

1 Solutions to the generalised alibi query in moving 2 object databases

3 Arthur Jansen¹ 

4 Hasselt University, Databases and Theoretical Computer Science Group and Data Science Institute
5 (DSI), Agoralaan, Building D, 3590 Diepenbeek, Belgium

6 Bart Kuijpers 

7 Hasselt University, Databases and Theoretical Computer Science Group and Data Science Institute
8 (DSI), Agoralaan, Building D, 3590 Diepenbeek, Belgium

9 — Abstract —

10 Space-time prisms provide a framework to model the uncertainty on the space-time points that a
11 moving object may have visited between measured space-time locations, provided that a bound on
12 the speed of the moving object is given. In this model, the *alibi query* asks whether two moving
13 objects, given by their respective measured space-time locations and speed bound, may have met.
14 An analytical solution to this problem was first given by Othman [5]. In this paper, we address
15 the *generalised alibi query* that asks the same question for an arbitrary number $n \geq 2$ of moving
16 objects. We provide several solutions (mainly via the spatial and temporal projection) to this query
17 with varying time complexities. These algorithmic solutions rely on techniques from convex and
18 semi-algebraic geometry. We also address variants of the generalised alibi query where the question
19 is asked for a given spatial location or a given moment in time.

20 **2012 ACM Subject Classification** Information systems → Spatial-temporal systems; Information
21 systems → Query languages

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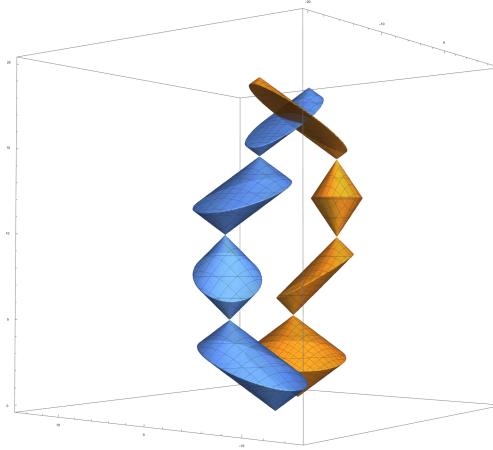
28 1 Introduction

29 In Moving Object Databases (MODs), various data models and query languages have been
30 proposed to deal with moving objects whose position is recorded by location-aware devices
31 (such as GPS), at not always regular moments in time [2]. The movement data of an object
32 is therefore discrete in nature and can be seen as a sequence $\langle (x_1, y_1, t_1), \dots, (x_n, y_n, t_n) \rangle$ of
33 measured space-time locations, which we call a *trajectory sample*. Between measured space-
34 time points, the trajectory of a moving object is unspecified and unknown and several models
35 have been proposed to deal with this uncertainty. Based on the assumption that moving
36 objects have some physically determined or law imposed speed bounds, the *space-time prism*
37 model delimits the region in space-time which a moving object may have visited between two
38 sampled points. This model, originating from the field of “time geography” in the 1970s, has
39 found its way into MOD research. The uncertainty on the movement of an object associated

¹ Corresponding author

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40 with a trajectory sample, is then modeled by a chain of space-time prisms. An illustration of
41 two chains is shown in Figure 1.



■ **Figure 1** An illustration of two intersecting chains of space-time prisms (one chain in yellow and the other in blue).

42 One query of particular interest in this context is the *alibi query*, which asks whether
43 two moving objects may have met, given their trajectory samples and speed bounds. The
44 difficulty of answering this query can be reduced to deciding whether two space-time prisms
45 intersect. Using a geometric argument, a solution to this problem was given by Othman et
46 al. [5]. We address the *generalised alibi query*, which asks the same question, but for any
47 (finite) number of moving objects. Because a space-time prism can be described by a system
48 of polynomial inequalities, the generalised alibi query can be expressed as an existential
49 first-order logic formula over the ordered field of real numbers. While there are algorithms for
50 deciding the truth of such sentences [1], existing implementations cannot solve the query in
51 practice (that is, within an acceptable amount of time). We provide several solutions (mainly
52 via the spatial and temporal projection) to this query with varying time complexities. These
53 algorithmic solutions rely on techniques from convex and semi-algebraic geometry, because
54 space-time prisms are both convex and semi-algebraic sets. We also address variants of the
55 generalised alibi query where the question is asked for a given spatial location or a given
56 moment in time. Additionally, some of our methods are capable of producing a *sample point*,
57 which is a point in the intersection of the n space-time prisms, if it exists. Finally, we give a
58 quantifier-free description of the spatial projection of the intersection of n space-time prisms,
59 which is exactly the region in space where the objects may have met (between two measured
60 points), and allows answering the spatial variant of the generalised alibi query in linear time.

61 Our main contributions are summarised in Tables 1 and 2, which give an overview of the
62 proposed methods or problems, their time complexity and their ability to produce sample
63 points. For clarity, the complexity results in this table refer to deciding the emptiness of the
64 intersection of n prisms. When the n moving objects are given by chains of prisms then the
65 time complexity results in the table need to be multiplied by $L - n + 2$, where L is the total
66 number of prisms in the n chains.

67 We give a brief summary of the workings of each of the proposed methods in the next
68 sections.

■ **Table 1** Time complexity (in terms of the number of prisms n) and sample points of the proposed methods for the generalised alibi query.

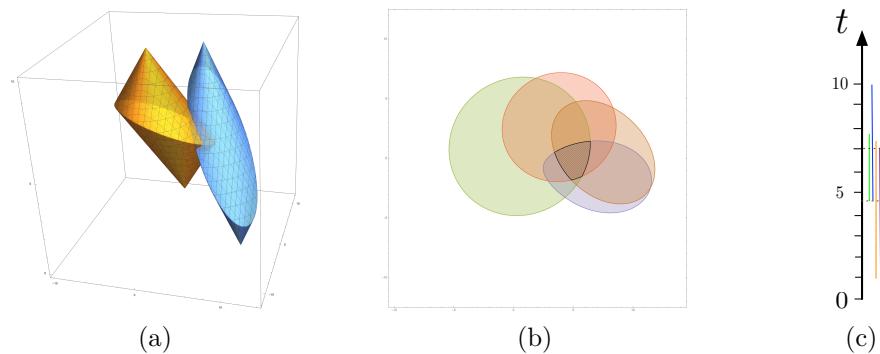
method	time complexity	sample points
via spatial projection	$O(n^5)$	yes
via spatial projection with Helly	$O(n^4)$	no
via temporal projection	$O(n^3)$	yes

■ **Table 2** Time complexity (in terms of the number of prisms n) and sample points of the variants for the generalised alibi query.

variants of generalised alibi query	time complexity	sample points
at a fixed location	$O(n)$	yes
at a fixed moment	$O(n^3)$	yes

69 2 Deciding the generalised alibi query via the spatial projection

70 The essence of this method is to test whether the spatial projection of the intersection of the
 71 n space-time prisms is empty. Because this projection is precisely the region in space where
 72 the n objects could theoretically have met, we call it the *meeting region*. We show, using
 73 Fourier-Motzkin elimination, that the meeting region can be characterised as the intersection
 74 of regions enclosed by curves called “Cartesian ovals”, which are generalisations of ellipses.
 75 An illustration of the meeting period as the intersection of such regions is shown in part
 76 (b) of Figure 2. This characterisation has two uses. On one hand, it provides a linear-time
 77 solution to the generalised alibi query at a fixed location, which includes the production of
 78 sample points. On the other hand, we can use it to test the emptiness of the meeting region,
 79 providing an answer to the generalised alibi query. To do this, we make use of the algebraic
 80 nature of Cartesian ovals, which allows us to compute a (finite) set of “candidate points”,
 81 with the property that the meeting region is not empty if and only if it contains one of those
 82 candidate points. This method requires $O(n^5)$ time and also produces sample points.



■ **Figure 2** An illustration of (a) two intersecting prisms; (b) regions enclosed by Cartesian ovals whose intersection (in black) is the meeting region of the two prisms; and (c) the meeting period (in red) of the two prisms, shown as the intersection of four projections of intersections of three cones.

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83 Due to Helly's theorem [3], which gives an equivalent condition for the non-emptiness of
84 the intersection of convex sets, we can also solve the generalised alibi query by applying the
85 above method to every combination of 4 of the n space-time prisms. The result of this is a
86 method that only requires $O(n^4)$ time. However, the disadvantage of that method is that it
87 cannot produce sample points.

88 3 Deciding the generalised alibi query via the temporal projection

89 The method described here works by testing the emptiness of the temporal projection of
90 the intersection of the space-time prisms. This temporal projection is called the *meeting*
91 *period*, as it is precisely the period of time during which the n moving objects could have met.
92 Because space-time prisms are closed and convex sets, the meeting period is a closed interval.
93 In fact, not only can we test its emptiness, we can explicitly compute the meeting period, that
94 is, compute its minimum and maximum. To do this, we give a new characterisation of the
95 meeting period, based on Helly's theorem. This characterisation tells us that the computation
96 of the meeting period can be reduced to the computation of the temporal projection of the
97 intersection of three cones. An illustration of this characterisation of the meeting period is
98 shown in part (c) of Figure 2. We have to compute such projection for every combination of
99 three out of a set of $2n$ cones. Because computing the temporal projection of three cones
100 obviously takes constant time, the meeting period can be computed in $O(n^3)$ time.

101 To compute the temporal projection of the intersection of three cones, we show that the
102 minimum and maximum of such interval are contained in a finite set of "candidate moments".
103 Then, we only have to test which of the candidate moments are contained in the temporal
104 projection, which is straightforward.

105 Finally we use a general property about the intersection of convex sets to show that,
106 given a moment in the meeting period, we can find a sample point in the intersection of the
107 space-time prisms, also in $O(n^3)$ time.

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