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Examples of simple transformations of the h-index: qualitative and quantitative conclusions and consequences for other indices

by

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ABSTRACT

General results on transformations on information production processes (IPPs), involving transformations of the h-index and related indices, are applied in concrete, simple cases: doubling the production per source, doubling the number of sources, doubling the number of sources but halving their production, halving the number of sources but doubling their production (fusion of sources) and, finally, special cases of general power law

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transformations. In each case we calculate concrete transformation formulae for the h-index h (transformed into h^*) and we discuss when we have $h^* < h$, $h^* = h$ or $h^* > h$.

These results are then extended to some other h-type indices such as the g-index, the R-index and the weighted h-index.

I. Introduction

A general information production process (IPP) where sources produce items is characterized by a size-frequency function $f:[a,\rho_m]\to \mathbb{R}^+$ or, equivalently, by a rank-frequency function $g:[0,T]\to \mathbb{R}^+$. Here ρ_m denotes the maximum item density (a is the minimum item density) and T denotes the total number of sources. For each $j\in[a,\rho_m]$, $f(j)$ denotes the density of sources with item density j and for each $r\in[0,T]$, $g(r)$ denotes the item density in the source at rank density r (see Egghe (2005) and many papers in the bibliography of Egghe (2005), e.g. Egghe (2004).

In Egghe (2007a) one studies general transformations of such an IPP: a transformation ψ on the sources:

$$\psi:[0,T]\to\left[0,T^*\right] \tag{1}$$

and a transformation φ on the items:

$$\varphi:[a,\rho_m]\to\left[a^*,\rho_m^*\right] \tag{2}$$

such that φ, ψ are increasing, $\psi(0)=0$, $\psi(T)=T^*$, $\varphi(a)=a^*$ and $\varphi(\rho_m)=\rho_m^*$, acting on g as follows: the transformed rank-frequency function g^* satisfies

$$g^*\left(r^*\right)=g^*\left(\psi(r)\right)=\varphi\left(g(r)\right) \tag{3}$$

In Egghe (2007a) one proves that a Lotkaian size-frequency function

$$f(j) = \frac{C}{j^{\alpha}} \qquad (4)$$

$C > 0$, $\alpha > 1$, $j \in [1, +\infty[$, is transformed into another Lotkaian size-frequency function

$$f^*(j^*) = \frac{G}{j^{*\delta}} \qquad (5)$$

where

$$\delta = 1 + \frac{b(\alpha - 1)}{c} \qquad (6)$$

and $j^* \geq \varphi(a) = Ba^c$, in case φ and ψ are increasing power functions

$$r^* = \psi(r) = Ar^b \qquad (7)$$

$$j^* = \varphi(j) = Bj^c \qquad (8)$$

$A, B, b, c > 0$

The power functions (7) and (8) are natural functions to describe evolution of an IPP: they comprise convex and concave growth of sources and items (if the exponents are > 1 of < 1 respectively). They are also logical to use in connection with the Lotkaian function (4) which is also a power law. Finally, from a pragmatic point of view: only for functions (7) and (8) a simple and exact result as (5), (6) can be proved.

The h-index was defined in Hirsch (2005) in the connection of papers and their citations. In the general IPP context the h-index can be defined as follows (cf. Egghe and Rousseau (2006a)): if we order the sources in decreasing order of their number of items then the h-index of this IPP is the largest rank h such that all the sources on ranks $\leq h$ have at least h items.

The h-index for a general Lotkaian IPP for which (4) is valid, is proved in Egghe and Rousseau (2006a) to be equal to

$$h = T^{\frac{1}{\alpha}} \tag{9}$$

We refer to Ball (2005), Bornmann and Daniel (2005), Braun, Glänzel and Schubert (2005, 2006), Glänzel (2006), Popov (2005), van Raan (2006) and, of course, to the introductory paper Hirsch (2005) for some background on the advantages and disadvantages of the h-index.

Under the above described transformations (7) and (8), we proved in Egghe (2007b) that the transformed h-index h^* equals

$$h^* = B^{\frac{\delta-1}{\delta}} T^{*\frac{1}{\delta}} \tag{10}$$

where B is as in (8) and δ as in (6) (here $T^* = \psi(T)$ is the transformed total number of sources).

It is clear that (10) can be further developed as follows. Since

$$T^* = \psi(T) = AT^b \tag{11}$$

by definition of ψ and by (7), we can put (11) in (10) yielding

$$h^* = B^{\frac{\delta-1}{\delta}} A^{\frac{1}{\delta}} T^{\frac{b}{\delta}} \tag{12}$$

which is the general equation for h^* in Lotkaian systems and where we have power functions (7), (8) as transformations.

It is proved in Egghe (2007a) (but it also follows from (6)) that $\delta = \alpha \Leftrightarrow b = c$. In this case we have

$$h^* = B^{\frac{\alpha-1}{\alpha}} A^{\frac{1}{\alpha}} T^{\frac{b}{\alpha}} \tag{13}$$

$$h^* = B^{\frac{\alpha-1}{\alpha}} A^{\frac{1}{\alpha}} h^{b} \tag{14}$$

by (9). Finally, if $b = c = 1$ we have by (14)

$$h^* = B^{\frac{\alpha-1}{\alpha}} A^{\frac{1}{\alpha}} h \tag{15}$$

In this case we have the simple linear transformations

$$\varphi(j) = Bj \tag{16}$$

$$\psi(r) = Ar \tag{17}$$

but these cases will give us quantitative and qualitative insight in the relation between h^* and h (e.g. when is $h^* > h$, $h^* = h$, $h^* < h$?). This will be done in the next section.

Section 3 studies the general relation (12) if $b = c = 1$ is not valid. We e.g. prove that $b,c > 1$ implies $h^* > h$ and that $0 < b,c < 1$ implies $h^* < h$. Other cases are also studied.

Section 4 then studies transformation properties of indices that are derived from h such as the g-index (Egghe (2006a,b,c)), the h_w-index (weighted h-index) (Egghe and Rousseau (2007)) and the R-index (Jin, Liang, Rousseau and Egghe (2007)) whose definitions will be repeated there.

II. h^* versus h in case b=c=1

Of course, all transformations φ and ψ are continuous but, in each case studied below, we will also indicate what the transformation means on practical tables. Say we start (before the transformation) with the standard Table 1.

Table 1. Starting table

r	#
1	y_1
2	y_2
3	y_3
$\vdots$	$\vdots$

Here sources are ranked $(r = 1,2,3,...)$ decreasingly according to the number $y_1, y_2, y_3,...$ of items they have.

For each transformation studied below we will also give the transformed table, in order to make clear what is happening in practice.

II.1 $\psi(r)= r$ and $\varphi(j)= 2j$

This is the case where sources remain the same but each source doubles its number of items, hence Table 2

Table 2. Transformed Table 1

in case $\psi(r)= r$, $\varphi(j)= 2j$

r^*	$\#^*$
1	$2y_1$
2	$2y_2$
3	$2y_3$
$\vdots$	$\vdots$

It is clear that h^* should be larger than h but here we can give the exact formula. By (15) we have

$$h^* = 2^{\frac{\alpha-1}{\alpha}} h \tag{18}$$

Since $\alpha > 1$ we have $h^* > h$, indeed. But since $\frac{\alpha-1}{\alpha} < 1$ we also have $h^* < 2h$ which is logical: doubling the items should not double the h-index. Conclusion

$$h < h^* < 2h \tag{19}$$

in this case. The reader can generalise this to the more general case $\varphi(j) = Bj$

$$\left(h < h^* = B^{\frac{\alpha-1}{\alpha}} h < Bh \right)$$

II.2 $\psi(r) = 2r$ and $\varphi(j) = j$

This is the case where all the sources are used twice with the same number of items. This is illustrated by Table 3.

Table 3. Transformed Table 1

in case $\psi(r) = 2r$, $\varphi(j) = j$

r^*	$\#^*$
1	y_1
2	y_1
3	y_2
4	y_2
5	y_3
6	y_3
⋮	⋮

Since also the sources with the highest number of items are “copied” we also expect h^* to be larger than h. We indeed have, by (15)

$$h^* = 2^{\frac{1}{\alpha}} h \tag{20}$$

We immediately see that, since $\alpha > 1$

$$h < h^* < 2h \tag{21}$$

as in the case (18) but it is interesting to see that the transformed h-index is different in these cases. We only have that (18) and (20) are the same if and only if

$$\frac{\alpha - 1}{\alpha} = \frac{1}{\alpha}$$

iff

$$\alpha = 2,$$

the famous "turning point" in Lotkaian informetrics (cf. mentioning of other applications where we see this turning point in Egghe (2005)).

For $\alpha < 2$ we have that h^* in this subsection is larger than the one in the previous subsection; for $\alpha > 2$, we have the opposite relation.

For general $\psi(r) = Ar$ it is clear that $h < h^* = A^{\frac{1}{\alpha}} h < Ah$.

II.3 $\psi(r) = 2r$ and $\varphi(j) = 2j$

Here we double the sources and we double the items in each source. This is depicted in Table 4.

Table 4. Transformed Table 1

in case $\psi(r) = 2r$, $\varphi(j) = 2j$

r^*	$\#^*$
1	$2y_1$
2	$2y_1$
3	$2y_2$
4	$2y_2$
5	$2y_3$
6	$2y_3$
$\vdots$	$\vdots$

Intuitively one should expect that the transformed h^* is twice the original value h. This is indeed so: by (15) we see

$$h^* = 2^{\frac{\alpha-1}{\alpha}} 2^{\frac{1}{\alpha}} h = 2h \tag{22}$$

This result can be generalized for $\psi(r) = Ar$, $\varphi(j) = Bj = Aj$ where we then have, by (15): $h^* = Ah$.

An interesting case, suggested by R. Rousseau (2007), is the following

II.4 $\psi(r) = 2r$ and $\varphi(j) = \frac{j}{2}$

This case describes the situation that an author writes more papers (double here) but he/she pays a price for these "shorter" (or less important) papers: these receive less citations (here halved). This is depicted in Table 5

Table 5. Transformed Table 1

in case $\psi(r) = 2r$, $\varphi(j) = \frac{j}{2}$

r^*	$\#^*$
1	$y_1/2$
2	$y_1/2$
3	$y_2/2$
4	$y_2/2$
5	$y_3/2$
6	$y_3/2$
$\vdots$	$\vdots$

From (15) it now follows that

$$h^* = \left(\frac{1}{2}\right)^{\frac{\alpha-1}{\alpha}} 2^{\frac{1}{\alpha}} h$$

$$h^* = 2^{\frac{2}{\alpha}-1} h \tag{23}$$

The question now is: if an author is doing this, will his/her h-index improve or not ? The answer depends on Lotka's α and again $\alpha = 2$ is a turning point: we have

$$h^* > h$$

iff

$$\alpha < 2$$

(and $h^* = h$ iff $\alpha = 2$ and $h^* < h$ iff $\alpha > 2$).

So "publicitis" or breaking down scientific results to their "least publishable unit" only pays (in this model) iff $\alpha < 2$.

Note that (23) is a composition of the transformations in the first subsection (for $\varphi(j) = \frac{j}{2}$) and the ones in the second subsection (for $\psi(r) = 2r$).

The next case is similar to the one above but is more realistic.

II.5 $\psi(r) = 2r$ and $\varphi(j) = \frac{2j}{3}$

This means that we double our articles, that they are less cited then but still keep more than 50% of the citations than before the doubling, which we think is more likely to be the case. This situation is depicted in Table 6

Table 6. Transformed Table 1

in case $\psi(r) = 2r$, $\varphi(j) = \frac{2j}{3}$

r^*	$\#^*$
1	$2y_1/3$
2	$2y_1/3$
3	$2y_2/3$
4	$2y_2/3$
5	$2y_3/3$
6	$2y_3/3$
$\vdots$	$\vdots$

From (15) it now follows that

$$h^* = \left(\frac{2}{3}\right)^{\frac{\alpha-1}{\alpha}} 2^{\frac{1}{\alpha}} h$$

so

$$h^* = \frac{2h}{3^{\frac{\alpha-1}{\alpha}}} \tag{24}$$

Now

$$h^* > h$$

iff

$$\alpha < \frac{1}{1 - \frac{\ln 2}{\ln 3}} = 2.7095113 \tag{25}$$

which is true in most cases (usually $1 < \alpha < 3$ - see Egghe (2005)). So, in most cases, "publicitis" pays, in this model. Replacing $\frac{2}{3}$ by $\frac{3}{4}$ in φ even yields for $h^* > h$ the condition $\alpha < 3.409421$ which is almost always the case.

Finally we simulate "fusion" of sources.

II.6 $\psi(r) = \frac{r}{2}$ and $\varphi(j) = 2j$

Here we fuse sources (halving the number of sources) and add (double) the number of items in each source before the fusion. This is depicted in table 7.

Table 7. Transformed Table 1

in case $\psi(r) = \frac{r}{2}$, $\varphi(j) = 2j$

r^*	$\#^*$
1/2	$2y_1$
2/2	$2y_2$
3/2	$2y_3$
$\vdots$	$\vdots$

Now (15) yields

$$h^* = 2^{\frac{\alpha-1}{\alpha}} \left(\frac{1}{2}\right)^{\alpha} h$$

$$h^* = 2^{1-\frac{2}{\alpha}} h \tag{26}$$

Again the value $\alpha = 2$ is a turning point:

$$h^* > h$$

iff

$$\alpha > 2$$

(and $h^* = h$ iff $\alpha = 2$ and $h^* < h$ iff $\alpha < 2$).

In Egghe and Rousseau (2006b) we showed that under a fusion operation as above we are expecting lower values of α. Hence we expect that this operation will lower the h-index: $h^* < h$. An application of this can be seen in the fusion of villages into larger villages or cities (or the consideration of this: looking at several city-areas separately or together as agglomerations): see Egghe and Rousseau (2006b).

This concludes the (simple) part $b = c = 1$ (linear transformations). Now we go back to the general case (12) but where we study another special case.

III. h^* versus h for general power transformations φ and ψ

In this section we return to the general case where (7) and (8) are valid, hence for $A, B, b, c > 0$. Note that $0 < b$, $c < 1$ imply a concavely increasing transformation for ψ, φ and that $b, c > 1$ imply a convexly increasing transformation ψ, φ. In this section, we hence use the general formula (12)

$$h^* = B^{\frac{\delta-1}{\delta}} A^{\frac{1}{\delta}} T^{\frac{b}{\delta}}$$

In order to limit this full generality (for reasons of simplicity) we will only investigate the case

$$B^{\frac{\delta-1}{\delta}} A^{\frac{1}{\delta}} = 1 \tag{27}$$

Hence, now (12) reduces to

$$h^* = T^{\frac{b}{\delta}} \tag{28}$$

$$h^* = \left(T^{\frac{1}{\alpha}}\right)^{\frac{b\alpha}{\delta}}$$

$$h^* = h^{\frac{b\alpha}{\delta}} \tag{29}$$

because of (9). So the relation between h^* and h is determined by the relation between $\frac{b\alpha}{\delta}$ and 1. But, by (6)

$$\frac{b\alpha}{\delta} = \frac{\alpha bc}{c + b(\alpha - 1)} \tag{30}$$

So

$$h^* > h$$

iff

$$\alpha b(c - 1) > c - b \tag{31}$$

(since $\alpha > 1$).

(i) Let $c \geq b \geq 1$ (but at least one inequality is strict). Then (31) is equivalent with

$$\alpha > \frac{c-b}{b(c-1)} \tag{32}$$

But $\alpha > 1$ and

$$\frac{c-b}{b(c-1)} \leq 1$$

in this case so that (32) is valid

(ii) Let $b \geq c \geq 1$ (but at least one inequality is strict). Then $c-1 \geq 0$ and $c-b \leq 0$ and at least one inequality is strict, hence (31) is satisfied.

We conclude:

$$h^* > h$$

in case $b, c \geq 1$ where we do not have $b = c = 1$ (this case is covered in the previous section in general and in the special case of (27) we have, since $\alpha = \delta$ that, by (29), $h^* = h$).

As above, we have

$$h^* < h$$

iff

$$\alpha b(c-1) < c-b \tag{33}$$

(iii) Let $b \leq c \leq 1$ (but at least one inequality is strict). Then $c-1 \leq 0$ and $c-b \geq 0$ (and at least one inequality is strict) so that (33) is satisfied.

(iv) Let $c \le b \le 1$ (but at least one inequality is strict). Then (33) is equivalent with (since $c - 1 < 0$)

$$\alpha > \frac{c - b}{b(c - 1)} \tag{34}$$

But $\alpha > 1$ and

$$\frac{c - b}{b(c - 1)} \le 1$$

so that (34) is valid.

We conclude:

$$h^* < h$$

in case $0 < b$, $c \le 1$ where we do not have $b = c = 1$ (same remark as above).

In case $b < 1 < c$ or $c < 1 < b$ we can have $h^* > h$ or $h^* < h$. Indeed, by (31), we must relate α with

$$\frac{c - b}{b(c - 1)} \tag{35}$$

But this value is > 1 in both cases $b < 1 < c$ and $c < 1 < b$. So a value of $\alpha > 1$ can be below or above (35). Concrete examples

(I) $b = \frac{1}{2} < 1 < c = 2$

Then

$$\frac{c - b}{b(c - 1)} = 3$$

For $\alpha = 2$ we have (by (6))

$$\delta = \frac{c + b(\alpha - 1)}{c} = \frac{5}{4},$$

hence, by (29) we have

$$h^* = h^{\frac{4}{5}} < h$$

(since $h > 1$ by (9)). For $\alpha = 5$ we have

$$\delta = \frac{c + b(\alpha - 1)}{c} = 2$$

Hence, by (29)

$$h^* = h^{\frac{5}{4}} > h$$

since $h > 1$.

(II) $c = \frac{1}{2} < 1 < b = 2$

Then

$$\frac{c - b}{b(c - 1)} = \frac{3}{2}$$

For $\alpha = \frac{5}{4}$ we have by (6)

$$\delta = \frac{c + b(\alpha - 1)}{c} = 2$$

and by (29)

$$h^* = h^{\frac{5}{4}} > h$$

For $\alpha = 2$ we have, by (6)

$$\delta = \frac{c + b(\alpha - 1)}{c} = 5$$

and hence, by (29)

$$h^* = h^{\frac{4}{5}} < h$$

From the above we also see that $h^* = h$ is possible in these cases.

We can conclude with the following Proposition.

Proposition III.1:
For the general transformations φ and ψ as in (7), (8) but for

$$B^{\frac{\delta - 1}{\delta}} A^{\frac{1}{\delta}} = 1$$

we have that

(i) If $b, c \geq 1$ (but not $b = c = 1$) we have $h^* > h$.

(ii) If $b, c \leq 1$ (but not $b = c = 1$) we have $h^* < h$.

(iii) If $b < 1 < c$ or $c < 1 < b$ we can have $h^* > h$ or $h^* < h$ or $h^* = h$.

(iv) If $b = c = 1$ we have $h^* = h$ in case (27). From the previous section we see that, even in this special case $b = c = 1$ (but where (27) is not valid), we can have $h^* < h$, $h^* > h$ or $h^* = h$.

This concludes all cases.

Note:

In case $b = c = 1$ we have (15) in general

$$h^* = B^{\frac{\alpha-1}{\alpha}} A^{\frac{1}{\alpha}} h$$

Hence h^* is h, multiplied with $B^{\frac{\alpha-1}{\alpha}} A^{\frac{1}{\alpha}}$, the generalized geometric mean of A and B. This result was also found, implicitely, in Egghe (2007c) where we also used linear (but time-dependent) transformations of the item and source densities, so that (15) is in accordance with these results.

IV. Transformation properties of other h-type indices

We do not go into the pros and cons of the h-index and of their alternatives g, R and h_w (to be defined below). For this we refer to their introductory papers.

In Egghe (2006a,b,c) we introduced the g-index as the unique number g such that

$$G(g) = g^2 \tag{36}$$

where

$$G(r) = \int_0^r g(r')dr' \tag{37}$$

(do not confuse between the g-index and the rank-frequency function g(r)). General transformation formulae (for general φ, ψ) where presented in Egghe (2007b).

In Jin, Liang, Rousseau and Egghe (2007), we presented the R-index, defined as follows:

$$R = \sqrt{\int_0^h g(r)dr} \tag{38}$$

where h is the h-index and $g(r)$ is the rank-frequency function. General transformation formulae for R are presented in the Appendix.

In Egghe and Rousseau (2007), we presented the weighted h-index h_w, defined as follows:

$$h_w = \sqrt{\int_0^i g(r)dr} \tag{39}$$

for this (unique) i such that

$$\frac{\int_0^i g(r)dr}{h} = g(i) \tag{40}$$

where, again, $g(.)$ denotes the rank-frequency function. General transformation formulae for h_w are also presented in the Appendix.

In the above mentioned articles we proved the following formulae for g, R and h_w, in case of Lotkaian systems (4), for $\alpha > 2$:

$$g = \left(\frac{\alpha - 1}{\alpha - 2}\right)^{\frac{\alpha-1}{\alpha}} h \tag{41}$$

$$R = \left(\frac{\alpha - 1}{\alpha - 2}\right)^{\frac{1}{2}} h \tag{42}$$

$$h_w = \left(\frac{\alpha - 1}{\alpha - 2}\right)^{\frac{1}{2(\alpha-1)}} h \tag{43}$$

Hence, using (5), we can apply (41), (42) and (43) to the transformed Lotka function (5) with δ as in (6) (provided that $\delta > 2$).

$$g^* = \left(\frac{\delta - 1}{\delta - 2}\right)^{\frac{\delta - 1}{\delta}} h^* \tag{44}$$

$$R^* = \left(\frac{\delta - 1}{\delta - 2}\right)^{\frac{1}{2}} h^* \tag{45}$$

$$h_w^* = \left(\frac{\delta - 1}{\delta - 2}\right)^{\frac{1}{2(\delta - 1)}} h^* \tag{46}$$

so that the properties of g^*, R^* and h_w^* follow from those of h^* (compared to h).

The condition $\delta > 2$ boils down to, by (6):

$$(\alpha - 1)b > c \tag{47}$$

(is e.g. satisfied for $b > c$ since $\alpha > 2$).

Note that (44)-(46) indicate that the transformations φ and ψ have their impact on h (via h^*) as well as on the factor before h^* since α is transformed into δ. If $\delta = \alpha$ (iff $b = c$) we clearly have

$$g^* = \left(\frac{\alpha - 1}{\alpha - 2}\right)^{\frac{\alpha - 1}{\alpha}} h^* \tag{48}$$

$$R^* = \left(\frac{\alpha - 1}{\alpha - 2}\right)^{\frac{1}{2}} h^* \tag{49}$$

$$h_w^* = \left(\frac{\alpha - 1}{\alpha - 2}\right)^{\frac{1}{2(\alpha - 1)}} h^* \tag{50}$$

so that here, the transformations φ and ψ have no impact on the factor before h^* but only in h^*. In this case, all the properties of h^* versus h, proved above (for $b = c$), are also valid for g^* versus g, R^* versus R and h_w^* versus h_w.

From (48)-(50), (41)-(43) and (15), in case $b = c = 1$ we even have:

$$g^* = B^{\frac{\alpha-1}{\alpha}} A^{\frac{1}{\alpha}} g \tag{51}$$

$$R^* = B^{\frac{\alpha-1}{\alpha}} A^{\frac{1}{\alpha}} R \tag{52}$$

and

$$h_w^* = B^{\frac{\alpha-1}{\alpha}} A^{\frac{1}{\alpha}} h_w \tag{53}$$

so that we have here the result

$$\frac{h^*}{h} = \frac{g^*}{g} = \frac{R^*}{R} = \frac{h_w^*}{h_w} = B^{1-\frac{1}{\alpha}} A^{\frac{1}{\alpha}}, \tag{54}$$

the generalized geometric mean of A and B (note that, if $\alpha = 2$, (54) equals $\sqrt{AB}$, the geometric mean of A and B).

V. Conclusions

In this paper we gave simple mathematical examples of transformations on an IPP and studied their influence on the h-index. We showed that doubling the items per source or doubling the sources lead to higher h-indices h^* such that $h < h^* < 2h$. "Publicitis" pays off when a double number of articles attract more than 50% of citations. We also showed that the fusion of sources in general leads to lower h^* values, when compared with h. In all these cases we found (once more) that $\alpha = 2$ (α = Lotka exponent) is a turning point, as is the case in many other informetric topics.

General power transformations are also studied where we limit our study to the case (27), comprising the cases $\psi(r) = r^b$ and $\varphi(j) = j^c$ (general $b, c > 0$). Here we proved that $h^* > h$ if $b, c > 1$ (convexly increasing functions φ, ψ) and that $h^* < h$ if $b, c < 1$ (concavely increasing functions φ, ψ, hence not so fastly increasing as in the convex case). The other cases are inconclusive.

We finally studied properties of the alternative indices g, R and h_w and showed, essentially, that they inherit the same transformation properties from the ones of h^* versus h.

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Appendix

A.1 General transformation formula for R

By (38) we have that the transformed index, denoted R^* equals

$$R^* = \sqrt{\int_0^{h^*} g^*(r^*)dr^*} \qquad \text{(A1)}$$

where h^* is the h-index of the transformed IPP and where $g^*(r^*)$ is given by (3). Since $r^* = \psi(r)$ and $\psi(0) = 0$ we hence have

$$R^* = \sqrt{\int_0^{\psi^{-1}(h^*)} \varphi(g(r))\psi'(r)dr} \qquad \text{(A2)}$$

where h^* is given by the general transformation formula for h as presented in Egghe (2007b).

A.2 General transformation formula for h_w

By (39) and (40), we have that the transformed index, denoted h_w^* equals

$$h_w^* = \sqrt{\int_0^{i^*} g^*(r^*)dr^*} \qquad \text{(A3)}$$

with i^* defined as

$$\int_0^{i^*} g^*(r^*)dr^* = h^* g^*(i^*) \qquad \text{(A4)}$$

Hence we also have

$$h_w^* = \sqrt{h^* g^*(i^*)} \qquad \text{(A5)}$$

$$h_w^* = \sqrt{h^* \varphi(g(\psi^{-1}(i^*)))} \qquad \text{(A6)}$$

with i^* following from (by (3), (A4) and since $r^* = \psi(r)$)

$$\int_0^{\psi^{-1}(i^*)} \varphi(g(r))\psi'(r)dr = h^*\varphi\left(g\left(\psi^{-1}(i^*)\right)\right) \tag{A7}$$

where again h^* is given by the general transformation formula for h as given in Egghe (2007b).