Visit probability in space-time prisms for moving object data

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Abstract

Space-time prisms have been extensively studied as a model to describe the uncertainty of the spatio-temporal location of a moving object in between measured space-time locations. In many applications, the desire has been expressed to provide an internal structure to these prisms, that includes what has been called "visit probability". Although several proposals have been studied in the past decades, a precise definition of this concept has been missing. The contribution of this paper is to provide such a specification by means of a formal framework for *visit probability*. Once this concept is established, we are able to derive on which parts of a prism, visit probability can be seen to give rise to a probability space.

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1 Introduction

In a wide range of applications that deal with moving objects (such as people, animals or vehicles), time-stamped location data are collected using location-aware devices (such as GPS) and these data are stored and managed in moving object databases (MODs) [3]. The actual space-time trajectories of the moving object may be reconstructed or estimated from these measured space-time locations (called anchor points) using, for example, linear interpolation [11]. *Space-time prisms*, originating from the field of time geography [1, 4, 7], are used in Geographical Information Systems (GIS) [9] and MODs [2, 5] to model the movement uncertainty of a moving object between anchor points, based on a known speed bound on the object's movement. For spatio-temporal anchor points (p^-, t^-) and (p^+, t^+), with $t^- < t^+$, and a speed bound v_{max}, the prism with these and anchors and speed bound is denoted $\mathcal{P}(p^-, t^-, p^+, t^+, v_{max})$. Figure 1 depicts space-time prisms for movement in a two- and a one-dimensional space, respectively. As shown in the figure, prisms can be seen there to be the intersection of a future and past cone. The spatial projection of the prism, also called the *potential path area*, is an envelope of the spatial whereabouts of the moving object between the measured spatial locations.

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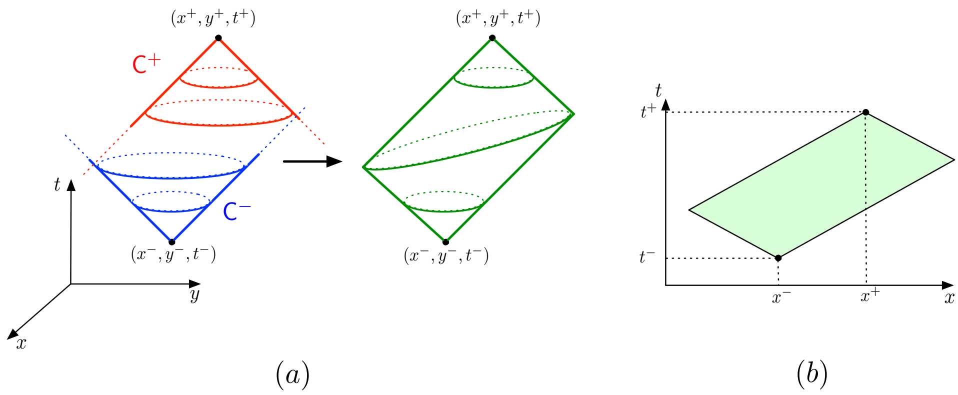

Figure 1 Part (a) of the figure shows the space-time prism (in green) for movement in the plane as the intersection of the past cone C^+ (in red) and the future cone C^- (in blue). Part (b) of the figure shows a prism for movement in a one-dimensional space.

In its basic form, a space-time prism lacks any internal structure and can be seen as a homogeneous geometric object, meaning that no two space-time points can be distinguished as more or less likely to have been visited. Conceptually, an infinite number of velocity-bound trajectories can be imagined within a prism and each point inside a prism is visited by infinitely many of them (except for some boundary points). This means that there is no a-priori reason to distinguish between space-time points inside the prism. Still, in many applications, such as animal or human movement [8], it is plausible that certain points in a prism, such as those on a linear interpolation path, should be considered more likely than other space-time points that are more towards the boundary of the prism (since they require a considerable detour). The notion of probability distributions in space-time prisms has become known as *visit probability* and has been studied extensively in the past decades (see e.g., [10, 12]). Several proposals have been made to assign probability values to space-time points or regions within a prism, thus providing the prism with an internal structure that expresses the unequal movement opportunities within the prism. Many of these proposals take an, often ad-hoc, approach towards the problem of establishing a visit probability for a particular application and each approach necessarily depends on a series of assumptions, which often remain obscure to a certain extend and the resulting visit probability therefore directly reflects these assumptions (in the best case, in a transparent way).

In this paper, we develop a general framework (or theory) of visit probability in the context of space-time prisms. In this approach, we start from the clear understanding that any definition of visit probability assumes a probability distribution on the possible velocity-bound trajectories within a prism. Once a probability space is defined on the set of trajectories (including a σ-algebra and a probability function), we can clearly determine for which parts of a space-time prims a visit probability can be derived. We also specify which conditions the σ-algebra on the set of trajectories must satisfy in order to be able to speak about a visit probability of certain classes of subsets of interest in the prism. Next, we address the question that asks on which subsets of a prism on which the derived visit probability is really a probability. We give a characterization of exactly those subsets of a prism for which this is the case. These sets are what we call “singleton-separators” and they are a wider class of subsets than just time slices of prisms. In the above mentioned literature, it is often the case that some notion of visit probability that has been defined is only considered on time slices of a prism. Our result shows that in fact it can be considered a probability on a wider class of subsets.

2 Towards a definition of visit probability in space-time prisms

In this section, we investigate how we can define the notion of "visit probability" on space-time prisms, thus providing some internal structure to an otherwise homogeneous prisms. Let $\mathcal{P} = \mathcal{P}(p^-, t^-, p^+, t^+, v_{max})$ be a space-time prism with anchors $(p^-, t^-), (p^+, t^+)$ and speed bound $v_{max} \geq 0$. A v_{max}-trajectory in the prism $\mathcal{P}$ is a (differentiable) mapping γ from the time interval $[t^-, t^+]$ to space, that connects the anchor points and whose velocity vector is bounded in size by v_{max} at all time. The set of v_{max}-trajectories in the prism $\mathcal{P}$ is denoted by $\Gamma_{\mathcal{P}}$. Space-time prisms are homogeneous in the sense that a-priori no higher likelihood can be assigned to a point (p, t) in a space-time prism $\mathcal{P} = \mathcal{P}(p^-, t^-, p^+, t^+, v_{max})$ as compared to another point (p', t') in $\mathcal{P}$. In general, the sets of v_{max}-trajectories passing through these points have the same cardinality (only points on the "rim" of a prism are exceptions [6]). Only by assigning a probability (or probability distribution) to the set of v_{max}-trajectories $\Gamma_{\mathcal{P}}$, we are be able to indicate certain points or parts of a prism as more likely than others. To obtain such a probability space on $\Gamma_{\mathcal{P}}$, we need to specify a σ-algebra $\mathcal{G}$ of subsets of $\Gamma_{\mathcal{P}}$ and to define a probability function P on this σ-algebra.

Once such a probability space $(\Gamma_{\mathcal{P}}, \mathcal{G}, P)$ has been specified, we can start working towards specifying a "visit probability": for a subset A of $\mathcal{P}$, for which $\Gamma_{\mathcal{P}}(A)$, which is the subset of v_{max}-trajectories that intersect A, belongs to $\mathcal{G}$, we define the *visit probability of A (relative to the given probability P on $\Gamma_{\mathcal{P}}$)* as $P(\Gamma_{\mathcal{P}}(A))$ and we denote it by $\mathsf{vp}_P(A)$.

However, the fact that $\Gamma_{\mathcal{P}}(A)$ belongs to the σ-algebra $\mathcal{G}$ is not guaranteed. Indeed, a σ-algebra $\mathcal{G}$ on $\mathcal{P}$ can range from very "poor" (or $\mathcal{G} = \{\emptyset, \Gamma_{\mathcal{P}}\}$) to very "rich" (or $\mathcal{G} = 2^{\mathcal{P}}$). In the former case, we can derive the visit probability of very few subsets of the prism, whereas in the latter case, we can derive it for all subsets. When we are interested in knowing the visit probability of a certain class $\mathcal{F}$ of subsets of the prism $\mathcal{P}$, we can only obtain this visit probability when for $F \in \mathcal{F}$, we have that $\Gamma_{\mathcal{P}}(F)$ belongs to the σ-algebra $\mathcal{G}$.

We turn to a visit probability for parts of the potential path area (PPA) of a prism. The following definition of visit probability of a part A of the the PPA reflects the idea that a moving object has visited A if at some point in time it has visited A. For a subset A of the PPA, we denote the cylindrical subset $(A \times \mathbb{R}) \cap \mathcal{P}$ of $\mathcal{P}$ by $\mathsf{Cyl}_{\mathcal{P}}(A)$ and when $\Gamma_{\mathcal{P}}(\mathsf{Cyl}_{\mathcal{P}}(A)) \in \mathcal{G}$, we can define the probability of visiting the part A of the the PPA as $\mathsf{vp}_P(\mathsf{Cyl}_{\mathcal{P}}(A))$.

3 When is visit probability a probability?

In this section, we determine in which circumstances the definition of visit probability on a prism, as given above, gives rise to a probability space within the prism. When we have a prism $\mathcal{P} = \mathcal{P}(p^-, t^-, p^+, t^+, v_{max})$ and consider the time slices $\mathcal{P}_{t_1}$ and $\mathcal{P}_{t_2}$ at two different moments $t^- \leq t_1 < t_2 \leq t^+$, then we clearly have $\mathsf{vp}_P(\mathcal{P}_{t_1}) = 1$ and $\mathsf{vp}_P(\mathcal{P}_{t_2}) = 1$, since every v_{max}-trajectory must intersect a time slice (that is $\Gamma_{\mathcal{P}}(\mathcal{P}_{t_1}) = \Gamma_{\mathcal{P}}(\mathcal{P}_{t_2}) = \Gamma_{\mathcal{P}}$). Since t_1 and t_2 are different, we have that the time slices $\mathcal{P}_{t_1}$ and $\mathcal{P}_{t_2}$ are disjoint subsets of $\mathcal{P}$. For disjoint unions, we would expect $\mathsf{vp}_P(\mathcal{P}_{t_1} \cup \mathcal{P}_{t_2})$ to be $\mathsf{vp}_P(\mathcal{P}_{t_1}) + \mathsf{vp}_P(\mathcal{P}_{t_2})$, but since this is $1 + 1 = 2$, that cannot be the case. This simple example shows that vp_P is not a probability on the complete prism $\mathcal{P}$. This raises the question: are there subsets of a prism on which the visit probability of Section 2 is a probability? The main contribution of this paper is a characterisation exactly those subsets of a prism for which this is the case. These subsets

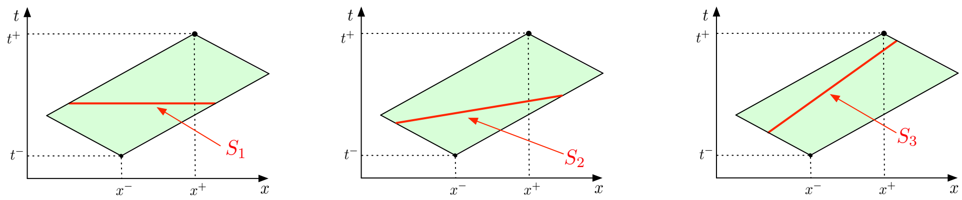

Figure 2 The subsets S_1 and S_2 are examples of singleton-separators, S_3 is not.

are “singleton-separators” in a prims, which are illustrated in Figure 2 for one-dimensional movement. A subset S of $\mathcal{P}$ is called a *singleton-separator*, if for any v_{max}-trajectory $\hat{\gamma} \in \Gamma_{\mathcal{P}}$, we have that the cardinality of $\hat{\gamma} \cap S$ equals one.

For a subset S of $\mathcal{P}$, we define $M_{\mathcal{G}}(S) := \{A \subseteq S \mid \Gamma_{\mathcal{P}}(A) \in \mathcal{G}\}$ and we have as a first result that S is a singleton-separator if and only if for every σ-algebra $\mathcal{G}$ on $\Gamma_{\mathcal{P}}$ we have that $M_{\mathcal{G}}(S)$ is a σ-algebra on S.

This means that the only subsets of a prism for which we can hope to obtain a probability space are the singleton-separators. The following theorem states how this probability space looks like.

▶ **Theorem 1.** *Let $\mathcal{P} = \mathcal{P}(p^-, t^-, p^+, t^+, v_{max})$ be a space-time prism and let S be a singleton-separator in $\mathcal{P}$. Then a probability space $(\Gamma_{\mathcal{P}}, \mathcal{G}, P)$ on the set of all v_{max}-trajectories induces a probability space $(S, M_{\mathcal{G}}(S), \mathsf{vp}_P)$ on S, when $\mathsf{vp}_P(A)$ is defined as $P(\Gamma_{\mathcal{P}}(A))$ for $A \in M_{\mathcal{G}}(S)$.*

References

1. L. Burns. *Transportation, Temporal, and Spatial Components of Accessibility.* Lexington Books, Lexington, MA, 1979.
2. Max J. Egenhofer. Approximation of geospatial lifelines. In Elisa Bertino and Leila De Floriani, editors, *SpadaGIS, Workshop on Spatial Data and Geographic Information Systems.* University of Genova, 2003.
3. R. Güting and M. Schneider. *Moving Object Databases.* Morgan Kaufmann, 2005.
4. T. Hägerstrand. What about people in regional science? *Papers of the Regional Science Association*, 24:7–21, 1970.
5. Kathleen Hornsby and Max J. Egenhofer. Modeling moving objects over multiple granularities. *Ann. Math. Artif. Intell.*, 36(1-2):177–194, 2002.
6. Bart Kuijpers and Walied Othman. Trajectory databases: data models, uncertainty and complete query languages. *J. Comput. Syst. Sci.*, 76(7):538–560, 2010.
7. B. Lenntorp. *Paths in Space-Time Environments: A Time-Geographic Study of the Movement Possibilities of Individuals.* Number 44 in Series B. Lund Studies in Geography, 1976.
8. Jed A. Long. Modeling movement probabilities within heterogeneous spatial fields. *Journal of Spatial Information Science*, 16:85–116, 2018. doi:10.5311/JOSIS.2018.16.372.
9. H.J. Miller. A measurement theory for time geography. *Geographical Analysis*, 2005. doi:10.1111/j.1538-4632.2005.00575.x.
10. Ying Song and Harvey J. Miller. Simulating visit probability distributions within planar space-time prisms. *International Journal of Geographical Information Science*, 28(1):104–125, jan 2014.
11. G. Trajcevski, O. Wolfson, K. Hinrichs, and S. Chamberlain. Managing uncertainty in moving objects databases. *ACM Trans. Database Syst.*, 29(3):463–507, 2004.
12. Stephan Winter and Zhang Cai Yin. Directed movements in probabilistic time geography. *International Journal of Geographical Information Science*, 2010.