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# CFT dual to gravitational non-locality in string theory

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**Abstract** We investigate non-perturbative bulk corrections arising from instantons in string theory and M-theory. By deriving non-local curvature corrections of the form $e^{-\gamma R} R_{\mu\nu} R^{\mu\nu}$, we demonstrate how these modifications emerge from wrapped brane instantons and their summation over multi-instanton configurations. Utilizing holographic techniques, we establish a direct connection between these non-perturbative effects and large-$N$ gauge theories, identifying the appropriate holographic dual conformal field theory (CFT). We further analyze this connection through resummations in the large-$N$ expansion. Additionally, we study black hole solutions in AdS backgrounds and show that these instanton-induced corrections significantly modify the near-horizon geometry. Finally, we explore the regularization of curvature singularities via these exponential damping terms, providing a natural resolution mechanism in quantum gravity. Our findings underscore the fundamental role of non-perturbative physics in shaping the structure of spacetime and its holographic duals.

## 1 Introduction

Non-locality in string theory is not an incidental feature but rather an essential aspect of its formulation, distinguishing it fundamentally from conventional local quantum field theories (QFTs). In QFT, locality is embedded in the structure of the theory: fields are assigned to definite spacetime points, and interactions are governed by local operators satisfying differential equations of finite order. The underlying assumption is that physical effects propagate causally through local interactions. String theory, however, modifies this framework in a fundamental way by replacing point-like degrees of freedom with extended objects, leading to a formulation in which non-locality is not merely a possibility but an intrinsic feature of the theory. This non-locality is not an auxiliary property that can be turned on or off but arises directly from the way strings propagate and interact. Unlike in local field theories, where interactions occur at specific spacetime points, the extended nature of strings introduces a fundamentally different notion of interaction, one that is intrinsically non-local and manifests in a variety of interrelated ways.

In local quantum field theory, interactions are localized at specific spacetime points, and the equations of motion (EoMs) are governed by differential operators that act locally on fields. The principle of locality ensures that the behavior of a given region is determined entirely by its immediate surroundings, with interactions mediated by fields propagating in a causal manner. String theory, however, fundamentally alters this framework, introducing non-locality in a way that is neither incidental nor adjustable but rather an intrinsic aspect of the theory. One of the clearest manifestations of this arises in string field theory (SFT) [1,2], where the second-quantized formulation of string interactions leads to kinetic and interaction terms that involve an infinite series of derivatives.

Unlike in conventional quantum field theories, where differential operators are typically of finite order, the structure of SFT naturally incorporates non-local differential operators, reflecting the extended nature of strings and the infinite tower of higher-spin modes present in the spectrum [1,3]. In open string field theory (OSFT), this non-locality is particularly evident in the formulation of the star product, which encodes string interactions in a way that fundamentally differs from the local vertex structure of field theory. The star product, defining a non-commutative algebra of string fields, ensures that interactions are not confined to specific spacetime points but rather integrate over the entire worldsheet, leading to a

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formulation in which non-locality is an unavoidable consequence of the theory's structure. The implications of this are particularly striking in the context of tachyon condensation in open bosonic string theory, where the effective action for tachyon fields includes exponential differential operators such as $e^{\Box}$, with $\Box$ the d'Alembertian operator [4,5].

The presence of these operators signals a departure from conventional locality, modifying short-distance behavior in a way that fundamentally alters the vacuum structure of the theory. Unlike in conventional field theories, where tachyon condensation is typically understood in terms of spontaneous symmetry breaking, the non-local structure of SFT suggests a far more intricate vacuum landscape, where an infinite series of derivative interactions collectively determine the fate of the tachyonic mode. This non-locality is not merely a technical feature but has deep physical significance, particularly in the high-energy regime, where it modifies the short-distance structure of interactions and suggests new insights into ultraviolet (UV) completion.

Since the presence of infinite derivatives naturally regularizes singularities in a way that differs from conventional renormalization, SFT has been proposed as a framework in which the problem of divergences in quantum gravity can be addressed in a fundamentally new way. Moreover, the interplay between non-locality and gauge symmetry in SFT suggests deeper connections to the background independence of string theory, hinting at structures that may be relevant for a non-perturbative formulation of the theory. In this sense, the non-locality inherent in string field theory is not just a feature that distinguishes it from conventional field theories but one that reshapes fundamental aspects of our understanding regarding spacetime, interactions, and vacuum structure.

Another significant manifestation of non-locality arises in non-commutative string theory, where the presence of a background $B$-field modifies the fundamental structure of open string interactions. In this setting, the endpoints of open strings no longer commute, leading to a deformation of the underlying algebraic structure of spacetime itself. This non-commutativity translates directly into a modification of the field equations, introducing non-local terms that fundamentally alter the low-energy effective description of the theory [6,7].

These effects are particularly important in the context of gauge theories and gravity, where non-commutativity modifies interaction vertices in a way that has no direct analog in conventional field theory. The presence of non-local structures in string theory is further reinforced by higher-derivative corrections in string effective actions, which give rise to non-local modifications of Einstein's equations [8,9]. These higher-curvature terms are of particular interest in black hole physics and early universe cosmology, where they play a crucial role in attempts to resolve singularities by modifying the short-distance behavior of the gravitational field.

Non-locality also appears naturally in double field theory (DFT), where T-duality is incorporated at a fundamental level. Unlike in conventional string theory, where T-duality acts as a discrete symmetry relating momentum and winding modes, DFT is formulated in an extended spacetime where this symmetry becomes manifest at the level of equations of motion. This requires the introduction of additional dual coordinates conjugate to winding modes, effectively doubling the degrees of freedom of the theory. However, physical consistency imposes a strong constraint, ensuring that only a subset of these degrees of freedom contribute to physical observables.

The presence of these extra coordinates leads to a reformulation of the underlying differential structures, requiring a generalized geometric framework in which conventional derivatives are replaced by covariant derivatives adapted to the doubled space. As a consequence, DFT provides a natural setting for describing string interactions in a way that maintains manifest T-duality, offering new insights into the role of non-commutativity and non-associativity in string theory [10].

A further, albeit structurally different, realization of non-locality appears in p-adic string theory, which provides an alternative mathematical formulation of string dynamics. In this approach, spacetime coordinates are replaced by p-adic numbers, leading to a kinetic term that involves non-local differential operators rather than conventional second-order derivatives. The resulting wave equation incorporates an infinite series of derivative terms, fundamentally altering the structure of dispersion relations and modifying the behavior of perturbations in a way that is not captured by standard local field theories.

The presence of an effective nonlocality scale in p-adic string theory determines its high-energy behavior, suggesting intriguing connections to ultraviolet (UV) completion [11,12]. Moreover, p-adic string models have played an important role in the study of tachyon condensation, where their structure allows for a precise analytic treatment of the dynamics in a manner that is difficult to achieve in conventional string field theory. These models also exhibit deep connections to the adelic formulation of quantum mechanics, further reinforcing the idea that non-locality is not merely a feature of specific string constructions but rather an essential ingredient in the broader framework of string theory.

These different manifestations of non-locality challenge conventional notions of locality and causality, raising fundamental questions about the nature of spacetime and interactions at the most basic level. In local quantum field theory, interactions are confined to well-defined regions of spacetime, with causality ensuring that physical effects propagate only within the light cone. String theory, however, introduces non-local effects that suggest a more subtle structure, where information can be encoded and transmitted in ways that transcend conventional spacetime locality. This poses a direct challenge to the standard formulation of effective field theory and raises deep questions about the consistency of quantum gravity. Beyond its role in perturbative string theory, non-locality is also a central feature of the holographic principle and the AdS/CFT correspondence.

The holographic nature of gravity implies that the degrees of freedom in an anti-de Sitter (AdS) bulk are fully encoded in a lower-dimensional conformal field theory (CFT) on its boundary. This encoding is inherently non-local in the sense that local bulk operators do not simply correspond to local operators in the CFT but are reconstructed from non-local boundary data through integral transformations involving smearing functions. The reconstruction procedure ensures that bulk physics is entirely determined by boundary correlation functions, reinforcing the idea that spacetime itself is not fundamental but rather an emergent construct arising from a deeper non-local framework.

The explicit reconstruction of bulk operators from boundary data follows a well-defined prescription in the context of the Hamilton–Kabat–Lifschytz–Lowe (HKLL) formalism, which expresses bulk fields in terms of non-local CFT operators [13,14]. This feature plays an essential role in understanding how gravitational observables in the bulk correspond to non-local quantities in the dual CFT. Moreover, the scale-radius duality in AdS/CFT, where radial evolution in AdS corresponds to renormalization group flow in the CFT, further illustrates the role of non-locality in the emergence of a higher-dimensional spacetime from a lower-dimensional field theory. These considerations suggest that spacetime itself may be a derived concept, emerging from deep non-local correlations in the underlying quantum theory, pointing to a perspective in which space, time, and gravity arise from a more fundamental non-local structure.

## 2 Derivation of the five-dimensional bulk action from heterotic string theory

The purpose of this section is to obtain the five-dimensional gravitational action within the framework of heterotic string theory, incorporating both perturbative $\alpha'$-corrections and non-perturbative worldsheet instanton effects. Our starting point is the ten-dimensional low-energy effective action, written in the string frame, given by [15,16]:

$$S_{10D} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}e^{-2\phi}\left(R + 4(\nabla\phi)^2 - \frac{1}{4}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) - \frac{\alpha'}{4}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right). \tag{2.1}$$

Here, $\phi$ is the dilaton, $F_{\mu\nu}$ represents the gauge field strength associated with the $E_8 \times E_8$ or $SO(32)$ gauge groups, while the last term encodes the leading $\alpha'$-correction, required by the Green–Schwarz anomaly cancellation mechanism. At the level of the ten-dimensional local effective action, the curvature-squared sector can be organized, after field redefinitions and including the dilaton coupling, into the Gauss–Bonnet (GB) invariant. This is the well-known ghost-free combination in $D > 4$ that yields second-order metric equations. In what follows, our five-dimensional reduction will isolate the $R_{\mu\nu}R^{\mu\nu}$ structure as a proxy for the overall curvature-squared sector; the crucial property controlling unitarity and the analytic structure in our construction is provided by the nonlocal completion introduced in Sect. (6), rather than by a particular choice of local basis at $\mathcal{O}(\alpha')$. To obtain a five-dimensional effective theory, we consider a compactification on a five-dimensional internal manifold $X_5$, adopting the metric ansatz:

$$ds_{10}^2 = g_{\mu\nu}^{(5)}dx^\mu dx^\nu + g_{mn}^{(5)}dy^m dy^n. \tag{2.2}$$

The ten-dimensional Ricci scalar decomposes as $R^{(10)} = R^{(5)} + R^{(X_5)}$, with the internal curvature $R^{(X_5)}$ contributing to an effective negative cosmological constant in the resulting five-dimensional theory. Integrating over the compact space, which has volume $V_5$, yields the five-dimensional Einstein-Hilbert action:

$$S_{5D} = \frac{V_5}{16\pi G_N}\int d^5x\sqrt{-g}e^{-2\phi}\left(R^{(5)} + \frac{12}{L^2}\right). \tag{2.3}$$

Throughout Sects. (2)–(6) we assume a stabilized, constant dilaton, $\phi = \phi_0$, and work in the 5D Einstein frame. Accordingly, the overall factor $e^{-2\phi_0}$ is absorbed into $16\pi G_N$, and no additional dilaton source terms appear in Eq. (5.7) (see [15,16]). At the same time, the $\alpha'$-correction descends as

$$S_{\alpha'} = -\frac{\alpha' V_5}{4}\int d^5x\sqrt{-g}e^{-2\phi}R_{\mu\nu}R^{\mu\nu}. \tag{2.4}$$

In heterotic string theory the leading curvature-squared correction is the Gauss-Bonnet combination. Upon reduction, this yields a specific linear combination of $R^2$-invariants. In this work we retain the $R_{\mu\nu}R^{\mu\nu}$ piece as a representative term to capture the qualitative effect [8,17]. Thus, within the perturbative sector of heterotic string theory, we find that the five-dimensional action acquires an $R_{\mu\nu}R^{\mu\nu}$ contribution. Beyond these perturbative effects, additional corrections can emerge from non-perturbative worldsheet instantons. In heterotic string theory, the worldsheet action is given by [18]:

$$S_{\mathrm{ws}} = \frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{h}h^{ab}g_{MN}\partial_a X^M\partial_b X^N + \text{fermionic terms}, \tag{2.5}$$

where $h_{ab}$ is the worldsheet metric, $g_{MN}$ is the target space metric, and $X^M$ denote the embedding coordinates. The worldsheet instanton action is controlled by the wrapped area, $S_{\rm ws} = V(\Sigma_2)/\alpha'$. In general $V(\Sigma_2)$ is a function of the internal moduli. In this work we assume a homogeneous (constant-scalar-curvature) internal geometry in which the internal scalar curvature

$$R_X \equiv R(X_5) \tag{2.6}$$

is constant over $X_5$ and depends smoothly on a finite-dimensional set of stabilized moduli, collectively denoted by $u$. We likewise assume that the calibrated 2-cycle volume $V(\Sigma_2)$ depends smoothly on the same modulus sector. Near a stabilized point $u = u_0$ with $R_X(u_0) = R_{X,0}$ and $R'_X(u_0) \neq 0$, one may eliminate the modulus locally and expand

$$V(\Sigma_2) = V_0 + \left.\frac{dV}{dR_X}\right|_{u_0} (R_X - R_{X,0}) + \mathcal{O}\Big((R_X - R_{X,0})^2\Big), \tag{2.7}$$

so that

$$e^{-S_{\rm ws}} = \exp\left[-\frac{V(\Sigma_2)}{\alpha'}\right] = e^{-V_0/\alpha'} \exp\Big[-\gamma\,(R_X - R_{X,0}) + \mathcal{O}\Big((R_X - R_{X,0})^2\Big)\Big],\quad \gamma \equiv \frac{1}{\alpha'}\left.\frac{dV}{dR_X}\right|_{u_0}. \tag{2.8}$$

The constant prefactor $e^{-V_0/\alpha'}$ is absorbed into the effective coupling multiplying the induced operator, while $R_{X,0}$ is fixed hence, to the order retained here we write the instanton factor as $\exp[-\gamma R_X]$ up to an overall constant. A detailed derivation in an explicit homogeneous example is given in Appendix (A). Consequently, the instanton-corrected effective action includes an exponentially suppressed higher-curvature term,

$$\lambda\, e^{-S_{\rm ws}}\, R_{\mu\nu}R^{\mu\nu} \simeq \lambda\, e^{-\gamma R_X}\, R_{\mu\nu}R^{\mu\nu}, \tag{2.9}$$

where $R_{\mu\nu}R^{\mu\nu}$ is the five-dimensional curvature invariant and $R_X$ denotes the internal scalar curvature. Such terms originate from a sum over worldsheet topologies in the heterotic string path integral, reinforcing the role of non-perturbative physics in string compactifications [19,20].

Incorporating both perturbative $\alpha'$-corrections and non-perturbative worldsheet instanton effects, we have arrived at a five-dimensional effective action in which the higher-curvature term is explicitly modified. The presence of the suppression factor $e^{-\gamma R}$ underscores the role of non-perturbative contributions in the heterotic string, offering a refined perspective on how instanton effects alter the gravitational sector in holographic frameworks. This exponential form is not an ad hoc insertion but the resummed manifestation of the infinite tower of worldsheet instanton corrections contributing to the heterotic effective action. To avoid ambiguity, we denote the internal Ricci scalar by $\mathcal{R} \equiv R(X_5)$ (assumed constant in this section) and the 5D spacetime Ricci scalar by $R \equiv R^{(5)}$. The invariant $R_{\mu\nu}R^{\mu\nu}$ always refers to the five-dimensional quantity.

## 3 Non-perturbative corrections in CFT

The AdS/CFT correspondence provides a strikingly precise relation between gravity in asymptotically AdS spaces and strongly coupled gauge theories. A particularly well-studied example is the entropy of an $AdS_5$ black hole, which, in the dual CFT, is known to scale as [21]

$$s \sim N^2, \tag{3.1}$$

where $N$ denotes the rank of the gauge group. This dependence immediately implies that any correction term of the form $e^{-s}$ exhibits an exponential suppression in $N^2$. More precisely, one finds

$$e^{-s} = e^{-cN^2}, \tag{3.2}$$

for some numerical coefficient $c$. This behavior stands in contrast to the more familiar $1/N$ expansion characteristic of holographic CFTs, where transport coefficients are typically organized as

$$\frac{\eta}{s_c} = \frac{1}{4\pi}\left(1 + \frac{c_1}{N} + \frac{c_2}{N^2} + \dots\right). \tag{3.3}$$

While corrections of order $1/N$ emerge naturally from quantum loop effects in the bulk, terms of the form $e^{-S}$ are qualitatively distinct, displaying an exponential suppression in $N^2$. This suggests a fundamentally different origin, one that is inherently non-perturbative. In standard AdS/CFT setups, finite-$N$ corrections appear in transport coefficients due to higher-loop contributions in the bulk, which affect the stress-energy tensor correlators. These corrections take the well-known power-law form

$$\frac{1}{N},\quad \frac{1}{N^2},\quad \dots \tag{3.4}$$

However, non-perturbative terms of the form $e^{-s}$ introduce a qualitatively new ingredient. Their scaling behavior,

$$e^{-s} = e^{-cN^2} \ll \frac{1}{N^k},\quad \forall k, \tag{3.5}$$

reveals that no finite order in $1/N$ can reproduce such effects. Thus, their very presence signals a departure from the perturbative framework, pointing instead to the role of non-perturbative physics. This divergence is not only an issue of formal expansion. While power-law suppressed components can be traced back to perturbative quantum effects in the bulk, exponentially suppressed corrections frequently show contributions from non-perturbative saddle points, instanton effects, or even topological configurations in gravitational theory. Their presence suggests a deeper structure behind quantum gravity corrections in holography, one that cannot be comprehended using ordinary perturbative approaches alone.

## 4 CFT dual of higher-curvature corrections in the bulk

The AdS/CFT correspondence implies that bulk metric fluctuations $g_{\mu\nu}$ are encoded in the stress-energy tensor $T_{\mu\nu}$ of the boundary theory. More generally, the presence of higher-curvature terms in the bulk action suggests the existence of corresponding higher-order corrections in the CFT. A natural candidate for the dual of the bulk term $R_{\mu\nu}R^{\mu\nu}$ is a multi-trace interaction involving $T_{\mu\nu}$, which modifies the CFT action as

$$S_{\rm CFT} = S^{(0)}_{\rm CFT} + \lambda \int d^4x \, e^{-\gamma \mathcal{O}_{\rm NP}} T_{\mu\nu} T^{\mu\lambda} T^{\nu}_{\lambda}. \tag{4.1}$$

The use of a triple-trace structure follows the general rule that a bulk term quadratic in curvature couples to the CFT operator cubic in $T_{\mu\nu}$, because functional differentiation of the CFT generating functional with respect to the boundary metric brings down one $T_{\mu\nu}$ per bulk graviton leg (see [22]). Lower double-trace deformations would correspond to $R$-linear bulk corrections, while $R^2$ terms require three stress-tensor insertions at leading order in $1/N$. In the language of holographic renormalization, such multi-trace deformations correspond to mixed boundary conditions for the bulk graviton, so the deformation in Eq. (4.1) is consistent with the variational problem and the AdS/CFT dictionary [22]. Here, $S^{(0)}_{\rm CFT}$ represents the undeformed large-$N$ CFT action, while $e^{-\gamma \mathcal{O}_{\rm NP}}$ introduces a non-perturbative correction. Given the structure of this interaction, we seek a CFT operator whose expectation value reproduces the bulk suppression factor $e^{-\gamma R}$. A well-motivated choice is the instanton-induced term

$$\mathcal{O}_{\rm NP} = e^{-S_{\rm inst}}, \tag{4.2}$$

where $S_{\rm inst}$ denotes the action of an instanton-like configuration in the CFT. In large-$N$ gauge theories, instanton effects typically scale as

$$e^{-S_{\rm inst}} \sim e^{-N^2}. \tag{4.3}$$

This suggests defining

$$\mathcal{O}_{\rm NP} = W_{\rm inst} = e^{-S_{\rm inst}}, \tag{4.4}$$

so that upon taking the expectation value,

$$\langle e^{-\gamma W_{\rm inst}} \rangle \approx e^{-\gamma N^2}, \tag{4.5}$$

which precisely matches the suppression factor appearing in the bulk. This provides a natural holographic origin for the exponential suppression of higher-curvature corrections.

The bulk effective action can now be obtained by integrating out high-energy modes in the CFT. In the large-$N$ expansion, the relevant CFT correlation function satisfies

$$\langle T_{\mu\nu} T^{\mu\lambda} T^{\nu}_{\lambda} \rangle_{conn} \sim N^2, \tag{4.6}$$

in agreement with the scaling of the central charge $c \sim N^2$ in holographic CFTs [23], leading to an effective bulk action of the form

$$S_{\rm eff} \approx \frac{N^2}{16\pi G_N} \int d^5x \sqrt{-g} \left( R + \frac{12}{L^2} \right) + \frac{N^4}{16\pi G_N} \lambda e^{-\gamma N^2} \int d^5x \sqrt{-g} R_{\mu\nu} R^{\mu\nu}. \tag{4.7}$$

The overall $N^4$ scaling in Eq. (4.7) then arises from combining this with the $N^2/16\pi G_N$ prefactor of the gravitational action. Note that despite the $N^4$ prefactor, the non-perturbative factor $e^{-\gamma N^2}$ renders this contribution smaller than any power $1/N^k$ in the large-$N$ limit, in agreement with the resurgent/trans-series structure discussed in Sect. 7. Comparing with the bulk expression,

$$\lambda e^{-\gamma R} R_{\mu\nu} R^{\mu\nu} \quad \longleftrightarrow \quad \lambda e^{-\gamma N^2} \langle T_{\mu\nu} T^{\mu\lambda} T^{\nu}_{\lambda} \rangle, \tag{4.8}$$

we confirm the emergence of the higher-curvature term from the non-perturbative structure of the CFT. This establishes an explicit link between the large-$N$ behavior of the stress tensor correlators and the gravitational corrections in the bulk. This construction shows that higher-derivative gravity naturally arises from non-perturbative effects in the CFT. The instanton-induced operator $W_{\rm inst}$ generates an expectation value of the form $e^{-N^2}$, leading to an exponentially suppressed correction in the bulk action. The presence of the multi-trace interaction $T_{\mu\nu} T^{\mu\lambda} T^{\nu}_{\lambda}$ provides a direct mechanism for generating the curvature-squared term in the gravitational theory.

We conclude by noting that these non-perturbative effects play a crucial role in the physics of transport coefficients in the CFT. The shear viscosity corrections induced by the higher-derivative terms in the bulk are not perturbative in $1/N$, but rather suppressed as $e^{-N^2}$. This aligns with the expected contributions from instanton effects, D-brane corrections, or resurgence phenomena. Thus, the structure of the modified CFT action provides new insight into the nature of non-perturbative holography and the emergence of higher-curvature terms in the bulk.

## 5 Instantons and non-perturbative bulk corrections

In string theory, non-perturbative effects naturally arise through Euclidean D-branes wrapping compact cycles in the extra-dimensional geometry [24]. These instanton effects contribute to the low-energy effective action via an exponential suppression factor of the form:

$$e^{-S_{\text{brane}}}, \tag{5.1}$$

where the instanton action, for a Euclidean $p$-brane wrapping a $(p+1)$-dimensional cycle $\Sigma$ in the bulk, takes the form:

$$S_{\text{brane}} = \frac{1}{g_s}\int d^{p+1}\xi \sqrt{\det G} + S_{\text{WZ}}, \tag{5.2}$$

in which $g_s$ is the string coupling, $G$ the induced metric on the brane, and $S_{\text{WZ}}$ accounts for the coupling to background fluxes. Holographic duality suggests a natural scaling behavior in the large-$N$ limit [25]. The string coupling satisfies:

$$\frac{1}{g_s} \sim N, \tag{5.3}$$

and since the brane volume scales as $N$, the instanton action behaves as:

$$S_{\text{brane}} \sim N^2. \tag{5.4}$$

Consequently, the contribution to the effective action is exponentially suppressed,

$$e^{-S_{\text{brane}}} \sim e^{-N^2}, \tag{5.5}$$

which is the expected form of non-perturbative corrections. The presence of a Euclidean D-brane necessarily modifies the background geometry, acting as a localized source in the gravitational equations. The induced correction to the effective action takes the form:

$$\delta S \sim e^{-S_{\text{brane}}}\int d^5x\sqrt{-g}\,\mathcal{O}_{\text{curvature}}, \tag{5.6}$$

where $\mathcal{O}_{\text{curvature}}$ denotes higher-curvature corrections arising from the backreaction of the instanton. More explicitly, the Einstein equations receive a correction:

$$R_{MN} - \frac{1}{2}g_{MN}R + \Lambda g_{MN} = 8\pi G_N\left(T^{\text{bulk}}_{MN} + T^{\text{brane}}_{MN}\right), \tag{5.7}$$

in which $T^{\text{bulk}}_{MN}$ contains the usual stress-energy contributions from fluxes, the dilaton, and metric fluctuations, while $T^{\text{brane}}_{MN}$ encodes the effects of the wrapped Euclidean D-brane. The brane stress tensor follows from the variation of the brane action:

$$T^{\text{brane}}_{MN} = -\frac{2}{\sqrt{-g}}\frac{\delta S_{\text{brane}}}{\delta g^{MN}}. \tag{5.8}$$

For a Euclidean Dp-brane wrapping a cycle in the internal space, the supergravity description of the action is given by:

$$S_{\text{brane}} = \mu_p\int d^{p+1}\xi\, e^{-\phi}\sqrt{\det P[g]_{ab}} + S_{\text{WZ}} + \cdots, \tag{5.9}$$

where $P[g]_{ab}$ is the pullback metric, and $S_{\text{WZ}}$ captures the interaction with background Ramond-Ramond fluxes. Since the Euclidean brane wraps a compact cycle $\Sigma$ in the extra-dimensional geometry, the dominant contribution to its action comes from the Born-Infeld term:

$$S_{\text{brane}} \sim \mu_p\int_\Sigma d^p\xi\, e^{-\phi}\sqrt{\det P[g]_{ab}}. \tag{5.10}$$

For large cycle volume, the Euclidean brane action scales linearly with the volume:

$$S_{\text{brane}} \sim \frac{\text{Vol}(\Sigma)}{g_s}, \tag{5.11}$$

so the corresponding instanton weight is $e^{-S_{\text{brane}}} \sim e^{-\text{Vol}(\Sigma)/g_s}$. The backreaction of the brane induces an additional curvature-dependent term in the effective action:

$$S_{\text{brane}} \sim \int d^5x\sqrt{-g}\,e^{-S_{\text{brane}}}\,\mathcal{R}_{\text{brane}}, \tag{5.12}$$

where $\mathcal{R}_{\text{brane}}$ represents the curvature correction sourced by the brane.

Taking the variation of $e^{-S_{\text{brane}}}$, we obtain:

$$T^{\text{brane}}_{MN} = -\frac{1}{\sqrt{-g}}\frac{\delta}{\delta g^{MN}}e^{-S_{\text{brane}}}. \tag{5.13}$$

Since $S_{\text{brane}}$ scales with the Ricci scalar near the brane, the resulting stress tensor is of the form:

$$T^{\text{brane}}_{MN} \sim e^{-S_{\text{brane}}}R_{MN}. \tag{5.14}$$

This scaling follows from the DBI action's coupling to the induced metric:

$$S_{\text{brane}} \propto \int_\Sigma \sqrt{P[g]} \;\approx\; \text{Vol}(\Sigma)\left(1 + \tfrac{1}{2}R_{mnmn}\,\ell_s^2 + \cdots\right), \tag{5.15}$$

so to leading order $S_{\text{brane}} \propto R$ for small curvature deformations. Thus, the leading-order contribution from brane backreaction enters through a curvature-dependent correction, exponentially suppressed by $e^{-S_{\text{brane}}}$. From a string-theoretic perspective, Euclidean D-branes generate non-perturbative modifications to the effective supergravity description, typically appearing in the Kähler potential, superpotential, or as higher-derivative interactions. The stress tensor structure suggests that the dominant correction in the effective action involves brane-induced curvature interactions of the form:

$$\delta S \sim e^{-S_{\text{brane}}}\int d^5x\sqrt{-g}\,\mathcal{R}_{\text{brane}}. \tag{5.16}$$

At leading order, this implies a modification to the Ricci curvature:

$$\delta R_{MN} \sim e^{-S_{\text{brane}}}R_{MP}R^P{}_N. \tag{5.17}$$

Thus, the instanton contribution to the effective action is of the form:

$$e^{-S_{\text{brane}}}R_{\mu\nu}R^{\mu\nu}. \tag{5.18}$$

Identifying $e^{-S_{\text{brane}}}$ with an effective exponential suppression $e^{-\gamma R}$, we conclude that the induced term in the five-dimensional action takes the form:

$$I_{\text{brane}} = \lambda e^{-\gamma R} \int d^5x \sqrt{-g} R_{\mu\nu} R^{\mu\nu}. \tag{5.19}$$

These results illustrate the emergence of non-perturbative higher-curvature corrections from Euclidean D-brane instantons, highlighting their role in the interplay between string-theoretic effects and AdS/CFT [16].

## 6 M-brane and non-perturbative bulk corrections

Higher-derivative corrections to gravitational effective actions are a well-known feature of string theory and M-theory compactifications. These terms naturally arise from a variety of sources, including bulk supergravity interactions, curvature effects induced by compactification, and non-perturbative contributions from Euclidean M-brane instantons. Our aim here is to explicitly derive the five-dimensional effective action, incorporating the contributions from wrapped Euclidean M-branes, and to analyze the emergence of curvature-dependent suppression factors.

The non-perturbative effects of M-brane instantons generate corrections to the five-dimensional action, with a characteristic prefactor appearing in the higher-derivative terms. The leading-order terms in this action correspond to the standard Einstein-Hilbert action with a cosmological constant, while additional terms proportional to $R_{\mu\nu}R^{\mu\nu}$ emerge as a consequence of quantum effects, in particular, from Euclidean M2- and M5-brane instanton contributions.

Instantons in M-theory arise when Euclidean M2-branes (EM2) wrap holomorphic two-cycles $C_2$ and Euclidean M5-branes (EM5) wrap holomorphic four-cycles $C_4$ in the internal compactification manifold [24,26,27]. The contributions of these instantons to the low-energy effective action are exponentially suppressed and take the general form $e^{-S_{\text{inst}}}$, where $S_{\text{inst}}$ denotes the instanton action. For an EM2-brane wrapping a two-cycle $C_2$, the leading contribution is dictated by its worldvolume action:

$$S_{\text{EM2}} = T_{\text{M2}} \int_{C_2} d^2\xi \sqrt{\det g}, \tag{6.1}$$

where the M2-brane tension is given by

$$T_{\text{M2}} = \frac{1}{(2\pi)^2 l_p^3}. \tag{6.2}$$

The volume of $C_2$ depends on the internal curvature, leading to a warping correction in the effective action. This dependence is manifest in the modified form of the instanton action:

$$S_{\text{EM2}} \propto \frac{1}{l_p^3} \int_{C_2} e^{-\gamma R} d^2\xi. \tag{6.3}$$

Thus, the instanton contribution to the partition function takes the form

$$e^{-S_{\text{EM2}}} = e^{-\frac{1}{g_s} e^{-\gamma R} \text{Vol}(C_2)}, \tag{6.4}$$

The factor $e^{-\gamma R}$ effectively encodes the curvature-induced warping of the internal metric in flux compactifications: in 11D supergravity with $R^4$ corrections, the background volume form acquires multiplicative corrections $1 + \gamma R + \cdots$ [17]. Resumming these into an exponential is a convenient way to represent the non-perturbative backreaction on the wrapped brane action. where the curvature-dependent suppression factor $e^{-\gamma R}$ appears as a direct consequence of the geometric deformation of $C_2$ induced by higher-curvature corrections.

A parallel analysis applies to EM5-branes wrapping four-cycles $C_4$, where the leading contribution to the instanton action is

$$S_{\text{EM5}} = T_{\text{M5}} \int_{C_4} d^4\xi \sqrt{\det g}, \tag{6.5}$$

with the M5-brane tension given by

$$T_{\text{M5}} = \frac{1}{(2\pi)^5 l_p^6}. \tag{6.6}$$

Once again, the dependence of the four-cycle volume on curvature results in a warping-dependent suppression factor:

$$S_{\text{EM5}} \propto \frac{1}{l_p^6} \int_{C_4} e^{-\gamma R} d^4\xi. \tag{6.7}$$

Thus, the corresponding instanton contribution takes the form

$$e^{-S_{\text{EM5}}} = e^{-\frac{1}{g_s} e^{-\gamma R} \text{Vol}(C_4)}. \tag{6.8}$$

A more complete understanding of these corrections is obtained by summing over multi-instanton configurations. The total instanton contribution to the partition function is given by

$$Z = \sum_n \frac{1}{n!} \left(e^{-S_{\text{EM5}}}\right)^n = \sum_n \frac{1}{n!} \left(e^{-\frac{1}{g_s} e^{-\gamma R} \text{Vol}(C_4)}\right)^n. \tag{6.9}$$

Expanding the exponential and summing over instanton numbers yields the final result:

$$Z = \exp\left(-\frac{1}{g_s} e^{-\gamma R} \text{Vol}(C_4)\right). \tag{6.10}$$

The volume of the four-cycle can be expressed in terms of the local curvature, incorporating higher-derivative corrections:

$$\text{Vol}(C_4) \approx V_0 + \alpha R_{\mu\nu} R^{\mu\nu}, \tag{6.11}$$

where $\alpha$ is a compactification-dependent coefficient. Substituting this into the partition function yields the modified effective action:

$$S_{\text{eff}} \supset \int d^5x\sqrt{-g}\, e^{-\gamma R}\text{Vol}(C_4). \tag{6.12}$$

Here, $V_0$ renormalizes the background terms, while the relevant curvature correction takes the form

$$S_{\text{eff}} \supset \int d^5x\sqrt{-g}\, e^{-\gamma R} R_{\mu\nu}R^{\mu\nu}. \tag{6.13}$$

While Eq. (6.13) has been written as an exponentially suppressed curvature-squared operator, its origin as a *resummed* instanton contribution implies a nonlocal completion of the quadratic graviton operator rather than a standalone local $R_{\mu\nu}R^{\mu\nu}$ deformation. Concretely, expanding around a maximally symmetric background with constant curvature $R = R_0$ and metric perturbation $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$, the linearized kinetic operator can be written schematically as

$$\mathcal{K}(\Box) \;=\; e^{-H(\Box/M^2)}\,\Box\left(\mathcal{P}^{(2)} - \tfrac{1}{2}\,\mathcal{P}^{(0-s)}\right) + \mathcal{O}(R_0/M^2), \tag{6.14}$$

where $\Box$ is the covariant d'Alembertian on the background, $\mathcal{P}^{(2)}$ and $\mathcal{P}^{(0-s)}$ are the usual spin projectors, $M$ is the nonlocal (stringy) scale, and $H(z)$ is an *entire* function determined by the multi-instanton resummation. The ghost-free entire-function form factor structure in Eq. (6.14) is standard in infinite-derivative/nonlocal gravity; see e.g. [28–30] for derivations of the exponential-of-entire-function completion and its pole/spectrum implications. The corresponding propagator in momentum space reads

$$\tilde{D}_{\mu\nu\rho\sigma}(p) \;=\; \frac{e^{-H(-p^2/M^2)}}{p^2}\left(\mathcal{P}^{(2)}_{\mu\nu\rho\sigma} - \tfrac{1}{2}\,\mathcal{P}^{(0-s)}_{\mu\nu\rho\sigma}\right) + \cdots, \tag{6.15}$$

so that, because $e^{-H}$ has no zeros for finite $p^2$, *no new poles* are introduced beyond the massless graviton pole at $p^2 = 0$. Correspondingly, the propagator form Eq. (6.15) realizes the usual condition for ghost-freedom: the exponential of an entire function does not introduce additional poles beyond the graviton pole; see [28–30]. Thus the would-be massive spin-2 ghost that afflicts a purely *local* $R_{\mu\nu}R^{\mu\nu}$ deformation never appears here. In particular, whereas a local quadratic theory yields

$$\tilde{D}_{\text{local}}(p) \sim \frac{1}{p^2} - \frac{1}{p^2 + m_2^2}, \tag{6.16}$$

with the second term the unphysical (wrong-residue) massive spin-2 pole, the nonlocal completion (6.15) replaces any such extra pole by an *entire-function form factor*, which softens the UV behavior without adding degrees of freedom. This mechanism is precisely what one expects from a string-theoretic resummation: instanton sums and string field theory both generate nonpolynomial (infinite-derivative) structures that are ghost-free when the form factors are exponentials of entire functions. In this sense, Eq. (6.13) is consistent with the stringy spectrum-there is no isolated "massive spin-2 ghost" mode to be matched-and it is complementary to the well-known local statement that, at $\mathcal{O}(\alpha')$, curvature-squared terms can be organized into the Gauss-Bonnet combination (ghost-free at the local level). Here, the *nonlocal* structure induced by wrapped M-brane instantons ensures ghost-freedom directly at the level of the full quadratic operator (6.14), while the overall $e^{-\gamma R}$ factor encodes the curvature dependence of the instanton action and reduces to a harmless constant multiplier in the linearized analysis about $R_0$. Consequently, Eq. (6.13) *eliminates* the unphysical massive tensor mode and remains fully consistent with string-theoretic expectations regarding the physical graviton content.

### 6.1 Cauchy problem and ghost-free nonlocal dynamics

The correction in Eq. (6.13), is not introduced as an isolated fourth-derivative interaction; rather, it is the resummed limit of an infinite series of higher-derivative terms generated by wrapped M-brane instantons. In the effective theory, this resummation is captured by entire form factors of the covariant d'Alembertian, so that the quadratic (in fluctuations) kinetic operator takes the schematic ghost-free form Eq. (6.14). In transverse-traceless (TT) gauge $h_{\mu\nu} \to h^{\text{TT}}_{\mu\nu}$, Eq. (6.14) implies the linearized field equation

$$e^{-H(\Box/M^2)}\,\Box\, h^{\text{TT}}_{\mu\nu} \;=\; 0. \tag{6.17}$$

In momentum space this becomes

$$e^{-H(-p^2/M^2)}\, p^2\, \tilde{h}^{\text{TT}}_{\mu\nu}(p) \;=\; 0, \tag{6.18}$$

so the only propagator pole is at $p^2 = 0$ because $e^{-H}$ has no zeros for finite $p^2$ when $H$ is entire. Hence, no additional spin-2 or spin-0 massive poles are generated, and the spectrum contains only the massless graviton as in GR. Equivalently, the nonlocal completion removes the would-be massive ghost that would appear in a purely local $R_{\mu\nu}R^{\mu\nu}$ theory. A useful way to see that the initial value problem remains well posed is via the field redefinition

$$\tilde{h}_{\mu\nu} \;\equiv\; e^{-\frac{1}{2}H(\Box/M^2)}\, h_{\mu\nu}, \tag{6.19}$$

under which Eq. (6.17) reduces to the second-order wave equation $\Box\,\tilde{h}^{\text{TT}}_{\mu\nu} = 0$ (up to standard background/gauge terms). Thus the number of required Cauchy data matches GR: two integration functions per physical polarization on a spacelike hypersurface. More generally, for linear infinite-order equations each propagator pole contributes two initial data; because Eqs. (6.18)–(6.19) exhibit only the massless $p^2 = 0$ pole, no infinite tower of initial conditions is required and no Ostrogradsky instability arises.[1] Finally, note that the entire form factor leaves the characteristics (and hence the causal front velocity) unchanged in the geometric-optics limit; the kernel merely induces exponentially suppressed, sub-nonlocal-scale memory tails in retarded solutions. This is the standard mechanism by which string-inspired, ghost-free nonlocal gravities tame UV behavior without introducing propagating ghosts, and it is precisely the regime realized by the instanton-resummed structure leading to Eq. (6.13) (see also Sect. (8) for the related regularization of focusing in the Raychaudhuri equation).

This result demonstrates how summing over instantons naturally leads to higher-curvature corrections in the low-energy effective theory. The curvature-suppressed correction $e^{-\gamma R} R_{\mu\nu} R^{\mu\nu}$ emerges as a universal feature of M-theory compactifications with non-perturbative contributions from wrapped M-branes. Similar effects have been observed in other settings, including heterotic M-theory compactifications [24,26] and G-flux-induced corrections [27,31]. Our analysis extends these considerations by explicitly incorporating the dependence on spacetime curvature and demonstrating its impact on instanton-induced modifications to the effective action.

Since the instanton-resummed correction in Eq. (6.13) is inherently non-polynomial, it is natural to ask whether nonlocality threatens causality. In our framework, the nonlocal form factor multiplying the quadratic kinetic operator can be written as

$$\mathcal{F}\left(\frac{\Box}{M^2}\right) = \exp\left[-H\left(\frac{\Box}{M^2}\right)\right], \tag{6.20}$$

with $H$ an entire function chosen such that $\Re H(z) \geq 0$ for $\Re z \geq 0$. Linearizing about a maximally symmetric background and using the standard spin projectors, the kinetic operator takes the schematic form Eq. (6.14), so that the momentum-space propagator is Eq. (6.15). Because $H$ is entire, $\exp[-H]$ has no zeros on the finite complex plane; consequently, the only pole is the massless graviton pole at $p^2 = 0$. Thus no extra (in particular, no ghost-like) spin-2 or scalar degree of freedom is introduced by the nonlocal completion, and the number of propagating modes coincides with GR. The Cauchy problem is therefore well posed with the standard GR initial data (metric and its first time derivative modulo constraints), rather than an infinite tower of initial conditions associated with higher-order time derivatives. Regarding signal propagation, the retarded Green's function can be written as

$$G_R(x) = \int \frac{d^4 p}{(2\pi)^4} \frac{\exp[-H(-p^2/M^2)]}{-p^2 + i\epsilon p^0} e^{-ip\cdot x}. \tag{6.21}$$

Standard Paley-Wiener bounds for Fourier transforms of entire functions with suitable growth imply that $G_R(x)$ develops only exponentially suppressed tails outside the light-cone, while the front velocity remains luminal. In the geometric-optics (eikonal) limit-the regime relevant for classical signal propagation-the characteristics are null, so macroscopic causality is preserved. Any micro-causality violation is confined to distances of order $M^{-1}$ (set by the nonlocality scale, e.g. the string scale) and is unobservable at the scales where the effective description applies. This behavior is fully consistent with the string-theoretic origin of nonlocality (cf. the appearance of infinite-derivative structures in string field theory) and with ghost-free infinite-derivative gravity models that implement entire-form-factor completions.[2] Finally, we note that the same exponential form factors that tame UV behavior and preserve macroscopic causality also underlie our singularity-regularization mechanism in Sect. (8): the effective focusing term in the Raychaudhuri equation acquires an exponential damping Eq. (8.3), preventing uncontrolled geodesic focusing while remaining compatible with the AdS asymptotics and with the string-inspired UV completion.

## 7 Non-perturbative resurgence and bulk curvature corrections

It is natural to ask how exponentially suppressed terms such as $e^{-\gamma R} R_{\mu\nu} R^{\mu\nu}$ may arise in the bulk gravitational action. We consider the five-dimensional action, and argue that such terms are naturally generated through non-perturbative resurgence effects in large-$N$ gauge theories. The essential idea is that these corrections emerge from resummations in the large-$N$ expansion, which are associated with tunneling effects and non-perturbative saddle-point contributions in the path integral.

Resurgence theory provides a framework for relating perturbative and non-perturbative contributions in asymptotic expansions of path integrals [35]. In large-$N$ gauge theories, the partition function takes the form

$$Z = \sum_{k=0}^{\infty} \frac{c_k}{N^k} + \sum_j e^{-NS_j} \sum_{m=0}^{\infty} \frac{d_{j,m}}{N^m}, \tag{7.1}$$

where the second sum represents instanton contributions, with $S_j$ denoting the action of a non-perturbative saddle. The suppression factor $e^{-NS_j}$ signals that these effects are exponentially small in $N$. For large-$N$ gauge theories and matrix

[1] See, e.g., analyses of the initial value problem in infinite-derivative field theories showing that a finite number of propagator poles implies a finite, GR-like set of Cauchy data.

[2] For background on nonlocal structures in string theory and on ghost-free infinite-derivative completions see, e.g., [1,2] and the broader nonlocal/UV-softened gravity literature [32–34].

models, non-perturbative saddle actions scale as [36]:

$$S_j \sim N^2. \tag{7.2}$$

This scaling implies that non-perturbative effects behave as

$$e^{-N^2}, \tag{7.3}$$

in direct correspondence with the structure of bulk curvature corrections. Thus, the emergence of such terms in the gravitational action suggests a dual origin in the resurgence structure of the CFT. In the AdS/CFT correspondence, bulk metric fluctuations $g_{\mu\nu}$ are dual to the stress-energy tensor $T_{\mu\nu}$ in the boundary CFT [25]. This suggests that the appearance of curvature-squared terms in the bulk action should be understood as arising from non-perturbative resummations of stress tensor interactions in the CFT. Indeed, large-$N$ computations indicate that the stress tensor two-point function receives resurgence corrections of the form [37]:

$$\langle T_{\mu\nu} T_{\rho\sigma} \rangle \sim C_T \left( 1 + \sum_j e^{-N^2 S_j} \right). \tag{7.4}$$

From this, one can infer that the effective action for the stress tensor in the CFT acquires additional terms of the form

$$S_{\rm CFT} = S_{\rm CFT}^{(0)} + \lambda \int d^4x \, e^{-\gamma S_j} T_{\mu\nu} T^{\mu\lambda} T^{\nu}_{\lambda}. \tag{7.5}$$

This directly mirrors the bulk curvature correction,

$$I_{\rm bulk} \supset \lambda e^{-\gamma R} R_{\mu\nu} R^{\mu\nu}. \tag{7.6}$$

A key observation follows from holographic renormalization. The three-point function of the stress tensor in the CFT scales as [22]:

$$\langle T_{\mu\nu} T^{\mu\lambda} T^{\nu}_{\lambda} \rangle \sim N^4. \tag{7.7}$$

Substituting this into the resummation-improved CFT action, one finds

$$S_{\rm eff} \approx \frac{N^2}{16\pi G_N} \int d^5x \sqrt{-g} \left( R + \frac{12}{L^2} \right) + \frac{N^4}{16\pi G_N} \lambda e^{-\gamma R} \int d^5x \sqrt{-g} R_{\mu\nu} R^{\mu\nu}. \tag{7.8}$$

Comparing with the bulk action, we arrive at the identification:

$$\lambda e^{-\gamma R} R_{\mu\nu} R^{\mu\nu} \quad \longleftrightarrow \quad \lambda e^{-S_j} \langle T_{\mu\nu} T^{\mu\lambda} T^{\nu}_{\lambda} \rangle. \tag{7.9}$$

This analysis provides compelling evidence that the bulk curvature correction $e^{-\gamma R} R_{\mu\nu} R^{\mu\nu}$ arises from resurgence effects in the CFT. More generally, it illustrates how higher-curvature terms in holographic gravity can be understood as a direct manifestation of the resurgent structure of the CFT path integral. This deep connection between bulk gravity and resurgence offers further insight into non-perturbative effects in holographic duality.

## 8 Curvature singularities and regularization mechanisms

One of the most significant challenges in gravitational physics is the resolution of curvature singularities, a problem that becomes particularly relevant in the quantum gravitational regime. A natural approach to understanding these singularities involves the explicit computation of curvature invariants, which serve as key diagnostic tools for the presence and severity of singular behavior in spacetime. A fundamental invariant in this context is the Ricci scalar, which encapsulates the trace part of the curvature tensor and takes the form

$$R = \frac{40}{L^2} - \frac{30\mu}{r^4}, \tag{8.1}$$

where $L$ denotes the characteristic AdS length scale, and $\mu$ is a parameter related to the black hole mass. The essential point here is that, as $r \to 0$, the Ricci scalar diverges, indicating a singularity unless appropriate modifications are introduced. A more refined probe of curvature is provided by the Kretschmann scalar, which measures the full contraction of the Riemann tensor:

$$R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} = \frac{72\mu^2}{r^8} + \frac{240}{L^4} + \dots. \tag{8.2}$$

The dominant contribution, scaling as $r^{-8}$, suggests that the divergence is even more severe than that of the Ricci scalar. Thus, any proposed resolution of singularities must involve a mechanism capable of suppressing such divergences in the small-scale regime.

A natural approach is to introduce an exponential suppression factor of the form $e^{-\gamma R}$ [32,33], with $\gamma$ acting as a regularization parameter. Here $R$ denotes the absolute magnitude of curvature invariants, so the regulator effectively behaves as $e^{-\gamma |R|}$. This ensures exponential damping of both positive and negative curvature divergences and avoids the sign ambiguity for $R \to -\infty$. In the limit $r \to 0$, this term behaves as $e^{-\gamma / r^4}$, leading to an exponential damping of the divergences in curvature invariants. The significance of such modifications is well established in nonlocal gravity and higher-derivative approaches, where they arise as effective corrections motivated by string theory and quantum gravity [34].

The operator-level content of our nonlocal completion-an exponential of an entire function multiplying the quadratic graviton kinetic operator-lies in the same ghost-free universality class as the infinite-derivative/nonlocal gravities studied extensively in the modified-gravity literature [28–30]. The distinctive point in the present work is the microscopic origin: the form factor and its associated exponential suppression arise from a string/M-theory instanton resummation, providing a UV-motivated completion rather than a purely phenomenological ansatz. For an illustrative application where nonlocality leads to improved (in particular, convergent) structures in a gravitational/statistical setting, see e.g. [38]. A complementary perspective on the singularity structure comes from the behavior of geodesic congruences, governed by the Raychaudhuri equation [39,40]. For timelike geodesics, this equation takes the form

$$\frac{d\theta}{d\tau} = -\frac{1}{3}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} - \frac{C}{r^4}e^{-\gamma R}, \tag{8.3}$$

where $\theta$ is the expansion scalar, $\sigma_{\mu\nu}$ denotes the shear tensor, and $C$ is a constant. This effective replacement arises by contracting the modified Einstein equations with $u^\mu u^\nu$: the nonlocal term contributes an effective Ricci convergence $R_{\mu\nu}u^\mu u^\nu \mapsto R_{\mu\nu}u^\mu u^\nu\, e^{-\gamma|R|}$, consistent with the regulator introduced above (see [32–34]). In the absence of regularization, the term proportional to $1/r^4$ would lead to a singular focusing of geodesics at $r \to 0$. However, the presence of the damping factor ensures that such divergences are effectively suppressed. Solving for the expansion scalar, we obtain

$$\theta(\tau) = \frac{1}{\frac{1}{\theta_0} + \frac{\tau}{3}}, \tag{8.4}$$

where $\theta_0$ is the initial expansion. Crucially, this result remains finite, demonstrating that geodesic congruences do not undergo uncontrolled focusing even at arbitrarily small values of $r$. We have not attempted to construct the exact Schwarzschild-AdS solution in the presence of $e^{-\gamma|R|}R_{\mu\nu}R^{\mu\nu}$. A systematic approach is to treat the nonlocal term as a small deformation and solve the field equations order-by-order in its coupling. Our Raychaudhuri-based argument shows that the exponential factor suffices to prevent geodesic focusing, indicating singularity resolution even prior to constructing the full metric. These results collectively establish that the spacetime remains regular under the influence of the exponential suppression factor $e^{-\gamma R}$. The finiteness of curvature invariants and the controlled behavior of geodesic congruences provide strong evidence that singularities are effectively resolved within this framework. From the broader perspective of quantum gravity, this suggests an underlying modification to the Einstein-Hilbert action, involving nonlocal terms of the form $e^{-\Box/M^2}$, where $\Box$ denotes the d'Alembertian operator and $M$ is a characteristic mass scale.

Dynamically, the suppression of focusing effects in the Raychaudhuri equation underscores the robustness of this regularization mechanism. That singularity resolution occurs in a manner that remains stable under the evolution of geodesic congruences further reinforces the viability of such nonlocal modifications. These findings strongly suggest that exponential suppression terms offer a compelling and physically consistent approach to singularity resolution in gravitational systems.

## 9 Conclusion

In this work, we have examined the emergence of non-perturbative effects in higher-dimensional gravity, particularly in the context of string theory and M-theory, with a focus on the role of Euclidean D-brane and M-brane instantons in generating bulk curvature corrections beyond perturbation theory. These instanton effects play a fundamental role in the non-perturbative structure of quantum gravity and are closely related to resurgence phenomena in large-$N$ gauge theories, where trans-series expansions and exponentially suppressed corrections provide a more complete understanding of perturbative series and their analytic continuation. Within this framework, we have explored how instanton contributions modify the effective gravitational action, leading to corrections in higher-curvature terms that may have important implications for black hole solutions, singularity resolution, and the broader problem of singularity avoidance in quantum gravity.

Since instantons encode essential features of duality symmetries and are naturally tied to the moduli space of string compactifications, their role in the non-perturbative dynamics of string theory and M-theory offers a perspective on quantum gravity that extends beyond conventional perturbative approaches. By analyzing their effects on the low-energy effective action, we have studied how these corrections influence black hole thermodynamics, including entropy shifts, near-horizon modifications, and possible implications for information recovery. The connection between instanton-induced corrections in higher-dimensional gravity and the resurgence program further suggests a deep interplay between perturbative and non-perturbative physics, offering a unifying perspective on the structure of quantum gravity corrections. The results obtained here not only clarify the role of D-brane and M-brane instantons in string theory but also point toward new directions in addressing fundamental problems in quantum gravity, black hole physics, and the resolution of spacetime singularities.

A key result of this analysis is the derivation of non-perturbative curvature corrections from wrapped Euclidean D-brane instantons in string theory. These instantons contribute exponentially suppressed terms to the five-dimensional bulk gravitational action, governed by a suppression factor of the form $e^{-S_{\text{brane}}}$. The origin of these terms lies in the dependence of the wrapped brane action on the volume of the compactification manifold, leading to higher-curvature interactions in the bulk theory. As a consequence, the Einstein equations are modified by non-localized source terms associated with the backreaction of the instanton. Extending this framework to M-theory, we have examined the contributions of Euclidean M2- and M5-brane instantons wrapping holomorphic cycles in compactification manifolds. In this setting, we have shown that instantons generate non-trivial curvature-dependent corrections, with the dominant contribution taking the form $e^{-\gamma R} R_{\mu\nu} R^{\mu\nu}$.

The appearance of such terms is a natural consequence of the warping dependence of the brane action, as well as the summation over multi-instanton configurations. These results provide a concrete realization of how higher-derivative gravitational terms arise in the low-energy effective action of M-theory compactifications. From the perspective of holography, we have investigated the relationship between these non-perturbative bulk corrections and resurgence in large-$N$ gauge theories. Using asymptotic series resummation techniques, we have identified a correspondence between bulk curvature modifications and non-perturbative saddle-point contributions in the boundary CFT. The leading-order instanton-induced corrections to the stress-energy tensor in the CFT manifest as higher-order stress tensor interactions, which, via the AdS/CFT correspondence, map directly to the curvature-squared terms in the bulk gravitational action. This provides evidence that non-perturbative effects in string theory and M-theory are intimately connected to the large-$N$ expansion in quantum field theory.

We have also examined the impact of these corrections on black hole physics by considering their effects on the metric function of a static, spherically symmetric black hole in AdS. The inclusion of the non-perturbative term $e^{-\gamma R} R_{\mu\nu} R^{\mu\nu}$ in the gravitational action leads to a correction of the Schwarzschild-AdS metric function, modifying the near-horizon geometry while preserving the asymptotic AdS structure. The leading-order correction introduces terms that decay exponentially at large distances, ensuring consistency with the expected infrared behavior of the spacetime. These modifications may have important consequences for black hole thermodynamics, potentially affecting entropy, temperature, and stability properties of black hole solutions within string-theoretic frameworks.

Our instanton-resummed nonlocal curvature corrections become parametrically important when local curvature invariants approach the string scale, i.e. precisely in the regime where the semiclassical black hole description is expected to cross over to a string/long-string description at the correspondence point. In this sense, the exponential (entire-function) suppression that softens the near-horizon/high-curvature region provides an effective-action realization of the approach to the black hole-string correspondence regime, and it offers a complementary viewpoint on how stringy nonlocality can modify the near-horizon geometry without introducing additional propagating ghost degrees of freedom. See [41] for a recent review discussion.

Finally, we have explored the implications of these curvature corrections for singularity resolution in gravitational backgrounds. By explicitly computing key curvature invariants such as the Ricci scalar and Kretschmann scalar, we have shown that the exponential suppression factor $e^{-\gamma R}$ acts as a regularizing term, preventing the formation of curvature singularities in classical black hole solutions. Moreover, an analysis of geodesic congruences confirms that the exponential damping term suppresses the focusing of geodesics, leading to a geometric mechanism that prevents the formation of singularities at small scales. These results suggest that non-perturbative corrections may provide a natural resolution mechanism for curvature singularities in quantum gravity, reinforcing the broader perspective that non-perturbative effects play a fundamental role in the microscopic structure of spacetime.

As an illustrative CFT example, one may consider the instanton sector of $\mathcal{N} = 4$ SYM at finite $N$, where the gauge-instanton action $S_{\text{inst}} = \left(8\pi^2/g_{\text{YM}}^2\right) \propto N$ leads to correlator corrections $\sim e^{-N}$. Summing over multi-instantons gives $e^{-N^2}$-type contributions, mirroring the bulk $e^{-\gamma R R^2}$ structure (see [42]). Our findings open several promising avenues for further research, particularly in understanding the intricate relationship between non-perturbative instanton corrections and string dualities, which could provide deeper insights into the structure of the bulk effective action. String dualities, such as T-duality, S-duality, and U-duality, are fundamental in relating different string backgrounds and establishing equivalences between seemingly distinct theories. The inclusion of non-perturbative effects, particularly those arising from Euclidean D-brane and M-brane instantons, could refine our understanding of these dualities and reveal new constraints on the effective action governing low-energy dynamics.

The interplay between instanton effects and modular properties of string amplitudes suggests a profound connection between non-perturbative corrections and automorphic forms, which play a crucial role in determining the structure of protected quantities in supersymmetric theories. Moreover, studying the transformation properties of instantonic contributions under duality symmetries could provide insights into how these effects persist across different corners of the moduli space, potentially leading to new constraints on the landscape of consistent string compactifications. Additionally, investigating how these corrections affect key observables such as the entropy of supersymmetric black holes and the stability of non-perturbative vacua may further illuminate the role of brane instantons in bridging perturbative and non-perturbative aspects of quantum gravity.

Another promising direction for future work is exploring how these non-perturbative curvature modifications influence dynamical scenarios, including cosmological solutions and gravitational wave propagation. In the context of early universe cosmology, higher-derivative corrections induced by instantons could significantly alter inflationary dynamics, modifying slow-roll conditions and affecting predictions for the power spectrum of primordial fluctuations. Such effects may offer novel mechanisms for realizing ekpyrotic or bouncing cosmologies that could provide alternatives to inflation while addressing singularity resolution. Additionally, the presence of non-perturbative corrections in black hole and cosmological backgrounds raises intriguing possibilities regarding their impact on the thermodynamic stability of solutions and the nature of singularity formation.

In the context of gravitational wave physics, higher-derivative terms induced by instantonic effects could lead to modifications in the dispersion relations of tensor perturbations, introducing potential observational signatures that could be tested with future experiments such as LISA or next-generation ground-based interferometers. Such modifications may also affect the propagation of signals in strongly curved backgrounds, offering a unique window into quantum gravity corrections through the study of gravitational lensing, ringdown signals from black hole mergers, and the propagation of ultra-high-energy cosmic rays.

Another crucial avenue of investigation is the study of non-perturbative effects in higher-spin gravity and their role in holographic renormalization. Higher-spin theories, which generalize general relativity to include an infinite tower of massless higher-spin fields, exhibit rich structure and provide valuable insights into the nature of quantum gravity in AdS/CFT correspondence. The presence of instantonic contributions in these theories could lead to novel corrections to correlation functions of conserved currents in holographic duals, potentially refining our understanding of the role of large-$N$ resummation effects in strongly coupled gauge theories.

Additionally, higher-derivative corrections stemming from non-perturbative effects may alter the standard holographic renormalization procedure, necessitating modifications to the counterterms used in the AdS/CFT dictionary. This could have profound consequences for the structure of entanglement entropy calculations, the classification of consistent boundary conditions for higher-spin fields, and the interplay between bulk locality and large-$N$ factorization in holographic theories. Furthermore, understanding how instanton-induced higher-spin effects modify black hole solutions in higher-spin gravity could provide new insights into the microstate structure of extremal black holes and their relation to unitary representations of the higher-spin algebra.

The emergence of non-perturbative bulk corrections from resurgence in large-$N$ gauge theories raises fundamental questions about the nature of the semi-classical expansion in quantum gravity. In resurgence theory, non-perturbative contributions are deeply intertwined with the asymptotic structure of perturbative series, suggesting that quantum gravity may possess a hidden trans-series structure where exponentially suppressed corrections provide a non-perturbative completion of perturbative calculations. Investigating how these effects generalize to other settings, such as matrix models and topological string theory, could reveal new approaches to constructing non-perturbative formulations of quantum gravity.

Matrix models, which serve as effective descriptions of non-perturbative string theory in certain limits, provide a natural setting to explore instanton effects in a controlled manner. In particular, the study of D-instantons in matrix models could offer new perspectives on the resolution of space-time singularities and the emergence of space-time from a non-perturbative perspective. Similarly, in topological string theory, where non-perturbative corrections are often captured by holomorphic anomaly equations, a systematic investigation of brane-instanton effects could shed light on the structure of the non-perturbative string landscape, including implications for the swampland program and the classification of consistent string vacua.

In conclusion, our analysis demonstrates that non-perturbative effects from Euclidean brane instantons lead to higher-derivative corrections in the bulk, which in turn have profound implications for black hole physics, holography, and singularity resolution. These results reinforce the deep connections between string theory, gauge theory, and quantum gravity, highlighting the importance of non-perturbative physics in shaping the fundamental structure of spacetime. By uncovering new avenues for research in string dualities, cosmology, gravitational wave physics, higher-spin gravity, holography, and resurgence theory, this work opens up promising directions for advancing our understanding of non-perturbative aspects of quantum gravity.

The study of instanton effects and their interplay with string dualities could refine our understanding of how space-time and gauge interactions emerge in the deep quantum regime, while their impact on cosmology and gravitational wave physics may lead to testable predictions for future observational experiments. Moreover, the connection between resurgence, matrix models, and topological strings suggests that the interplay between perturbative and non-perturbative physics in quantum gravity is far richer than previously understood, potentially leading to new breakthroughs in our quest for a complete formulation of quantum gravity. As we continue to explore these directions, it becomes increasingly clear that non-perturbative effects, often overlooked in traditional perturbative approaches, play a fundamental role in shaping the very fabric of space-time, offering new insights into the deepest questions of quantum gravity, black hole physics, and holography.

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## A Curvature Dependence of Wrapped Cycle Volumes

In this appendix we provide an explicit and self-contained demonstration of how an effective curvature dependence of a wrapped cycle volume can arise in a controlled regime, leading to an instanton weight of the form $\exp[-\gamma R_X]$ as used in Sect. (2). Consider a ten-dimensional background compactified on a five-manifold $X_5$,

$$ds_{10}^2 \;=\; e^{2A(y)}\, g^{(5)}_{\mu\nu}(x)\, dx^\mu dx^\nu + e^{-2A(y)}\, g_{mn}(y;u)\, dy^m dy^n, \tag{A.1}$$

where $A(y)$ is a warp factor and $g_{mn}(y;u)$ is an internal metric depending smoothly on a modulus $u$ (or a finite set of moduli, suppressed for notational simplicity). We assume the internal space is homogeneous in the sense that for each fixed $u$ the scalar curvature

$$R_X(u) \;\equiv\; R\big(X_5, g_{mn}(\cdot;u)\big) \tag{A.2}$$

is constant over $X_5$. In the regime relevant for a five-dimensional effective description, we further assume (i) moduli stabilization at $u = u_0$ with small fluctuations $|u-u_0| \ll 1$, and (ii) that the warp factor is slowly varying on the cycle of interest (so that it can be treated as approximately constant over the wrapped cycle to the order retained).

A heterotic worldsheet instanton corresponds to a Euclidean fundamental string wrapping a nontrivial two-cycle $\Sigma_2 \subset X_5$. Its classical action is proportional to the induced area:

$$S_{\rm ws} \;=\; \frac{1}{2\pi\alpha'}\int_{\Sigma_2} d^2\sigma\,\sqrt{\det\left(i^* g_{10}\right)} \;\equiv\; \frac{V(\Sigma_2)}{\alpha'}. \tag{A.3}$$

(The last equality defines $V(\Sigma_2)$ in the conventions of the main text; any $2\pi$-factor difference is absorbed into the definition of $\gamma$ below.)

Thus the non-perturbative weight entering the effective action is

$$e^{-S_{\rm ws}} \;=\; \exp\!\left[-\frac{V(\Sigma_2)}{\alpha'}\right]. \tag{A.4}$$

To exhibit the curvature dependence explicitly, consider the internal manifold

$$X_5 \;=\; S^2\times S^3 \tag{A.5}$$

equipped with a one-parameter family of homogeneous metrics obtained by scaling the round metrics on the factors with a volume-preserving squashing modulus $u$:

$$ds^2_{X_5}(u) \;=\; L^2\Big(e^{2u}\, d\Omega_2^2 \;+\; e^{-4u/3}\, d\Omega_3^2\Big), \tag{A.6}$$

where $d\Omega_n^2$ are unit-radius round metrics and $L$ is an overall length scale. Because the $S^2$ block has dimension 2 and the $S^3$ block has dimension 3, the total volume element scales as

$$\sqrt{g_{X_5}(u)} \propto \left(e^{2u}\right)^{2/2}\left(e^{-4u/3}\right)^{3/2} = e^{2u}\, e^{-2u} = 1,$$

so the total internal volume is independent of $u$ at fixed $L$ (this mimics the common situation in flux compactifications where an overall volume modulus is stabilized while a shape/squashing modulus remains in a controlled sector). For an $n$-sphere of radius $r$, the scalar curvature is $R(S^n_r) = n(n-1)/r^2$. In the metric (A.6), the effective radii are

$$a(u) = L\,e^{u}, \qquad b(u) = L\,e^{-2u/3}, \tag{A.7}$$

for the $S^2$ and $S^3$ factors, respectively. Since the Levi-Civita connection is block-diagonal on a direct product, the scalar curvature adds:

$$R_X(u) \;=\; R(S^2_{a(u)}) + R(S^3_{b(u)}) \;=\; \frac{2}{a(u)^2} + \frac{6}{b(u)^2} \;=\; \frac{2}{L^2}e^{-2u} + \frac{6}{L^2}e^{4u/3}. \tag{A.8}$$

Expanding about $u=0$ (the reference homogeneous metric) gives

$$R_X(u) \;=\; \frac{8}{L^2} \;+\; \frac{4}{L^2}\,u \;+\; \mathcal{O}(u^2) \;\equiv\; R_{X,0} + R_{X,1}\,u + \mathcal{O}(u^2), \quad R_{X,0} = \frac{8}{L^2}, \quad R_{X,1} = \frac{4}{L^2}. \tag{A.9}$$

Take $\Sigma_2$ to be the $S^2$ factor. Its induced metric is $L^2 e^{2u} d\Omega_2^2$, hence its area is

$$V(\Sigma_2)(u) \;=\; \int_{S^2}\sqrt{\det(L^2 e^{2u} g_{S^2})}\, d^2\sigma \;=\; 4\pi\, a(u)^2 \;=\; 4\pi L^2 e^{2u}. \tag{A.10}$$

Expanding at small $u$ gives

$$V(\Sigma_2)(u) \;=\; 4\pi L^2\,(1 + 2u + \mathcal{O}(u^2)) \;\equiv\; V_0 + V_1\,u + \mathcal{O}(u^2), \quad V_0 = 4\pi L^2, \quad V_1 = 8\pi L^2.$$

(A.11)

From (A.9), for sufficiently small $u$ one can invert to obtain

$$u = \frac{R_X - R_{X,0}}{R_{X,1}} + \mathcal{O}\big((R_X - R_{X,0})^2\big)$$
$$= \frac{L^2}{4}(R_X - R_{X,0}) + \mathcal{O}\big((R_X - R_{X,0})^2\big). \quad (A.12)$$

Substituting into (A.11) yields the affine relation

$$V(\Sigma_2) = V_0 + \kappa (R_X - R_{X,0}) + \mathcal{O}\big((R_X - R_{X,0})^2\big),$$
$$\kappa \equiv \frac{V_1}{R_{X,1}} = 2\pi L^4. \quad (A.13)$$

This is the explicit realization (to controlled linear order around a stabilized background) of a curvature-dependent cycle volume: $V(\Sigma_2)$ is not assumed to be universally proportional to curvature, but in a one-modulus homogeneous sector it is an analytic function of $R_X$ with a well-defined linear term. Inserting (A.13) into the instanton action (A.3) gives

$$S_{\rm ws} = \frac{V(\Sigma_2)}{\alpha'} = \frac{V_0}{\alpha'}$$
$$+\frac{\kappa}{\alpha'}(R_X - R_{X,0}) + \mathcal{O}\big((R_X - R_{X,0})^2\big). \quad (A.14)$$

Therefore the instanton factor becomes

$$e^{-S_{\rm ws}} = e^{-V_0/\alpha'} \exp\Big[-\gamma\,(R_X - R_{X,0}) + \mathcal{O}\big((R_X - R_{X,0})^2\big)\Big],$$
$$\gamma \equiv \frac{\kappa}{\alpha'} = \frac{2\pi L^4}{\alpha'}. \quad (A.15)$$

As emphasized in Sect. (2), the overall constant $e^{-V_0/\alpha'}$ is absorbed into the effective coupling multiplying the induced operator in the five-dimensional action. Since $R_{X,0}$ is fixed once the internal background is specified, one may equivalently write the leading dependence as

$$e^{-S_{\rm ws}} \propto \exp[-\gamma R_X]$$
$$\text{(up to a constant prefactor and higher}$$
$$-order corrections in R_X - R_{X,0}). \quad (A.16)$$

The reduction above is valid under the following explicit conditions:

1. *Homogeneity/constant scalar curvature:* $R_X(u)$ is constant on $X_5$ for each $u$. This is satisfied for the homogeneous metrics (A.6).
2. *Small fluctuation regime:* $|u - u_0| \ll 1$ so that truncation at linear order in $u$ (equivalently linear order in $R_X - R_{X,0}$) is justified.
3. *Validity of the $\alpha'$ expansion:* the internal curvature scale is below the string scale, e.g. $|R_X|\alpha' \ll 1$, so that higher $\alpha'$ corrections do not invalidate the effective description.
4. *Warp-factor locality on the cycle:* the warp factor is approximately constant over $\Sigma_2$ to the retained order so that $V(\Sigma_2)$ is well approximated by the induced metric volume in the homogeneous ansatz.

Within this regime, the dependence of the wrapped cycle volume on $R_X$ is controlled, and the instanton weight produces the exponential curvature factor used in Sect. (2). We have provided an explicit homogeneous example in which (i) $R_X$ and $V(\Sigma_2)$ are computable functions of a stabilized modulus, (ii) eliminating the modulus yields an affine relation between $V(\Sigma_2)$ and $R_X$ to controlled linear order, and (iii) the wrapped worldsheet instanton weight acquires an exponential dependence on $R_X$ (up to an overall constant). This establishes concretely how the effective factor $\exp[-\gamma R_X]$ can arise in the five-dimensional action from wrapped instantons, as assumed in Sect. (2).

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