

**07212 Abstracts Collection**  
**Constraint Databases, Geometric Elimination and**  
**Geographic Information Systems**  
— Dagstuhl Seminar —

Bernd Bank<sup>1</sup>, Max J. Egenhofer<sup>2</sup> and Bart Kuijpers<sup>3</sup>

<sup>1</sup> HU Berlin, DE

bank@mathematik.hu-berlin.de

<sup>2</sup> Univ. of Maine, US

max@spatial.maine.edu

<sup>3</sup> Hasselt Univ. - Diepenbeek, BE

bart.kuijpers@uhasselt.be

**Abstract.** From 20.05. to 25.05., the Dagstuhl Seminar 07212 “Constraint Databases, Geometric Elimination and Geographic Information Systems” was held in the International Conference and Research Center (IBFI), Schloss Dagstuhl. During the seminar, several participants presented their current research, and ongoing work and open problems were discussed. Abstracts of the presentations given during the seminar as well as abstracts of seminar results and ideas are put together in this paper. The first section describes the seminar topics and goals in general. Links to extended abstracts or full papers are provided, if available.

**Keywords.** Constraint databases, geometric elimination, quantifier elimination algorithms, geographic information systems

**07212 Manifesto – Constraint Databases, Geometric Elimination and Geographic Information Systems**

During the last two decades the topic of constraint databases has evolved into a mature area of computer science with sound mathematical foundations and with a profound theoretical understanding of the expressive power of a variety of query languages. Constraint databases are especially suited for applications in which possibly infinite sets of continuous data, that have a geometric interpretation, need to be stored in a computer. Today, the most important application domains of constraint databases are geographic information systems (GIS), spatial databases and spatio-temporal databases. In these applications infinite geometrical sets of continuous data are finitely represented by means of finite combinations of polynomial equality and inequality constraints that describe these data sets (in mathematical terms these geometrical data sets are known as semi-algebraic sets and they have been extensively studied in real algebraic geometry). On the other hand, constraint databases provide us with a new view on classic (linear and nonlinear) optimization theory.

*Keywords:* Constraint databases, elimination procedures, geographical information systems

*Joint work of:* Bank, Bernd; Egenhofer, Max; Heintz, Joos; Kuijpers, Bart; Revesz, Peter

*Extended Abstract:* <http://drops.dagstuhl.de/opus/volltexte/2007/1282>

## **Relevance of Elimination Theory for Constraint Data Bases and Geographic Information Systems**

*Marc Giusti (Ecole Polytechnique - Palaiseau, F)*

The talk was a survey on the contributions of the TERA group to elimination theory, in the most illustrative and geometric way, trying to avoid as most as possible mathematical jargon.

The purpose was to convince the attending members of the communities of CDB and GIS of the relevance of this theory. See: <http://tera.medicis.polytechnique.fr>

I presented an algorithm whose idea goes back to Kronecker, performing a "geometric solving" of a system of polynomial equations as encountered in CDBs for example. It can be presented as a "generic" projection of the solution set on a linear space of dimension just 1 more.

Different data structures to encode polynomials were discussed, as well as upper and lower bounds of complexity, in the complex and real case.

Eventually I exhibited a situation with much more worst complexity, and another one warning that truncation of analytic equations to polynomials is a wild operation.

*Keywords:* Elimination, generic solving, Kronecker

## **A lower bound for the complexity of linear optimization from a quantifier-elimination point of view**

*Rafael Grimson (Hasselt University, B)*

We discuss the impact of data structures in quantifier elimination.

We analyze the arithmetic complexity of the feasibility problem in linear optimization theory as a quantifier-elimination problem. For the case of polyhedra defined by  $2n$  halfspaces in  $\mathbb{R}^n$  we prove that, if dense representation is used to code polynomials, any quantifier-free formula expressing the set of parameters describing nonempty polyhedra has size  $\Omega(4^n)$ .

*Keywords:* Quantifier elimination, dense representation, intrinsic, lower bound

*Extended Abstract:* <http://drops.dagstuhl.de/opus/volltexte/2007/1283>

## Constraint Database Complexity and Programming

*Joos Heintz (University of Cantabria, E)*

This talk focuses on software engineering (programming) and complexity issues in FO-querying of constraint data bases.

There exists a close connection between constraint database and elimination theory. On one side efficient elimination algorithms are needed for efficient querying of constraint data bases, on the other hand the constraint database base approach may be used in order to specify fundamental tasks in effective elimination theory and (semi-) algebraic geometry.

The notion of geometric query is introduced for the characterization of tasks like the resultant computation in effective algebraic geometry.

There exist two traditions in elimination theory, coming from the very beginning of algebraic geometry (geometric elimination) and logics (quantifier elimination).

The geometric tradition starts with Faulhaber (1622), Newton, Bézout (1779) and finds its general expression in Kronecker's paper "Grundzüge einer arithmetischen Theorie der algebraischen Grössen" (1882).

The logic tradition starts with Tarski in 1930 (first published in 1948).

(Standard) constraint database theory relies on the logical tradition (quantifier elimination). The generality of this approach leads to highly inefficient query algorithms. This fact inhibits the development of software which meets the complexity requirements of real world applications. Moreover, many important tasks, where constraint databases might be applied, remain out of the scope of the traditional theory (e. g. the computation of extremal points in optimization tasks).

This practical inadequacy of traditional constraint database theory is due to the fact that (polynomial) equation and inequality solving is not captured by the standard notion of quantifier elimination in logics. Moreover, in this context canonical elimination expressions do neither exist. However, the geometric tradition of elimination theory covers both aspects. In particular, the aspect of canonical elimination expressions becomes relevant when elimination procedures are iteratively used.

In order to remedy this situation in constraint database theory, the new feature of (generalized) sample point query is introduced.

In this talk I exhibit a series of elimination tasks (including a quite natural interpolation task which models realistically existing constraint database applications), which require intrinsically exponential time, even when alternative data structures and types (like circuit encoding of polynomials) are used. This exponential time behavior is also an intrinsic feature of geometric queries.

The proof is based on the numerically meaningful algorithmic restriction that the procedures under consideration are "branching parsimonious" (this means that unavoidable divisions have to be replaced by de l'Hôpital's rule - like limits).

Since 1995 the first incremental geometric elimination algorithms were developed by the international TERA (Turbo Evaluation and Rapid Algorithms)

group. The final outcome was the "Kronecker" software package written by G. Lecerf and collaborators (<http://www.math.uvsq.fr/lecerf/software/kronecker>).

It turns out that all known approaches to elimination and polynomial equation solving (based on rewriting, linear algebra or incremental techniques like the TERA algorithms) rely on principles that unavoidably lead to intrinsically exponential time complexities in the worst case.

Joint work with B. Kuijpers (Hasselt University, Belgium), G. Matera and P. Solernó (Argentina)

*Keywords:* Quantifier elimination, geometric elimination theory, sample point query

## The Dedale Constraint Database System

*Bart Kuijpers (Hasselt University, B)*

We present the Dedale system, developed at INRIA, Paris in the second half of the 1990s. Dedale is a prototype of a constraint database system with application to spatial databases.

At the heart of the Dedale implementation is a relational algebra which works on the symbolic representation of linear spatial relations. We discuss the implementation of this algebra and look at the algorithms from computational geometry used in there.

A description of Dedale can also be found in the recent textbook by Güting and Schneider.

## References

1. S. Grumbach, Implementing linear constraint databases, in Proceedings of the 2nd Workshop on Constraint Databases and Applications, vol. 1191 of Lecture Notes in Computer Science, Berlin, 1997, Springer-Verlag, pp. 105-115.
2. S. Grumbach, P. Rigaux, M. Scholl, and L. Segoufin, DEDALE, a spatial constraint database, in Sixth International Workshop on Database Programming Languages, vol. 1369 of Lecture Notes in Computer Science, Springer-Verlag, 1998, pp. 124-135.
3. S. Grumbach and J. Su, Towards practical constraint databases, in Proceedings of the 15th ACM SIGACT-SIGMOD-SIGART Symposium on Principles of Database Systems, New York, 1996, ACM Press, pp. 28-39.
4. R. Güting and M. Schneider. Moving Object Databases. Morgan Kaufmann, 2005.

*Keywords:* Constraint database, Dedale

## Rotations, Shifts and Wavelet Filter Banks

*Lutz Lehmann (HU Berlin, D)*

The aim of this talk is to demonstrate the practical utility of algebraic elimination procedures on the example of the layout of wavelet filter banks. To this end, the method of Groebner basis - eliminating all variables at once - and Kronecker's method - eliminating one variable at a time - were compared on several instances of the example.

The application of the results of such a computation was then explained in the context of the process of wavelet image compression. Finally, to make the connection between the two parts, the structure of a wavelet filter bank was exhibited as a composition of several shifts and rotations. As a consequence, optimizing the wavelet filters is a polynomial problem in the rotation parameters.

*Keywords:* Computational algebraic geometry, signal filter constructions

## Spatiotemporal Interpolation and Constraint Databases for a GIS Application: Ozone in the Contiguous U.S.

*Lixin Li (Georgia Southern University, USA)*

It is important to conduct research on the connection between air pollution such as ozone and human health on a large scale with respect to area and population. A GIS-based geostatistical approach has been used to do spatial interpolation for environmental exposure analysis in the work of (Liao et al. 2006). However the geostatistical analyst in ArcGIS could not handle the spatiotemporal interpolation, and computationally this approach is not efficient for large datasets. In our paper, using a set of spatiotemporal data with annual ozone concentration measurements in the contiguous U.S. during 1994 and 1999, we address the following challenging issues in conducting such research: spatiotemporal interpolation, comparison of spatiotemporal interpolation methods, data representation, visualization, and analysis of population exposure to ozone.

The shape function based spatiotemporal interpolation method has been used in this paper to estimate the ozone concentrations at any unmeasured location and time. Using the leave-one-out cross-validation, we compute error statistics to compare the shape function and IDW (Inverse Distance Weighting) methods. It is shown that the shape function method is better than IDW in terms of MAPE (Mean Absolute Percentage Error) and algorithm complexity. We illustrate how to use constraint databases to represent the interpolation results efficiently and accurately. For generating maps of annual ozone concentrations, we propose a new approach to select locations to interpolate and visualize: picking U.S. census block centroids as sample locations. The advantage of this approach is to generate more sample points in the areas with more intensive human activities.

In our experiment, there were about 8,000,000 sample points selected per year. Traditional GIS techniques are insufficient in handling such kind of spatiotemporal data. The visualization results of ozone concentration distribution at the census tract level in the contiguous U.S. from 1994 to 1999 are illustrated. We also analyze the population exposure to ozone in the year of 1999 according to different ozone concentration levels following the recommendations given by the U.S. EPA on air quality. Our finding is that in the year of 1999, 9.8% total population in the contiguous U.S. has been exposed to a high risk ozone level, 78.7% to a moderate risk, and only 11.5% to a low risk.

*Keywords:* Shapefunctions, spatiotemporal interpolation, constraint databases, ozone, population exposure

*Joint work of:* Li, Lixin; Zhang, Xingyou; Piltner, Reinhard

## Checking the Integrity of Spatial Integrity Constraints

*Stephan Mäs (Univ. der Bundeswehr - Neubiberg, D)*

Integrity constraints play a major role when the quality of spatial data is checked by automatic procedures. Nevertheless the possibilities of checking the internal consistency of the integrity constraints themselves are hardly researched yet. This work analyses the applicability of reasoning techniques like the composition of spatial relations and constraint satisfaction in networks of constraints to find conflicts and redundancies in sets of spatial semantic integrity constraints. These integrity rules specify relations among entity classes. Such relations must hold to assure that the data is fitting to the semantics intended by the data model. For spatial data many semantic integrity constraints are based on spatial properties described for example through qualitative topological or metric relations. Since integrity constraints are defined at the class level, the reasoning properties of these spatial relations can not directly be applied. Therefore a set of class relations has been defined which, combined with the instance relations, enables for the specification of integrity constraints and to reason about them.

*Keywords:* Semantic Integrity Constraints, Spatial Relations, Class Level Relations, Reasoning, Consistency of Constraints, Constraint Networks

*Full Paper:* <http://drops.dagstuhl.de/opus/volltexte/2007/1285>

*See also:* Mäs, Stephan (2007): Reasoning About Spatial Semantic Integrity Constraints. Accepted for publication at the Cosit 2007

## **An analytic solution to the alibi query in the bead model for moving object data**

*Walied Othman (Hasselt University, B)*

Moving objects produce trajectories, which are stored in databases by means of finite samples of time-stamped locations. When also speed limitations in these sample points are known, beads can be used to model the uncertainty about the object's location in between sample points.

In this setting, a query of particular interest, that has been studied in the literature of geographic information systems (GIS), is the alibi query. This boolean query asks whether two moving objects can have physically met. This adds up to deciding whether the necklaces of beads of these objects intersect. This problem can be reduced to deciding whether two beads intersect.

Since, existing software to solve this problem fails to answer this question within a reasonable time, we propose an analytical solution to the alibi query, which can be used to answer the alibi query in constant time, a matter of milliseconds or less, for two single beads and in time proportional to the product of their lengths for necklaces of beads.

*Keywords:* Beads, uncertainty, alibi, query, solution, quantifier elimination, constraint database

*Joint work of:* Kuijpers, Bart; Othman, Walied

*Full Paper:* <http://drops.dagstuhl.de/opus/volltexte/2007/1286>

## **Constraint Databases and Geographic Information Systems**

*Peter Revesz (University of Nebraska, USA)*

Constraint databases and geographic information systems share many applications. However, constraint databases can go beyond geographic information systems in efficient spatial and spatiotemporal data handling methods and in advanced applications. This survey mainly describes ways that constraint databases go beyond geographic information systems. However, the survey points out that in some areas constraint databases can learn also from geographic information systems.

*Keywords:* Constraint databases, geographic information systems, moving objects, spatiotemporal data, visualization

*Full Paper:* <http://drops.dagstuhl.de/opus/volltexte/2007/1281>

## Geospatial Phenomena and Queries Modeled by GIS

*Maria Andrea Rodriguez-Tastets (Universidad de Concepción, RCH)*

An overview of data and queries modeled by GIS were presented. Issues about space modeling, spatial data modeling, qualitative spatial reasoning and spatio-temporal applications are discussed. Special attention were paid to field versus object modeling of space associated with constraint representations. For field-based modeling, the representation of spatio-temporal as continuous functions is a challenging area for constraint representation. In addition, using concepts borrowed from the theory of constraint satisfaction problems leads us to use constraints not only for data representation but for qualitative spatial reasoning.

*Keywords:* Geographic Information Systems, Spatial Data Modeling, Spatial reasoning

## Minimum work path problem

*Takeshi Shirabe (TU Wien, A)*

A new variant of the shortest path problem is introduced. It is one of finding a path that takes the least amount of pedaling work to cycle from an origin to a destination through a network situated on a hilly geography, where the bicycle is subject to a conservative force, gravity, as well as a nonconservative force, friction. In this setting, the cyclist's pedaling work to overcome the friction of each arc varies with the bicycle's kinetic energy and gravitational potential energy, which transform to one another. Thus while geometric characteristics of the network are invariable, arc weights representing required pedaling work are variable. The problem is formulated as a quadratic integer program and an approximation procedure is presented.

*Keywords:* Shortest path, physical constraints, dynamic programming

## REDLOG and its Possible Application for CDB/GIS

*Thomas Sturm (Universität Passau, D)*

The first part of the presentation gives overview on the REDUCE-based computer logic system REDLOG. We discuss real quantifier elimination methods their implementation in RDLOG and list successful applications of these implementations. REDLOG provides quantifier elimination for many more domains including integers, p-adics, quantified propositional calculus, term algebras, and differential algebras. The REDLOG example management and information system REMIS is an online database with a web frontend containing input for

REDLOG computation examples and scientific publications, where these examples are discussed. The second part of the presentation dicusses computational experiments together with Jan van den Bussche during the seminar week.

These computations include the automatic determination of crossings in streetmaps. The presentation closes with a list of questions and suggestions for possible further cooperation and discussion.

*Keywords:* Quantifier elimination, constraint solving, implementation, application

## **Introduction to Constraint Databases**

*Jan Van den Bussche (Hasselt University, B)*

We present an introduction to constraint databases. Topics discussed include quantifier elimination; topological and geometric queries; and expressivity questions.