Controlling for cross-sectional and spatial heterogeneity in the hedonic property value model using entropy econometrics

Elsevier Editorial System(tm) for Journal of Urban Economics
Manuscript Draft

Manuscript Number:

Title: Controlling for cross-sectional and spatial heterogeneity in the hedonic property value model using entropy econometrics.

Article Type: Research Paper

Section/Category: Empirical

Keywords: hedonic property value model; generalized maximum entropy; unobserved heterogeneity

Corresponding Author: dr. Marc Tiri, Ph.D.

Corresponding Author's Institution: Kizok

First Author: Marc Tiri, Ph.D.

Order of Authors: Marc Tiri, Ph.D.; Ludo Peeters

Manuscript Region of Origin:

Abstract:

Controlling for cross-sectional and spatial heterogeneity in the hedonic property value model using entropy econometrics

MARC TIRI* AND LUDO PEETERS

*KIZOK, Research Centre for Entrepreneurship and Innovation, Hasselt University, Agoralaan (Building D), BE-3590 Diepenbeek, Belgium, Tel.: ++32-11-26 86 06, Fax: ++32-11-26 87 00, E-mail: marc.tiri@uhasselt.be

Abstract

Hedonic pricing models that apply data from many different metropolitan areas are subject to considerable *cross-sectional* and *spatial* heterogeneity. When disregarded, this heterogeneity can have important consequences for estimation and inference. In the present paper we develop a flexible entropy-based estimator that allows controlling for several sources of (unobserved) heterogeneity in the hedonic price function. In the empirical application, we examine the performance of the new estimator applying data from the seminal Harrison and Rubinfeld paper on hedonic pricing. Our empirical results show how failure to control for cross-sectional and spatial heterogeneity may result in biased coefficients and a loss of explanatory power. Furthermore, analysis of the individual sensitivities reveals important dynamics regarding the operation of the housing markets.

Key words: hedonic property value model, generalized maximum entropy, unobserved heterogeneity

JEL classification: C21; C31; R21

1 Introduction: cross-sectional and spatial heterogeneity in hedonic property value models

Hedonic property value (HPV) models are widely used to study the urban housing market. Central in the theory underlying the HPV model, is the premise that houses are multi-attribute commodities comprising various bundles of heterogeneous characteristics. HPV models then attempt to obtain implicit prices for these characteristics that can be of *quantitative* or *qualitative* nature. Specific sources of product-differentiating features include among others *structural house characteristics* (*e.g.*, number of rooms, presence of a pool, quality of materials used, year of construction, *etc.*), *local amenities* (*e.g.*, neighborhood attributes such as accessibility to highways, school quality, crime rate, *etc.*) and *environmental characteristics* (*e.g.*, proximity of industrial activity, pollution, noise, *etc.*).

Seemingly homogeneous houses are often characterized by *unmeasured (quality) differences*. In fact, a house can possess some unique, yet possibly unobserved (unmeasured) characteristic or an unusual combination of characteristics that radically determines its price in a bizarre way (*e.g.*, Haurin's [16] 'atypicality'-effect). Since often only 'rudimentary' data are used in empirical HPV models, this effect will occur frequently. For example, simply considering *the number* of rooms does not reveal anything about the *quality* (finishing) of these rooms. Furthermore, in empirical modeling, geographical or environmental influential factors (*e.g.*, noise, pollution, *etc.*) are typically not measured at the individual property lot, but at a regional or municipal level in a central monitoring station. In practice, these factors can change within very short distance. Even when restricting the analysis to a relatively small region, application of these single-point measurements hence will possibly lead to unobserved heterogeneity (due to measurement error). Two otherwise 'identical' houses, moreover, might be valued differently when located in a different neighborhood due to (unmeasured) neighborhood composition effects (Turnbull et al. [33]).

Standard linear regression models have only limited ability to deal with *cross-sectional* and *spatial* heterogeneity in housing prices (Clapp et al. [8]). For this reason, in practical applications, neighborhood effects are often introduced into the regression through varying intercepts, hereby (inaccurately) conceptualizing metropolitan areas as single unified markets. The implicit assumption of stability of the hedonic relation across submarkets, and hence the use of constant marginal prices of housing attributes within a single metropolitan area, however, is to be questioned due to non-constancy of the level of neighborhood (dis)amenities. Consequently, the effects of structural attributes may very well be non-constant yet spatially dependent (*i.e.*, spatial heterogeneity). In this case, stationary coefficient models will produce parameter estimates that are in essence an 'average' value of the parameter over all locations (Bitter et al. [5]). The incorrect application of a too parsimonious specification that neglects existing heterogeneity, however, can have important consequences for estimation and inference. In fact, it may result in biased coefficients and a loss of explanatory power, and in obscuring important dynamics regarding the operation of housing markets (Cameron [6]; Bitter et al. [5]).

Recent research based on micro-level housing data shows that *heteroskedasticity* is prevalent in HPV models. Alleged causes of heteroskedasticity include among others *market-segmentation heteroskedasticity* (Chung [7]), *age-related heteroskedasticity* (Goodman and Thibodeau [13]; [14]) and *external area heteroskedasticity* (Fletcher et al. [9]; Stevenson [31]). It has been established a long time ago (*e.g.*, Sims [30]) that even relatively small amounts of heteroskedasticity can have very large effects on the kurtosis of the empirical frequency distribution of the residuals. In this case, point estimators and hypothesis tests become inefficient (Robinson [27]). In practice, however, the precise form of heteroskedasticity is often unknown (Kiefer et al. [17]) and thus hard to control for.

In view of these difficulties, the aim of this paper is to incorporate *individual-level* heterogeneity directly in the econometric specification and to exploit the (cross-section) information more fully than standard econometric approaches that only present broad, average tendencies. To this end we propose a varying coefficient variation of the classical Generalized Maximum Entropy estimator that allows coefficients to vary randomly from a common sample mean across observations in order to account for unobserved heterogeneity in the sample. In addition, we show that this specification can be easily adapted to incorporate spatial heterogeneity (*group-specific effects*) in the hedonic price function as well.

To demonstrate the performance of the GME-based estimators, we apply data from the seminal Harrison and Rubinfeld [15] paper on hedonic pricing. Being the archetype of the HPV model, abundant research has been performed using this dataset which makes it an ideal benchmark.

The remainder of this paper is organized as follows. In Section 2 we discuss briefly the empirical framework of the HPV model and its consequences for estimation. Specifically, we develop a Varying Coefficients Maximum Entropy based specification to account for *cross-sectional* (GME-VC) as well as *spatial* (GME-VCGE) heterogeneity in the hedonic price function. In Section 3, we present the basic characteristics of the Harrison and Rubinfeld [15] dataset. Next, we present and discuss the empirical results. Finally, in Section 4 we present the main conclusions.

2 Empirical framework and implications for estimation

2.1 Assumption of homogeneity

In the HPV model, the house price (*PRICE*) is typically regressed against a large set of covariates that represent (among others) various *structural house characteristics* (*STRUC), local amenities* (*LOC*) and *environmental characteristics* (*ENVIR*). The basic OLS based price equation hence can be expressed as:

$$PRICE_i = \alpha + \sum_{k=1}^{K} \beta_k STRUC_{i,k} + \sum_{l=1}^{L} \gamma_l LOC_{i,l} + \sum_{m=1}^{M} \varphi_m ENVIR_{i,m} + u_i \qquad (1)$$

In this model specification, the parameters α, β_k, γ_l and φ_m are 'fixed' for all observations implying that each identified attribute has the same intrinsic contribution. Such a model hence disregards potential *cross-sectional* heterogeneity in house prices. Furthermore, it is assumed that the data exhibit no aggregation bias and that the relationship of interest is invariant over all sub-regions (*i.e.*, *'ecological inference'* hypothesis, *e.g.* Freedman et al. [10]; Peeters and Chasco [25]). From the discussion in Section 1, however, it is clear that the implicit assumption of stability of the hedonic relation over all sub-regions is subject to substantial skepticism due to non-constancy of the level of neighborhood (dis)amenities among others. Furthermore, both the uniqueness of each property and potential measurement issues lead to pervasive individual-level price heterogeneity.

2.2 Controlling for cross-sectional heterogeneity (individual-specific heteogeneity)

In order to control for *individual-specific* behavior, it is possible to rewrite the 'fixed' or 'stationary' coefficients model that was specified in Eq. (1) as a 'varying' coefficients model in which the coefficients are allowed to vary across observations. Consequently, the original parameters α, β_k, γ_l and φ_m are replaced by a new set of *individual-specific parameters* α_i, $\beta_{i,k}$, $\gamma_{i,l}$ and $\varphi_{i,m}$. Formally, these coefficients can be written as consisting of a fixed (mean) part and an individual–specific component, such that:

$$PRICE_i = (\alpha' + a_i) + \sum_{k=1}^{K} (\beta_k' + b_{i,k}) STRUC_{i,k} + \sum_{l=1}^{L} (\gamma_l' + c_{i,l}) LOC_{i,l} + \sum_{m=1}^{M} (\varphi_m' + d_{i,m}) ENVIR_{i,m} + u_i \tag{2}$$

While the varying coefficients specification mitigates the (incorrect) 'constancy assumption', it also allows for dealing with (multiplicative) disturbance heteroskedasticity in an appealing way (Peeters [24]).

2.3 Controlling for both cross-sectional and spatial heterogeneity (individual- and group-specific heterogeneity)

The specification developed in Eq. (2) interprets all (unexplained) heterogeneity as being *individual-specific*. Some part of it, however, might in fact be *group-specific* rather than *individual-specific*. That is, linked to (unexplained) neighborhood effects for example (*i.e.*, spatial heterogeneity). In order to eliminate potential common spatial heterogeneity from the individual-specific part, Eq. (2) can be adapted to account not only for *cross-sectional* heterogeneity but also for *spatial* heterogeneity, *i.e.*, *group-specific* effects, in the hedonic price function. To this end, the signal effect can easily be extended once more and being rewritten as consisting of (a) a fixed (mean) coefficient, (b) a *group-specific* coefficient, representing metropolitan or some other common effects, and (c) an *individual–specific* component, representing individual heterogeneity. In accordance with Eq. (2), we can model potential *group-specific* components through A_g, $B_{g,k}$, $C_{g,l}$ and $D_{g,m}$ such that we obtain:

$$PRICE_i = (\alpha'' + A_g + a_i) + \sum_{k=1}^{K} (\beta_k'' + B_{g,k} + b_{i,k}) STRUC_{i,k} + \sum_{l=1}^{L} (\gamma_l'' + C_{g,l} + c_{i,l}) LOC_{i,l} + \sum_{m=1}^{M} (\varphi_m'' + D_{g,m} + d_{i,m}) ENVIR_{i,m} + u_i \tag{3}$$

2.4 Modeling procedure: A Generalized Maximum Entropy based estimator for the HPV model

The models developed in Eq. (2) and (3) are clearly underdetermined, that is, the number of unknown parameters is larger than the number of observations. In this case traditional estimation methods (such as OLS) become inconsistent and run into the incidental parameter problem (see Neyman and Scott [21]). For that reason, we propose the Generalized Maximum Entropy (GME) based estimator in order to avoid the over-parameterized estimation problem. Next to this, GME has many advantages over the classical estimation techniques since it performs well in situations where traditional methods may fail to produce stable and/or efficient estimates. Moreover, GME avoids strong distributional assumptions and is well-suited for small samples, even when the covariates are highly correlated (Golan et al. [12]). A description and full specification of the newly developed GME-based estimators is presented in the Appendix.

3 Empirical application

3.1 Harrison and Rubinfeld data[1]

The performance of the GME-based estimators is examined using data from the well-known Harrison and Rubinfeld [15] (H&R) paper on hedonic pricing that investigates the impact of a fairly large set of house characteristics and location amenities on housing values in the Boston Standard Statistical Metropolitan Area. In particular, their interest is on the impact of air pollution on the price of owner-occupied homes and on the demand for clean air.

The H&R dataset contains a sample of 506 observations. The dependent variable is the median value of the owner-occupied homes, while the list of regressors includes the basic air pollution variable (*NOXSQ*), two structural house characteristics and ten variables relating to local amenities. A summary list of the variables along with some descriptive statistics is presented in Table 1.

The H&R data are published in several econometric textbooks (*e.g.* Wooldridge [34]; Baltagi [3]). Furthermore, over the years, these data have been used in a number of empirical studies to demonstrate various properties and issues of the OLS-based HPV model and to serve as a benchmark model to demonstrate the performance of various semi- and non-parametric estimation techniques.

From this, it appears that the H&R data contain some observations that are unusual or inordinately influential (*i.e.*, outliers) (Belsley et al. [4])[2]. Furthermore, when estimated by OLS, the error terms appear to be non-normal (Belsley et al. [4]; Lange and Ryan [18]) and heteroskedastic since the variance of the error terms is related to the crime rate, the number of rooms and the tax rate (Subramanian and Carson [32]). In addition, analysis of the collinearity diagnostics and calculation of the Condition Index shows that some covariates exhibit fairly high correlations while at the same time only limited variability is found in some of the independent variables. Finally, abundant evidence is given of the presence of group-specific effects in the data (*e.g.*, Moulton [20]; Subramanian and Carson [32]; Lange and Ryan [18]; Baltagi and Chang [2]; Baltagi [3]) implying that the equality of the coefficients over different metropolitan areas can be rejected.

Precisely these properties make that the H&R dataset is an ideal benchmark to demonstrate the flexibility and performance of the GME-based estimators.

<< Insert Table 1: Variable Description>>

[1] The data in the present study are taken from HEDONIC.xls by Baltagi [3].

[2] Furthermore, it has been demonstrated by Gilley and Pace [11] that the H&R data have some incorrectly coded observations. Since most empirical applications apply the *original* H&R data, we will do likewise.

3.2 Estimation results

3.2.1 Basic OLS results

As a benchmark model in our study we replicate the original Harrison and Rubinfeld hedonic property value model. Based on the data that is described in Table 1, this specification uses least squares regression to estimate the following model:

$$\begin{aligned} Log(MV)_i = &\beta_1 + \beta_2 CRIM_i + \beta_3 ZN_i + \beta_4 INDUS_i + \beta_5 CHAS_i + \beta_6 NOXSQ_i + \beta_7 RM_i + \beta_8 AGE_i \\ &+ \beta_9 LDIS_i + \beta_{10} RAD_i + \beta_{11} TAX_i + \beta_{12} PTRATIO_i + \beta_{13} B_i + \beta_{14} STAT_i + \varepsilon_i \end{aligned} \quad (4)$$

To start with, Table 2 exhibits the *ex-ante* expected sign of the parameters to be estimated as argued by Pace [22]. Next, in Column 1, we show the original H&R OLS based estimation results that will serve as a meaningful performance and validity check. In order to facilitate comparison across methods, all relevant inferential statistics (standard errors) are calculated asymptotically.[3] Based on these results, it appears that the OLS model performs reasonably well, with most variables being statistically significant and R^2 being rather high for a cross-sectional dataset.

A comparison of the H&R results with the *ex-ante* expectation of Pace [22] reveals, however, that the OLS results exhibit an apparent sign violation for the (non-significant) variables age (*AGE*) and industry effect (*INDUS*).

<< Insert Table 2: Estimation results for (mean) coefficients >>

3.2.2 A Generalized Maximum Entropy approach to estimate the HPV model[4]

In this section we re-estimate the H&R model by using the Generalized Maximum Entropy (GME) method that was presented in Section 2 and in the Appendix. First, we present the results of the (basic) fixed coefficients approach. Next, we demonstrate the performance of the GME estimator while controlling for several sources of (unobserved) heterogeneity.

3.2.2.1 Assumption of homogeneity: fixed coefficients approach (GME-FC)

First, we apply the Fixed Coefficients Generalized Maximum Entropy (GME-FC) formulation of the general linear model. In the GME-FC estimation, the estimated parameters are 'fixed' for all observations, implying stability of property prices over all metropolitan areas and all cases, *i.e.*, just as in the OLS approach.

In Table 2, Column 2 it is shown that the results obtained from the GME-FC and OLS estimation are nearly alike. In particular, the magnitude of the coefficients and their standard errors differs only

[3] When the true error values and parameters are contained in their support bounds, the GME estimator is consistent and asymptotically normal (Golan et al. [12]; Mittelhammer and Cardell [19]).
[4] All GME models are estimated using the GAMS 22.5 PATHNLP solver.

marginally, the signs of the coefficients are in both instances identical and the variables *ZN*, *INDUS* and *AGE* are non-significant in both approaches.

Similarly as in the OLS approach, the signs of the variables *AGE* and *INDUS* are opposite to the *ex-ante* predicted effect that was argued by Pace [22]. In the following sections, however, we argue that these *hypothesized average* effects may *not* hold for all cases.

Specifically, concerning the *ex-ante* expected effect of *AGE* one can reasonably assume that - in general - *AGE* tends to decrease the condition of a house for reasons of (normal) tear and wear. This then will reflect in the price that will decrease. This effect, however, is only an expected *average* effect. In fact, *AGE* will not automatically exert a negative influence on price in all instances. For example, it is quite likely that (well kept) historical properties will sell for more than younger properties do. In these instances, the effect of *AGE* actually will be positive!

Concerning the *ex-ante* expected effect of *INDUS* we refer to Table 1 which shows that *INDUS* measures 'the proportion of non-retail business acres per town'. This, however, is not informative about the type of industrial activities that are carried out, if and how these industrial activities might affect the quality of life, or how much job opportunities they offer for people living nearby. In our opinion, the latter factors are the true factors that affect the price. As a consequence, in the present application the *ex-ante* expected effect of *INDUS* might rather be *undetermined* instead of negative.

Moreover, for all variables we argue that the use of stationary coefficients obscures important dynamics regarding the operation of the housing market.

3.2.2.2 Controlling for cross-sectional heterogeneity through varying coefficients (GME-VC)

In order to mitigate the (incorrect) 'constancy assumption' we have adapted the fixed-coefficients GME specification to a varying coefficients specification (GME-VC) in order to capture individual (*i.e.*, cross-sectional) heterogeneity in the sample.

As is shown in Table 2, Column 3, both the model performance and the (mean) parameter estimates of the GME-VC model are quite similar to those of the OLS and GME-FC specifications. In the latter models, emphasis is on estimating the (fixed) parameter coefficients (along with their variances). These coefficients, however, provide no information about the individual sensitivities. In the GME-VC model on the other hand, cross-sectional heterogeneity is implicitly accounted for. Moreover, the GME-VC specification allows for *directly estimating* the individual-specific coefficients rather than predicting them from some distribution as in the random-coefficients modelling approach. This provides powerful opportunities for an in-depth analysis of the individual-specific estimates.

For example, in the H&R paper, focus was on estimating the impact of pollution (*NOXSQ*) on the price of a property. Applying GME-VC, in Table 2 it is shown that the estimated (mean) effect is negative and identical to the estimated effect when applying OLS (-0.638). Importantly, using OLS (or GME-FC) the estimated coefficient is equal for all observations, while the GME-VC method provides information about the individual pollution sensitivities. In order to illustrate visually the variability of the estimated individual sensitivities of the pollution variable (*NOXSQ*), we present the kernel density of the estimates in Figure 1, Panel A. As is shown, the *mean* pollution effect is -0.638 (see also Table 2!), but this effect varies between -0.667 and -0.611. On the whole, the negative effect of *NOXSQ* hence appears to be relatively similar over all observations.

<< Insert Figure 1: GME-VC kernel densities of estimated individual sensitivities >>

In contrast, Panel B and C of Figure 1 exhibit that the effect of the (non-significant) variables *INDUS* and *AGE* varies substantially over all observations, with very large deviations from the *mean* effect. For example, while the (mean) *AGE* effect is 0.0137, the maximum effect is found to be 1000% larger! Furthermore, while the estimated *mean* coefficient is positive for both variables, the individual sensitivities appear to be negative in some cases! Specifically, *INDUS* (resp. *AGE*) exerts a positive effect on the price of a property in 91.1% (resp. 80.8%) of the sample, and a negative effect in the rest of the sample! These results hence confirm our remarks in the previous section. Specifically, the effect of *AGE* on the price of a property is difficult to predict *ex ante* due to individual heterogeneity and measurement issues. The same holds true for *INDUS*.

For all variables in Figure 1, it appears that the individual-specific coefficients are relatively symmetrically distributed around their mean value, yet with a relatively large spread (*i.e.*, large variability) around this mean.

Furthermore, these individual sensitivities can be used to analyze possible influential observations or to analyze differences between specific groups of observations. As an example, we refer to an application of influential-data diagnostics by Belsley et al. [4]. Replicating the OLS estimation of the H&R HPV model, they show that the H&R dataset contains some very influential observations and that the distribution of the error terms is significantly non-normal. From their analysis, it appears that the influential observations tend to be quite heavily concentrated in a few neighborhoods, mostly in the central city of Boston. This suggests that the central city of Boston and the suburbs are fundamentally different housing markets (Belsley et al. [4]; Subramanian and Carson [32]).

Now, turning to the results of the GME-VC estimation, it is possible to examine this proposition more carefully by investigating the variability of the obtained individual parameter estimates separately for the central city of Boston (132 cases) and the suburbs (374 cases).

<< Insert Figure 2: GME-VC kernel densities of estimated individual sensitivities (City vs. Suburb) >>

In Figure 2 we present the kernel densities separately for the central city of Boston and the suburbs for the variables *NOXSQ*, *INDUS* and *AGE*. Panel A shows with respect to the parameter results of the variable *NOXSQ* that relatively little difference exists concerning its effect on the price of houses in the central city of Boston and these in the suburbs. Not only is the *mean* impact virtually the same for both groups, also the minimum and maximum sensitivities are quite similar. Despite this, the shape of the distribution of the individual sensitivities is rather different for the central city of Boston and these of the suburbs. In particular, the kernel density of the suburb fraction has a strong leptokurtic distribution with a more acute "peak" around the mean while the kernel density of the city suburb has a platykurtic distribution.

Panel B and C exhibit that the situation for the variables *INDUS* and *AGE* is more complicated. For example with respect to *AGE*, it is shown that the difference between the *mean* values of both the central city and the suburbs is relatively small. For both subgroups, the effect of *AGE* on price turns out to be positive (yet not significant). On the other hand, the differences between the minimum and maximum sensitivities are more pronounced, with the maximum sensitivity in the city sample (0.1587) being almost *double* of the maximum sensitivity in the suburb sample (0.0894). Furthermore, the city fraction has a more platykurtic distribution with many of the individual sensitivities relatively far from the mean value. Finally, it is shown that for the city sample, 65% of the cases have an individual-specific parameter that is positive, while for the suburbs this amounts to 86%.

These results corroborate those of Belsley et al. [4] who found that the (basic) H&R OLS based housing equation is not so well specified due to (substantial) differences in the housing market in Boston city *vs.* the suburbs.

3.2.2.3 Controlling for both cross-sectional and spatial heterogeneity (GME-VCGE model)

While in the previous section the existence of a group effect was studied by analyzing *ex-post* the estimation results for houses from the central city of Boston and those from the suburbs, alternatively, the GME approach allows for incorporating a (systematic) group effect *implicitly* into the specification (see Section 2.3).

3.2.2.3.1 GME-VCGE(1): city *vs.* suburbs

First, assuming that the central city of Boston and the suburbs are fundamentally different housing markets, a group effects coefficient $w_{g,k}$ was added to the specification with $g \in (city, suburb)$. The results from this specification are presented in Table 2, Column 5. From this, it appears that in all instances, the predicted (mean) coefficients are rather similar with respect to sign and magnitude to these of the previous GME-based estimators as well as the OLS results.

As demonstrated before, these *mean* coefficients, however, obscure the heterogeneity that exists in the sample. For that reason, we carry out a more in-depth analysis of the individual-specific coefficients from the GME-VCGE(2) specification by showing the kernel densities for the variables *NOXSQ*, *INDUS* and *AGE* for both the central city of Boston and the suburbs (see Figure 3). In these figures, we also show the estimated *average* effect for the whole sample (see also Table 2, Column 5!) as well as the estimated *average* effects for both subsamples.

<< Insert Figure 3: GME-VCGE kernel densities of estimated individual sensitivities (City vs. Suburb) >>

For the pollution variable *NOXSQ*, the estimated *average* effect is equal to -0.671. Drawing the kernel densities for the central city of Boston and the suburbs, however, clearly shows that this is *only* an average effect which obscures important dynamics in the different housing markets. In fact, the difference between the estimated *mean* effects for the central city of Boston and the suburbs is larger than in Figure 2, with both kernel densities being very differently shaped and exhibiting only minor overlap.

Furthermore, relatively large differences exist between the kernel density distributions of the central city of Boston and the suburbs for the variables *INDUS* and *AGE* as well. Hence, while in the fixed coefficients GME specification, the differences between the calculated *average* effects for the central city of Boston and the suburbs are small (see Figure 2), they are more pronounced in the current specification. Additionally, for both variables the minimum and maximum sensitivities are quite different for the city and the suburbs. This holds true for the shape of the kernel densities as well.

In all, this clearly demonstrates that both neighborhoods represent fundamentally different housing markets, which needs to be accounted for in empirical modeling.

3.2.2.3.2 GME-VCGE(2): 92 towns

Previously, it was taken into account that the dataset contains 132 observations from the central city and 374 from the suburbs. Yet, as a matter of fact, the H&R dataset contains data that are actually drawn from a population with a grouped structure since it relates to 92 different towns in the Boston area.

For 17 of these 92 towns, there is only 1 observation (*i.e.*, 1 census tract) in the dataset. Furthermore, inspection of the basic data reveals that only limited variability exists for several variables. For example, all tracts from the same town have the same variable value for the variables *ZN*, *RAD*, *TAX*, *PTRATIO* and *INDUS*. Atkinson and Crocker [1] show that this may increase the likelihood of unreliable parameter estimates with unexpected signs.

Given the specific nature of the H&R dataset, Moulton [20], Baltagi and Chang [2] and Baltagi [3] use it to examine group-specific heterogeneity under both fixed and random effects assumptions. Applying various unbalanced variance components methods, they find evidence of considerable *inter-metropolitan heterogeneity* such that the equality of the coefficients in the central city of Boston and the suburbs can be rejected for most of the variables in the specification. For illustrative purposes, Panel B, Column 4 presents the original Baltagi [3] results that are based on an unbalanced one-way maximum likelihood model with random group effects (*i.e.*, 92 towns).

This aspect of the data explains why the results of the GME-VCGE(1) are closer to the OLS results than to these from the ML estimation. In fact, while the latter uses a random effects model to account for 92 towns, the GME-VCGE(1) specification controls for a group effect that captures spatial heterogeneity that originates from the central city and the suburbs.

Consequently, we estimate the GME-VCGE(2) model taking into account that the data is drawn from a population with a grouped structure relating to 92 different towns within the greater Boston area (see Section 2.3). In contrast with the previous model, spatial heterogeneity hence is captured at the town-level and not at the city/suburb level.[5]

The results of this specification are presented in Table 2, Column 6. From this, it appears that in general, the results of the GME-VCGE(2) model are *between* those of the OLS and the ML model. This

[5] Obviously, *ex post*, in our approach the kernel densities can be calculated separately for the central city and the suburbs as well.

is most obvious for the variables *CHAS* and *AGE* for which the results differ significantly in the OLS and the ML model (with a sign change from positive to negative). In the GME-VCGE(2) model, the (average) effect of bounding the Charles River (CHAS) remains positive, yet becomes insignificant. The (average) *AGE* effect becomes negative (insignificant).

Since our parameters exist of a mean part, a group-specific (*i.e.*, town) part and an individual-specific part, this provides abundant possibilities for investigating the parameters in more detail. As an example, we focus on the pollution variable *NOXSQ* in Figure 4.

<< Insert Figure 4: NOXSQ GME-VCGE kernel densities of estimated individual sensitivities (92 towns) >>

In Panel A, we show the kernel density that is based on the *full* parameter, that is, for every observation we take the sum of the mean part, the group-specific part and the individual-specific part. Evidently, the *mean* effect of the full parameter (-0.587) is equal to the result that is shown in Table 2, Column 6. In the same panel it is shown that this effect varies between -0.6402 and -0.5470. Next, in Panel B, we focus on the group-specific (town) part as special element of the GME-VCGE(2) specification. Isolating the town effect from the *full NOXSQ* parameter, reveals that this effect ranges between -0.0412 and 0.0258. Furthermore, we use this information to draw separate kernel densities for the city centre and the suburbs. From this, it is shown that while the average effect for both neighborhoods is almost identical, more heterogeneity exists among towns from the city than from the suburbs. Finally, we show that the information from the GME-VCGE(2) specification can be applied to analyze the effect of *NOXSQ* at the level of a specific town as well. For example, in Panel C we compare the effect of pollution on the price for a house within a town from the city centre (Boston Savin Hill) and a house within a town from the suburbs (Cambridge). Despite the relatively small difference in the *mean* effect of *NOXSQ* on housing prices within these towns, it appears that only minor overlap exists between the kernel densities of the individual sensitivities. Hence, both are essentially different.

4 Conclusions

Previous research on hedonic property valuation (HPV) models has demonstrated several pitfalls in using the commonly applied OLS specification. Importantly, standard OLS specifications have only limited ability to deal with *cross-sectional* and *spatial* heterogeneity.

In view of this problem, we proposed a maximum entropy based alternative to the general HPV model. Starting from the Generalized Maximum Entropy (GME) estimator of the general linear model, we developed a Varying Coefficients GME (GME-VC) specification to account for *cross-sectional* heterogeneity in the sample. Furthermore, we extended this model to account for *spatial* heterogeneity in housing prices as well (GME-VCGE specification). Importantly, while the use of fixed coefficients obscures important dynamics regarding the operation of the housing market, application of varying coefficients allows for careful investigation of the variability of the individual parameter estimates.

In order to evaluate the performance of the various GME specifications in the hedonic setting, we applied the GME-FC, GME-VC and GME-VCGE approaches to the well-known Harrison and Rubinfeld [15] (H&R) dataset. The present analysis clearly demonstrates that both the central city of Boston and the suburbs represent fundamentally different housing markets. Even more, it has been demonstrated that the spatial heterogeneity is not so much related to the distinction between the central city of Boston and the suburbs, yet needs to be captured at the town level.

References

[1] S.E. Atkinson, T.D. Crocker, A Bayesian Approach to Assessing the Robustness of Hedonic Property Value Studies, Journal of Applied Econometrics 2 (1987) 27-45.

[2] B.H.Baltagi, Y.J. Chang, Incomplete panels: A comparative study of alternative estimators for the unbalanced one-way error component regression mode, Journal of Econometrics 62 (1994) 67-89.

[3] B.H. Baltagi, Econometric analysis of panel data, New York, Wiley, 2005.

[4] D.A.Belsley, E. Kuh, R.E. Welsch, Regression Diagnostics, New York, Wiley, 1980.

[5] C. Bitter, G.F. Mulligan, S. Dall'erba, Incorporating spatial variation in housing attribute prices: a comparison of geographically weighted regression and the spatial expansion method, Journal of Geographical Systems 9 (2007) 7-27.

[6] T.A. Cameron, Directional heterogeneity in distance profiles in hedonic property value models, Journal of Environmental Economics and Management 51 (2006) 26-45.

[7] K.-S. Chung, Hedonic models of housing prices: A critical appraisal of the assumptions, Dissertation, The University of Texas at Dallas, 1994.

[8] J.M. Clapp, H.-J. Kim, A.E. Gelfand, Predicting Spatial Patterns of House Prices Using LPR and Bayesian Smoothing, Real Estate Economics 30 (2002) 505-532.

[9] M. Fletcher, P. Gallimore, J. Mangan, Heteroscedasticity in hedonic house price models, Journal of Property Research 17 (2000) 93-108.

[10] D.A. Freedman, S.P. Klein, M. Ostland, M.R. Roberts, Review of "A solution to the ecological inference problem", Journal of the American Statistical Association 93 (1998) 1518-1522.

[11] O.W. Gilley, R.K. Pace, On the Harrison and Rubinfeld Data, Journal of Environmental Economics and Management 31 (1996) 403-405.

[12] A. Golan, G. Judge, D. Miller, Maximum Entropy Econometrics: Robust Estimation with Limited Data, New York, John Wiley & Sons, 1996.

[13] A.C. Goodman, T.G. Thibodeau, Age-Related Heteroskedasticity in Hedonic House Price Equations, Journal of Housing Research 6 (1995) 25-42.

[14] A.C. Goodman, T.G. Thibodeau, Dwelling-Age-Related Heteroskedasticity in Hedonic House Price Equations: An Extension, Journal of Housing Research 8 (1997) 299-317.

[15] D. Harrison, D.L. Rubinfeld, Hedonic Housing Prices and the Demand for Clean Air, Journal of Environmental Economics and Management 5 (1978) 81-102.

[16] D. Haurin, The Duration of Marketing Time of Residential Housing, AREUEA Journal 164 (1988) 396-410.

[17] N.M. Kiefer, T.J. Vogelsang, H. Bunzel, Simple Robust Testing of Regression Hypotheses Econometrica 68 (2000) 695-714.

[18] N. Lange, L. Ryan, Assessing Normality in Random Effects Models, The Annals of Statistics 17 (1989) 624-642.

[19] R. Mittelhammer, N.S. Cardell, On the Consistency and Asymptotic Normality of Dataconstrained GME Estimation in the GLM, Mimeo, Washington State University, Pullman, WA, 1997.

[20] B.R. Moulton, Diagnostics for Group Effects in Regression Analysis, Journal of Business & Economic Statistics 5 (1987) 275-282.

[21] J. Neyman, E.L. Scott, Consistent Estimation from Partially Consistent Observations, Econometrica 16 (1948) 1-32.

[22] R.K. Pace, Nonparametric Methods with Applications to Hedonic Models, Journal of Real Estate Finance and Economics 7 (1993) 185-204.

[23] Q. Paris, M.R. Caputo, Sensitivity of the GME Estimates to the Support Bounds, Working Paper No. 01-008 (August), Department of Agricultural and Resource Economics, University of California, Davis, 2001.

[24] L.M.K. Peeters, Estimating a Random-Coefficients Sample-Selection Model Using Generalized Maximum Entropy, Economics Letters 84 (2004) 87-92.

[25] L. Peeters, C; Chasco, Ecological inference and spatial heterogeneity: an entropy-based distributionally weighted regression approach, Papers in Regional Science 85 (2006) 257-276.

[26] F. Pukelsheim, The three sigma rule, American Statistician 48 (1994) 88-91.

[27] P.M. Robinson, Asymptotically Efficient Estimation in the Presence of Heteroskedasticity of Unknown Form, Econometrica 55 (1987) 875-91.

[28] C.E. Shannon, A Mathematical Theory of Communication, Bell System Technical Journal 27 (1948) 379-423.

[29] B.W. Silverman, Density Estimation for Statistics and Data Analysis, Champan & Hall, NY, 1986.

[30] C.A. Sims, Linear Regression with Non-Normal Error Terms: A Comment, The Review of Economics and Statistics 53 (1971) 204-205.

[31] S. Stevenson, New empirical evidence on heteroscedasticity in hedonic housing models, Journal of Housing Economics 13 (2004) 136-153.

[32] S. Subramanian, R.T. Carson, , Robust Regression in the Presence of Heteroskedasticity, in: G.F. Rhodes, T.B. Fomby, (Eds.), Advances in Econometrics, Volume 7, Greenwich, Conn.: JAI Press, 85-138, 1988.

[33] G.K. Turnbull, J. Dombrow, C.F. Sirmans, Big House, Little House: Relative Size and Value, Real Estate Economics 34 (2006) 439-456.

[34] J. M. Wooldridge, Introductory Econometrics: A Modern Approach, Cincinnati, South-Western College Publishing, 2005.

Appendix: A flexible Maximum Entropy based estimator for the HPV model

1. Generalized Maximum Entropy estimation (GME-FC)

Following Golan et al. [12] the Generalized Maximum Entropy (GME) formulation of the general linear model can be represented as follows (matrix notation):

$$y = X\beta + e \tag{A1}$$

with y a $N \times 1$ vector of sample observations on the dependent variable, X a $N \times K$ matrix of explanatory variables, β a $K \times 1$ vector of unknown parameters and e a $N \times 1$ vector of unknown errors. Using the principle of maximum entropy, both the parameter coefficients and the noise component of the model are reparametrized as sets of unknown probabilities over some range of support values. For each parameter β_k we hence define a support vector z of dimension M, with $M \geq 2$. This support vector contains a discrete set of points, representing prior knowledge about the parameters to be estimated. Furthermore, we define a $KM \times 1$ vector p of unknown weights such that $p_{km} > 0$ and $\sum_{m=1}^{M} p_{km} = 1$ for all k. Through this, β_k can be written as shown in (A2). The same approach is applied for reparametrizing the error terms e_i. Leaving the distribution of the error terms otherwise unspecified, using a vector w of unknown weights, we can write:

$$\beta_k = \sum_{m=1}^{M} z_{\beta_{km}} p_{\beta_{km}} \tag{A2}$$

$$e_i = \sum_{j=1}^{J} z_{e_{ij}} p_{e_{ij}} \tag{A3}$$

Applying Shannon's [28] information criterion, the probabilities p_k and w_i can be calculated using the following dual objective function:

$$\underset{p_{\beta_{km}} p_{eij}}{Max} \ H(p_\beta, p_e) = -\sum_{k=1}^{K} \sum_{m=1}^{M} p_{\beta_{km}} \ln(p_{\beta_{km}}) - \sum_{i=1}^{I} \sum_{j=1}^{J} p_{eij} \ln(p_{eij}) \tag{A4}$$

subject to the data-consistency (information-moment) condition for $(i = 1, \ldots, I)$:

$$y_i = \sum_{k=1}^{K} \sum_{m=1}^{M} x_{ik} z_{\beta_{km}} p_{\beta_{km}} + \sum_{j=1}^{J} z_{eij} p_{eij} \tag{A5}$$

and the adding-up (*i.e.*, normalization) constraints:

$$\sum_{m=1}^{M} p_{\beta_{km}} = 1 \ \text{ for } \ k = 1, \ldots, K \tag{A6}$$

$$\sum_{j=1}^{J} p_{eij} = 1 \quad \text{ for } \ i = 1, \ldots, I \ . \tag{A7}$$

In the present GME specification, the parameters β_k are 'fixed' for all observations; hence we can refer to this model as the Fixed Coefficients GME specification (β_k^{GMEFC}).

2. Cross-sectional heterogeneity: a Varying Coefficients GME specification (GME-VC)

Following Peeters [24], we adapt the fixed-coefficients GME specification by adding an individual-specific term to the signal effect. Specifically, the original parameter β_k^{GMEFC} is replaced by a parameter β_k^{GMEVC} consisting of fixed (mean) β_k and an individual–specific component $v_{i,k}$ that represents the individual heterogeneity (*i.e.*, $\beta_k^{GMEVC} = \beta_k + v_{i,k}$). Analogously as in the GME-FC specification, we define a prior vector z_v and a probability vector p_v, both with dimension S, to formalize the *individual–specific* component $v_{i,k}$:

$$v_{i,k} = \sum_{s=1}^{S} z_{v_{iks}} p_{v_{iks}} \tag{A8}$$

Introducing $v_{i,k}$ in Shannon's [28] information criterion, we now obtain the following *constrained optimization problem*:

$$\underset{p_{\beta_{km}} p_{eij} p_{vikm}}{Max} \quad H(p_\beta, p_e, p_v) = -\sum_{k=1}^{K}\sum_{m=1}^{M} p_{\beta_{km}} \ln(p_{\beta_{km}}) - \sum_{i=1}^{I}\sum_{j=1}^{J} p_{eij} \ln(p_{eij}) - \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{s=1}^{S} p_{viks} \ln(p_{viks}) \tag{A9}$$

Subject to the *data-consistency (information-moment) condition for* $i = 1,\dots,I$:

$$y_i = \sum_{k=1}^{K}\sum_{m=1}^{M}\sum_{s=1}^{S} x_{ik}\left(z_{\beta_{km}} p_{\beta_{km}} + z_{viks} p_{viks}\right) + \sum_{j=1}^{J} z_{eij} p_{eij} \tag{A10}$$

With *adding-up constraints*:

$$\sum_{m=1}^{M} p_{\beta_{km}} = 1 \;\; \textit{for} \;\; k = 1,\dots,K \tag{A11}$$

$$\sum_{j=1}^{J} p_{eij} = 1 \;\; \textit{for} \;\; i = 1,\dots,I \tag{A12}$$

$$\sum_{s=1}^{S} v_{iks} = 1 \;\; \text{for} \;\; i = 1,\dots,I \text{ and } k = 1,\dots,K\,. \tag{A13}$$

Finally, we define the *preservation-of-mean constraints*:

$$\sum_{i=1}^{I} v_{ik} = 0 \;\; \textit{for} \;\; k = 1,\dots,K \tag{A14}$$

such that:

$$E\left(\beta_k^{GMEVC}\right) = E\left(\beta_k + \sum_i v_{ik}\right) = E(\beta_k)\,. \tag{A15}$$

3. Spatial heterogeneity: a Varying Coefficients Group Effects GME specification (GME-VCGE)

Being a very flexible and straightforward modeling approach, the GME-VC specification presented above can be adapted to account not only for *cross-sectional heterogeneity* but for *spatial* *heterogeneity* (*i.e.*, group-specific effects) in the hedonic price function as well. To this end, we extend the signal effect and rewrite it as consisting of a fixed (mean) coefficient β_k, a group-specific coefficient $w_{g,k}$ (representing metropolitan effects) and an individual–specific component $v_{i,k}$ (representing the individual heterogeneity). Consequently, the former parameter β_k is extended to:

$$\beta_k^{GMEVCGE} = (\beta_k + w_{g,k} + v_{i,k}) \quad \text{(A16)}$$

Formally, we define a prior vector and a probability vector, both with dimension T, to formalize the group–specific component $w_{g,k}$, such that:

$$w_{g,k} = \sum_{t=1}^{T} z_{w_{gkt}} p_{w_{gkt}} \quad \text{(A17)}$$

In the empirical specification, the new component is also integrated in the constrained optimization problem and the data-consistency (information-moment) condition. Besides the constraints that have been defined before, for the group-specific component $w_{g,k}$ we formulate the following adding-up constraint:

$$\sum_{t=1}^{T} p_{w_{gkt}} = 1 \text{ for } g = 1,\dots,G \text{ and } k = 1,\dots,K \quad \text{(a.15)}$$

and the preservation-of-mean constraints:

$$\sum_{g=1}^{G} w_{gk} = 0 \text{ for } k = 1,\dots,K \quad \text{(A18)}$$

such that:

$$E\left(\beta_k^{GMEVCGE}\right) = E\left(\beta_k + \sum_g w_{gk} + \sum_i v_{ik}\right) = E(\beta_k). \quad \text{(A19)}$$

4. Choice of the support values

In the literature, only limited guidance is offered with respect to the choice of the support bounds for the parameters and the error terms, and the effect of this choice on the parameter estimates (a notable exception is Paris and Caputo [23]). Broadly speaking, two approaches can be distinguished as to set values (boundaries) for the support values. First, priors can reflect the situation that no prior knowledge is available about the sign and size of the parameters to be estimated, *i.e.*, *non-informative priors*. A second, and more realistic approach, uses *informative priors*. In these priors, previous knowledge about the parameters to be estimated is reflected. This knowledge can come from different sources, *e.g.*, previous related work, or from results generated from other techniques on the same dataset.

Applying the latter approach, we set the vector of support values for the (mean) parameter coefficients equal to $z_\beta \in (-100\hat{\beta}_{OLS}, -50\hat{\beta}_{OLS}, 0, +50\hat{\beta}_{OLS}, +100\hat{\beta}_{OLS})'$. This should provide an interval that is wide enough to include the 'true' β.[6] In doing so, we do not put too much subjective knowledge about the parameter values in the model, while at the same time, we do not completely disregard previous knowledge about the parameters to be estimated. For the support vector of the error term (z_e), we apply the 'three-σ rule' (Pukelsheim [26]; Golan et al. [12]), setting $z_e = (-3\sigma, 0, +3\sigma)'$ where σ is the empirical standard deviation of y. For the support values of the varying components, the only comparable study (Peeters [24]) sets these as about 1% of the absolute size of the support vector for the mean coefficients. In our approach, we define the boundaries of the support vector of the individual effect (z_v) and the group effect (z_w) as $z_v \in (-3se(\hat{\beta}_{OLS}), 0, +3se(\hat{\beta}_{OLS}))'$ and $z_w \in (-3se(\hat{\beta}_{OLS}), 0, +3se(\hat{\beta}_{OLS}))'$.

[6] Applying a nonparametric alternative to the H&R data, Pace [22] finds coefficient estimates that are well within these boundaries.

Figure 1

Figure 1: GME-VC kernel densities of estimated individual sensitivities

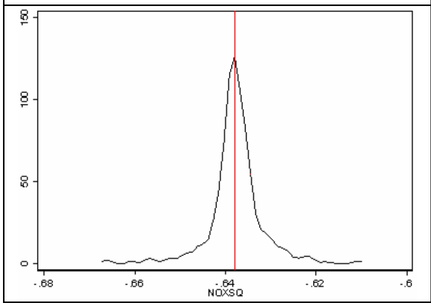
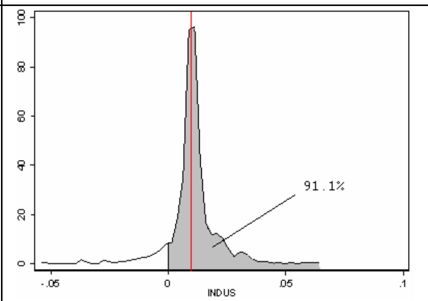
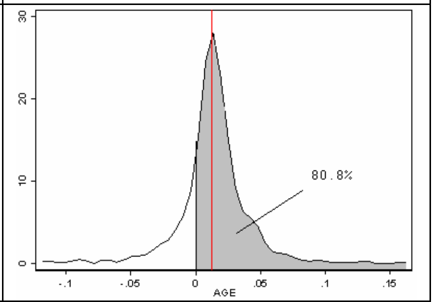
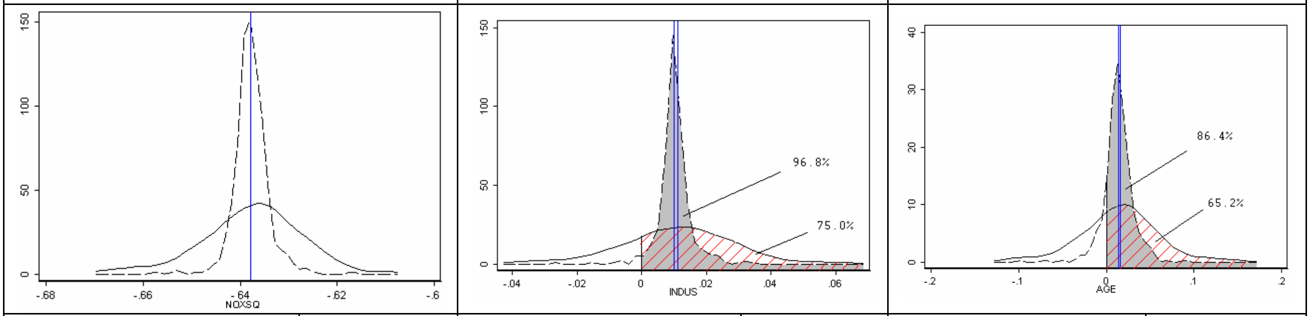
<table>
<tr><th>PANEL A (NOXSQ)</th><th>PANEL B (INDUS)</th><th>PANEL C (AGE)</th></tr>
<tr>
<td>
</td>
<td>
</td>
<td>
</td>
</tr>
<tr>
<td>Mean: -0.638
Min.: -0.667
Max.: -0.611
Std. dev.: 0.00611</td>
<td>Mean: 0.0102
Min.: -0.0532
Max.: 0.0632
Std. dev.: 0.0106</td>
<td>Mean: 0.0137
Min.: -0.115
Max.: 0.159
Std. dev.: 0.0277</td>
</tr>
</table>

Figure 2

Figure 2: GME-VC kernel densities of estimated individual sensitivities (City vs. Suburb)

PANEL A (NOXSQ)			PANEL B (INDUS)			PANEL C (AGE)		

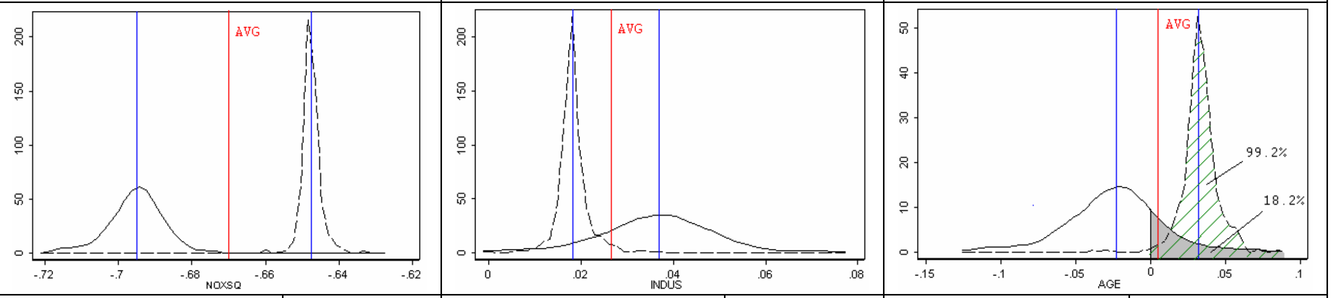


	City (—)	Suburb (---)		City (—)	Suburb (---)		City (—)	Suburb (---)
Mean:	-0.6377	-0.6378	Mean:	0.0110	0.0100	Mean:	0.0148	0.0133
Min.:	-0.6665	-0.6572	Min.:	-0.0371	-0.0532	Min.:	-0.1145	-0.0978
Max.:	-0.6107	-0.6171	Max.:	0.0632	0.0335	Max.:	0.1587	0.0894
Std. dev.:	0.0103	0.0036	Std. dev.:	0.0181	0.0062	Std. dev.:	0.0460	0.0171
Kurtosis:	0.617	6.745	Kurtosis:	0.657	33.366	Kurtosis:	1.045	7.166

Figure 3

Figure 3: GME-VCGE kernel densities of estimated individual sensitivities (City vs. Suburb)

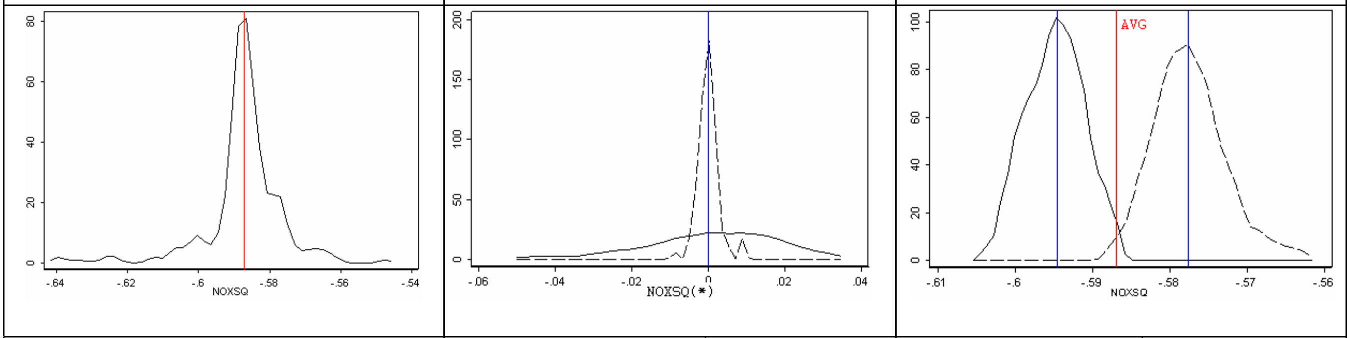
PANEL A (NOXSQ)			PANEL B (INDUS)			PANEL C (AGE)		



	City (—)	Suburb (---)		City (—)	Suburb (---)		City (—)	Suburb (---)
Mean:	-0.6952	-0.6476	Mean:	0.0364	0.0180	Mean:	-0.0241	0.0330
Min.:	-0.7156	-0.6600	Min.:	0.0024	-0.0224	Min.:	-0.1168	-0.0381
Max.:	-0.6761	-0.6330	Max.:	0.0738	0.0365	Max.:	0.0788	0.0845
Std. dev.:	0.0072	0.0025	Std. dev.:	0.0126	0.0042	Std. dev.:	0.0321	0.0116
Kurtosis:	0.7994	6.9520	Kurtosis:	0.8550	27.8285	Kurtosis:	1.2460	6.7561

Figure 4

Figure 4: NOXSQ GME-VCGE kernel densities of estimated individual sensitivities (92 towns)



PANEL A (NOXSQ)		PANEL B (NOXSQ)			PANEL C (NOXSQ)		
			City (—)	Suburb (---)		Boston Savin Hill (—)	Cambridge (---)
Mean:	-0.5874	Mean:	0.0003	-0.00005	Mean:	-0.5948	-0.5775
Min.:	-0.6402	Min.:	-0.0412	-0.0088	Min.:	-0.6016	-0.5851
Max.:	-0.5470	Max.:	0.0258	0.0090	Max.:	-0.5890	-0.5655
Std. dev.:	0.0110	Std. dev.:	0.0170	0.0028	Std. dev.:	0.0035	0.0043
Kurtosis:	6.335	Kurtosis:	1.224	13.697	Kurtosis:	-0.674	0.866

Table 1

Table 1: **Variable Description**

Variable	Definition	Mean	Standard Deviation
Dependent variable			
Log(MV)	Log (Median value of owner-occupied homes)	9.942	0.409
Air pollution variable			
NOXSQ	Annual average nitrogen oxide concentration in parts per hundred million (squared)	32.109	13.921
Structural house characteristics			
RMSQ	Average number of rooms per dwelling (squared)	39.990	9.079
CHAS	Charles River dummy variable (= 1 if tract bounds river; 0 otherwise)	0.070	/
Local amenities (neighborhood characteristics)			
CRIM	Per capita crime rate by town	3.61	8.601
ZN	Proportion of residential land zoned for lots over 25,000 sq.ft	11.360	23.322
INDUS	Proportion of non-retail business acres per town.	11.137	6.860
AGE	Proportion of owner units built prior to 1940	68.575	28.149
TAX	Full value property tax rate ($/$10,000)	408.24	168.537
PTRATIO	Pupil / teacher ratio by town school district	18.455	2.165
B	1000(Bk - 0.63)^2 where Bk is the proportion of blacks by town	0.357	0.091
Log(STAT)	Log(Proportion of population that is lower status)	-2.234	0.601
Log(DIS)	Log(Weighted distances to five employment centers in the Boston area)	1.188	0.539
Log(RAD)	Log(Index of accessibility to radial highways)	1.870	0.875

Table 2

Table 2: Estimation results for (mean) coefficients

		PANEL A			PANEL B		
		Fixed coefficients		Individual-specific varying coefficients		Group- and individual-specific varying coefficients	
	Ex ante sign	(1) OLS Original H&R results	(2) Maximum Entropy GME-FC	(3) Maximum Entropy GME-VC	(4) Maximum Likelihood Original Baltagi results	(5) Maximum Entropy GME-VCGE(1) city *vs.* suburb	(6) Maximum Entropy GME-VCGE(2) 92 towns
CONST		9.756*** (0.150)	9.767*** (0.150)	9.776*** (0.150)	9.676*** (0.207)	9.950*** (0.159)	9.720*** (0.153)
Environmental characteristics (Air pollution variable)							
NOXSQ x 10^{-2}	-	-0.638*** (0.123)	-0.647*** (0.123)	-0.587*** (0.123)	-0.587*** (0.123)	-0.671*** (0.120)	-0.587*** (0.116)
Structural house characteristics							
RMSQ x 10^{-2}	+	0.633*** (0.131)	0.614*** (0.131)	0.664*** (0.131)	0.920*** (0.116)	0.627*** (0.139)	0.835*** (0.134)
CHAS x 10^{-1}	+	0.914*** (0.332)	0.931*** (0.332)	0.908*** (0.332)	-0.120 (0.290)	0.899*** (0.353)	0.306 (0.340)
Local amenities (neighborhood characteristics)							
CRIM x 10^{-2}	-	-1.187*** (0.124)	-1.193*** (0.124)	-1.200*** (0.124)	-0.719*** (0.103)	-1.236*** (0.132)	-0.860*** (0.128)
ZN x 10^{-3}	+	0.0803 (0.506)	0.0706 (0.506)	0.0785 (0.506)	0.029 (0.689)	0.0271 (0.537)	0.107 (0.518)
INDUS x 10^{-2}	(-)	0.0241 (0.236)	0.0130 (0.236)	0.0102 (0.236)	0.00222 (0.440)	0.0272 (0.251)	0.00163 (0.242)
AGE x 10^{-3}	(-)	0.0898 (0.526)	0.103 (0.526)	0.0137 (0.526)	-0.943 (0.461)	0.0045 (0.559)	-0.405 (0.539)
TAX x 10^{-3}	-	-0.420*** (0.123)	-0.422*** (0.123)	-0.424*** (0.123)	-0.374* (0.190)	-0.582*** (0.130)	-0.401*** (0.126)
PTRATIO x 10^{-2}	-	-3.112*** (0.501)	-3.123*** (0.501)	-3.116*** (0.501)	-2.980*** (0.980)	-3.307*** (0.533)	-2.971*** (0.514)
B	+	0.364*** (0.103)	0.353*** (0.103)	0.364*** (0.103)	0.578*** (0.100)	0.364*** (0.109)	0.490*** (0.106)
Log(STAT) x 10^{-1}	-	-3.710*** (0.250)	-3.746*** (0.250)	-3.627*** (0.250)	-2.838*** (0.241)	-3.726*** (0.266)	-3.188*** (0.256)
Log(DIS) x 10^{-1}	-	-1.913*** (0.334)	-1.938*** (0.334)	-1.942*** (0.334)	-1.299*** (0.470)	-1.896*** (0.355)	-1.762*** (0.342)
Log(RAD) x 10^{-1}	+	0.957*** (0.191)	0.968*** (0.191)	0.953*** (0.191)	0.971*** (0.284)	0.740*** (0.203)	0.938*** (0.196)
R^2		0.80586	0.80584	0.80581	n.a.	0.79757	0.79968
MSPE		0.03237	0.03238	0.03238	n.a.	0.03656	0.03398

***,**,* Significant at the 0.01, 0.05, 0.10 level (2-tailed). Standard error between brackets.