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Application of Chaitin's Incompleteness Theorem to Quantum Gravity

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Abstract

In this paper, we explore the applications of Chaitin's incompleteness theorem to quantum gravity. Chaitin's theorem, grounded in the concept of Kolmogorov complexity, asserts that within any formal mathematical system, there exist true statements whose Kolmogorov complexity surpasses the system's capacity to encode or prove them. Since quantum gravity must be described by a formal mathematical framework, it follows that there are true statements about the universe, arising from quantum gravity, whose Kolmogorov complexity exceeds the computational capabilities of the underlying formal system. Consequently, this implies that the universe or multiverse cannot be the computational outcome of a formal system describing quantum gravity. We discuss an explicit application of this conclusion to the spacetime foam.

1 Introduction

Quantum gravity seeks to reconcile the principles of quantum mechanics and general relativity within a unified theoretical framework and hence should be a complete and consistent theory of everything. Despite major advancements in various approaches, such as string theory [1, 2] and loop quantum gravity (LQG) [3, 4], a fully consistent and complete theory of quantum gravity has not yet been discovered. As there are different approaches to quantum gravity, multiple axiomatic systems have been proposed to explain quantum gravity [5, 6, 7, 8, 9, 10]. Now any such viable candidate for quantum gravity must adhere to fundamental axioms that ensure both physical and mathematical coherence. Although a full set of axioms is yet to be established, certain foundational principles must be satisfied by any such approach. For the theory to be computationally well-defined, it should be effectively axiomatizable. So, the axioms of quantum gravity should be finite. It should reduce to general relativity and quantum mechanics in appropriate approximation, and hence it should be sufficient expressiveness. It should be capable of encoding basic arithmetic operations over natural numbers, including addition and multiplication. Which is needed as it has to reproduce the calculations done using general relativity and quantum mechanics, in appropriate limits. Finally, it should be consistent. So, any theory of quantum gravity must be internally consistent, and so it should be free of any internal contradiction.

In this paper, we will analyze the applications of Chaitin's incompleteness theorems [11, 12] to quantum gravity. Our work will be independent of the details of specific approaches, and hence our results will hold for any axiomatic system that describes quantum gravity. We will only assume that the axiomatic system describing quantum gravity is effectively axiomatizable, sufficient expressiveness, and consistent. Here, we would like to remark that Chaitin's incompleteness theorem is closely related to Gödel's incompleteness theorems [13, 14]. In fact, Chaitin's incompleteness theorem can be used to derive the Gödel's incompleteness theorems [15]. Now Gödel's incompleteness theorems has been used to show that universe/multiverse cannot be a computational consequences of axioms of quantum gravity [16, 17, 18]. Furthermore, it was argued that non-algorithmic understanding in the hyperuranion is the only way for quantum gravity to be a complete and consistent theory of everything. This was done by generalizing the original Lucas-Penrose argument [19, 20] to the axioms of quantum gravity in the hyperuranion [16, 17, 18]. So, it is natural to analyze similar results using Chaitin's incompleteness theorem. Thus, here we will demonstrate that the universe cannot be a computational outcome of quantum gravity using Chaitin's incompleteness theorem.

There exist four potential scenarios concerning the nature of reality: (1) Physical laws emerge as consequences of matter and spacetime, with spacetime and matter within the universe/multiverse being fundamental; (2) physical laws and a computational algorithm exist within a hyperuranion, giving rise to spacetime and matter; (3) the universe is a simulation;

and (4) a non-computational, non-algorithmic understanding exists in the hyperuranion, producing both physical laws and computational algorithms. In this paper, we critically examine these possibilities. We will demonstrate the inadequacy of the first three possibilities, concluding that the universe/multiverse cannot arise from them. We start with the principle of sufficient reason, which asserts that everything in nature must have a reason for its occurrence [21, 22]. Without this foundational assumption, the structure of logical arguments and the entirety of scientific thought collapses, leaving us with metaphysical absurdities.

We will first argue that spacetime is an emergent structure in quantum gravity, and hence axioms of quantum gravity cannot exist in spacetime, but rather spacetime emerges due to them. Then using Chaitin's incompleteness theorem, we show that axioms of quantum gravity, and a computational algorithm within a hyperuranion cannot give rise to a complete and consistent description of the universe or even the multiverse. We also use this argument to show that the universe cannot be a simulation. Finally, generalizing Lucas-Penrose argument to the axioms of quantum gravity in the hyperuranion, we show that this is the only way to obtain a complete and consistent description of reality, including not only the universe but even the multiverse. We will give an explicit example for the application of this conclusion. We will do that using the Planck scale physics. Quantum fluctuations at the Planck scale lead to the formation of spacetime foam [23, 24]. It has been argued that spacetime foam can have measurable consequences [25, 26]. In fact, it has been proposed that spacetime foam could be detected using gravitational waves [27, 28]. We will show how this present work has direct and measurable consequences for such Planck scale phenomena.

Now we shall discuss each case separately. We will show that the first three cases cannot provide a complete and consistent description of reality, and only the fourth provides such a complete and consistent description.

2 Case 1

Let us first analyze case (1): the structure of spacetime is described by general relativity, which serves as the theoretical framework for spacetime. A notable feature of general relativity is the prediction of its limitations, particularly at singularities, where the spacetime description of reality fails. Penrose-Hawking theorems further indicate that these singularities are inherent to general relativity rather than mathematical artefacts present in certain solutions [29, 30]. Consequently, spacetime cannot be fundamental but rather an emergent phenomenon. Moreover, the two cornerstones of modern theoretical physics-quantum mechanics and general relativity-suggest that quantum effects would preclude the formation of singularities [31, 32].

Although a consistent quantum theory of gravity remains elusive, various approaches have been proposed, with some universal, model-independent predictions emerging. Notably, examining spacetime at smaller scales requires increasingly higher energy levels, and at the

Planck scale, this results in the formation of mini black holes, effectively preventing further measurement [33]. This implies that quantum gravity theory must incorporate a fundamental minimal length and time [34, 35], which could also address issues in quantum field theory such as divergences or infinities, by naturally introducing a cutoff at this minimal length and time scale [36].

In perturbative string theory, the minimal length arises because spacetime cannot be probed below the string length scale, which represents the smallest possible probe size [37]. According to quantum mechanics, an unmeasurable object does not physically exist, implying that spacetime at scales below the string length does not exist. This principle holds even when considering point-like objects, such as D0-branes, in non-perturbative string theory due to dualities. T-duality, for instance, connects string theory at large and small scales, demonstrating that the spectrum above and below the string length scale is identical, confirming that no new information is accessible beyond this minimal distance [38]. Other discrete spacetime models, including causal sets [39], quantum graphity [40], and causal dynamical triangulation [41], also predict a minimal length below which geometry cannot be probed. This geometric cutoff is a recurring theme in quantum gravity theories and is observed in noncommutative [42] and nonlocal quantum field theories [43]. Loop quantum cosmology introduces a polymer length through a background-independent quantization method called polymer quantization [44, 45]. Minimal areas and volumes naturally arise in loop quantum gravity [46] and spin foam models [47], further indicating that a geometric cutoff is a universal feature of any quantum gravity approach. Thus, quantum gravity cannot merely be a theory of matter within spacetime but must be a theory from which spacetime and matter fields dynamically emerge as an emergent phenomenon, breaking down at the Planck scale.

This geometric cutoff addresses issues in general relativity, such as singularities [31, 32]. In string theory, T-duality introduces a minimal length that also avoids singularities [48], while loop quantum cosmology prevents singularities through a geometric cutoff [49]. The absence of singularities appears to be a universal feature of quantum gravity theories, as the Penrose-Hawking singularity theorems are connected to Bekenstein-Hawking entropy [50], and a bound on this entropy, imposed by the geometric cutoff, prevents singularities. Bekenstein-Hawking entropy is directly linked to geometry through the Jacobson formalism [51], and modifying this entropy alters spacetime geometry. It has been shown that a bound on this entropy, derived from the minimal length in quantum gravity, prevents singularity formation in spacetime [52, 53]. Singularities are avoided because a minimal value for a geometric quantity suggests spacetime geometry is an emergent structure that breaks down below this threshold. In general relativity, singularities arise when the theory is applied in contexts where spacetime descriptions become invalid. Interestingly, this geometric bound is closely tied to limits on quantum information [52, 53], indicating that spacetime geometry itself may emerge from quantum information. The critical role played by Bekenstein-Hawking entropy seems to indicate that information is more fundamental than spacetime and that spacetime emerges from the information of quantum states.

The holographic principle best illustrates how spacetime geometry emerges from non-geometric quantum states. This principle connects a theory within a volume to one on its boundary [54], widely applied in string theory, where it relates a gravitational theory in anti-de Sitter spacetime to a quantum field theory on its boundary [55]. It shows that the entanglement of quantum states in the boundary theory generates the geometric structure, and removing this entanglement destroys the geometry in the dual theory.

In loop quantum gravity [46] and spin foam models [47], macroscopic geometry emerges from loops at the microscopic scale. Similarly, spacetime geometry only emerges at scales larger than the Planck scale, much like how a table's geometry arises from atomic physics at scales larger than atoms. This emergence of geometry also occurs in other quantum gravity models with discrete degrees of freedom, such as causal sets [39], quantum graphity [40], and causal dynamical triangulation [41]. Despite fundamental differences from string theory, these models also suggest that geometry emerges from quantum information theory [56, 57, 58, 59]. Therefore, the emergence of spacetime geometry from information appears to be a model-independent feature of quantum gravity. Consequently, the laws of physics cannot be considered as properties of matter moving through spacetime; instead, matter fields and spacetime itself arise due to these laws of physics. These laws exist as a set of rules or a formal axiomatic system, with a computational algorithm that actualizes the consequences of this system. Spacetime and matter fields (in the universe or even the multiverse) exist as a consequence of these laws of physics, making scenario (1) untenable.

3 Case 2 and 3

These laws of physics and computational algorithms cannot exist within spacetime, as spacetime emerges from them, thus they have to exist in some hyperuranion, a possibility we have labelled as (2). However, it is challenging to distinguish between this possibility (2) and the idea that the laws of physics and a computational algorithm reside on a computer in an alien universe, making our universe a simulation [60], which we have labelled as (3). Thus, as (1) is not possible, we have to consider (2) and (3).

The possibility (3) and its indistinctively from (2), i.e., the possibility that our universe/multiverse could be a simulation has been a captivating thought, sparking debates across various disciplines. Often referred to as the "Simulation Argument," this hypothesis suggests that our reality could be an artificial construct, similar to a computer simulation created by an advanced civilization. Nick Bostrom has articulated the origins of this hypothesis, proposing that at least one of the following is true: (a) civilizations are likely to go extinct before developing the technology to create such simulations; (b) advanced civilizations are not interested in running simulations of their ancestors; or (c) if such simulations are possible and common, we are almost certainly living in a computer simulation [61]. Proponents argue that certain features of our universe, such as its mathematical laws, might indicate an underlying computational framework [62]. Additionally, advances in computational physics, where increasingly complex systems-including entire physical environments-are already being simulated on a much smaller scale than our universe, lend credence to this idea [63].

Thus, it seems that if (2) is correct, then it is possible for (3) to also be correct. Given that scenarios (2) and (3) are indistinguishable, we will treat them together, assuming that the universe/multiverse is a computational consequence of a formal system. We then explore whether everything true within that formal system is actualized by a computational algorithm. In that case, it is plausible that the universe/multiverse is a computational result of a formal system and that it exists within either the hyperuranion or on the computer of an advanced alien civilization. But to be perfectly fair, one needs to acknowledge that if this universe/multiverse is a simulation, the same line of argument can be used against the alien civilisation that is running the simulation, and at least one of those civilisations would have to be a computational result of some formal system in the hyperuranion. But now let us try to demonstrate, however, both (2) and (3) are impossible.

From the theory of algorithms, and more particularly on Kolmogorov complexity; we will derive this result from Chaitin's incompleteness theorem [11, 12]. From the theory of algorithms, particularly Kolmogorov complexity, we derive a result based on Chaitin's incompleteness theorem [11, 12]. The Kolmogorov complexity $K(s)$ of a binary string s is defined as the size of the shortest binary program p on a universal Turing machine U that produces s and halts afterward [64, 65]. Formally:

$$K(s) = \min\{|p| : U(p) = s\}, \tag{3.1}$$

where $|p|$ is the length of the program p. This provides a quantitative measure of the information content of s.

Let $\mathcal{QG}$ be a consistent and sufficiently powerful formal system representing quantum gravity. Chaitin's incompleteness theorem asserts the existence of a constant $c_{\mathcal{QG}}$ such that no string s can be proven to have Kolmogorov complexity greater than $c_{\mathcal{QG}}$ within $\mathcal{QG}$. Formally:

$$\forall s\, \forall n \left(\mathcal{QG} \vdash K(s) \geq n \implies n \leq c_{\mathcal{QG}}\right). \tag{3.2}$$

Consider the definition of Kolmogorov complexity. For any string s, there exists a program p such that $U(p) = s$ and $|p| = K(s)$. Define a statement φ_s as:

$$\varphi_s : K(s) \geq n, \tag{3.3}$$

which asserts that the Kolmogorov complexity of s is at least n. Assume $\mathcal{QG}$ can prove φ_s for some arbitrarily large n.

Now, construct a specific string s^* such that $K(s^*) > c$, where c is a constant. Define s^* as:

$$s^* = \arg\max_s \left\{s : K(s) > |s| \text{ and } |s| \leq c_{\mathcal{QG}} + 1\right\}. \tag{3.4}$$

This construction ensures that s^* is not the output of any program shorter than a certain threshold length, guaranteeing $K(s^*) > c_{\mathcal{QG}}$. Suppose $\mathcal{QG}$ could prove $K(s^*) > n$ for some $n > c_{\mathcal{QG}}$. The existence of such a proof implies that s^* can be described by the proof itself, whose length is bounded by $c_{\mathcal{QG}}$. This contradicts the construction of s^*, which ensures $K(s^*) > c_{\mathcal{QG}}$. Hence, $\mathcal{QG}$ cannot prove $K(s^*) > n$ for $n > c_{\mathcal{QG}}$. It follows that there exists a bound $c_{\mathcal{QG}}$ such that:

$$\forall s\, \forall n\, \big(K(s) > n \implies \neg(\mathcal{QG} \vdash K(s) \geq n)\big). \tag{3.5}$$

This result demonstrates that for any consistent formal system $\mathcal{QG}$, there are strings with Kolmogorov complexity exceeding $c_{\mathcal{QG}}$, but the Kolmogorov complexity of these strings cannot be proven within $\mathcal{QG}$. Thus, $\mathcal{QG}$ is inherently incomplete with respect to statements about Kolmogorov complexity. This conclusion is a direct consequence of Chaitin's incompleteness theorem, rooted in the interplay between provability and Kolmogorov complexity [11, 12, 64, 65].

4 Case 4

This proof suggests that within any sufficiently powerful formal system, there exist true statements about the Kolmogorov complexity of certain strings that cannot be proven within the system. The bound $c_{\mathcal{QG}}$ depends on the formal system itself, and while it exists, it limits the system's ability to capture the full Kolmogorov complexity of strings, leading to a form of incompleteness directly linked to algorithmic information theory. This result is analogous to Gödel's incompleteness theorems [13, 14], but from the perspective of information theory and computation rather than pure logic. However, it has already been argued, using the Lucas-Penrose argument, that such limitations of Gödel's incompleteness theorems can be overcome by the non-computational, non-algorithmic understanding of the human brain [19, 20]. Furthermore, it has been argued that such non-computational, non-algorithmic understanding operating in the hyperuranion can transcend the formal system and overcome Gödelian limitations [16, 17, 18] . Hence, they could produce a consistent theory of quantum gravity. Here, we shall argue that such non-computational, non-algorithmic understanding operating in the hyperuranion also overcomes the limitations of Chaitin's incompleteness theorem on $\mathcal{QG}$. By analyzing $\mathcal{QG}$, such non-computational, non-algorithmic understanding can deduce the existence of strings s with $K(s) > c_{\mathcal{QG}}$, even though $\mathcal{QG}$ cannot formally prove this.

To demonstrate, consider the definition of Kolmogorov complexity. For any string s, there exists a program p such that $U(p) = s$ and $|p| = K(s)$. Define a statement φ_s as:

$$\varphi_s : K(s) \geq n, \tag{4.1}$$

which asserts that the Kolmogorov complexity of s is at least n. Assume $\mathcal{QG}$ can prove φ_s for some arbitrarily large n.

Now, construct a specific string s^* such that $K(s^*) > c_{\mathcal{QG}}$. Define s^* as:

$$s^* = \arg\max_s \big\{s : K(s) > |s| \text{ and } |s| \leq c_{\mathcal{QG}} + 1\big\}. \tag{4.2}$$

This ensures s^* is not the output of any program shorter than $c_{\mathcal{QG}}+1$, guaranteeing $K(s^*) > c_{\mathcal{QG}}$. Now H in the hyperuranion allows the deduction that $K(s^*) > c_{\mathcal{QG}}$. While $\mathcal{QG}$ satisfies:

$$\forall s\, \forall n > c_{\mathcal{QG}},\ \neg(\mathcal{QG} \vdash K(s) \geq n), \tag{4.3}$$

the hyperuranion-based non-computational, non-algorithmic understanding concludes:

$$K(s^*) > c_{\mathcal{QG}} \quad \text{is true.} \tag{4.4}$$

Let H represent this non-computational, non-algorithmic understanding within the hyperuranion, and so H is non-computational and non-algorithmic. Define H as a mapping:

$$H : S \to \{\text{true}, \text{false}\}, \tag{4.5}$$

where S is the set of mathematical statements. For a statement $\varphi_s : K(s) \geq n$, non-computational, non-algorithmic understanding in the hyperuranion satisfies:

$$\exists s\, \big(K(s) > c_{\mathcal{QG}} \wedge H(\varphi_s) = \text{true}\big), \tag{4.6}$$

even though:

$$\neg(\mathcal{QG} \vdash \varphi_s). \tag{4.7}$$

This demonstrates that H transcends the limitations of $\mathcal{QG}$. While $\mathcal{QG}$ cannot prove statements of the form $K(s) > c_{\mathcal{QG}}$, H in the hyperuranion accesses truths beyond the system, resolving the incompleteness posed by Chaitin's theorem.

As such, non-computational, non-algorithmic understanding operating in the hyperuranion transcends the computational aspects of the formal system $\mathcal{QG}$, they also overcome the limitation imposed by Chaitin's incompleteness theorems. Thus, scenario (4) appears to be the only viable theory of the universe and even the multiverse. However, by the very definition (4) cannot be simulated due to the Lucas-Penrose argument [19, 20]. Therefore, the universe/multiverse cannot be a computational consequence of a formal system [16, 17, 18]. Thus, this universe cannot be a simulation.

5 Spacetime Foam

Now we will give an explicit example of this construction. At the Planck scale, spacetime exhibits quantum fluctuations, leading to the formation of spacetime foam [23, 24]. Here, it is important to note that spacetime foam can have measurable consequences [25, 26]. It is also possible to observe effects of spacetime foam using gravitational waves [27, 28]. We will explicitly analyze measurable effects produced from spacetime foam. Then the consequences of this present work on those effects will also be investigated. The geometry of spacetime at this scale can be described by a metric $g_{\mu\nu}$

$$g_{\mu\nu} = g_{\mu\nu}^{(\mathrm{c})} + \delta g_{\mu\nu}, \tag{5.1}$$

where $g_{\mu\nu}^{(\mathrm{c})}$ is the classical spacetime metric, and $\delta g_{\mu\nu}$ represents Planck-scale quantum fluctuations. The quantum states of these fluctuations are described by a wavefunction $\Psi[g]$, encoding the probabilities of different spacetime configurations. To delve into detailed calculations, assume that the fluctuation term $\delta g_{\mu\nu}$ has a typical amplitude characterized by $\delta g_{\mu\nu} \sim l_P/L$, with l_P denoting the Planck length and L a macroscopic length scale [66]. In a perturbative expansion about the classical metric, the wavefunction can be written as [67, 68]

$$\Psi[g] = \Psi\left[g_{\mu\nu}^{(\mathrm{c})}\right] + \sum_{n=1}^{\infty} \frac{1}{n!} \int \prod_{i=1}^{n} d^4x_i \, \delta g_{\mu_i\nu_i}(x_i) \, \frac{\delta^n \Psi}{\delta g_{\mu_1\nu_1}(x_1) \ldots \delta g_{\mu_n\nu_n}(x_n)} \Bigg|_{g=g^{(\mathrm{c})}} \tag{5.2}$$

which quantifies the contributions of higher-order fluctuations.

In string theory the fundamental objects are not point-like but extended one-dimensional strings with a characteristic length scale set by the string tension (or equivalently by $\sqrt{\alpha'}$). This extended nature modifies the usual Heisenberg uncertainty relations so that attempts to probe distances shorter than $\sqrt{\alpha'}$ instead excite additional string modes rather than yielding sharper localization [69]. In this way, the theory itself dynamically enforces a minimal length below which the notion of spacetime distance loses meaning [70]. Moreover, T-duality, a hallmark symmetry of string theory, shows that a string moving on a circle of radius R is physically equivalent to one moving on a circle of radius α'/R. This duality implies the existence of a self-dual point, typically of order $\sqrt{\alpha'}$, below which the concepts of "small" and "large" distances become interchangeable, thereby preventing any operational measurement of distances below this threshold [71, 72]. In loop quantum gravity the quantization of geometry leads to discrete spectra for operators corresponding to areas and volumes. The eigenvalues of these operators are bounded from below by scales on the order of the Planck length, which implies the existence of a fundamental minimal length in the theory. This discreteness arises directly from the canonical quantization of general relativity and the SU(2) gauge structure that underpins LQG [3]. Interestingly, as argued in [73], it is possible to view spacetime as simultaneously continuous and discrete mathematically equivalently via sampling theory. In this picture, physical fields that are bandlimited by a natural ultraviolet cutoff (such as the minimal length scale) can be perfectly reconstructed from discrete sets of sample points without loss of information. This dual description suggests that the minimal length emerging from the extended structure of strings and the discreteness inherent in loop quantum gravity might be two facets of a single underlying quantum gravitational reality. Due to discrete spacetime in quantum gravity, it is possible to use the methods of Regge calculus and causal dynamical triangulations. Regge calculus [74] approximates a continuous spacetime manifold by a simplicial complex (a collection of flat simplices), effectively replacing integrals over spacetime with discrete sums over lattice points. This framework, along with the developments in causal dynamical triangulations [75], provides a basis for discretizing the gravitational field and its quantum fluctuations, leading naturally to the summation expression shown above. So, in such a discrete approach, if spacetime is sampled on a grid of N points, the cumulative effect of fluctuations can be approximated by summing their contributions:

$$\Psi[g] \approx \Psi\left[g^{(\mathrm{c})}\right] + \sum_{i=1}^{N} \delta g_{\mu\nu}(x_i)\, \Delta V_i, \tag{5.3}$$

where ΔV_i is the volume element at point x_i.

The Kolmogorov complexity $K(g_{\mu\nu})$ of a spacetime configuration measures the size of the shortest algorithm capable of reproducing $g_{\mu\nu}$. For highly fluctuating spacetime foam, $K(g_{\mu\nu})$ grows due to the randomness inherent in quantum fluctuations. In a simplified model, consider that the Kolmogorov complexity of each grid point is given by [76]

$$K_i \approx \log_2\left(1 + \frac{|\delta g_{\mu\nu}(x_i)|}{\epsilon}\right), \tag{5.4}$$

with ϵ representing a precision threshold. Thus, the total Kolmogorov complexity is approximated by

$$K(g_{\mu\nu}) \approx \sum_{i=1}^{N} K_i = \sum_{i=1}^{N} \log_2\left(1 + \frac{|\delta g_{\mu\nu}(x_i)|}{\epsilon}\right). \tag{5.5}$$

From Chaitin's incompleteness theorem, in a formal system $\mathcal{QG}$ (representing quantum gravity), there exists a Kolmogorov complexity limit $c_{\mathcal{QG}}$

$$\forall g_{\mu\nu},\, \forall n > c_{\mathcal{QG}}, \quad \neg(\mathcal{QG} \vdash K(g_{\mu\nu}) \geq n). \tag{5.6}$$

For instance, if we set $c_{\mathcal{QG}} = 10^6$ bits, any configuration that requires more than 10^6 bits for its algorithmic description lies beyond the provability limits of $\mathcal{QG}$.

Gravitational waves carry information about spacetime dynamics. Suppose an experiment measures gravitational waves generated by Planck-scale fluctuations. The relation

$$h(t) = F[g_{\mu\nu}(t)], \tag{5.7}$$

expresses that the gravitational wave signal $h(t)$ is obtained as a functional F of the spacetime metric $g_{\mu\nu}(t)$. This concept is rooted in the linearized theory of general relativity where the metric perturbations yield the observable gravitational waves [77]. In the linearized regime of general relativity, where $\delta g_{\mu\nu}$ is small compared to $g^{(\mathrm{c})}_{\mu\nu}$, one can derive an approximate relation for the transverse-traceless component [78, 79]

$$h_{ij}^{\mathrm{TT}}(t, \mathbf{x}) \approx \frac{2G}{c^4 R} \ddot{Q}_{ij}^{\mathrm{TT}}(t - R/c), \tag{5.8}$$

is the well-known quadrupole formula derived from the linearized Einstein field equations. It relates the transverse-traceless component of the gravitational wave strain to the second-time derivative of the reduced mass quadrupole moment.

The Kolmogorov complexity $K(h(t))$ measures the shortest description of the gravitational wave signal. When the signal $h(t)$ is digitized, say by sampling at a rate f_s over a time T, the Kolmogorov complexity may be estimated as

$$K(h(t)) \approx \sum_{k=1}^{f_s T} \log_2 \left(1 + \frac{|h(t_k)|}{\delta}\right), \tag{5.9}$$

where δ characterizes the detector's noise floor. This sum indicates how fine details in the waveform (due to quantum fluctuations) contribute to the overall algorithmic description. provides a heuristic model for quantifying the Kolmogorov complexity of a digitized gravitational wave signal. Here, the Kolmogorov complexity is approximated by summing the contributions from each sampled data point, where each term reflects the information content of the corresponding sample relative to a baseline precision δ. This approach is inspired by the foundational ideas of algorithmic information theory, which relate the minimal description length of a dataset to its inherent Kolmogorov complexity. For a comprehensive introduction to these concepts, see [76, 80].

Now $g_{\mu\nu}$ can exhibit high Kolmogorov complexity due to quantum fluctuations, such that $K(h(t)) > c_{\mathcal{QG}}$. As $\mathcal{QG}$ cannot prove $K(h(t)) > c_{\mathcal{QG}}$, it will fail to computationally describe the signal $h(t)$ entirely. In practice, if advanced detectors record $h(t)$ with high precision, one might observe features that require, for example, $K(h(t)) \sim 10^7$ bits to fully describe, whereas $c_{\mathcal{QG}}$ is limited to 10^6 bits. The discrepancy, $\Delta K = K(h(t)) - c_{\mathcal{QG}} \approx 10^7 - 10^6 = 9 \times 10^6$ bits, quantifies the failure of $\mathcal{QG}$ to capture all the necessary details of the waveform. The formal system, therefore, oversimplifies the signal by missing critical features arising from the underlying quantum fluctuations. However, non-algorithmic understanding recognizes that $K(h(t)) > c_{\mathcal{QG}}$ by analyzing the intricate structure of $h(t)$ beyond the computational constraints of $\mathcal{QG}$. The formal system $\mathcal{QG}$ is incomplete in its capacity to algorithmically account for all details of spacetime foam. So, due to the non-algorithmic understanding operating in the hyperuranion, the explanation of such phenomena remains both complete and consistent. Without this non-algorithmic understanding, Planck-scale physics could not be explained, and this would contradict the principle of sufficient reason [21, 22]. So, gravitational wave signals due to spacetime foam cannot be explained using algorithmic predictions from $\mathcal{QG}$, leaving phenomena arising from spacetime foam unexplained. In contrast, due to non-algorithmic understanding operating in the hyperuranion, Planck-scale physics is consistently and completely explained.

Here we emphasize that our thesis does not contradict the fact that virtually all verified predictions in physics are obtained through standard, Turing-computable calculations. The situation is entirely analogous to mathematics, where Gödel's incompleteness theorems delimit-but do not disrupt-the everyday practice of proving theorems within arithmetic. We contend only that beyond the algorithmically tractable domain of effective field theory there lies a residual stratum of intrinsically non-computable phenomena-tied here to Planck-scale dynamics-that no finite computational procedure can exhaustively describe. Furthermore, such Planckian effects can be observed in gravitational-wave experiments.

6 Conclusion

In this paper, we have analyzed the implications of Chaitin's incompleteness theorem for quantum gravity, demonstrating that the universe, or even the multiverse, cannot be the computational outcome of an axiomatic system describing quantum gravity. Our argument is independent of the specific details of various approaches, and applies to any formal system that unifies quantum mechanics and general relativity into a quantum theory of gravity. By extending previous work based on Gödel's incompleteness theorems and the Lucas-Penrose argument, we have established that any computational framework is fundamentally inadequate for providing a complete and consistent theory of quantum gravity.

Through a critical examination of four potential scenarios concerning the nature of physical reality, we have shown that the first three: where physical laws emerge from matter and spacetime, where they exist as part of a computational algorithm within a hyperuranion, or where the universe is a simulation, are insufficient to account for a complete and consistent description of the reality. Instead, we have argued that the only viable resolution requires a non-computational, non-algorithmic understanding in the hyperuranion, which generates both physical laws and computational algorithms. Finally, we have discussed the application of this to spacetime foam. We have also discussed how this can have measurable consequences. This is because gravitational waves signals will be effected due to the Planck scale physics, and this cannot be explained using algorithmic predictions from quantum gravity. It would require non-algorithmic understanding in the hyperuranion.

Data Availability Statement

No data were generated or analyzed in this study. Therefore, no datasets are available.

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