Master's Thesis Engineering Technology 2024-2025

Statistical evaluation of the envelope geometry for efficiency calibration in gamma spectrometry

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Introduction

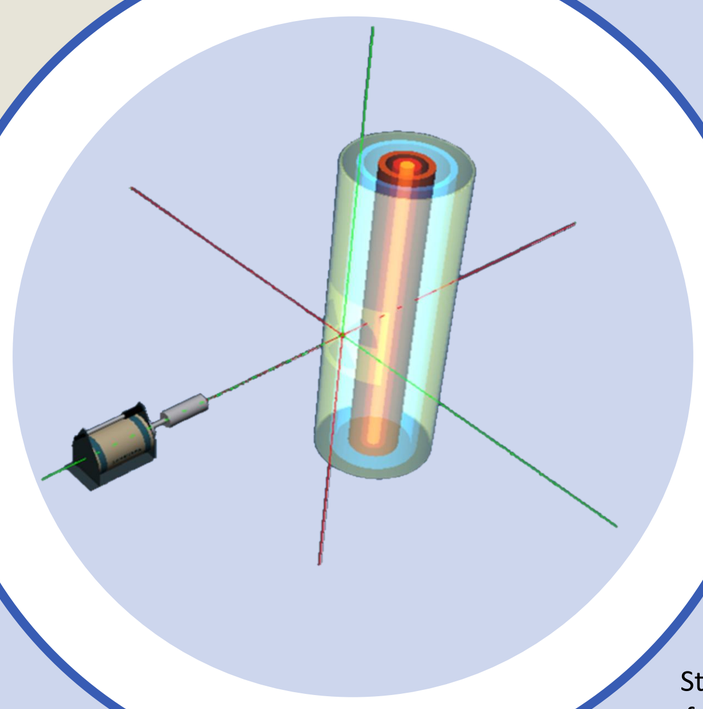
Figure 1: Large Hadron Collider at CERN [1]

At CERN, **gamma spectrometry** faces challenges due to samples with irregular geometry, such as screws and cables from particle accelerators (Figure 1). To account for these variations, **efficiency calibration** is required.

Problem statement & objective

Defining exact sample geometries is time-consuming. CERN employs the **envelope method,** an efficiency calibration approach using **ISOCS/LabSOCS software** with predefined standard geometries (central figure). **Discrepancies** between real and modelled geometries can introduce systematic errors.

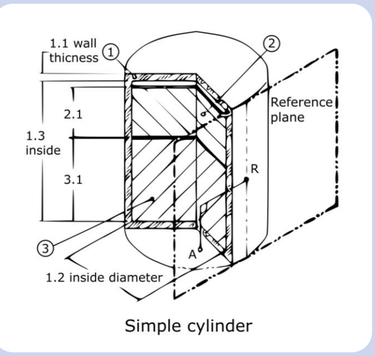
This study assesses the accuracy of the envelope method and estimates its error, uncertainty, and bias.



Method

A **database** was compiled containing gamma spectrometry measurement data for samples analysed by CERN using the envelope method and by an external reference laboratory (Jacobs) [2]. The latter is considered the **reference standard.**

An example of a standard geometry template in the ISOCS/LabSOCS software is shown in figure 2.



Simple cylinder

Figure 2: Standard geometry simple cylinder template in LabSOCS software [3]

Co-60 and Ti-44 were selected for performing statistical analysis based on the availability of high-quality data. Figure 3 shows the **ratio** of the activities measured by CERN (C) and Jacobs (J) for every sample. A ratio equal to 1 indicates perfect accordance between the methods.

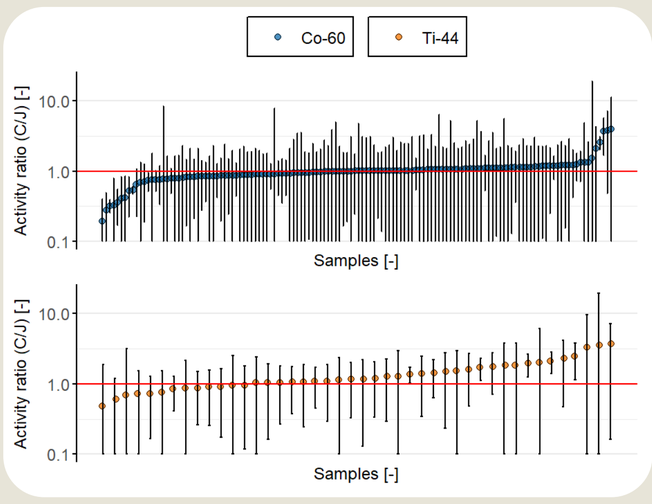


Figure 3: Scatter plot of the ratio of measured activity from CERN to reference activity from Jacobs (CERN/Jacobs)

Statistical tests are used to quantitatively assess the accuracy of the envelope method against the reference standard:

- the Wilcoxon signed-rank test and the paired t-test verify if there is a **statistically significant difference** between the activity results of the two methods;
- the Kullback-Leibler divergence quantitatively evaluates the **difference** between the methods;
- the statistical properties of the distribution of the activity ratio (C/J) estimate the **bias** introduced by the envelope method and its **uncertainty**.

Results

To estimate uncertainty in activity ratios, the **Full Width at Half Maximum** (FWHM) of their distributions is evaluated. Several parametric models were fitted, with the best fit identified via R^2. For Ti-44 and Co-60, a **lognormal distribution** provided the best fit (for Ti-44 R^2 = 0.9728 and 0.6226 for Co-60). Based on this model, the FWHM of the Ti-44 activity ratio (C/J) yielded **1.1567** (**0.7728** for Co-60), serving as a preliminary uncertainty estimate.

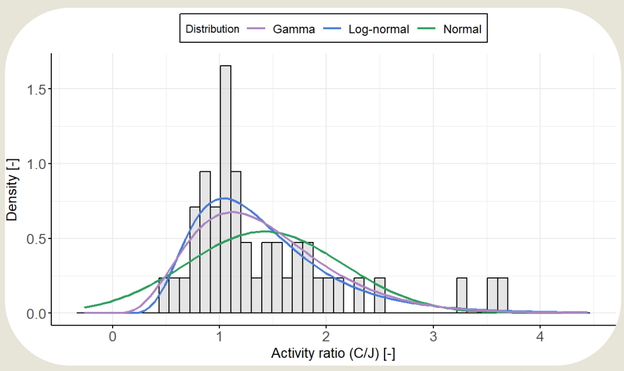


Figure 4: Histogram of the Ti-44 activity ratios between CERN and Jacobs (C/J) with parametric fits

A more robust, non-parametric method for determining the FWHM of the distribution is the **bootstrap resampling method**, as shown in figure 5. This method yielded a more reliable FWHM of **1.0395** for the Ti-44 activity ratio (C/J) distribution (**0.3455** for Co-60).

Figure 5 also shows the **bias** in the Ti-44 activity ratio (C/J) distribution, computed as median minus 1. Its deviation from 1 indicates a **systematic overestimation** (**0.1633** for Ti-44 and **0.0023** for Co-60) by the envelope method.

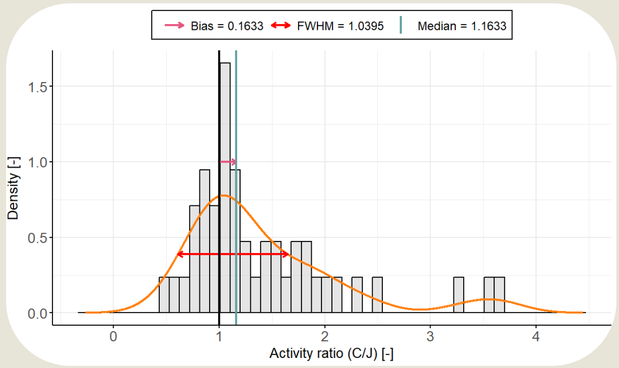


Figure 5: Histogram of the Ti-44 activity ratios between CERN and Jacobs (C/J) with a density distribution estimated with the bootstrap method

Conclusions

The results reveal **statistically significant differences** between the two approaches, particularly for Ti-44. Nevertheless, the overall agreement may be acceptable given the operational advantages of the envelope method. While no formal acceptance criteria currently exist, this study offers a **technical basis** to guide CERN in defining performance thresholds and assessing implementation potential.

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[1] 'The Large Hadron Collider', CERN. Accessed: Apr. 28, 2025. [Online]. Available: https://home.web.cern.ch/science/accelerators/large-hadron-collider
[2] 'Radiochemical Analysis | Jacobs'. Accessed: Feb. 20, 2025. [Online]. Available: https://www.jacobs.com/solutions/markets/energy-environment/nuclear-lifecycle/radiochemical-analysis
[3] 'ISOCS™ / LabSOCS™ Calibration Methodology', Mirion. Accessed: Nov. 17, 2024. [Online]. Available: https://www.mirion.com/isocs

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