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Peer-reviewed author version

EGGHE, Leo \& ROUSSEAU, Ronald (2000) Partial orders and measures for language preferences. In: Journal of the American Society for Information Science, 51(12). p. 1123-1130.

DOI: 10.1002/1097-4571(2000)9999:9999<::AID-ASI1014>3.0.CO;2-4
Handle: http://hdl.handle.net/1942/789

# Partial orders and measures for language preferences 

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#### Abstract

Relative own-language preference depends on two parameters: the publication share of the language and the self-citing rate. Openness of language $L$ with respect to language $J$ depends on three parameters: the publication share of language $L$, the publication share of language J , and the citation share of language J among all citations given by language L . It is shown that the relative own-language preference and the openness of one language with respect to another one, can be represented by a partial order. This partial order can be represented by a polygonal line (for the relative own-language preference) or a three-dimensional solid (for openness), somewhat in the same spirit as the Lorenz curve for concentration and evenness. Any function used to measure relative own language preference or openness of one language with respect to another one should at least respect the corresponding partial orders. This is a minimum requirement for such measures. Depending on the use one wants to make of these measures other requirements become necessary. A logarithmic dependence on the language share(s) seems a natural additional requirement. This would correspond with the logarithmic behavior of psychophysical sensations. We give examples of normalized functions satisfying this additional requirement. It is further shown that openness partial orders can not be used to express the relative own-language preference.


## 1. Introduction

Is it true that articles written in English hardly ever cite articles written in another language, or is it just a false impression due to the fact that most articles in the Western world are written in English? It is certainly true that, whatever the language of an article, most citations are given to articles written in the same language. For such situations we use the term 'language self-citation' and the fraction of references to articles written in the same language as the citing one is called the self-citing rate of this language.

It is clear that the study of language self-citation is important since it reflects, certainly for smaller languages, the degree of the existence of a foreign language barrier (foreign with respect to the used language). For further discussions about a possible growing language barrier and functions to measure it we refer the reader to Yitzhaki (1997), Bookstein and Yitzhaki (1999) and Egghe, Rousseau and Yitzhaki (1999) and to the references therein. It is evident that it is an expression of parochialism if the self-citing rate is high in case of a small language. Conversely, if a language's publication share is large (e.g. English) it is much more common (and expected) to have a high self-citing rate. This shows that the self-citing rate is not a perfect measure of own-language preference. Such a measure should take into account the share of this language in the total set of publications under study. The above mentioned publications present attempts to introduce good, so-called ROLP-measures: functions that measure the relative own language preference. Here the term 'own language' refers to the same language as that used in the publication. It has nothing to do with the mother tongue of the author.

This article extends these results by defining ROLP partial orders, represented by curves in the plane. Every such curve reflects a language self-citation and an order relation between any two of these curves determines the 'ROLP difference' between two situations. Any function respecting this order relation is then an acceptable ROLP measure, hence extending the existing measures considerably.

We then continue our research with a similar (but in a sense opposite) study of how to measure the degree to which a language L cites another language J. Three parameters are now involved: the relative citing rate of $L$ to $J$ and the relative sizes of the languages $L$ and $J$. Again it is clear that using only the relative citing rate of $L$ to $J$ is not enough to measure the 'openness (or preference) of language $L$ with respect to language $\mathrm{J}^{\prime}$. Indeed, the larger the citing language L is, the larger this measure should be (all other parameters being equal), while the larger the language J , the smaller this measure should be.

As far as we know, this problem has never been tackled before (the 'openness indices' introduced in (So, 1990) are just revised self-citing rates, or their complements). As in the case of ROLP we start by defining RO (relative openness) figures (2- or 3dimensional) on which order relations determine, in a general way (as in the ROLP case), the RO difference between two situations. As in the ROLP case, we hence have a powerful machinery to determine acceptable RO measures, namely functions that respect this order relation.

Next, we will fix the notation. Consider the following citation matrix: $\mathrm{C}=\left(\mathrm{c}_{\mathrm{LJ}}\right)_{\mathrm{LJ}}$, where $C_{L J}$ denotes the number of citations given by language $L$ to publications written in language J . Assume further that we consider n different languages: $\mathrm{J}_{1}, \mathrm{~J}_{2}, \ldots, \mathrm{~J}_{n}$ (where
language $L$ is one of the $J s)$. The self-citing rate of language $L$ is then defined as: $c(L)=$ $\mathrm{CLL}_{\mathrm{LI}} / \mathrm{L}_{\mathrm{c}}$, where

$$
\begin{equation*}
L_{c}=\sum_{k=1}^{n} c_{L J_{k}} \tag{1}
\end{equation*}
$$

If the number of publications in language $L$ is denoted as $P(L)$ (in the population under study), and $P$ denotes the total number of publications in the system, then the publication share of language $L$ is given as:

$$
\begin{equation*}
\alpha_{L}=\frac{P(L)}{P} \tag{2}
\end{equation*}
$$

If the target language $L$ is fixed we will denote $\alpha_{L}$, simply by $\alpha$. Similarly, the self-citing rate of language $L$, i.e. $c(L)$, will be denoted by $c$.

Further, the relative citation rate of language $L$ with respect to language $J$, denoted as $\gamma_{\mathrm{LJ}}$, is defined as:

$$
\begin{equation*}
\gamma_{\mathrm{LJ}}=c_{\mathrm{LJ}} / \mathrm{L}_{\mathrm{c}} \tag{3}
\end{equation*}
$$

If it is clear which languages are meant $\gamma_{L J}$ is simply denoted as $\gamma$.

## 2. A partial order for the relative own-language preference (ROLP)

The relative own-language preference depends on two parameters: $\alpha$ (the publication share of the language) and $c$ (the self-citing rate). With a given pair $(\alpha, c),(\alpha, c) \in] 0,1[\times$ $[0,1] \cup\{(1,1)\}$, we associate the polygonal line linking $(0,0),(\alpha, 0),(\alpha, c),(1, c)$ and $(1,1)$. This curve is called a ROLP-curve.

If two points yield the same curve they are considered to be equivalent. For points with different coordinates this only happens for points of the form $(\alpha, 0), 0<\alpha<1$, which are all equivalent, and are equivalent with $(1,1)$. This means that, taking equivalences into account:

$$
\begin{gathered}
\left(\alpha_{1}, c_{1}\right)=\left(\alpha_{2}, c_{2}\right) \\
\text { if and only if } \\
\left(\alpha_{1}=\alpha_{2} \text { and } c_{1}=c_{2}\right) \text { or } c_{1}=c_{2}=0 \text { or } \\
\left(c_{1}=0 \text { and }\left(\alpha_{2}, c_{2}\right)=(1,1)\right) \text { or }\left(c_{2}=0 \text { and }\left(\alpha_{1}, c_{1}\right)=(1,1)\right)
\end{gathered}
$$

In the set of all ROLP-curves, denoted as $\left\{\mathrm{R}_{\mathrm{i}}\right\}$, we define a partial order, $-<$, by the requirement

$$
R_{1}-<R_{2} \text { if } R_{2} \text { is at no point situated under } R_{1} \text {. }
$$

This ROLP-partial order is transferred to the set of all equivalence classes of ( $\alpha, \mathrm{c}$ )-pairs.
Fig. 1 shows the ROLP-curves corresponding to ( $\alpha_{L}, c_{L}$ ), ( $\alpha_{M}, c_{M}$ ) and ( $\alpha_{N}, c_{N}$ ) where $\left(\alpha_{L}, c_{L}\right)-<\left(\alpha_{M}, c_{M}\right),\left(\alpha_{L}, c_{L}\right)-<\left(\alpha_{N}, c_{N}\right)$, while $\left(\alpha_{M}, c_{M}\right)$ and $\left(\alpha_{N}, c_{N}\right)$ are not comparable. The smallest curve for this partial order is the polygonal line connecting ( 0,0 ), ( 1,0 ) and $(1,1)$. As $\alpha \neq 0$, there is no largest one, but it is clear that curves corresponding to ( $\alpha, 1$ )pairs can become larger than any other given curve (let $\alpha$ tend to zero). We note that in this partial order all curves corresponding to ( $\alpha, c$ ) with $\alpha=c$ (but different from 1) are incomparable.

Insert Fig. 1 about here

This ROLP-partial order can also be expressed without reference to ROLP-curves. Then

$$
\begin{gather*}
\left(\alpha_{1}, c_{1}\right)-<\left(\alpha_{2}, c_{2}\right) \text { if and only if } \\
\left(\alpha_{2} \leq \alpha_{1} \text { and } c_{1} \leq c_{2}\right) \text { or } c_{1}=0 \text { or }\left(\alpha_{1}, c_{1}\right)=(1,1) \tag{5}
\end{gather*}
$$

A ROLP-function $f$ is defined as any real-valued continuous function that maps ( $\alpha, c$ )pairs to non-negative numbers and respects the ROLP partial order. This means: if ( $\alpha_{1}, \mathcal{C}_{1}$ ) corresponds to $R_{1}$ and $\left(\alpha_{2}, c_{2}\right)$ corresponds to $R_{2}$, with $R_{1}-<R_{2}$, (strictly) then

$$
\begin{equation*}
f\left(\alpha_{1}, c_{1}\right)<f\left(\alpha_{2}, c_{2}\right) \tag{6}
\end{equation*}
$$

Properties of ROLP-functions
Based on Fig. 1 we note the following properties.
$1^{\circ}$ ) For fixed $0<c<1$ a ROLP-function is decreasing in $\alpha$.
$2^{\circ}$ ) For fixed $0<\alpha<1$ a ROLP-function is increasing in c .
$3^{\circ}$ ) A ROLP-function attains its smallest value for all situations of the form ( $\alpha, 0$ ), $0<\alpha<1$. The same value is attained for ( 1,1 ).

These properties, derived from the properties of the corresponding partial order for ROLP-curves, correspond exactly to the basic requirements introduced in (Egghe et al., 1999).

An obvious ROLP-function is the area under the ROLP-curve given as (1- $\alpha$ )c. This is the third function studied in (Egghe et al., 1999). Of course, as is proposed in that article, one usually wants to add sensitivity requirements. This leads to functions such

$$
\begin{equation*}
c \ln \left(\frac{1}{\alpha}\right) \text { or } c^{2} \ln \left(\frac{1}{\alpha}\right) \tag{7}
\end{equation*}
$$

which are, clearly, also ROLP-functions. Normalization (leading to functions that attain values between zero and one) yields:

$$
\begin{equation*}
\frac{2}{\pi} \arctan \left(c \ln \left(\frac{1}{\alpha}\right)\right) \text { or } \frac{2}{\pi} \arctan \left(c^{2} \ln \left(\frac{1}{\alpha}\right)\right) \tag{8}
\end{equation*}
$$

## 3. A partial order for relative openness

Now we fix a language $L$ and want to study its openness for one specific language or group of languages, denoted as J . The following 3-vector will determine the openness of one language with respect to another one (this could also be termed the relative specific language preference rate) : $\left(\alpha_{L}, \alpha_{J}, \gamma\right), 0<\alpha_{L}<1,0<\alpha_{J}<1,0 \leq \gamma \leq 1$ or $\left(\alpha_{L,} \alpha_{J}, \gamma\right)=(1,0,0)$. Here $\alpha_{L}$ denotes the publication share of language $L, \alpha_{J}$ denotes the publication share of language J , and $\gamma$ denotes the citation share of language J among all citations given by language L (cf. (3)). Note also that always $0 \leq \alpha_{L}+\alpha_{J} \leq 1$. As we have one parameter more than in the case of the relative own-language preference rate it is natural to work with solids in three-dimensional space (instead of curves in a twodimensional plane). By the term 'solid' we mean any subset of three-dimensional space that can be written as the union (possibly an infinite union) of non-degenerate (or proper) blocks. A block, spanned by the vectors $\mathrm{v}_{1}=\left(\mathrm{x}_{1}, \mathrm{y}_{1}, \mathrm{z}_{1}\right), \mathrm{v}_{2}=\left(\mathrm{x}_{2}, \mathrm{y}_{2}, \mathrm{z}_{2}\right), \mathrm{v}_{3}=$ $\left(x_{3}, y_{3}, z_{3}\right)$, in the three-dimensional space is the set of points

$$
\begin{equation*}
t_{1} v_{1}+t_{2} v_{2}+t_{3} v_{3} \quad \text { with } 0 \leq t_{i} \leq 1 \tag{9}
\end{equation*}
$$

A block is degenerate if the vectors $\mathrm{v}_{1}, \mathrm{v}_{2}, \mathrm{v}_{3}$ are linearly dependent (leading to a 'block' that lies in a plane), otherwise it is non-degenerate (Lang, 1968).

Two points ( $\left.\alpha_{L 1}, \alpha_{J 1}, \gamma_{1}\right)$ and $\left(\alpha_{L 2}, \alpha_{J 2}, \gamma_{2}\right)$ are said to be equivalent if they either coincide, i.e. $\left(\alpha_{L 1}, \alpha_{J 1}, \gamma_{1}\right)=\left(\alpha_{L 2}, \alpha_{J 2}, \gamma_{2}\right)$, or if $\gamma_{1}=\gamma_{2}=0$.

With a given 3 -vector $\left(\alpha_{L}, \alpha_{J}, \gamma\right), \gamma \neq 0$, we associate the block spanned by the vector $\left(\alpha_{L}, 1-\alpha_{J}, \gamma\right)$, cf. Fig.2. These blocks are called openness solids. With a 3-vector of the form ( $\alpha_{\mathrm{L}}, \alpha_{\mathrm{J}}, 0$ ), we associate the degenerate block spanned by the vectors $(0,0,0),(0,0,0),(0,0,0)$. For completeness sake this degenerate block is added to the set of openness solids.

## Insert Fig. 2 about here

In the set of all openness solids, denoted as $\left\{S_{i}\right\}$, we define a partial order by the requirement

$$
S_{1}-\ll S_{2} \text { if } S_{2} \text { is at no point situated under } S_{1} \text {. }
$$

This openness partial order is transferred to the set of all equivalence classes of 3vectors ( $\alpha_{\mathrm{L},}, \alpha_{\mathrm{J}}, \gamma$ ). The smallest solid is the degenerate one. There is no largest one, but it is clear that solids corresponding to 3 -vectors ( $\alpha_{\mathrm{L}}, \alpha_{\mathrm{J}}, 1$ ) can become larger than any other given one (let $\alpha_{L}$ tend to 1 , while $\alpha_{j}$ tends to zero).

Without reference to solids this partial order ( $-\ll$ ) can be defined as follows:

$$
\begin{gather*}
\left(\alpha_{L 1}, \alpha_{J 1}, \gamma_{1}\right)-\ll\left(\alpha_{L 2}, \alpha_{J 2}, \gamma_{2}\right) \text { if and only if } \\
\left(\alpha_{L 1} \leq \alpha_{L 2} \text { and } \alpha_{J 2} \leq \alpha_{J 1} \text { and } \gamma_{1} \leq \gamma_{2}\right) \text { or }\left(\gamma_{1}=\gamma_{2}=0\right) \text { or } \alpha_{L 1}=1 \tag{10}
\end{gather*}
$$

A relative openness function g is defined as any continuous real-valued function that maps 3-vectors ( $\alpha_{L}, \alpha_{J}, \gamma$ ) to non-negative numbers and respects the openness partial order. This means: if ( $\alpha_{L 1}, \alpha_{J 1}, \gamma_{1}$ ) corresponds to $S_{1}$ and ( $\alpha_{L 2}, \alpha_{J 2}, \gamma_{2}$ ) corresponds to $S_{2}$, with $\mathrm{S}_{1} \ll \mathrm{~S}_{2}$ (strictly) then

$$
\begin{equation*}
g\left(\alpha_{L 1}, \alpha_{J_{1}}, \gamma_{1}\right)<g\left(\alpha_{L 2}, \alpha_{J_{2}}, \gamma_{2}\right) \tag{11}
\end{equation*}
$$

Properties of relative openness functions
Based on Fig. 2 we note the following properties
$1^{\circ}$ ) For fixed $0 \leq \gamma<1$, and $0<\alpha_{J}<1$ a relative openness function is increasing in $\alpha_{L}$.
$2^{\circ}$ ) For fixed $0 \leq \gamma<1$, and $0<\alpha_{L}<1$ a relative openness function is decreasing in $\alpha_{J}$.
$3^{\circ}$ ) For fixed $0<\alpha_{J}<1$, and $0<\alpha_{L}<1$ a relative openness function is increasing in $\gamma$.
$4^{\circ}$ ) A relative openness function attains its smallest value for all situations of the form

$$
\left(\alpha_{L}, \alpha_{J}, 0\right), 0<\alpha_{J}<1 \text {, and } 0<\alpha_{L}<1 \text {, and for }(1,0,0)
$$

These properties are the ones one would require for a bona fide openness function.

An obvious openness function is the volume of the openness solid given as

$$
\begin{equation*}
\gamma \alpha_{L}\left(1-\alpha_{J}\right) \tag{12}
\end{equation*}
$$

We note, however, that all solids with $\alpha_{L}=\alpha_{J}$ and $\gamma$ fixed are incomparable for $-\ll$.

## 4. Another approach to relative openness of a language with respect to another one

Although three parameters are involved in the determination of relative openness, it would be attractive to find a partial order that satisfies the following list of requirements at the same time:
$1^{\circ}$ ) it can be represented in two dimensions
$2^{\circ}$ ) it leads to the same properties for a relative openness function
$3^{\circ}$ ) when used to find the preference (or openness) of a language towards itself, it leads to the ROLP-partial order of Section 2, or at least one with the same (or similar) properties.

A possible approach is the following. Put the $\alpha_{L}$ as well as the $\gamma$ value on the vertical axis, and $\alpha_{J}$ on the horizontal one. Then connect the following points, as shown in Fig. $3:$

$$
(0,0)-\left(\alpha_{J}, \min \left(\alpha_{L}, \gamma\right)\right)-\left(\alpha_{J}, \max \left(\alpha_{L}, \gamma\right)\right)-(1,1)
$$

Insert Fig. 3 about here

Such a curve will be called a 2-RO-curve (two-dimensional relative openness curve). In the set of all 2-RO-curves denoted as $\left\{\mathrm{E}_{\mathrm{i}}\right\}$, we define a partial order by the requirement

$$
\mathrm{E}_{1} \angle \mathrm{E}_{2} \text { if } \mathrm{E}_{2} \text { is at no point situated under } \mathrm{E}_{1} \text {. }
$$

Without reference to these curves, we can define the partial order $\angle$ as follows:

$$
\begin{gather*}
\left(\alpha_{L 1}, \alpha_{J 1}, \gamma_{1}\right)<\left(\alpha_{L 2}, \alpha_{J 2}, \gamma_{2}\right) \text { if and only if } \\
\left(\alpha_{L 1} \leq \alpha_{L 2} \text { and } \alpha_{J 2} \leq \alpha_{J 1} \text { and } \gamma_{1} \leq \gamma_{2}\right) \text { or }\left(\alpha_{L 1}=\alpha_{J 1}=\gamma_{1} \text { and } \alpha_{L 2}=\alpha_{J 2}=\gamma_{2}\right) \tag{13}
\end{gather*}
$$

Equality in this partial order occurs if and only if

$$
\begin{equation*}
\left(\alpha_{\mathrm{L} 2}=\alpha_{\mathrm{L} 1} \text { and } \alpha_{\mathrm{J} 1}=\alpha_{\mathrm{J} 2} \text { and } \gamma_{1}=\gamma_{2}\right) \text { or }\left(\alpha_{\mathrm{L} 1}=\alpha_{\mathrm{J1}}=\gamma_{1} \text { and } \alpha_{\mathrm{L} 2}=\alpha_{\mathrm{J} 2}=\gamma_{2}\right) \tag{14}
\end{equation*}
$$

This 2-RO-partial order is transferred to the set of all equivalence classes of 3-vectors $\left(\alpha_{L}, \alpha_{J}, \gamma\right)$. There is no smallest curve but the curves corresponding to 3 -vectors $\left(\alpha_{L}, \alpha_{J}, 0\right)$ can become smaller than any other given one (let $\alpha_{L}$ tend to 0 , while $\alpha_{J}$ tends to one). Similarly, there is no largest one, but again, curves corresponding to 3-vectors $\left(\alpha_{L}, \alpha_{J}, 1\right)$ can become larger than any other given one (let $\alpha_{L}$ tend to 1 , while $\alpha_{J}$ tends to zero). Note, however, that we have to exclude the case $(1,0,0)$ as this would yield the largest curve, which is against intuition.

A 2-RO-function $h$ is defined as any continuous real-valued function that maps 3 vectors $\left(\alpha_{\mathrm{L}}, \alpha_{\mathrm{J}}, \gamma\right)$ to non-negative real numbers and respects the $2-\mathrm{RO}$ partial order. This means: if ( $\alpha_{L 1}, \alpha_{J 1}, \gamma_{1}$ ) corresponds to $E_{1}$ and ( $\alpha_{L 2}, \alpha_{J 2}, \gamma_{2}$ ) corresponds to $E_{2}$, with $E_{1}$ $\angle \mathrm{E}_{2}$, (strictly) then

$$
\begin{equation*}
h\left(\alpha_{L 1}, \alpha_{J 1}, \gamma_{1}\right)<h\left(\alpha_{L 2}, \alpha_{J 2}, \gamma_{2}\right) \tag{15}
\end{equation*}
$$

Properties of 2-RO-functions
Based on Fig. 3 we note the following properties
$1^{\circ}$ ) For fixed $0 \leq \gamma<1$, and $0<\alpha_{J}<1$ an 2-RO-function is increasing in $\alpha_{L}$.
$2^{\circ}$ ) For fixed $0 \leq \gamma<1$, and $0<\alpha_{L}<1$ an 2-RO-function is decreasing in $\alpha_{J}$.
$3^{\circ}$ ) For fixed $0<\alpha_{J}<1$, and $0<\alpha_{L}<1$ an 2-RO-function is increasing in $\gamma$.

An obvious 2-RO-function is the area under the 2-RO-curve given as

$$
\begin{equation*}
0.5\left(\alpha_{J} \cdot \min \left(\alpha_{L}, \gamma\right)+\left(1-\alpha_{J}\right)\left(1+\max \left(\alpha_{L,} \gamma\right)\right)\right) \tag{16}
\end{equation*}
$$

What happens if $\alpha_{L}=\alpha_{J}$ ? This is illustrated in Fig.4. The partial order derived from these curves is clearly not the same as the ROLP partial order studied in section 2. Yet, it has a lot of good properties.

We note though that restricting $\angle$ to the subset with $\alpha_{L}=\alpha_{\nu}$, yields that all curves with $\alpha_{L}=\alpha_{J}=c$ are the same. So they all have the same value for a function respecting this restricted poset. This leads to a problem as the value in $(1,1)$ must be at a minimum. Thus a function respecting the restriction of $\angle$ to the diagonal set can never be continuous at the point $(1,1)$ and, moreover, attain the lowest value there.

This observation together with the fact that we had to exclude the case $(1,0,0)$ leads us to the following problem. Is it possible to find an acceptable poset for the relative openness problem (in 2 or 3 dimensions), which restricted to the diagonal set yields an acceptable poset of the ROLP problem? In the next section we will show that this is not possible.

## 5. The restriction of a relative openness partial order, to the set $\alpha_{L}=\alpha_{J}$ is never a ROLP-partial order.

We first define in general terms what we mean by the terms RO (relative openness) and ROLP partial orders. A general RO partial order, here denoted as $\check{\varepsilon}$, is a partial order defined on the set of all $\left(\alpha_{L}, \alpha_{J}, \gamma\right), 0<\alpha_{L}<1,0<\alpha_{J}<1,0<\gamma<1$, such that

$$
\begin{gather*}
\left(\alpha_{L 1}, \alpha_{J 1}, \gamma_{1}\right) \sqsubset\left(\alpha_{L 2}, \alpha_{J 2}, \gamma_{2}\right) \text { if and only if } \\
\left(\alpha_{L 1} \leq \alpha_{L 2} \text { and } \alpha_{J 2} \leq \alpha_{J 1} \text { and } \gamma_{1} \leq \gamma_{2}\right) \tag{17}
\end{gather*}
$$

This means that other triples are either incomparable or that the opposite relation holds. Similarly, we define a general ROLP partial order as a partial order defined on the set $(\alpha, c), 0<\alpha<1,0<c<1$, here denoted as $\subseteq$, such that

$$
\begin{gather*}
\left(\alpha_{1}, c_{1}\right) \subseteq\left(\alpha_{2}, c_{2}\right) \text { if and only if } \\
\left(\alpha_{2} \leq \alpha_{1} \text { and } c_{1} \leq c_{2}\right) \tag{18}
\end{gather*}
$$

Other pairs are either incomparable or are related according to the opposite relation. Particular RO and ROLP posets are usually defined on a larger set. Yet, they will only differ in the way they treat special cases, i.e. parameter values equal to zero or one. We will next show that restricting a RO poset to the diagonal set ( $\alpha_{L}, \alpha_{L}, \gamma$ ) and identifying ( $\alpha_{L}, \alpha_{L}$ ) with $\alpha_{L}, 0<\alpha_{L}<1$, in which case $\gamma(0<\gamma<1)$ becomes $c$, never yields a ROLP poset. Indeed, considering the restricted set $\left(\alpha_{L}, \alpha_{L}, c\right), 0<\alpha_{L}<1,0<c<1$, we have:

$$
\begin{gathered}
\left(\alpha_{L 1}, \alpha_{L 1}, \gamma_{1}\right) \sqsubset\left(\alpha_{L 2}, \alpha_{L 2}, \gamma_{2}\right) \text { if and only if } \\
\left(\alpha_{L 1} \leq \alpha_{L 2} \text { and } \alpha_{L 2} \leq \alpha_{L 1} \text { and } \gamma_{1} \leq \gamma_{2}\right)
\end{gathered}
$$

$$
\begin{equation*}
\text { hence: } \alpha_{L 1}=\alpha_{L 2} \text { and } \gamma_{1} \leq \gamma_{2} \tag{19}
\end{equation*}
$$

This implies that the restriction of $\check{c}$ to the diagonal set is (after identification, and writing $\gamma$ as $c$ ) only defined for pairs $\left[\left(\alpha_{1}, c_{1}\right),\left(\alpha_{2}, c_{2}\right)\right]$ with $\alpha_{2}=\alpha_{1}$ and $c_{1} \leq c_{2}$. This means that, for the restricted relation, all pairs of the form $\left[\left(\alpha_{1}, c_{1}\right),\left(\alpha_{2}, c_{2}\right)\right]$ with $\alpha_{2}<\alpha_{1}$ or $\alpha_{2}>\alpha_{1}$ are incomparable. This is not allowed in a bona fide ROLP poset.

## 6. Sensitivity aspects

As we have shown that the restriction of a RO poset to the diagonal can never be a ROLP poset, the third requirement mentioned at the beginning of Section 4 can be removed. Moreover, we were not able to find a two-dimensional representation that yields the cases $\left(\alpha_{L}, \alpha_{\lrcorner}, 0\right)$ and ( $1,0,0$ ) as smallest ones. It seems that this can only be attained using solids in three dimensions. It is, however, important to find more sensitive measures than the volume of solids.

A good candidate is:

$$
\begin{equation*}
R O_{1}\left(\alpha_{L}, \alpha_{J}, \gamma\right)=\sqrt{\alpha_{L}} \gamma \ln \left(\frac{1}{\alpha_{J}}\right) \tag{20}
\end{equation*}
$$

Another acceptable candidate is:

$$
\begin{equation*}
R O_{2}\left(\alpha_{L}, \alpha_{J}, \gamma\right)=\ln \left(1+\alpha_{L}\right) \gamma \ln \left(\frac{1}{\alpha_{J}}\right) \tag{21}
\end{equation*}
$$

These functions are increasing in $\alpha_{L}$ and $\gamma$ and decreasing in $\alpha_{J}$. Moreover, they are zero for $\gamma=0$ (putting $0 \cdot \ln (0)=0$, as is also done in the definition of the Theil or the entropy index (Egghe \& Rousseau, 1990)). Finally,

$$
\begin{equation*}
\left|\frac{\partial R O_{1}}{\partial \alpha_{J}}\right|=\frac{\sqrt{\alpha_{L}} \gamma}{\alpha_{J}} \tag{22}
\end{equation*}
$$

and

$$
\begin{equation*}
\left|\frac{\partial R O_{1}}{\partial \alpha_{L}}\right|=\frac{\gamma \ln \left(\frac{1}{\alpha_{J}}\right)}{2 \sqrt{\alpha_{L}}} \tag{23}
\end{equation*}
$$

showing that $\mathrm{RO}_{1}$ is such that the larger $\alpha_{\mathrm{L}}$ or $\alpha_{J}$ the less differences become important. Similarly,

$$
\begin{equation*}
\left|\frac{\partial R O_{2}}{\partial \alpha_{L}}\right|=\frac{\gamma \ln \left(\frac{1}{\alpha_{J}}\right)}{1+\alpha_{L}} \tag{24}
\end{equation*}
$$

and

$$
\begin{equation*}
\left|\frac{\partial R O_{2}}{\partial \alpha_{J}}\right|=\frac{\gamma \ln \left(1+\alpha_{L}\right)}{\alpha_{J}} \tag{25}
\end{equation*}
$$

We note that, if required, this function can be normalized, yielding values between 0 and 1. This is obtained through an arctan-transformation:

$$
\begin{equation*}
\frac{2}{\pi} \arctan \left(\sqrt{\alpha_{L}} \gamma \ln \left(\frac{1}{\alpha_{J}}\right)\right) \tag{26}
\end{equation*}
$$

or

$$
\begin{equation*}
\frac{2}{\pi} \arctan \left(\ln \left(1+\alpha_{L}\right) \gamma \ln \left(\frac{1}{\alpha_{J}}\right)\right) \tag{27}
\end{equation*}
$$

## 7. Conclusion

We have shown that similar to the notions of concentration, diversity and evenness, the relative own-language preference and the openness of one language with respect to another one, can be represented by a partial order. The partial order for the relative own-language preference can be represented by a polygonal line, somewhat in the same spirit as the Lorenz curve for concentration and evenness (Lorenz, 1905; Nijssen et al., 1998; Rousseau, 1998; Taillie, 1979) and the 'intrinsic diversity profiles' or 'kdominance curves' for diversity (Patil \& Taillie, 1979; Lambshead et al., 1983; Rousseau et al., 1999). Openness, on the other hand, is best represented by three-dimensional solids. Any function used to measure relative own language preference or openness of one language with respect to another one should at least respect the corresponding partial orders. This is a minimum requirement for such measures. Depending on the use one wants to make of these measures other requirements become necessary. A logarithmic dependence on the language share(s) seems a natural additional requirement. Thus would correspond with the logarithmic behavior of psychophysical sensations (Roberts, 1979). We have shown that such functions do exist. It is further shown that openness partial orders can not be used to express the preference (openness?) of a language with respect to itself.

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Fig. 1 ROLP-curves representing $\left(\alpha_{L}, c_{L}\right),\left(\alpha_{M}, c_{M}\right)$ and $\left(\alpha_{N}, c_{N}\right)$ where $\left(\alpha_{L}, c_{L}\right)-<\left(\alpha_{M}, c_{M}\right),\left(\alpha_{L}, c_{L}\right)-<\left(\alpha_{N}, c_{N}\right)$, while $\left(\alpha_{M}, c_{M}\right)$ and $\left(\alpha_{N}, c_{N}\right)$ are not comparable


Fig. 2 Openness solid representing a given 3-vector $\left(\alpha_{L}, \alpha_{J}, \gamma\right), \gamma \neq 0$


Fig. 3 Two-dimensional relative openness curve (2-RO-curve),
representing the vector $\left(\alpha_{L}, \alpha_{J}, \gamma\right), \gamma \leq \alpha_{L}$


Fig. 4 Two-dimensional relative openness curve representing the vector $\left(\alpha_{L}, \alpha_{J}, \gamma\right), \quad c=\gamma \leq \alpha_{J}=\alpha_{L}$

