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Time-dependent Lotkaian informetrics incorporating growth of sources and items

by

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ABSTRACT

In a previous article, static Lotkaian theory was extended by introducing a growth function for the items. In this article, a second general growth function – this time for the sources – is introduced. Hence this theory now comprises real growth situations, where items and sources grow, starting from zero, and at possibly different paces. The time-dependent size- and rank-frequency functions are determined and, based on this, we calculate the general, time-dependent, expressions for the h- and g-index. As in the previous article we can prove that both indices increase concavely with a horizontal asymptote, but the proof is more complicated: we need the result that the generalized geometric average of concavely increasing functions is concavely increasing.

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I. Introduction

Let us have the size-frequency function of Lotka type

$$f(j) = \frac{C}{j^\alpha} \quad (1)$$

with $j \in [1, +\infty[$ and $\alpha > 1$. The theory could be extended to general α provided j is limited to a finite closed interval but this complication would make the arguments in this paper more cumbersome while, essentially, nothing new is added; therefore we limit ourselves to $j \geq 1$, $\alpha > 1$. Hence, for every $j \geq 1$,

$$\sum_{j=1}^{\infty} f(j) = r \quad (2)$$

denotes the total number of sources with item-density j or larger. Transforming (1) into a probability density φ leads to

$$\varphi(j) = \frac{\alpha - 1}{j^\alpha} \quad (3)$$

as is readily seen (since $\alpha > 1$).

In Egghe (2006a), we let the items grow in time t , using a general growth function $G(t)$ as follows: define the new item-densities at time t as

$$jG(t) = k \quad (4)$$

Based on this we could prove in Egghe (2006a) that the time-dependent size-frequency density, derived from φ is the function

$$\varphi(k,t) = G(t)^{\alpha-1} \varphi(k) \quad (5)$$

for $k \geq G(t)$. Here $\varphi(k)$ is considered to be $\varphi(k, \infty)$, i.e. for $t = \infty$.

The rank-frequency function γ , associated with f , is generally defined as (cf. Egghe (2005))

$$r = \gamma^{-1}(j) = \int_j^{\infty} f(j') dj' \quad (6)$$

(cf. (2)), where γ^{-1} denotes the inverse of the function γ (note that γ^{-1} strictly decreases so that $\gamma = (\gamma^{-1})^{-1}$ is defined). Note also that $r \geq 0$.

The time-dependent rank-frequency function was proved in Egghe (2006a) to be

$$\gamma(r,t) = G(t)\gamma(r) \quad (7)$$

The Hirsch-index (or h-index, shortly) is defined on a ranked list of sources (ranked in decreasing order of the number of items they produce or have), cf. Ball (2005), Bornmann and Daniel (2005), Braun, Glänzel and Schubert (2005), Egghe (2006b,c,d,e), Egghe and Rousseau (2006), Glänzel (2006a,b), Hirsch (2005), van Raan (2006). It is the highest rank h such that all sources on rank $1, \dots, h$ have h or more items. In the above time dependent framework it was proved that (see Egghe (2006a) – see also Egghe (2006d)) the time-dependent h-index is

$$h(t) = G(t)^{\frac{\alpha-1}{\alpha}} h \quad (8)$$

where h is the h-index for $t = \infty$.

The g-index (introduced in Egghe (2006b,c,e)) is a solution for the drawback of the h-index that it does not increase even when the sources with the most items increase their number of items. We define the g-index to be the highest rank g such that the sources on rank $1, \dots, g$,

together, have g^2 or more items (note that h satisfies this, by definition, hence $g^3 h$). In the above framework, in Egghe (2006a), we proved that the time-dependent g -index is

$$g(t) = G(t)^{\frac{\alpha-1}{\alpha}} g \quad (9)$$

where g is the g -index for $t = \forall$.

In the next section we will extend this theory by allowing for two general growth functions: one is the function G as in (4) (from now on called G_1) and another function G_2 acting on source ranks. We then determine the general time-dependent Lotkaian theory by proving formulae for the time-dependent size- and rank-frequency functions (using both growth functions G_1 and G_2)

Formula (5) is extended to

$$\varphi(k, t) = G_2(t) G_1(t)^{\alpha-1} \varphi(k) \quad (10)$$

and formula (7) is extended to

$$\gamma(s, t) = G_1(t) \gamma(r) \quad (11)$$

with

$$r = G_2(t) = s \quad (12)$$

(the defining relation for G_2).

In the third section we show that the general time-dependent formulae for the h -index and the g -index are:

$$h(t) = G_1(t)^{\frac{1}{\alpha}} G_2(t)^{\frac{1}{\alpha}} h \quad (13)$$

$$g(t) = G_1(t)^{1-\frac{1}{\alpha}} G_2(t)^{\frac{1}{\alpha}} g \quad (14)$$

where h and g are the h -index and g -index for $t = \mathbb{Y}$. Hence both $h(t)$ and $g(t)$ are the values at $t = \mathbb{Y}$, multiplied by a time factor which is a generalized geometric mean of the functions $G_1(t)$ and $G_2(t)$. We prove that such a generalized geometric mean of concavely increasing functions G_1 and G_2 is concavely increasing, hence the same is true for the functions $h(t)$ and $g(t)$, growing to their values h and g (formulae proved earlier will be repeated here) for $t = \mathbb{Y}$.

II. Time-dependent Lotkaian theory, incorporating general growth models for items and sources

II.1 Basic relations

Basic in this theory is the “static” law of Lotka (1), normalized so that we have a size-frequency distribution (see (3))

$$\varphi(j) = \frac{\alpha - 1}{j^\alpha} \quad (15)$$

where $j \geq 1$ and $\alpha > 1$. This distribution is considered as the “limiting” situation at time $t = \mathbb{Y}$, where $\varphi(j)$ denotes the density of sources with item density j (see Egghe (2005)).

For the growth in time $t \geq 0$ we assume the following simple relations:

- (i) Item growth: Let $G_1(t)$ be the cumulative growth distribution of the item densities expressed by (cf. (4) with $G = G_1$)

$$jG_1(t) = k \quad (16)$$

Hence, since $j \geq 1$, we have that $k \geq G_1(t)$.

- (ii) Source growth: Let $G_2(t)$ be the cumulative growth distribution of the rank densities (of the sources) expressed by (new equation)

$$r \cdot G_2(t) = s \quad (17)$$

Hence $s \geq 0$ since $r \geq 0$.

We underline that G_1 and G_2 are arbitrary cumulative growth distributions and, of course G_1 and G_2 can be different. As cumulative growth distributions we can (and will) suppose that G_1 and G_2 are strictly increasing concave functions such that $G_1(0) = 0$, $G_2(0) = 0$, $\lim_{t \rightarrow \infty} G_1(t) = 1$, $\lim_{t \rightarrow \infty} G_2(t) = 1$. A classical example of such a function is

$$G_i(t) = 1 - a_i^t \quad (18)$$

where $0 < a_i < 1$, $t \geq 0$, but this is only an example (we do not need (18) for our model further on).

II.2 Size-frequency functions, in function of time t

Let T denote the total number of sources at $t = \infty$. Since (17) implies

$$\frac{r}{T} G_2(t) = \frac{s}{T} \quad (19)$$

and since

$$\frac{r}{T} = \int_0^{\infty} \varphi(j') dj' \quad (20)$$

denotes the fraction of sources (at $t = \infty$) with item-density j or more (cf. (2) divided by T , hence using (3)), we have, by (19) and (20) that

$$G_2(t) \int_0^{\infty} \varphi(j') dj' \quad (21)$$

denotes the fraction of sources (with respect to $t = \mathbb{Y}$) with k or more as item-density, where

$$jG_1(t) = k \quad (22)$$

by (16). Let $\Phi(k, t)$ denote this fraction of sources. Hence

$$\Phi(k, t) = G_2(t) \int_k^{\mathbb{Y}} \varphi(j') dj'$$

$$\Phi(k, t) = G_2(t) \int_k^{\mathbb{Y}} \frac{\alpha - 1}{j'^{\alpha}} dj'$$

$$\Phi(k, t) = G_2(t) \frac{G_1(t)^{\alpha-1}}{k^{\alpha}} \quad (23)$$

since $\alpha > 1$. Since

$$\Phi(k, t) = \int_k^{\mathbb{Y}} \varphi(k', t) dk' \quad (24)$$

by definition of the time-dependent Lotka function $\varphi(k, t)$, we clearly have

$$\varphi(k, t) = -\Phi'(k, t) \quad (25)$$

where $\Phi'(k, t)$ denotes $\frac{\partial \Phi(k, t)}{\partial k}$

Consequently, by (23)

$$\varphi(k, t) = G_2(t)(\alpha - 1) \frac{G_1(t)^{\alpha-1}}{k^{\alpha}} \quad (26)$$

$$\varphi(k, t) = G_2(t)G_1(t)^{\alpha-1} \varphi(k) \quad (27)$$

where $k^3 G_1(t)$ and by (15). Note that, here, we extend (15) to $k^3 G_1(t)$ (by (26)).

Note that $\varphi(k, t)$ is the density of source fractions (with respect to $t = \mathbb{Y}$) (cf. (20) and (21)) with item-density k . Hence, denoting by T the total number of sources at $t = \mathbb{Y}$,

$$f(k, t) = T\varphi(k, t) \quad (28)$$

$$f(k, t) = G_2(t)G_1(t)^{\alpha-1} T\varphi(k) \quad (29)$$

$$f(k, t) = G_2(t)G_1(t)^{\alpha-1} f(k) \quad (30)$$

denotes the actual density of sources (with respect to $t = \mathbb{Y}$) with item density k at time t and where $f(k) = T\varphi(k)$ denotes the same at $t = \mathbb{Y}$. Note that also here (15) is extended to $k^3 G_1(t)$.

II.3 Total number of sources and items at time t

The above function $f(k, t)$ already allows to calculate

$T(t)$ = total number of sources at time t

$A(t)$ = total number of items at time t .

We have, by definition of $f(k, t)$ (cf. (29)) and since $k^3 G_1(t)$:

$$T(t) = \int_{G_1(t)}^{\mathbb{Y}} f(k, t) dk$$

$$T(t) = G_2(t)G_1(t)^{\alpha-1} T \int_{G_1(t)}^{\mathbb{Y}} \varphi(k) dk$$

$$T(t) = G_2(t)G_1(t)^{\alpha-1}TG_1(t)^{1-\alpha}$$

$$T(t) = G_2(t)T \quad (31)$$

a logical result. In the same way, for $\alpha > 2$, we have (cf. (29))

$$\begin{aligned} A(t) &= \int_{G_1(t)}^{\Psi} kf(k,t)dk \\ &= TG_2(t)G_1(t)^{\alpha-1} \int_{G_1(t)}^{\Psi} k\varphi(k)dk \\ &= TG_2(t)G_1(t)^{\alpha-1} \frac{\alpha-1}{\alpha-2} G_1(t)^{2-\alpha} \end{aligned} \quad (32)$$

But, as is readily seen, on $t = \Psi$ (cf. Egghe (2005), Chapter 2), since $\alpha > 2$

$$\begin{aligned} \mu &= \frac{A}{T} = \frac{\int_{G_1}^{\Psi} k\varphi(k)dk}{\int_{G_1}^{\Psi} \varphi(k)dk} \\ &= \frac{\alpha-1}{\alpha-2} \end{aligned} \quad (33)$$

(T = total number of sources at $t = \Psi$, A = total number of items at $t = \Psi$).

Hence, (33) in (32) yields

$$A(t) = AG_1(t)G_2(t),$$

also a logical result since items at t are determined by the items in the already existing sources; the former one is determined by G_1 and the latter one by G_2 .

Since rank-frequency functions are equivalent with size-frequency functions (by the general relation (6), but now t-dependent) we are now in a position to calculate the time-dependent rank-frequency function $\gamma(s,t)$.

II.4 The rank-frequency function, in function of time t

Since, at t, the rank-densities are denoted by s via (17) and by (6), interpreted at time t, we have (the inverse refers to the variable k)

$$\begin{aligned} s = s(t) &= \gamma^{-1}(k,t) = \int_k^{\infty} f(k',t) dk' \\ &= TG_2(t)G_1(t)^{\alpha-1} \int_k^{\infty} \varphi(k') dk' \end{aligned} \quad (34)$$

, by (29). Hence, using (15)

$$s = s(t) = \frac{C}{\alpha-1} \frac{G_2(t)G_1(t)^{\alpha-1}}{k^{\alpha-1}}$$

denoting $T(\alpha-1) = C$ (hence $f(k) = \frac{C}{k^\alpha}$). This yields

$$k = \gamma(s,t) = \frac{C^{\frac{1}{\alpha-1}}}{((\alpha-1)s)^{\frac{1}{\alpha-1}}} G_2(t)^{\frac{1}{\alpha-1}} G_1(t)$$

Invoke (17), yielding

$$k = \gamma(s,t) = \frac{C^{\frac{1}{\alpha-1}}}{((\alpha-1)r)^{\frac{1}{\alpha-1}}} G_1(t)$$

$$\gamma(s,t) = \gamma(r)G_1(t) \quad (35)$$

with r and s related as in (17). Note that (35) follows from the fact that $f(k) = \frac{C}{k^\alpha}$ and by applying the same calculation as performed here (but for $t = \mathbb{Y}$, hence $G_1(\mathbb{Y}) = G_2(\mathbb{Y}) = 1$). In fact the result can also be found in Egghe (2005), Exercise II.2.2.6 (p. 134) and its proof is also given in Egghe and Rousseau (2006), Appendix.

In the next section, we will apply these results in the calculation of the general forms of the time-dependent h - and g -indices.

III. Formulae for the time-dependent h - and g -indices and properties of these functions of time

III.1 The time-dependent h -index

The h -index in this model is defined to be $h(t)$ such that

$$\int_{h(t)}^{\mathbb{Y}} f(k, t) dk = h(t) \quad (36)$$

Hence by (29) and (15) we have

$$TG_2(t)G_1(t)^{\alpha-1} \int_{h(t)}^{\mathbb{Y}} \frac{\alpha-1}{k^\alpha} dk = h(t)$$

So

$$h = h(t) = (TG_2(t))^{\frac{1}{\alpha}} G_1(t)^{\frac{\alpha-1}{\alpha}} \quad (37)$$

Invoke that, for $t = \mathbb{Y}$

$$h = h(\mathbb{Y}) = T^{\frac{1}{\alpha}} \quad (38)$$

(see Glänzel (2006b), Egghe and Rousseau (2006)). So (37) and (38) yield

$$h(t) = G_1(t)^{\frac{1}{\alpha}} G_2(t)^{\frac{1}{\alpha}} h \quad (39)$$

Note that the time-dependent h-index is proportional to the generalized geometric mean of the growth functions $G_1(t)$ and $G_2(t)$. In the sequel we will study properties of this generalized geometric mean.

III.2 The time-dependent g-index

There are two, equivalent, ways to define the g-index (at time t), denoted $g(t)$: if

$$\int_0^{g(t)} k' f(k', t) dk' = g(t)^2 \quad (40)$$

then

$$g(t) = \gamma^{-1}(k, t) \quad (41)$$

Equivalently, $g(t)$ is defined as

$$\int_0^{g(t)} \gamma(r, t) dr = g(t)^2 \quad (42)$$

The latter approach has been followed in Egghe (2006a) (but only using the growth function G_1 - there denoted G). Here we will follow the former approach. Formula (40) yields, using (15) and (19) (and the notation $C = T(\alpha - 1)$, already used above):

$$G_2(t) G_1(t)^{\alpha-1} \int_0^{g(t)} \frac{C}{k^{\alpha-1}} dk' = g(t)^2$$

$$G_2(t) G_1(t)^{\alpha-1} \frac{C}{\alpha-2} k^{2-\alpha} = g(t)^2 \quad (43)$$

, supposing $\alpha > 2$. But (34) yields (using (15) again)

$$\gamma^{-1}(k, t) = G_2(t)G_1(t)^{\alpha-1} \int_0^k \frac{C}{k'^{\alpha}} dk'$$

$$g(t) = \gamma^{-1}(k, t) = G_2(t)G_1(t)^{\alpha-1} \frac{C}{\alpha-1} k^{1-\alpha} \quad (44)$$

by (41). So, solving (44) for k , yields

$$k = \left(\frac{\alpha-1}{C} \frac{g(t)}{G_2(t)} \right)^{\frac{1}{\alpha-1}} G_1(t) \quad (45)$$

(45) in (43) yields, after some calculation and using that $T = \frac{C}{\alpha-1}$:

$$g(t) = G_1(t)^{1-\frac{1}{\alpha}} G_2(t)^{\frac{1}{\alpha}} \left(\frac{\alpha-1}{C} \right)^{\frac{\alpha-1}{\alpha}} T^{\frac{1}{\alpha}} \quad (46)$$

Invoke that, for $t = \mathbb{Y}$,

$$g = g(\mathbb{Y}) = \left(\frac{\alpha-1}{C} \right)^{\frac{\alpha-1}{\alpha}} T^{\frac{1}{\alpha}} \quad (47)$$

(see Egghe (2006c)). So (47) in (46) yields

$$g(t) = G_1(t)^{1-\frac{1}{\alpha}} G_2(t)^{\frac{1}{\alpha}} g \quad (48)$$

Hence $g(t)$ has the same proportionality factor as $h(t)$ (see (39)). We will now study this proportionality factor which determines the evolution of $h(t)$ and $g(t)$ in time.

III.3 Properties of $G_1(t)^{1-\frac{1}{\alpha}} G_2(t)^{\frac{1}{\alpha}}$

Since $G_1(t)$ and $G_2(t)$ are cumulative growth distributions we have that

$$G_1(0) = G_2(0) = 0$$

$$\lim_{t \rightarrow \infty} G_1(t) = \lim_{t \rightarrow \infty} G_2(t) = 1$$

and that G_1 and G_2 are strictly increasing in t and this increase is concave (example: (18) but we do not need this special case here)

So it is then trivial that $h(t)$ strictly increases from 0 to h (its maximal value, being $h(\infty)$) and that $g(t)$ strictly increases from 0 to $g(\infty)$.

The Lemma in the Appendix shows that the generalized geometric mean of strictly increasing concave functions is strictly increasing and concave so that also $h(t)$ and $g(t)$ are strictly increasing. Of course, since the g -index is larger than or equal to the h -index we also have, for every t :

$$g(t) \geq h(t). \quad (49)$$

IV. Conclusions and open problems

In this paper we studied the aspect of time-dependence in the evolution of a Lotkaian system. Source and item densities are growing in time using two general cumulative growth distributions. Using this general framework yields formulae for the time-dependent size- and rank-frequency functions in function of the final situation at $t = \infty$, where we suppose to have a Lotkaian system. We show that at every time t , we have a Lotkaian system with the same Lotka exponent but where the growth is dependent on – besides the two growth distributions – this Lotka exponent.

Similar results are proved for the time-dependent h- and g-indices. Here we prove that both indices are proportional to their values at $t = \forall$ and where the time-dependent proportionality factor is a generalized geometric mean of the two growth distributions. Also here the Lotka exponent is involved. We prove in a Lemma in the Appendix that a general geometric mean of strictly increasing concave functions is strictly increasing and concave. Hence the same is true for the time-dependent h- and g-indices.

We formulate as an open problem to define other growth processes of sources and items, where the growth functions are acting on the source- and item-densities in a nonlinear way and to derive from this framework the time-dependent size- and rank-frequency functions and the time-dependent h- and g-indices.

Appendix

Let x and y be two positive real numbers. We say that z is a generalized geometric average of x and y if there exists a $\lambda \in]0,1[$ such that

$$z = x^\lambda y^{1-\lambda} \quad (\text{A1})$$

The classical geometric mean is given when $\lambda = \frac{1}{2}$: then $z = \sqrt{xy}$.

In Section III we found that

$$h(t) = G_1(t)^{1-\frac{1}{\alpha}} G_2(t)^{\frac{1}{\alpha}} h \quad (\text{A2})$$

$$g(t) = G_1(t)^{1-\frac{1}{\alpha}} G_2(t)^{\frac{1}{\alpha}} g \quad (\text{A3})$$

hence $h(t)$ and $g(t)$ are equal to their values at $t = \lambda$ multiplied by a generalized geometric mean of the growth functions $G_1(t)$ and $G_2(t)$ (note, indeed, that, since $\alpha > 1$ we have that

$$\lambda = 1 - \frac{1}{\alpha} \hat{p}, 1].$$

We have the following general Lemma.

Lemma:

Let G_1, G_2 be two functions of the variable t which are strictly increasing and concave. Then their generalized geometric average (for $\alpha > 1$)

$$G(t) = G_1(t)^{\frac{1}{\alpha}} G_2(t)^{\frac{1}{\alpha}} \quad (\text{A4})$$

is strictly increasing and concave.

Proof:

That G is strictly increasing in t is trivial. For the concavity we need to calculate G'' . We have

$$G'(t) = \frac{\alpha - 1}{\alpha} \frac{G_1(t)^{\frac{1}{\alpha}} G_2(t)^{\frac{1}{\alpha}}}{G_2(t)} G_1'(t) + \frac{1}{\alpha} \frac{G_1(t)^{\frac{1}{\alpha}} G_2(t)^{\frac{\alpha-1}{\alpha}}}{G_2(t)} G_2'(t) \quad (\text{A5})$$

$$\begin{aligned} G''(t) = & \frac{\alpha - 1}{\alpha} \frac{1}{G_2(t)} \frac{G_1(t)^{\frac{1}{\alpha}}}{G_2(t)^{\frac{1}{\alpha}}} \frac{G_2(t)G_1'(t) - G_1(t)G_2'(t)}{G_2^2(t)} G_1'(t) + \frac{\alpha - 1}{\alpha} \frac{1}{G_2(t)} \frac{G_1(t)^{\frac{1}{\alpha}}}{G_2(t)^{\frac{1}{\alpha}}} G_1''(t) \\ & + \frac{1}{\alpha} \frac{\alpha - 1}{\alpha} \frac{1}{G_2(t)} \frac{G_1(t)^{\frac{\alpha-1}{\alpha}}}{G_2(t)^{\frac{\alpha-1}{\alpha}}} \frac{G_2(t)G_1'(t) - G_1(t)G_2'(t)}{G_2^2(t)} G_2'(t) + \frac{1}{\alpha} \frac{1}{G_2(t)} \frac{G_1(t)^{\frac{\alpha-1}{\alpha}}}{G_2(t)^{\frac{\alpha-1}{\alpha}}} G_2''(t) \end{aligned}$$

The second and fourth terms are clearly strictly negative since G_1 and G_2 are concave. The first and third term, together, equal

$$\frac{\alpha - 1}{\alpha^2} \frac{G_1(t)G_2'(t) - G_1'(t)G_2(t)}{G_2^2(t)} \frac{G_2(t)}{G_1(t)} G_1'(t) + G_2'(t)$$

$$= \frac{\alpha - 1}{\alpha^2} \frac{G_1(t)G_2'(t) - G_1'(t)G_2(t)}{G_1(t)G_2^2(t)} (G_2(t)G_1'(t) - G_1(t)G_2'(t))^2$$

< 0, since $\alpha > 1$. So $G''(t) < 0$ for all t and hence $G(t)$ is concave. \square

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