sample size for the tail of the distribution for quarterly and yearly observations complicates the modeling and the parameter estimation. In the case of a limited amount of observations, it is hard to argue that the data follow a Power Law. Huisman et al. (2001) suggest caution when using tail index estimates in small samples. Their paper highlights that the tail index estimates can be severely biased in small samples, resulting in an overestimation of the tail thickness or heaviness. The authors demonstrate that the Hill estimator, commonly used to estimate the tail index, is particularly prone to overestimation bias when the sample size is small.

As in the case of building dikes, the underestimation of an extreme risk can be disastrous. A famous saying in the investment world is: "Your biggest drawdown is the one that is yet to come." The distributions modeled in this paper confirm that observations very far from the expected average may occur and is a reason for cautiousness. The drawdown distribution functions remind us that in an area such as finance, where human behavior plays a key role, heavy tails, and extreme behavior are present, which might not always seem in line with rational behavior. Whether it is linked to behavioral biases or pure randomness, it is clear that extreme drawdowns over various time frames are present. Hence being aware of and protecting against them is vital for appropriate risk management for investors, fund managers, traders, and regulators.

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